

**BIDDING STRATEGY AND EMPIRICAL ANALYSIS OF BIDDING IN  
ELECTRICAL POWER MARKET**

By

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To the Faculty of Washington State University:

The members of the Committee appointed to examine the dissertation of  
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Abstract

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Electric restructuring was started in the early 1990s as a way to increase electric power industry' efficiency and lower the energy cost. The traditional integrated system has now been separated in many parts of the country and some degree of competition introduced throughout the power industry. This thesis focused on how market participants (primarily generators) react under this new market operation mechanism. Specifically, this work contributed with the following three investigations:

1. Transmission system congestion influence on market clearing price and market participant bidding behavior in the framework of game theory was analyzed. The conclusion was drawn that deviation from idealized price-taker behavior is more serious when some market participants suffer disproportionately from the congestion. Due to the complexity of the calculations in the theoretical approach, this thesis suggests that a statistical analysis methodology is more appropriate. An intuitive probabilistic bidding methodology was proposed for the bidding problem to demonstrate feasibility.
2. A detailed statistical analysis has been carried out on the California real time imbalance energy market. A linear regression model was applied to a zonal energy price prediction

process and a non-linear estimator based on a neural network was applied to predict bidding behavior. Sensitivity analysis was applied to understanding each factor's influence on market participant bidding behavior.

3. Statistical analysis results were applied to the optimal bidding strategy problem. The empirical conjecture approach was adopted using these results. Including risk as either an objective to be minimized or a constraint to be satisfied, a portfolio selection approach was applied. This method combines the statistical analysis technique with the optimal bidding problem. Although the results shown here are in the initial stage of development, it appears that this approach is more promising than an idealized game theoretic formulation.

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## **Dedication**

This dissertation/thesis is dedicated to my mother and father  
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## **CHAPTER ONE**

### **INTRODUCTION**

The last two decades, with the development of electric power markets and the introduction of competition, has initiated radical change for the power industry. These changes are still relatively poorly understood and this thesis addresses some basic questions around market operations. The main contributions of this thesis concern understanding bidding strategies of suppliers in a market with the possibility of transmission congestion.

#### **1.1 Power Markets**

Cope [1] divided the electric power industry in United State into three significant periods: the steady growth industry before 1973; the more turbulent and less predictable period between 1973 and 1992; and the deregulation period beginning in 1992. The electric power industry was generally considered not suitable for competition due to economies of scale, which is certainly true before the 1970s. The electricity price consistently decreased for almost 50 years before 1970s due in part to the advantage of integrated control over the entire system with such functions as: the Economic Dispatch (ED), Unit Commitment (UC) and other techniques. The traditional monopoly has an obvious advantage in coordination of the security control/reliability and economic operation. In the early 1980s, there was near universal agreement that the electricity supply was naturally vertically integrated (i.e., generation, transmission and dispatching should belong to same owner). The cornerstones of this assumption were: (1) that there were increasing economics of scale in building generation plants; and (2) that as a consequence of the physics of the electricity product, close

coordination was required between production and transmission and, accordingly, separation of these functions should result in inordinate transaction costs [2].

In the 1970s, the successful deregulation of the gas/fuel industry caused people to think more seriously about deregulation in the power industry. Technological advances in gas turbines and the falling price of natural gas made way for small generation units, which, coupled with the challenge posed by the changing institutional arrangements by Chile beginning in 1982 and in the UK beginning in 1988, led the way to commercialization and restructuring of the electric power industry [3]. Throughout the early 1990s many countries began the process of deregulation, including: Australia, Norway, and New Zealand among others. In the United States, following the publication by FERC (Federal Energy Regulatory Commission) of rules 888 and 889 in 1996, several regions began the deregulation process, most notably, New England, New York and California. Although some of states have slowed their steps toward deregulation since the 2000-2001 crisis in California electricity market, the overall trend has not changed. Currently, there are 17 states, who have taken steps toward full or partial deregulation.

## **1.2 Market Based Operation**

The objective of market operation is to introduce competition into the supply and delivery of electric power. Under market operations, the integrated system of generation, distribution and transmission is typically separated into different entities. Electricity price is determined not by a regulatory agency but by the supply and demand relationship. In a centrally operated market, all entities, who wish to participate in the power market need, to submit bids. The market operator will clear the market based on these bids. The price of



electricity is found similar to other common goods, such as agricultural products. When supply exceeds the demand, the price will decrease, while when the supply is less than demand, the price should increase until a balance between supply and demand is met. In practice, the operation tends to be far more complex.

Market operation has experienced tremendous change from the first power markets in Chile and the UK. The UK electrical power market could be divided into three stages: first stage, England and Wales first, then Scotland and finally Northern Ireland [4]. The day-ahead spot market acts similar to a power pool, which is the heart of the UK power market. All market participants submit bids indicating the quantity and price at which they desire to buy and/or sell. The market operator will determine the energy price by clearing the market so that the aggregate supply equals the aggregate demand. In the uniform price auction, the highest successful bidder received price is paid by all consumers. There can also be several markets beyond simply the energy market. For example, the Norwegian power market, which is administered by the state-owned Statnett Market, consists of three distinct markets: day-ahead market, regulating market (also spot market, with shorter time span) and the weekly market.

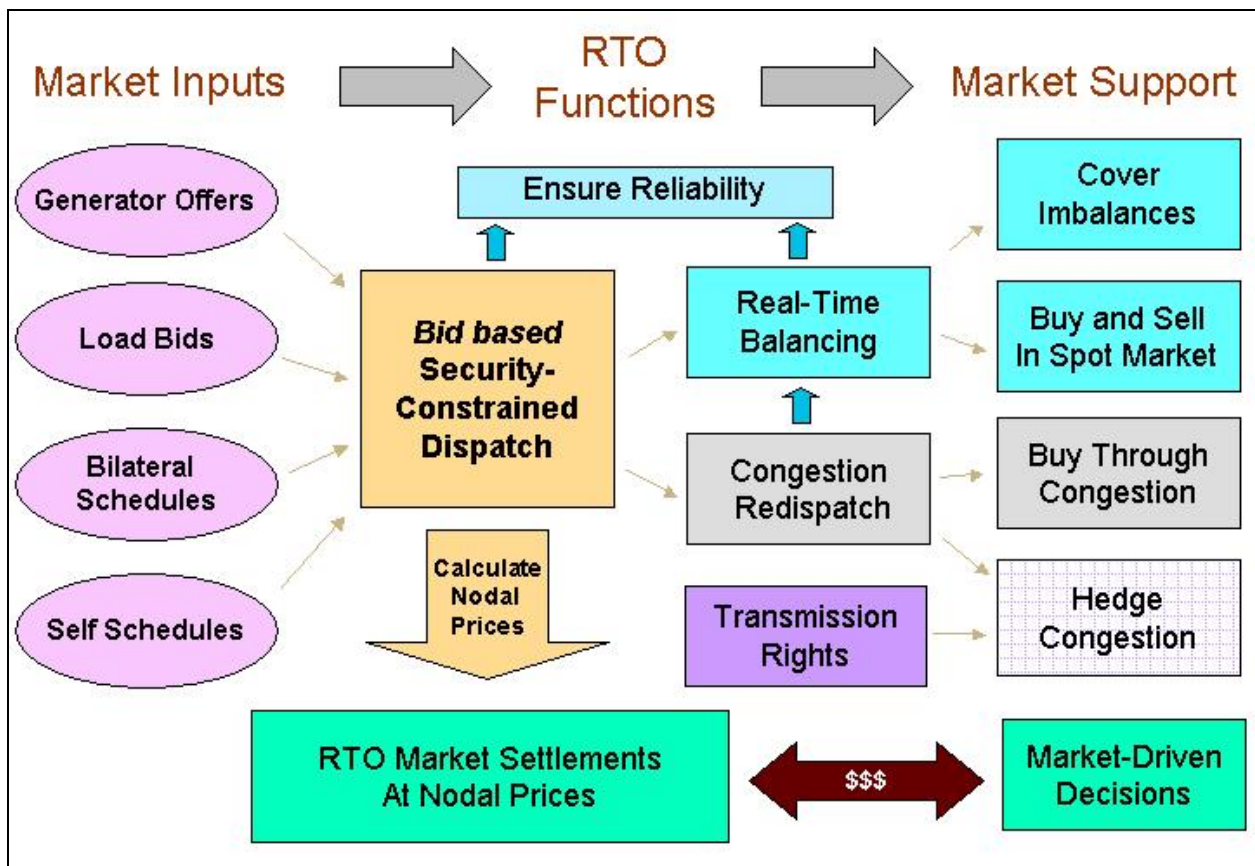
Both the UK and Norwegian markets have encountered difficulties. For example, early on the UK electrical power market did not show significant competition [5]. The supply side in UK is duopoly which has only two major company: National Power (70%) and PowerGen (30%). Thus, the electricity price is determined non-competitively by these two companies facilities and these two companies can easily exert the market power. As Thomas has pointed out, the new system displays a crucial weakness: “There is no explicit mechanism to balance supply and demand for power plants” [5]. Woo, et al. [6] evaluated the UK, Norway and California power markets as failures based on three criteria: Is the market competitive? Does

the market function properly? And does restructuring lead to marginal cost pricing? These questions about the value of market operation continue to be a source of much debate.

In the US, there were two major types of market operation in the mid 1990s, represented most characteristically by the Pennsylvania-Jersey-Maryland (PJM) and California Independent System Operator (CAISO) systems. PJM adopted a Locational Marginal Price (LMP) system that requires a system wide optimization problem and allows for different prices at different locations in the transmission network. CAISO created a simplified Zonal price system. The price spikes during 2000-2001 in the CAISO proved that the Zonal model as implemented by the CAISO has a number of problems and has since undergone major revisions. On July 31, 2002, FERC issued a Notice of Proposed Rulemaking (NOPR”) in Docket No. RMOI-12-000 and proposed a mandatory Standard Market Design (SMD) [7] that would be based upon: (1) the transfer of control of all transmission assets to Independent Transmission Providers (ITP); (2) a single transmission service known as Network Access Service; (3) security constrained bid-based dispatch for day-ahead and real-time spot markets with LMP and Congestion Revenue Rights; and (4) uniform market power monitoring and mitigation provisions. The SMD is controversial, and has yet to be universally adopted, but it described in detail in the following since it is certain to be influential within the US.

Under the SMD, a sample market operation could be represented by the diagram shown in Fig. 1.1. The diagram can be divided into three parts: market inputs, Regional Transmission Operator (RTO) or Independent System Operator (ISO) functions and market support. Market inputs refers to the all participants who will take part in market operations so not only generators and loads can be market participants but also transmission owners and

third party representatives. By definition [8], a Market Participant (MP) is: (i) any entity that, either directly or through an affiliate, sells or brokers electric energy, or provides ancillary services to the RTO, unless the Commission (FERC) finds that the entity does not have economic or commercial interests that would be significantly affected by the RTO's actions or decisions; and (ii) any entity that the Commission finds has economic or commercial interests that would be significantly affected by the RTO's actions or decisions.



*Fig.1.1 Market Operations under SMD*

The RTO has four important characteristics [9]: independence; scope and regional configuration; operational authority; and short-term reliability; that leads to eight major functions: (a) tariff administration and design; (b) congestion management; (c) parallel path

flow; (d) ancillary services; (e) OASIS, Total Transfer Capability and Available Transfer Capability; (f) market monitoring; (g) planning and expansion; and (h) interregional coordination. The ISO is similar to an RTO, but the RTO terminology is preferred in this thesis.

Most RTOs introduce centralized short-term real-time hourly markets and day-ahead markets for energy (i.e., spot markets) where sellers sell into the market and buyers buy from the market without matching a particular seller with a particular buyer. In this kind of market operation, market participants submit orders based on the business rules of RTO/ISO, and then based on the offers, the independent operator will run the security constrained algorithm to clear the market and ensure reliable operation of the system.

The day-ahead market provides a level of price certainty by clearing the market several hours prior to the start of the operating day. The benefit is that only a portion of the energy required is exposed to the uncertainty of the real time price. The market is cleared after receiving key inputs such as transmission outages, demand bids, resource offers, physical schedules and the proposed network model. The outputs are the cleared supply and demand, a set of constraints and the ex-ante LMP (ex-ante LMP is LMP generated during day ahead/ real-time market clearing process and ex-post LMP is generally refer to LMP calculated based on SE data after market clear) values.

The real-time market refers to the time period following the close of the hour-ahead market during which the RTO, or the control area operator, balances the system by deployment of energy from energy service, regulation service, operating reserve–spinning, and operating reserve–supplemental.

Different markets have different business rules. As a typical example, the rules from the Midwest ISO market operation [10] are given in the following for the day-ahead and real-time markets.

### **Day-Ahead Market Timeline**

- By 10:00 a.m.: Operations engineers complete the preliminary transmission security assessment, such as generation outages, scheduled transmission outages, and expected transmission constraints that may occur for the next market day.
- By 11:00 a.m.: Operations engineers begin to create the day-ahead market base case. At this time, the engineers will validate the input data to ensure the day-ahead market base case accurately reflects scheduled transmission system topology and limitations expected for the next day.
- At noon: Day-ahead offers and bids are locked.
- By Noon: Operations engineers begin clearing the day-ahead market. The market is cleared using a least-cost transmission security-constrained economic dispatch program. This dispatch determines the generation offers and virtual supply offers that satisfy the fixed demand bids, cleared price sensitive demand bids, and virtual demand bids, while minimizing the total production costs.
- By 4:00 p.m.: Hourly day-ahead energy market results are posted, including hourly MW schedules and LMPs.
- From 4:00 to 6:00 p.m.: Generation owners submit an hourly schedule of operation for each generator for the upcoming market day. Generators must also update their availability data for the next seven days, and submit start-up and minimum load offer prices to be used in the current day's reliability commitment.

- By 6:00 p.m.: The day-ahead resource adequacy assessment begins. This is done to ensure that sufficient generation resources are scheduled to satisfy the MISO load forecast and the control area reserve and regulation requirements. As a result, a revised schedule of operation may be created for the next market day. The schedule of operation will be communicated to each generation owner for the units for which it has submitted offers or schedules.
- By 8:00 p.m. up to operating day: Additional reliability commitment runs may be performed, as necessary, based on updated load forecasts and updated unit availability. MISO will send out individual generation schedule updates to the schedule of operation for specific generation owners only, as required.

### **Real-Time Market Timeline**

- From Day-Ahead 4:00 p.m. until 30 minutes prior to the start of the Operating Hour: MPs submit offers for the real-time energy spot market. Market Responsible Entities (MREs) may submit resource offers for the MISO real-time energy market. The energy offers cannot be revised after the hour-ahead market closes. Energy offers are selected through a security-constrained economic dispatch calculation with prices calculated using an LMP algorithm.
- 10 minutes prior to the start of the Operating Hour: Participants submit load offers and other data for the operating hour. The resource supply curves, generation operational data, resource plans, and external schedules are loaded into the market system that will dispatch instructions and ex-ante LMP.

- Every five-minutes of the Operating Hour: Every five minutes, MISO accepts updates to the external schedules, forecasts generation needs, determines current generator output, uses the data loaded prior to the operating hour to calculate the dispatch set points for each dispatchable generator, then sends any necessary instructions to the appropriate market participants.

### **1.3 Pricing**

A major issue faced by the power industry is how to price ancillary services and other transmission service as well as energy prices. There are essentially two different kinds of markets: contract markets and spot markets. In a contract market, the supplier and demander reach an agreement to sell/buy a certain kind of goods at a certain price. Both the price and quantity is specified. For example in the Australia power market, the energy price is settled mainly by contract price [11]. In the spot market, a MP will submit bids as a set of price/quantity pairs that represents the offer to provide energy. Generally, for a supply curve, prices must be monotonically increasing, i.e., the price of the first block offered must be less than the price of the second block offered, and so on. The participant may submit up to a given number of price/quantity pairs for each settlement interval to represent its incremental energy offer for the trading interval from each injection point. Prices must be expressed per MWh for each MW block submitted and the energy price and MW are cleared dynamically. The energy price is set as the Market Clearing Price (MCP) at which the aggregate demand is equal to the aggregate supply. Pricing of transmission and ancillary services tends to be far more complicated and there is still relatively little agreement on the best approach. Still, a

good pricing system should be able to give the right economic signals to all MPs, should be non-discriminating to all market participants, transparent and sufficient to cover costs [12].

Based on the transmission cost, the transmission pricing systems can be divided into three major groups: rolled-in transmission price, incremental transmission price and composite embedded/incremental price. In the rolled-in pricing system, all cost components are included. The total cost is allocated to the various system users. For example in the postage stamp pricing method [13], the transmission price is decided only by the MW to be transferred in the market. The charge of transmission usage can be represented by the following equation:

$$R_t = TotalCost \cdot \frac{P_t}{P_{peak}} \quad (1.1)$$

where  $R_t$  represents the transmission usage charge due to transaction  $t$ ,  $P_t$  the MW to be transferred due to transaction  $t$ , and  $P_{peak}$  the system peak MW value.

The postage stamp approach is the simplest transmission pricing method, but it has major shortcomings, including, providing incorrect economic signals by dividing the cost among all transmission customers. An alternative is the contract path method [14]. The contract path method attempts to consider the location information. Unfortunately, this method still may send incorrect economic signals since the transmission path is in fact complex, and a fictitious path between source and sink must be defined for contract purposes. The MW-mile [12] pricing method takes into account the distance between supply and demand. In [14] based on different voltage levels, the maximum distance is given. If the distance between the source and sink exceed this maximum value, then an excess charge will be incurred. This method considers the location's influence based on a theoretical analysis, it



still does not take into account the actual power flow on the grid. To reflect the real power flow caused by each transaction, it is necessary to include the power flow equation. A power flow based MW-mile method introduced in [14] was among the first methods to consider the real transmission system conditions. The transmission charge to each transaction is represented by the following equation:

$$R_t = \sum_k C_k \frac{|f_k(t)|}{f_{k \max}} \quad (1.2)$$

where  $C_k$  is the cost of transmission line  $k$ ,  $f_k(u)$  is the flow on line  $k$  caused by the transaction  $t$ , which is decided by the power flow and  $f_{k \max}$  is the maximum allowable flow on line  $k$ . This method considers the real power flow, but since the power flow on each transmission line generally is less than the transmission capacity, the price cannot fully cover transmission costs.

Many methods have been proposed to improve this cost recovery situation. For example, the sum of absolute MW flow caused by all transactions instead of the transmission capacity is proposed in [15]. Others have tried to reward flow that reduces congestion. For example, by “zeroing out” counter flow the transmission charge related to this transaction can be set to be zero [16]. The advantage of these power flow based methods is that they take into account the real transfer path of each transaction.

All of the above methods above are cost-based methods. Thus, no incentives are set for new investment in the transmission system. This shortcoming can be overcome by adding a penalty factor on the transmission usage charge, i.e., when the flow on transmission line approaches the limit, the charge increases to reflect the scarcity of the transmission line.

Further, it is not sufficient to consider only the embedded cost in the transmission price, the operating cost must be included. The incremental cost pricing system is one approach to deal with this problem.

Schweppe [17] first transferred the marginal pricing scheme to the electricity market. The generally idea of his work is model the electricity market as an optimization problem with the objective to maximize the overall social welfare subject to a number of limits, such as, power flow and generation limits, which could be represented by the following equations:

$$\begin{aligned} \max_{p_i} \quad & \sum_{i \in L} B_i(p_i) - \sum_{i \in G} C_i(p_i) \\ \text{s.t.} \quad & \sum_{i \in L} p_i = \sum_{i \in G} p_i - \text{losses} \\ & p_i^{\min} \leq p_i \leq p_i^{\max}, \quad \forall i \in G \\ & |p_{ij}| \leq p_{ij}^{\max}, \quad \forall i, j \text{ belong to transmission lines} \end{aligned} \quad (1.3)$$

The outcome of the optimization problem is the nodal price, i.e., the spot prices/nodal prices. Under this schema, the energy price is priced not only based on the generation cost, it also considers the delivery cost. This is the basis for the LMP schema, which has been adopted by the SMD. The solution can be represented by the following equation:

$$LMP = MEC + MCC + MLC \quad (1.4)$$

where *MEC* is marginal energy costs, *MCC* is marginal congestion cost and *MLC* is marginal loss cost as explained below:

- **Marginal Energy Cost:** This is the cost to generate one more unit of energy, say one MWh, which indicates the energy generation marginal cost for a particular operation scenario. If there is no congestion and no loss, then the energy price is the same at all locations.

- **Marginal Congestion Cost:** This cost relates to congestion in the transmission system. If there is congestion, then this cost should reflect how much would be saved if this transmission line's capacity increases incrementally. This will result in price differences between different locations.
- **Marginal Loss Cost:** This cost arises due to losses in the transmission system.

This method is sometimes called short-term marginal cost method since the generator output limit and transmission line capacities are assumed to be fixed and only the operation cost is reflected in the price signal. If there is no congestion in the transmission system, the total transmission usage charge is zero. Some complementary methods [18, 19] have been proposed to adjust the spot price accordingly. In [20], the long-term marginal cost is introduced in which the investment cost such as system expansion is reflected in the prices.

Still, all methods discussed are essentially cost-based methods. The embedded price system has advantages in terms of simplicity and cost recovery, but it fails to set an incentive for the new investment. Incremental prices provide the correct economic signals for the transmission scarcity but fail to recover the total system cost. In [14], auction-based alternatives to the cost-based approach have also been developed. These have yet to gain wide consideration and are not reviewed here.

## **1.4 Congestion Management**

Congestion in a power system is a consequence of network constraints characterizing a finite network capacity that limits the simultaneous transfer of power from all required transactions [21]. The limits are from maximum current flows, bus voltage requirements,

equipment ratings and so on. In addition, these limitations involve system security, i.e., the ability of the system to withstand disturbances. In a traditionally integrated power system, the utilities control both the generation and transmission system and maintaining the system security is a trade off between the system wide economic operation and security. In market operation, the challenge of congestion management is to create a set of rules that ensure sufficient control over producers and consumers to maintain an acceptable level of power security and reliability in both the short term and the long term, while maximizing market efficiency [22].

Generally, congestion management involves the economic re-dispatch of available resources based on the agreement among all MP and market rules. When re-dispatch is unable to solve the problem, curtailment is necessary. Curtailment is a reduction in firm or non-firm transmission service in response to a transmission capacity shortage or as a result of system reliability conditions. Curtailment is typically considered an integral component of congestion management. There are two major congestion management systems: LMP based congestion management and Zonal Price based congestion management. Based on the SMD, all congestion management in the US should be based on the LMP in the future. In LMP based congestion management, the LMP is used as price signal to mitigate the congestion. Zonal price based congestion management was adopted by CAISO, which will be discussed in detail in chapter 4. There are several other forms of congestion management, which do not involve optimization, for example, by allowing or disallowing bilateral transmissions to alleviate the congestion. The reader is referred to [22] for more details.

For curtailment, most ISO's choose the least cost strategy, including CAISO, MISO and PJM [23- 25]. All of them allow MPs to submit bids that reflect their willingness to

accept cutbacks. For example in CAISO, their objective of the re-dispatch is to minimize the total re-dispatch cost while main each scheduler coordinator's total MW change equal to zero.

$$\begin{aligned}
& \min_{\Delta p_i} \Delta P^T \cdot W \cdot \Delta P \\
& s.t. \quad f(p, u) = 0 \\
& \quad g(p, u) \leq g^{\max}(p, u) \\
& \quad \sum_{i \in SC_j} \Delta p_{ij} = 0
\end{aligned} \tag{1.5}$$

where  $p$  is the MW output,  $\Delta p$  is the MW to be curtailed,  $f(p, u) = 0$  is the power flow equation constraints,  $g(p, u)$  represents the security constraints such as line flow, voltage, etc.,  $w$  is the cost related matrix which represent an MP's willingness to be curtailed, and  $SC_j$  represent  $j^{th}$  schedule coordinator.

When all MPs's willingness to get cut are equal, i.e,  $w$  could be represented by a diagonal matrix with all diagonal elements equal to 1. This means that the cost of re-dispatch is only related to the MW value to be cut, which is the least curtailment method. In the least curtailment method, the objective is the amount of MW value to be curtailed, so the objective is to the make the new operation point as close to the original one as possible. From a mathematical point of view, the lease curtailment method is a simplified version of the least cost method.

## 1.5 Market Analysis and Monitor

From economics theory, in a completely competitive market, the marginal cost is the optimal equilibrium point. No single buyer/seller can influence the price since there are many buyers/sellers. The market ensures an overall efficient outcome where the price is equal to the marginal cost. Unfortunately, at this stage of the electricity power market, the market is far

from completely competitive. Two significant structural flaws are the lack of price-responsive demand (high degree of inelasticity of demand) and generation concentration in transmission-constrained load pockets [7]. Given these flaws, it is very possible, or even likely, that prices will deviate from the ideal. For example in the CAISO, market analysis results showed costs in excess of competitive levels of over \$6 billion for the period May 2000 through February 2001 [26]. Sheffrin [27] examined bids by individual suppliers in the real-time imbalance energy market of CAISO, and the results showed that most of the five in-state suppliers and many of the large importers displayed bidding patterns which were consistent with the exercise of market power. This highlights the importance of market monitoring as a necessary function for the RTO.

A major objective of market monitoring and market power mitigation measures is to deal with the consequences of major structural defects in wholesale electric markets by approximating the outcomes that a competitive market would produce. These measures should function in markets that are not workably competitive, but should not inhibit market operation in more competitive markets. The market power mitigation measures consist of the following 3 basic parts: local market power problem, safety-net bid cap and resource adequacy [7]. These should be conducted on an on-going basis by a market monitoring unit. Market monitoring requires using a set of questions and analytical techniques to assess market structure, participant behavior, market design and market power mitigation.

Several researchers have addressed market monitoring issues. For example in [28], the authors focused on developing a simulated competitive benchmark that can serve as a reasonable measure of the market's overall efficiency. Others have examined where specific generator bidding behavior has been consistent with profit maximization under competitive

conditions [29]. Some monitors estimate whether average generator profitability would cover costs of a gas-fired peaking unit and provide sufficient inducement for entry. Others measures try to track bidding patterns so that sudden, inexplicable changes can be investigated promptly to evaluate whether market power is the cause of change. Also, there are monitors that track changes using concentration measures (summation based on probability), unplanned generator and transmission outages and changes in various operating parameters that may signify market power problems [30]. Based on an approach from the American Antitrust Institute, the concentration statistics methods relied on by FERC in deciding the competitive problem are absent of consistency, and simulation models appear to be better choices [43]. Wolak [31] proposed a method using financial hedge contracts as a means to mitigate market power in the Australian power market.

A successful market monitoring tool requires a good market analysis technique. An examination of electrical power price for 14 deregulated markets reached the conclusion that North American markets show an unusual monotonic diurnal weekday price pattern while all other markets studied show more than one price peak [32]. A method for decomposing wholesale electricity payments into production costs, inframarginal competitive rent and payments resulting from the exercise of market power is presented in [33]. Borenstein, Bushell and Wolak draw the conclusion from 1998 to 2000 California experience, where the prices showed significant departure from competitive pricing, particularly during summer months. Still, it is very difficult to distinguish between source scarcity and market power and further research is needed. Currently, market monitoring by the ISO or RTO generally involves several indices which are used to compare market behavior. For example, PJM

considers hourly LMPs, frequency of LMPs, and the comparison of day ahead LMP with real time LMP.

## **1.6 Focus of Thesis**

There are many open questions concerning market-based operation for electric power, but from the MP perspective the central concern is the bidding strategies problem. This is the main focus of this thesis. This section summarizes the existing work on the bidding strategies and highlights illustrate the main contribution of this thesis.

### **1.6.1 Bidding Strategies**

For generator entities, the main operations concern is how to bid their resources. Traditionally, the power industry is an integrated system in which generation, distribution and transmission belong to the same company. The system operator decides which generator should be committed and at what MW output based on a traditional integrated optimization problem, i.e., the unit commitment (UC) and economic dispatch (ED) problems. The retail electricity price is decided by the government regulations. The objective of the generators is to minimize the generation cost given a set of set of reliability constraints.

In market operation, MPs (suppliers or loads) interact with the market through the submission of buy or sell prices for blocks of energy for specific periods of time, which are called “bids”. The participants achieve their respective performance goals by employing strategies with their bids. The market must be “impartial”, i.e., it cannot favor any particular participant’s performance, but instead has overall system savings optimization as its goal.



The bidding problem of each participant is formulated to earn maximum return from their local, or independent, perspective. A strategy is a complete contingent plan, or decision rule, that specifies how the player will act in every possible distinguishable circumstance, in which one might be called upon to move. So, strategy is the way each participant chooses to take part in the competition.

In a competitive market, a central feature of multi-player interaction is the potential for the presence of strategic interdependence. In multi-player situations with strategic interdependence, each agent recognizes that the payoff received (in utility or profit) depends not only on one's own actions but also on the actions of other individuals. The actions that are best may depend on actions that other individuals have already taken, are taking at the same time, and or on future action that they may take. This problem can be represented in a game theoretic framework [34]. The classic formulation refers to a Nash Equilibrium [34], where each player's strategy choice is a best response to the strategies actually played by his rivals so that

$$U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i}) \quad (1.6)$$

where  $s_i, s_{-i}$  represents player  $i$  and its rivals' strategies, while  $U_i$  represents player  $i$ 's utility function, and  $s_i^*$  represent the player  $i$ 's optimal strategy.

In energy markets, participants have a number of different strategies from which to choose. For example, some participants may choose to bid prices above marginal costs in order to enlarge their profits by selling at a high price. Still, the strategies must consider the principles of power system operation even if the participants have the freedom to price away from marginal production costs in the short-run.

Several researchers have looked at how to develop bidding strategies. Weber, et al. [35] developed a model in which a central operator solves an Optimal Power Flow (OPF) based on the maximization of social welfare to determine the generation/load dispatch and system price. The market chooses bids in order to maximize profit under the constraints that the dispatch and price are determined by the OPF. The approach uses price and dispatch sensitivity information available from the OPF to determine how a market participant should vary its bid portfolio in order to maximize its overall profit.

Chao-an Li, et al. [36] developed a revenue maximization bidding strategy in which they treat the energy trading problem under a competitive market structure as a centralized economic dispatch problem with a set of decentralized bidding sub problems for bidders participating in energy auction. Lamont and Rajan [37] present a framework in which strategies may be developed for individual participants in an energy brokerage. A key point of energy brokerage is the amount of information that is made available to a participant regarding the state of the market. Dividing the different types of participant into investor owned utilities (IOU), independent power producers (IPP), generation companies (Genco), distribution companies(Disco), large industrial customers (LIC), and power marketers (PM), a sub-optimal bidding strategy is formulated. The strategy uses the expected value of the lower bound on the saving achieved by buyers as and objective.

$$E_{lb}^{\max} = S(x) \cdot (c - x) = \frac{(x - b_{\min})^2}{d^2} (c - x) \quad (1.7)$$

where  $x$  is the competitor's buy bid;  $b_{\min}$  is the lower bound of quantities which the competitor won't willing to bid;  $d$  is the distance between the competitor's quantity lower bound the upper bound  $b_{\max}$ ;  $S(x)$  gives the probability of the competitor win the buy bid

given the buy bid  $x$  and  $c$  is the marginal generating cost. According to the first order condition, the sub-optimal bid is  $x_{lb}^* = \frac{2c + b_{\min}}{3}$ . The same approach might be used for a seller to determine a sub-optimal bid.

Dekrajangpetch et al. [38] discuss problems arising when an LR method is used to implement auctions in energy markets. They point out two main categories of difficulties: identical units (units with identical cost characteristics) and multiple optimal solutions. When identical units exist, there are two primary possibilities either LR will find only sub-optimal solutions (possibly in the sense of inequitable) or it may be unable to find any feasible solutions. This can be viewed as a problem of the economic interpretation of the LR iterations. If an energy market is considered, the LR algorithm proposes a sequence of hourly prices ( $\lambda$ ) to buy energy from GENCOs. GENCOs, each one independently, plan their output power in response to the price sequence, meeting their respective constraints. This results in a surplus of power in some hours and deficit of power in some other hours. They propose a rotating penalty for each company (unit) that should allow for equitable distribution among similar units.

In Richter et al. [39], trading agents use a genetic algorithm to evolve appropriate bidding strategies for current market conditions. The objective of the strategies used in the paper is strictly profit maximization. These strategies are coded in the form of finite automata coupled with genetic programming, which allows complex adaptive strategies to develop. Rau [42] used a mixed integer LP formulation to model the dispatch problem under competitive markets. The minimum up/down times of generating units are factored into the offer price by the intended supplier.

Game theory [34] has been proposed by several researchers. Bai et al. [40] used game theory to analyze the transmission system. Similarly, Ferrero et al. [41] used game theory to simulate the decision making process for defining offered prices in a deregulated environment. In this model, pool participants interact by means of price signals. Each participant is introduced as a player, economic benefits constitute payoffs and player's options are treated as strategies. Two classes of games are considered: a non-cooperative game and cooperative game. Spot price of electricity was introduced to simulate the player's strategy:

$$\text{Benefit}(r) = \sum_{i \in \Omega} \left\{ \left[ a(i) + b(i)p_0(i) + c(i)p_0(i)^2 \right] - \left[ a(i) + b(i)p(i) + c(i)p(i)^2 \right] + T(i)\rho \right\} \quad (1.8)$$

where  $a(i)$ ,  $b(i)$ , and  $c(i)$  are generation cost parameters for player  $i$ , and  $\rho$  is spot price. After transactions are defined, each seller receives  $T(i)\rho$  for generating excess power and each buyer pays  $-T(i)\rho$  for imports. Finally,  $\Omega$  is the set of generators for participant  $r$ . Assume each player is a utility (here the system is separated to three utilities for simplicity) that can either supply its local load or sell power to the pool depending on the market price. Three strategies are analyzed in paper [41]: 1) bid high (H), trading power at 1.15 times the marginal cost; 2) cooperate with pool, trading power at the marginal cost (M); and 3) bid low (L), trading power at 0.85 times the marginal cost. Based on the marginal price, two utilities, say A and C, sell power in a grand coalition while the third, utility B, buys power. Given perfect competition (i.e, no capacity limits and no additional constraints, such as, transmission limits), the dominant strategic equilibrium is ((M, M), H) of the game. Interestingly, if a transmission constraint of the tie-line between utility A and C is considered, utility C will change from selling power to buying power and the grand coalition A-C will broken.

## 1.6.2 Thesis Contribution

As discussed in above sections, there are many publications on the bidding strategy problem. Still, much of the research on bidding strategies in an electricity market has focused on idealized situations where participants have limited market power and the transmission system is not constrained. In practice, congestion often acts to effectively give a bidder market power, and consequently the ability to influence the market clearing price. In such a non-competitive situation, the bidding strategies of market participants will change. Thus, it is important to consider congestion's influence in bidding problem.

As will be discussed in a later chapter, it is not a trivial task to analyze the bidding problem via a pure mathematical model. Even for a small system, this involves extensive computation. This thesis seeks a more empirical approach to the bidding problem. As discussed in the market monitoring section, there are number of approaches that could be termed empirical approaches to market analysis. This work seeks to take advantage of these analysis results as most such analysis involves system price and bidding behavior. A market participant may take these analysis results as reference in his bidding decision making process. Relatively few efforts apply the empirical analysis technique in bidding behavior analysis for market participant.

Thus, the primary contributions of this thesis are outlined in the following.

- **Congestion's influence on bidding strategies** - In this work, the electrical power market is modeled as an oligopoly market and the Cournot quantity model is applied to the bidding strategy problem. The bidding process with congestion management is modeled as a three level optimization problem. A statistical methodology is then proposed as a solution for large systems.

- **Empirical analysis of bidding in California ISO real-time energy market** – Bidding in the California ISO real-time energy is analyzed by an empirical technique. Through a combination of correlation analysis, linear regression models and neural network, the impacts of gas price, system load and congestion on the market overall supply's bidding behavior and zonal energy price are analyzed. First, correlation coefficient and significant test technique was applied to analysis the supply behavior in CAISO real-time energy market, and the conclusion that the linear model is not suitable for the bids forecast has been drawn. Based on this conclusion, the neural network is applied in average bidding price forecast process. Sensitivity analysis is carried out for application to the optimal bidding strategy problem. Similarly, a linear regression method is applied to zonal price prediction, where a test and try method is introduced to divide the data set into several groups for linear price predicting. When load is light, the linear method is adequate to model the zonal energy price but under heavier loading conditions this does not hold.
- **Empirical conjecture model application to bidding problem** – The bidding strategy problem is a decision making problem that involves numerous factors. Primarily the objective is to maximize its profit, but MPs need to consider not only system conditions but other MPs activity. The empirical analysis results are applied to the decision making process by modeling the optimal bidding strategy problem via a conjecture model. In addition, since risk plays a central role in the decision making process in practice, risk is controlled through constraining the standard deviation of profit. Thus, the optimal bidding strategy problem is modeled as a mean maximization

while minimizing standard deviation. The numeric result shows the feasibility of the methods proposed.

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## **CHAPTER TWO**

### **CONGESTION INFLUENCE ON BIDDING STRATEGIES IN AN ELECTRICITY MARKET**

Much of the research on bidding strategies in an electricity market has focused on idealized situations where participants have limited market power and the transmission system is not constrained. Yet, congestion may act to effectively give a bidder market power, and consequently the ability to influence the market clearing price. In such a non-competitive situation, the bidding strategies of market participants will change. In this chapter, the electrical power market is modeled as an oligopoly market and the Cournot quantity model is applied to the bidding strategy problem. The bidding process with congestion management is modeled as a three level optimization problem. A statistical methodology is then proposed as a solution for large systems.

#### **2.1 Introduction**

Understanding how market participants bid into the electricity market is of fundamental importance for designing electricity markets. Generally, the objective of market participants is to maximize their expected profit. Since the expected profit of each participant depends upon the joint actions of others, effective decision-making requires that each participant evaluate the effects not only of their own actions, but also of the actions undertaken by the others. The complexity of these interactions makes it difficult to determine a priori the strategies that market players will employ in bidding.

There have been numerous attempts to model bidding strategies using optimization methods. For example in [1], a Lagrangian relaxation method was used to determine the utilities' optimal bidding and self-scheduling, and based on the New England ISO, a closed form solution was found assuming a simple bidding model. In [2], the power market was treated as an oligopoly, and by "guessing" competitor's bidding curves, a stochastic optimization model was built. Richter and Sheble applied a genetic algorithm to GENCO strategies and schedules, in which an intelligent bidding strategy was developed using a GP-Automata algorithm [3]. In [4], the combination of different pricing systems and curtailment methods was analyzed so as to understand methods to prevent taking advantage of network congestion. Hao studied the bidding strategies in a clearing price auction [5]. Based on the clearing price auction, the author drew the conclusion that the market participants have incentives to mark up their bids above their production cost and the amount of mark up depends on the probability of how frequently they win the bid.

Ni, et al., presented a unified optimization algorithm for the bidding strategy problem given a mix of hydro, thermal and pumped storage units [6]. Their algorithm manages bidding risk and self-scheduling requirements. Gan and Bourcier modeled the market as a single-period auction oligopoly market and examined the influence of suppliers' capacity constraints [7]. Shrestha, et al., analyzed the effect of minimum generator output [8]. Others, such as Ferrero, et al. [9], applied game theory to analyze transactions. In their work, spot price was used to calculate the payoff matrix and both Nash equilibrium and characteristic functions were applied to the bidding analysis. Congestion charges were not considered in their work. Hobbs and Kelley applied game theory to electric transmission pricing [10]. Bai, et al. applied the Nash Game equilibrium concept to the transmission system [11]. Yu, et al. [12] investigated

transmission limits and the influence of wheeling charges on competitive and gaming behavior. It was shown that wheeling charges and transmission line limits stimulate gaming phenomena.

While providing valuable insight into transmission system impacts, none of these efforts have fully incorporated transmission constraints into the bidding strategies. In practice, congestion management is separate from the bidding process and as such difficult to analyze in a single bidding framework. When congestion occurs, a non-competitive situation, i.e., deviation from price-taking behavior, is far more likely to occur. Much of the literature has ignored congestion or included it as part of the bidding process, as in [13]. That is, most researchers have included the congestion as constraints within the market clearing process. This is not representative of typical market rules.

In this paper, the bidding strategy problem is modeled as a three level optimization problem, and the congestion's influence is explicitly expressed in the profit function. Game theory is applied to the optimal bidding strategies problem based on a UK pricing system. Congestion's influence is modeled and the curtailment due to congestion is calculated via a separate least curtailment method [14]. Numerical examples clarify congestion's influence on price and bidding strategies. Subsequently, these results are modified to reflect behavior based on a statistical study of bidding in the California market.

## **2.2 Hierarchical Model of Bidding Process**

In a power clearing market, each participant submits a bidding curve to a power exchange, or similar organization. The exchange will decide the market clearing price (MCP)

based on these bids. The security coordinator then checks to ensure that the resulting bidding schedule is feasible. When there is a security problem, curtailment will be performed. If the uniform price and least curtailment algorithm are used, the bidding problem can be represented by a hierarchical optimization problem developed in this section. Other congestion management approaches may differ in details but the approach is similar.

The electricity power market is in practice an oligopoly [15]. In an oligopoly market, competition among the market participants is inherently a setting of strategic interaction. Thus, the appropriate tool for analysis is game theory. In the electric power market, the participants submit their bids first, and then the MCP is found by matching the aggregate demand to the aggregate supply. The bidding strategies have a clear influence on the MCP and price cannot be treated as a simple function of demand. The Cournot quantity model [16] is applied to the bidding strategy decision problem here. Other oligopoly market models, such as Bertrand's price competition model, may be more appropriate in specific situations. The Cournot model assumes that generators compete more by quantity than by price and generally holds well when capacity changes more slowly than price.

### 2.2.1 Market Clearing Price

To determine MCP, the exchange looks at the aggregated supply bid curve and the aggregated demand curve with the highest accepted bid the MCP. Assume for simplicity, the bidding curves are given as continuous curves of the form:

$$IC_i(p_i) = b_i + a_i p_i \quad (2.1)$$

where  $IC_i(p_i)$  is the incremental price for generating at  $p_i$  by the  $i^{th}$  generator, and  $a_i$  and  $b_i$  are the bidding coefficients. Here, we further assume that these two parameters have the



following relations with the generator costs,

$$k_i = \frac{b_i}{b_{ic}} = \frac{a_i}{a_{ic}} \quad (2.2)$$

where  $a_{ic}$  and  $b_{ic}$  are parameters from the generator's actual cost function. The true costs are given by:

$$C_{ic}(p_i) = c_{ic} + b_{ic}p_i + \frac{1}{2}a_{ic}p_i^2 \quad (2.3)$$

Thus, the bidding parameter  $k_i$  represents the proportion above (or below) marginal cost that a generator  $i$  decides to bid, i.e., the markup. Certainly, more complex functions for strategies are possible, e.g., the use of random variables [17]. Here, the focus is on the congestion's influence and mark-up provides more insight to the direct impact. Further, the strategies that might pursued by consumers are ignored and instead a simple demand benefit function  $B_i(p_i)$  is used to model their role as:

$$B_i(p_i) = b_i p_i - 0.5a_i p_i^2 \quad (2.4)$$

where  $p_i$  is the load consumed at bus  $i$ . The market clearing problem is represented by the following social welfare maximization problem (ignoring losses):

$$\begin{aligned} \max_{p_i} \quad & \sum_{i \in L} B_i(p_i) - \sum_{i \in G} C_i(p_i) \\ \text{s.t.} \quad & \sum_{i \in L} p_i = \sum_{i \in G} p_i \\ & p_i^{\min} \leq p_i \leq p_i^{\max}, \quad \forall i \in G \end{aligned} \quad (2.5)$$

where  $L$  and  $G$  represent the set of loads and generators, respectively; and  $p_i$  is the load in MW the  $i^{\text{th}}$  player delivers or receives in the bidding. The cost function  $C_i(p_i)$  here is derived from bidding curves:

$$C_i(p_i) = b_i p_i + 0.5a_i p_i^2 \quad (2.6)$$

Solving (2.5) yields the MCP, the generator outputs  $p_i^*$  and demands that provides maximum benefit. The MCP is simply:

$$\text{MCP} = \max_{i \in G} IC_i(p_i^*) \quad (2.7)$$

### 2.2.2 Congestion Management

When the bidding process is finished, the system security is analyzed. If there exists a security problem, curtailments must be carried out, either by modifying the generation dispatch or reducing load. While there are many different kinds of curtailment algorithms, here, the separate curtailment algorithm [14] is applied. Assuming a DC load flow model [18] (those equations are omitted for brevity) and no load curtailment (since demand side bidding is not considered here), this is formulated as:

$$\begin{aligned} \min_{\Delta p_i} \quad & \Delta P^T \cdot W \cdot \Delta P \\ \text{s.t.} \quad & \sum_{i \in G} \Delta p_i = 0 \\ & |P_{ij}| \leq P_{ij}^{\max} \end{aligned} \quad (2.8)$$

where  $\Delta P = [\Delta p_1, \Delta p_2, \dots, \Delta p_n]^T$  is the vector of the supplier's curtailment, so that  $\Delta p_i > 0$  means the  $i^{\text{th}}$  supplier must increase its output, the  $P_{ij}$  are the line flows; and  $W$  is a diagonal weight matrix whose elements denote the participant's willingness to pay to avoid curtailment. In this paper, the weights are set to 1, i.e., the objective of curtailment is the least curtailment. When a generator's output is reduced, it should be compensated for possible lost profits by receiving some payment. This is found here as

$$\text{RC}_i = \Delta p_i [\text{MCP} - (a_i + b_i p_i + b_i \Delta p_i)] \quad (2.9)$$

The supplier is compensated based on the philosophy that their bid represents their actual costs and so this payment will account for the actual loss of profit. Again, there are

other approaches to compensation but the approach here can accommodate such methods. The assignment of costs to consumers and the transmission company is not germane to the development here.

### 2.2.3 Bidding Strategy

When uniform pricing is applied in the system, all power originally purchased and actually run is paid at MCP. Thus, the profit function of participant  $i$  is:

$$\text{Profit}_i = \text{MCP} \cdot (p_i + \Delta p_i) - C_{ic}(p_i + \Delta p_i) + \text{RC}_i \quad (2.10)$$

MCP used here is the solution to (2.5),  $\Delta p_i$  is the curtailment due to the congestion from (2.8) and  $\text{RC}_i$  is found from (2.9) and  $C_{ic}(p_i + \Delta p_i)$  is the generator's production cost. For participant  $i$ , the best strategy is the bidding parameter  $k_i$  that will maximize profit. When the congestion problem is taken into account, the  $i^{\text{th}}$  player's problem is represented by the following maximization problem:

$$\max_{k_i} \text{Profit}_i \quad (2.11)$$

while satisfying (2.5)-(2.8). Note, the true production costs from (2.3) should be used in the solution.

### 2.2.4 Problem Formulation

The bidding strategy problem is now seen more clearly as a hierarchical optimization problem. For simplicity, the generator capacity limits are omitted at first. The inner solution for (2.7) is

$$\begin{aligned} \text{MCP} &= b_i + a_i p_i, \forall i \in G, \\ &= b_j - a_j p_j, \forall j \in L \end{aligned} \quad (2.12)$$

Simple algebraic manipulations shows:

$$\text{MCP} = \frac{\sum_{i \in L} \frac{b_i}{a_i} + \sum_{i \in G} \frac{b_{ic}}{a_{ic}}}{\sum_{i \in G} \frac{1}{k_i a_{ic}} + \sum_{i \in L} \frac{1}{a_i}} \quad (2.13)$$

with

$$p_i = \frac{\text{MCP} - k_i b_{ic}}{k_i a_{ic}} \quad (2.14)$$

The revenue from curtailment simplifies to:

$$\text{RC}_i = k_i a_{ic} \Delta p_i^2 \quad (2.15)$$

The individual's bidding problem (2.11) can be solved directly by substituting (2.12)-(2.15) if  $\Delta p_i$  is known. Unfortunately,  $\Delta p_i$  will not be known until after congestion management. In many power markets, the PTDF (Power Transfer Distribution Factor) [19] is used to decide the curtailment/redispach. Here, we use the GSF (Generation Shift Factor) [19], which is essentially same except the focus is on the sensitivity between the generation and transmission line. Assume the GSF is denoted by  $\rho_{jk,i}$ :

$$\rho_{jk,i} = \frac{\Delta P_{jk}}{\Delta p_i} \quad (2.16)$$

where  $\Delta P_{jk}$  is the flow change on line  $j$ - $k$ , and  $j$  and  $k$  are the initial bus and terminal bus of the line. When the DC power flow is employed, the GSFs are constants related to the system topology parameters. The curtailment of each generator can be represented as a linear function of overflow of the congested line and the GSFs. Let's take one congested path as an

example, assume there is only congested path with  $P_{jk}^{\max} < P_{jk}$ , then (2.8) can be solved and rewritten as (the derivation is given in Appendix A):

$$\Delta p_i = \frac{\rho_{jk,i} - \frac{1}{n} \sum_{j=2}^n \rho_{jk,j}}{\sum_{i=2}^n (\rho_{jk,i}^2 - \frac{1}{n} \rho_{jk,i} \sum_{j=2}^n \rho_{jk,j})} \cdot (P_{jk}^{\max} - P_{jk}) \quad (2.17)$$

Without congestion, there is no curtailment, i.e., the  $\Delta p_i = 0$ , so we can rewrite (2.17) in general form as:

$$\Delta p_i = \frac{\rho_{jk,i} - \frac{1}{n} \sum_{j=2}^n \rho_{jk,j}}{\sum_{i=2}^n (\rho_{jk,i}^2 - \frac{1}{n} \rho_{jk,i} \sum_{j=2}^n \rho_{jk,j})} \cdot \min(0, P_{jk}^{\max} - P_{jk}) \quad (2.18)$$

### 2.3 Solution Method

The difficulty in this problem stems from the conditional constraint (2.18). This differs from an ‘either-or’ type constraint that can be modeled as mixed integer problem since the existence of the constraint depends on the solution of the problem. Here, a method similar to branch and bound is employed. Given the rivals response, a series of ranges that divide a player’s response into congestion and non-congestion situations are found. Thus, the problem divides into a series of relaxations. A simple example illustrates the approach.

Consider the system from [13], there are two supplies and one demand whose parameters are shown in the Fig. 2.1. The power flow on the only line will be  $P_{12} = q_1$ . With a limit of power flow on this line of  $P_{ij}^{\max}$  (MW), then the conditional constraint can be written as:

$$\Delta p_1 = -\Delta p_2 = \min(0, P_{12}^{\max} - p_1) \quad (2.19)$$

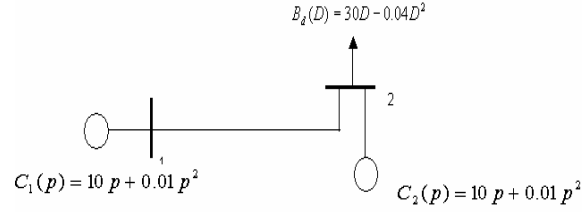


Fig. 2.1 Example System 1

For player 1, simple substitution (2.11) in (2.12) yields:

$$p_1 = \frac{MCP - k_1 a_{1c}}{k_1 b_{1c}} = \frac{\frac{d_1 + a_{1c} + a_{2c}}{d_0} + \frac{a_{1c}}{b_{1c}} + \frac{a_{2c}}{b_{2c}}}{1 + k_1 b_{1c} \left( \frac{1}{k_2 b_{2c}} + \frac{1}{d_0} \right)} - \frac{a_{1c}}{b_{1c}} \quad (2.20)$$

If  $p_1 > P_{12}^{\max}$ , then there is a congestion problem, otherwise, there is no congestion problem.

Solving  $p_1 = P_{12}^{\max}$ , yields

$$k_1^{\max}(k_2) = \frac{\left( \frac{d_1 + a_{1c} + a_{2c}}{d_0} + \frac{a_{1c}}{b_{1c}} + \frac{a_{2c}}{b_{2c}} - 1 \right)}{P_{12}^{\max} + \frac{a_{1c}}{b_{1c}}} \Bigg/ b_{1c} \left( \frac{1}{k_2 b_{2c}} + \frac{1}{d_0} \right) \quad (2.21)$$

This function  $k_1^{\max}(k_2)$  divides the problem into the congested and non-congested strategies.

That is, if  $k_1 \leq k_1^{\max}(k_2)$ , then there is congestion. Thus, the bidding problem is now the

following two optimization problems:

$$\begin{aligned} & \max_{k_1} MCP \cdot p_1 - C_1(p_1) \\ \text{s.t. } & MCP = \frac{\sum_{i \in L} \frac{d_{1i}}{d_{0i}} + \sum_{i \in G} \frac{a_{ic}}{b_{ic}}}{\sum_{i \in G} \frac{1}{k_i b_{ic}} + \sum_{i \in L} \frac{1}{d_{0i}}} \\ & p_1 = \frac{MCP - k_1 a_{1c}}{k_1 b_{1c}} \\ & k_1 \geq k_1^{\max}(k_2) \end{aligned} \quad (2.22)$$

$$\begin{aligned}
& \max_{k_1} MCP(p_1 + \Delta p_1) - C_1(p_1 + \Delta p_1) + k_1 b_{1c} \Delta p_1^2 \\
& \text{s.t. } MCP = \frac{\sum_{i \in L} d_{0i} + \sum_{i \in G} a_{ic}}{\sum_{i \in G} k_i b_{ic} + \sum_{i \in L} d_{0i}} \\
& p_1 = \frac{MCP - k_1 a_{1c}}{k_1 b_{1c}} \\
& 0 \leq k_1 \leq k_1^{\max}(k_2)
\end{aligned} \tag{2.23}$$

Given player 2's bidding parameters, both (2.22) and (2.23) can be solved. The more profitable solution of these two solutions is player 1's best response. Repeating for all of player 2's possible strategies will determine player 1's optimal responses. If this is duplicated for determine player 2's optimal strategies, then the market equilibrium point can be found by comparing solutions. While this procedure appears to be viable, even for larger systems with many players, complex relationships in  $k_i^{\max}(k_j)$  may arise that render finding market equilibrium points extremely difficult.

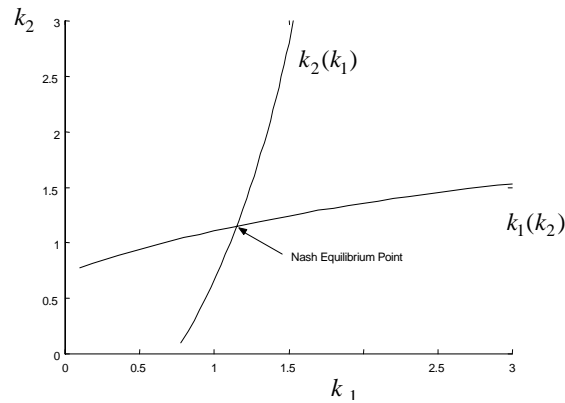
## 2.4 Numerical Results

To analyze congestion's influence on the bidding strategy and price, we first look at the situation when no congestion management is included. Subsequently, transmission system limits are included in the calculation. Comparing these two results highlights the influence of congestion on the optimal bidding strategy.

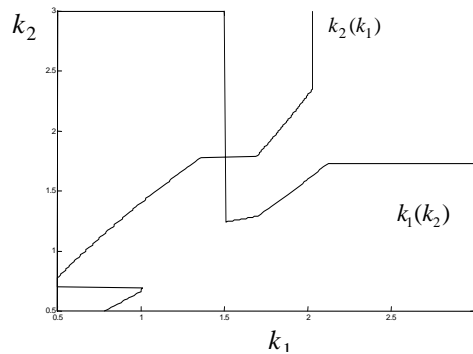
### 2.4.1 Example 1

Consider the system as shown in Fig. 2.1 but neglecting transmission line capacity. Fig. 2.2 and 2.3 plot the optimal values for  $k_1$  vs.  $k_2$ , with and without transmission constraints, respectively. The maximum for  $k_1$  and  $k_2$  is assumed to be 3. A maximum value

acts similarly to a price cap and is needed since the demand is relatively inelastic leading to unbounded mark up without the constraint. With no transmission constraints, the pure Nash equilibrium is for both players to choose to bid at 1.15 times marginal cost. When an 80 MVA transmission line capacity is included, the optimal strategies change radically.



*Fig. 2.2 Example 1 – Optimal strategies without transmission limit*



*Fig. 2.3 Example 1 – Optimal strategies with transmission limit*

As the constraint comes into force, this translates into sudden changes in strategy, i.e., a large variation in both  $k_1$  and  $k_2$ . A pure Nash equilibrium does not exist. For player 1, values of  $k_1$  in  $[1.36, 1.69]$  result in identical profit when player 2 chooses to play at  $k_2=1.78$ . Similarly, for player 2, values of  $k_2$  in  $[1.255, 3.0]$  result in identical profit when player 1 chooses to play at  $k_1=1.553$ . This is similar to the result in [1]. Thus, one should consider the



possibility of a mixed strategy equilibria. The mixed strategy for this problem is: player 1 will choose to play at  $k_1=1.36$  with probability 0.53 and  $k_1=1.69$  with probability 0.47, and player 2 will choose to play at  $k_2=3.0$  with probability of 0.80 and  $k_2=1.25$  with probability of 0.20. An approach to computation of the mixed strategy equilibrium point is given in Appendix B.

The above simple example shows that generator 2 should bid at the maximum feasible price most of the time. This means that player 2 is willing to forego any sale in the first round bid and take profits from the congestion round. Notice in this system, only player 1 faces a congestion problem, i.e., since  $P_{12} = q_1$ , the maximum output of generator 1 can only be  $P_{12}^{\max}$ . There is no transmission limit for generator 2. Thus, no matter how high generator 2 bids; it will finally win some bid when the curtailment is taken into account. When generator 2 expects congestion, the higher bid will tend to increase MCP. The system's potential congestion guarantees player 2 wins  $D - P_{12}^{\max} MW$ . This "biased" congestion situation (i.e., the congestion imparts more constraints on certain players) gives player 2 significant market power.

### 2.4.2 Example 2

In this example, both of the generators face transmission limits. Let the parameters of generators and loads remain the same, but the network is now the system [20] shown in Fig. 2.4. When the transmission system limits are not included, the system will have same Nash Equilibrium at  $k_1 = k_2 = 1.15$  as the former system. The line flows will be:

$$P_{12} = 0.6q_1 - 0.1q_2$$

$$P_{23} = 0.6q_1 + 0.9q_3$$

$$P_{13} = 0.4q_1 + 0.1q_2$$

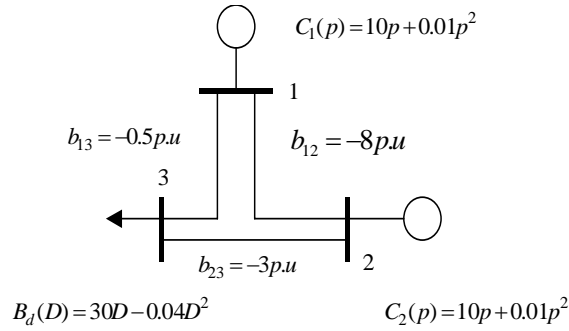


Fig. 2.4 Example System 2

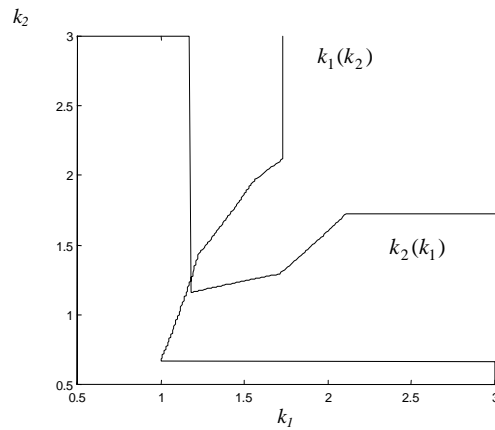


Fig. 2.5 Example 2 – Optimal strategies with transmission limit

with limits of:

$$P_{12}^{\max} = 78$$

$$P_{13}^{\max} = 225$$

$$P_{23}^{\max} = 300$$

The result of the response of a player versus its rival is shown in Fig 2.5. There is a jump in  $k_2$  from 3 to 1.16 when player 1 plays at 1.18. Again, there is no pure Nash Equilibrium, so a mixed strategy equilibrium point is sought. Since  $k_1$  is continuous,  $k_1=1.18$  with probability 1.0. The best response of player 2 is to choose to play at 3 with probability of only 0.09, and at 1.16 with probability of 0.91.

Relative to the first example, both generators tend to decrease their bidding price as they are both at risk of losing a sale due to the congestion problem. Both generators will increase their bidding price if there is any possibility of congestion. Notice also that the more serious the congestion, the higher the bidding price. When the transmission system is “fair” to each market participant, i.e., there is no obvious congestion problem for some participants, the market participants will have more incentive to bid at their marginal cost.

### 2.4.3 Example 3

A modified IEEE-30 bus system from [9] is applied in this example with two dominant market participants. System data and line limits can be found in [21]. Table 2.1 lists the respective cost functions.

TABLE 2.1  
Generator Cost Functions

Market Participant	Bus	Cost Coefficients			Max	Min
		$a(i)$	$b(i)$	$c(i)$		
A	1	0	2	0.02	0	80
	2	0	1.75	0.0175	0	80
	22	0	1	0.0625	0	50
B	13	0	3	0.025	0	30
	23	0	3	0.025	0	40
	27	0	3.25	0.00834	0	55

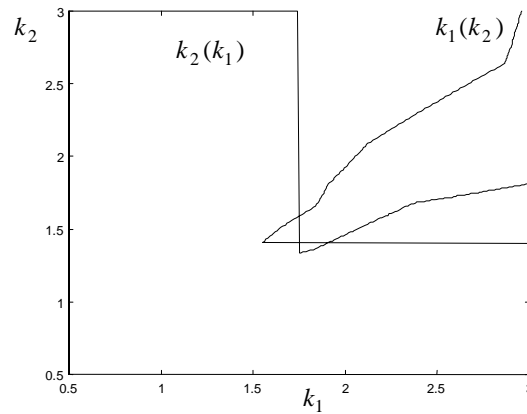


Fig. 2.6 Example 3 – IEEE-30 bus system with line limits

For simplicity, the benefit function for all demands is assumed to be identical.

Specifically:

$$B_i(D_i) = 18.136D_i - 0.02D_i^2$$

When the congestion is not included in the bidding process, there is a pure Nash equilibrium point at (1.27, 1.19). Considering congestion, the result is as shown in Figure 2.6. Similar to the previous examples, the pure Nash Equilibrium point disappears with the introduction of the congestion's influence. The mixed strategy equilibrium point is that player A chooses to play at  $k_1 = 1.41$  with probability 1.0; player B chooses to play at  $k_2 = 3.0$  with probability equal to 0.23, and at  $k_2 = 1.13$  with probability 0.73.

Comparing these results with the simpler cases, player A has more incentive to play high under the influence of congestion while player B tends to remain near the pure Nash equilibrium point. An examination of the congestion at the Nash equilibrium points shows that the transmission line (2-6) is overloaded. The generator at bus 2 belongs to player A while bus 6 is an intermediate bus, which connects with several load buses. Thus in this

situation, player A has more possibilities to force congestion and incentive to increase mark up.

Understandably as the system becomes more complex, finding the precise influence of congestion on the bidding strategy becomes more difficult. The possibility of more than one mixed strategy equilibrium point arises and other influences arise which make it is more difficult to apply the results. Thus, the next section introduces a new bidding strategy using statistics but following the basic form as the previous.

## **2.5 New bidding strategy**

The above examples show analytically how congestion influences the bidding strategy problem, and at least for these scenarios, shows pure Nash equilibrium points are less likely. Unfortunately, even for these idealized problems, the optimal strategies are difficult to find. For a larger system with many participants and where precise information about transmission limits is more difficult to determine, it may not be feasible to construct a practical formulation. The authors' analysis of actual bidding behavior in the California market will be used to modify the approach in the previous section. Specifically, the optimal strategy problem is simplified to reflect the information that would be most readily available for all participants. A few observations help clarify the approach.

- Due to the complexity, and limited knowledge of the transmission limits by most participants, congestion is modeled as the probability of congestion. This probability is based on the percentage of time that congestion exists during an operating day. The participants are assumed to be aware of this general risk of congestion, and in fact, this can be determined from historical data.

- The generators have different relative locations to the congestion zones. So a given congested path will tend to influence some generators more than others and that may be reflected by either higher or lower bids.

The analysis here looks at a base line when the possibility of congestion is low and compares this to congested time periods. The average bidding price is adopted as the index of bidding strategy and then the correlation coefficient between this index and congestion percent based on the day-ahead market are calculated. This coefficient can then be used as the indicator of adjustments due to the congestion. Here, we assume that a participant seeking to take advantage of congestion will modify  $k$  based on a linear function of the probability of congestion. For the examples here this is given as

$$\Delta k = 0.275P(\text{congestion}) \quad (2.26)$$

The following strategy is then employed. The bid will decrease  $k$  for all those bids less than the optimal output  $P^*$  and increase  $k$  for all those bids greater than  $P^*$ . By doing do, the bidding output (including  $MCP$  and  $P^*$ ) without considering congestion's influence will remain unchanged, i.e., the optimal strategy is chosen. When there is congestion, compensation will increase due to the difference between  $MCP$  and the bid price increase, and hence there will be greater profit. Also, since the higher the congestion possibility, the larger  $\Delta k$ , greater profits are realized at times of high congestion. Fig. 2.7 shows the new bidding strategy. Notice the result has a similar characteristic to the earlier example.

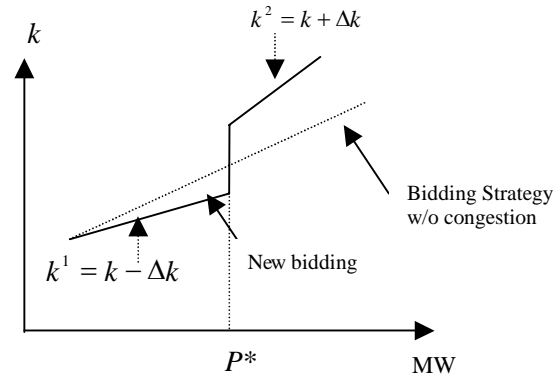


Fig. 2.7 New bidding strategy

From the earlier examples, the original optimal point is seen to be  $k_1^* = k_2^* = 1.15$  and the corresponding outputs are  $P_1^* = P_2^* = 101.08$  MW. When the 80 MVA line power limit is introduced and assuming a simple uniform distribution, the probability of congestion in the system is

$$P(\text{congestion}) = \frac{P_1^* - P_{12}^{\max}}{P_{12}^{\max}} = \frac{101.0781 - 80}{80} = 26.53\% \quad (2.27)$$

Thus,

$$\Delta k = 7.3\%$$

$$k^1 = k_1^* - \Delta k = 1.08 \quad (2.28)$$

$$k^2 = k_1^* + \Delta k = 1.22$$

This new bidding strategy is compared with the theoretical mixed Nash equilibrium and shown in Table 2.2. The results show that in the “biased” congestion case, when player 2 chooses to bid at 3.0, the profits of both players will be significantly higher than in the proposed probabilistic approach. This case also requires a significant amount of load curtailment so the result is not surprising. The statistical approach shows similar results to that

obtained in the “fair” congestion case.

TABLE 2.2  
STRATEGIES FOR ALL FOUR APPROACHES

	Player 1			Player 2		
	$k^*$	$P^*(MW)$	Profit	$k^*$	$P^*(MW)$	Profit
No congestion	1.15	80	252.42	1.15	122.17	278
“Biased” Congestion (mixed-strategy, two possibilities)	1.36	67.3	320	1.255	114.74	491.39
	1.36	80	761.71	3.0	72.99	833.68
“Fair” Congestion	1.18	94.19	290.18	1.16	105.53	313.16
Statistical (discrete strategy)	(1.08,1.22)	80	271.5	(1.08,1.22)	122.17	350.5

Note: We list both possibilities for player2 since they have very similar probability in case 2; while in case 4, the bidding strategy is a discrete strategy.

## 2.6 Conclusion

Congestion in the transmission system may allow some participants to enjoy effective market power, resulting in higher prices. This chapter analyzed this mechanism in the framework of game theory. We show that the deviation from idealized price-taker behavior is more serious when some market participants suffer disproportionately from the congestion problem. Based on this theoretical analysis, a probabilistic bidding methodology is proposed that shows similar profits to the game theoretic approach. Due to the complexity of the calculations in the theoretical approach, the statistical analysis methodology has clear advantages. We also believe these strategies reflect actual behavior in existing markets. Our on-going research is focusing on how bids change given the likelihood of congestion.



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## **CHAPTER THREE**

### **ANALYSIS OF PRODUCER BIDDING STRATEGIES FROM ELECTRICITY MARKET DATA**

Modeling the bidding problem by the proposed mathematical model required extensive simplifications and numerous assumptions. It is necessary to consider historical data for verification and deeper understanding of the decision-making process. This chapter focuses on the analysis of producer bidding strategies by an empirical technique. Bidding in the California ISO (CAISO) real-time energy is analyzed statistically and through a combination of correlation analysis, linear regression models and a non-linear estimator (neural network). The impact of gas price, system load and congestion on the market overall supplier bidding behavior and zonal energy price are investigated. This analysis provides some insight to the optimal bidding strategy approach to be developed in the Chapter IV.

#### **3.1 Introduction**

Among the important functions in an electricity market is the importance of modeling and forecasting electricity prices. Under regulation, retail prices are set by state public utility commissions (PUC's) so price variation is minimal and under the strict control of regulations. Under deregulation, price levels and variation become a concern and the number of energy based financial products has exploded. The recent experience in California has illuminated the importance of understanding electricity price volatility and the need for implementing hedging strategies.

Before a market participant takes action in the market, it is necessary to predict what other market participants will do, perhaps collectively, and then, according to this prediction,

choose the best response to maximize profit. Market conditions such as, load level and the possibility of congestion, affect energy prices and profit. Successfully predicting rival behavior as well as the market condition is the foundation of the optimal bidding strategy problem.

A number of researchers have empirically investigated energy price and bidding behavior prediction. For example, Knittel and Roberts [1] present an empirical analysis of deregulated electricity prices by investigating the behavior of California's deregulated electricity prices. They draw the conclusion that the existing financial models of asset prices fail to capture the extremely erratic nature of electricity prices. Non-Markovian specification in conjunction with exogenous information, such as, weather, is a necessary starting point for practical application. This may be unsurprising to the power engineers who are very familiar with the importance of weather, particularly temperature, on system operating conditions.

Wolak [2] presented an empirical method to derive a model of bidding behavior in a competitive electricity market that incorporated various sources of uncertainty and the impact of the electricity generator's position in the financial hedge contract market on its expected profit-maximizing bidding behavior. Borenstein, et al. [3], suggest that traditional prediction and estimation by concentration measures is not suitable in the electrical power market. An alternative method based on market simulations and the use of plant level data is proposed in their work. They found that traditional reliance on concentration measures is likely to be inadequate for the task. This is in part because concentration measures frequently depend upon historical data, such as, energy sales and transmission congestion, which are of questionable value since the incentives of many firms will change significantly after restructuring.

Borenstein, et al. [4], examine the degree of competition in the market from June 1998 to October 2000, just before the market effectively ceased operation. They found that significant departures from competitive pricing and observed that these departures were most pronounced during the highest demand periods, which tend to occur during summer. They found that 60% of the change in wholesale market expenditures, which rose from about \$2.1 billion in summer 1999 to over \$9 billion in summer 2000, could be attributed to market power. Jerko, et al. [5], studied dynamic interactions between six electricity spot markets in the western United States using time series analysis and directed graphs. Results show the western trading region to be highly integrated. The California market appears to be the driving force for prices in contemporaneous time. Seasonal analyses suggest there are seasonal differences in the short-run price discovery mechanisms. In the longer run, price dynamics appear to be similar between seasons. The mid-Columbia spot market appears to be the dominant market in the long run in both seasons.

Herguera [6], in examining spot markets in England and Wales and the Nordic countries, notes there is significant differences in the evolution of prices and volume traded. Yet in the United States, prices may not be as transparent because most trades are bilateral and futures trading volume is limited. DeVany and Walls [7] examined daily, peak and off-peak electricity spot prices during 1994 and 1996 using an error correction model on 11 regional markets in the western United States. They find spot markets are generally non-stationary and co-integrated. Peak prices at Palo Verde, conversely, are co-integrated with the off-peak price at only one other market, suggesting transfer capacities are limited during peak periods in this part of the western grid.

DeVany and Walls [8] use a vector autoregressive model to obtain impulse response

function and variance decomposition. Their impulse responses indicate shocks to a market impact neighboring markets first and then more distant locations along the transmission network. Variance decompositions suggest the California-Oregon border as one of the most important spot markets in determining prices throughout the network. Szkuta [9] used a three-layered Artificial Neural Network (ANN) paradigm with back-propagation for short-term system marginal price prediction. Stoft [10] analyzed the operation reservation and price cap's influence on the price spike. Stoft suggested that the high prices observed in present-day power markets do not reflect the desire of consumers for reliability, but reflect short-run regulatory and engineering policy. Munksgaard [11] analyzed the external environmental cost's such as CO<sub>2</sub> tax influence on the energy prices. They proposed a common producer tax method on power production to increase market efficiency.

Most studies of electricity pricing have investigated market structure and power, reasons for deregulation, or impact of deregulation on price. These studies tend to focus on how to forecast the energy price according to the marginal generation cost. This requires information of the system operation, which in practice can only be performed by the system operator. But to understand bidding strategy, it is necessary to see the problem from the MP point of view. That is, an MP makes decisions based on the limited information they are able to obtain. Here, the analysis is separated into two processes: prediction of the price and rivals' strategy and the determination of one's own bid.

This chapter provides a brief summary of the CAISO market processes. This is followed by analysis of generator bidding behavior and price prediction. All analysis is based on the CAISO real-time energy market.

## **3.2 California Electricity Market**

The competitive electric power market of the State of California began operation on March 31, 1998 with the CAISO and the now bankrupt Power Exchange (PX) as the main operationally independent market facilitators. The three major utilities Pacific Gas and Electric Company (PG&E), Southern California Edison and San Diego Gas and Electric Company continued to own and maintain their transmission systems. In California, the ISO market participants are called Scheduling Coordinators (SCs). SCs are the sole point of contact with the ISO and coordinate all the scheduling activities [12].

### **3.2.1 CAISO Market model**

There are two main market structures: bilateral model and pool model. The bilateral model is based on the principle that a free market structure is the best way to harvest the benefits of competition for consumers of electricity [13]. In this model, consumers and suppliers independently arrange the transaction with one another based on their own interests. In the pool model, all generators and customers submit their bid to the pool, and the pool operator determines the price and quantity. The fundamental difference between the pool model and the bilateral model is the how the MPs bid their generation and demand. In the pool model, the transactions are through the pool operator, while in the bilateral model, the transactions are made directly through suppliers and demands. California chose the bilateral model, in which MPs or the SCs manage their own portfolios (resources and loads) and ensure that this portfolio is balanced. They also participate in the congestion management system on a voluntary basis.



The CAISO operates three markets, a day-ahead market consisting of 24 hourly schedules, an hour-ahead market pertaining to a specific operating hour and a real-time energy imbalance. The ISO's Real Time Market for Imbalance Energy is an essential mechanism whereby the ISO controls the actual dispatch of resources to ensure the reliability of the transmission grid that it operates.

### **3.2.2 Pricing in CAISO**

In CAISO, the entities directly participating in the wholesale market are Scheduling Coordinators. The schedules submitted by SCs to the Day-Ahead and Hour-Ahead market are "balanced". California ISO's primary mechanism for maintaining a balance between loads and generation in real time is the real-time market, which involves the dispatch of generation based on the real time Energy bid prices through the Balance Energy and Ex-post Pricing (BEEP) system. If increased supply is needed to match actual loads with generation, bids for additional generation are selected in increasing order of price and dispatched through the BEEP system. If decreased supply is needed to match actual loads with generation (i.e., supply exceeds demand in real time), bids to decrease generation are selected in decreasing order of price through BEEP system. Bids available for dispatch through the BEEP system include the bids for incremental energy and ancillary services such as spinning, non-spinning and replacement reserve as well as supplemental energy bids for incremental and decremental energy submitted within and outside ISO system. The BEEP system ranks all such bids in merit order based on the price in order to create the aggregate supply curve of real time energy. The highest bid for incremental imbalance energy or lowest bid for decremental imbalance energy actually selected by the BEEP system for dispatch is the MCP.

In the absence of real-time congestion between the ISO's active Zones (SP15, NP15 and ZP26), the BEEP prices apply to all real-time imbalance system-wide. When real-time inter-zonal congestion occurs, the BEEP stack is constructed and applied separately for each zone and produces different prices for the zones on either side of the constrained interface. In this situation, the ISO frequently needs to decrement resources in one zone, while incrementing resources in the other zones to relieve the congestion. The adjustment bid is used to calculate the optimal curtailment for the system but the adjustment bid does not affect the MCP.

### **3.2.3 Congestion Management**

CAISO congestion management was divided into two parts: inter-zonal congestion management and intra-zonal congestion management. Zones are defined as areas where congestion is infrequent and prices can easily be computed on an average cost basis. By definition, intra-zonal congestion is infrequent and impossible to predict while inter-zonal congestion is frequent, easy to predict and has great impact [13]. For example, the path 15 connecting two zones NP15 and SP 15 is frequently constrained. CAISO's congestion management process can define new zones when intra-zonal congestion becomes frequent and inefficiently prices at average cost [13]. In this way, path 26 was created around 2002 by dividing the old SP 15 into two zones [14]. Similarly, zones can be combined if the inter-zonal congestion becomes infrequent and the average price could efficiently reflect the cost.

In intra-zonal congestion management, only the resources inside the zone are treated as a resource in congestion mitigation, and a simplified DC power flow equation is used in the optimization. Intra-zonal congestion management reschedules the resources within each zone

using the SC's incremental and decremental bids. In inter-zonal congestion management, a centralized optimization problem is formed and only the interconnections between zones are included in the objective to mitigate the congestion. CAISO relieves inter-zonal congestion by reducing scheduled energy production on one side of the interface and increasing generation (or decreasing load) on the other side while keeping the SC profile separate, i.e., each SC's total load/generation MW remains the same and the SC manages its own reschedule to keep load and generation balanced.

### **3.3 Generator bidding strategy: analysis and prediction**

#### **3.3.1 Modeling bids**

Before beginning any analysis, it is necessary to discuss how to model an MP's bidding strategy. As discussed before for the CAISO, a bid is represented by MW-Price pairs, i.e., a set of data indicating an MP's willingness to supply energy at given prices. To limit the number of inputs in the price forecast process, the price segments for a bid must be restricted. A straightforward approach is to measure or estimate some central tendency measure of an MP's bidding. Statistically, there are three common ways to represent a "center" of a sample: mean, median and mode. Mean is a good index for a normal distribution while median tends to be better for skewed samples (a distribution is skewed if one of its tails is longer than the other [15a]). In this thesis, the bidding curve is represented by MW-Price pairs (up to 10 pairs in CAISO) and the weighted average bidding price (ABP) is used to represent the bidding behavior:

$$ABP = \frac{\sum_{i=1, \dots, 10} (Price_i * MW_i)}{\sum_{i=1, \dots, 10} MW_i} \quad (3.1)$$

There are two possibilities in a real time energy market, either the supply exceeds the demand and or demand exceeds supply. If supply exceeds demand then one applies the incremental MCP and otherwise one must employ the decremental bids. In this thesis, only incremental bids are used in the ABP calculation as in practice, the scheduled load is typically less than actual load (e.g., from Feb 1, 2000 to Dec 24, 2000, this held for 6113 out of 7872 hours).

### **3.3.2 Bidding Behavior Analysis**

In order to allow any analysis to be manageable given the large number of generators in the market, it would be helpful if the suppliers could be grouped or classified based on similar behavior. Intuitively, one expects a link between different MP's bidding behavior but this should be verified. As a simple test, correlation coefficients and a significance test were calculated for a randomly selected set of generators. Since normal distributions are not assumed, the distribution-free rank correlation coefficient [15] and F-test are applied here (and throughout this chapter). Bidding data is based on the ISO published offers. When CAISO publishes this data, a pseudo ID called resource ID (ResID) is used to represent each generator/load to obscure the actual entity. As we do not know the actual generation units, 20 ResID are randomly chosen and the ABP is calculated. Using these ABP to perform correlation calculations, there are total of 190 correlation coefficients. Among the 190 correlation coefficients calculated, only 44 failed to pass a significance test. This simple test indicates significant correlation among bids (full results are listed in Appendix C) and suggests that it may be possible to group generators based on this relationship.

Further, the generators are group based on the correlation coefficient relation without relying on detailed knowledge of the generators, i.e., since we assume no specific information (e.g., fuel type, location) about these generators. If more information is available, then it could be used to improve the bids. Accordingly, the correlation coefficients between all bids and the four major inter-connection transmission lines (COI, Path 15, Palo Verde and Path 26) are calculated. The natural gas price is also included to help isolate the gas price influence. The following tables show the relevant correlations and p-values:

Table 3.1: Statistical analysis of bids and gas prices

Res ID	Hour 2		Hour 9		Hour 17	
	R	P	R	P	R	P
199871	-0.618	0	-0.425	0	-0.212	0
192115	-0.121	0.045	-0.003	<b>0.963</b>	<b>0.036</b>	<b>0.554</b>
282606	-0.471	0	-0.558	0	-0.24	0
108576	-0.578	0	-0.324	0	<b>-0.001</b>	<b>0.985</b>
106100	-0.696	0	-0.675	0	-0.683	0
599841	-0.692	0	-0.672	0	-0.679	0
715337	-0.318	0	<b>-0.004</b>	<b>0.948</b>	<b>0.101</b>	<b>0.094</b>
104351	-0.454	0	-0.261	0	0.131	0.03
142494	-0.267	0	-0.288	0	<b>0.024</b>	<b>0.692</b>
494629	-0.723	0	-0.698	0	-0.707	0
918588	0.455	0	0.494	0	0.496	0
102611	-0.434	0	-0.122	0.043	<b>-0.058</b>	<b>0.34</b>
205249	0.256	0	-0.541	0	-0.545	0
206887	-0.486	0	-0.487	0	-0.519	0
453332	-0.225	0	<b>0.027</b>	<b>0.662</b>	<b>0.076</b>	<b>0.212</b>
194543	-0.257	0	-0.259	0	-0.141	0.02
168177	-0.342	0	-0.25	0	-0.142	0.018
136212	<b>-0.056</b>	<b>0.355</b>	0.287	0	0.364	0
461530	0.746	0	0.695	0	0.699	0
475056	-0.222	0	0.648	0	0.533	0

Note in this and all subsequent tables in this chapter: 1) a p-value >0.05 assumed non-significant and is shown in boldface; and 2) all data is rounded to three decimals.

Most generators show significant correlation with the gas price, especially during off-peak hour. It is interesting to see that during the peak (hour 9 and 17), more bids tend not to reflect the gas price influence. This appears reasonable since off-peak, more generators would

be likely to bid at the marginal cost reflecting the gas price. During peak hour, other factors may determine the bid. Interestingly, the correlations are mostly negative. Considering that hydro constitutes around 50% of total capacity and that hydro power has little relation to gas price, one has to assume this reflects some strategizing by these units. Specifically, the hydro units may tend to bid lower than gas units to ensure sales during peak times, thus, earning more overall profit. To test this hypothesis, we further examine the correlation coefficient of bids with total load (i.e., system daily total load as published by CAISO website).

Table 3.2: Statistical analysis of bids and daily total load

ResID	Hour 2		Hour 9		Hour 17	
	R	P	R	P	R	P
199871	-0.232	0	<b>-0.018</b>	<b>0.77</b>	0.129	0.032
192115	<b>0.056</b>	<b>0.358</b>	0.206	0.001	0.247	0
282606	-0.288	0	-0.385	0	<b>-0.1</b>	<b>0.097</b>
108576	-0.224	0	<b>0.059</b>	<b>0.329</b>	0.225	0
106100	-0.379	0	-0.294	0	-0.285	0
599841	-0.509	0	-0.392	0	-0.396	0
715337	-0.197	0.001	0.134	0.027	0.265	0
104351	-0.338	0	-0.15	0.013	0.151	0.012
142494	-0.141	0.02	-0.174	0.004	0.129	0.032
494629	-0.48	0	-0.351	0	-0.36	0
918588	0.24	0	0.283	0	0.198	0.001
102611	-0.249	0	<b>0.046</b>	<b>0.45</b>	0.138	0.023
205249	0.413	0	-0.263	0	-0.348	0
206887	<b>-0.053</b>	<b>0.385</b>	<b>-0.052</b>	<b>0.393</b>	<b>-0.088</b>	<b>0.144</b>
453332	<b>-0.09</b>	<b>0.138</b>	0.188	0.002	0.278	0
194543	<b>-0.013</b>	<b>0.837</b>	<b>-0.042</b>	<b>0.493</b>	<b>-0.02</b>	<b>0.739</b>
168177	<b>-0.105</b>	<b>0.084</b>	<b>-0.044</b>	<b>0.469</b>	<b>-0.02</b>	<b>0.743</b>
136212	<b>-0.034</b>	<b>0.576</b>	0.324	0	0.381	0
461530	0.611	0	0.579	0	0.568	0
475056	<b>-0.076</b>	<b>0.208</b>	0.336	0	0.38	0

This simple test shows that on-peak bids tend to be influenced by the total load in peak load hours. Moreover, these bids are positively correlated so that prices increase with total load and clearly the MPs are considering total load when submitting bids.

Next the impact on bids of congestion on the four major interconnections (COI, Path 15, Palo Verde and Path 26) is analyzed. No significance was found relative to Palo Verde and Path 26 congestion so only Path 15 and COI are shown here. Full results are shown in Appendix C.

From Tables 3.3 and 3.4, we see that the relation between bids and congestion probability on Path 15 and the COI is strong and consistent for most generators. Note that especially for Path 15 bids show a strong relation with the congestion possibilities in all three hours. One can also discern a peak/off-peak pattern. For example, the resource 136212 shows a significant positive relation in both peak hours but nothing significant during off peak (hour 2). This holds, with a few more exceptions, for the COI as well.

Table 3.3 Statistical analysis of bids and path 15 congestion

ResID	Hour 2		Hour 9		Hour 17	
	R	P	R	P	R	P
199871	-0.571	0	-0.322	0	-0.249	0
192115	-0.277	0	-0.119	0.048	<b>-0.023</b>	<b>0.7</b>
282606	-0.502	0	-0.444	0	<b>-0.095</b>	<b>0.115</b>
108576	-0.544	0	-0.287	0	-0.134	0.026
106100	-0.5	0	-0.411	0	-0.404	0
599841	-0.634	0	-0.605	0	-0.555	0
715337	-0.341	0	<b>-0.103</b>	<b>0.088</b>	<b>0.016</b>	<b>0.789</b>
104351	-0.438	0	-0.198	0.001	0.199	0.001
142494	-0.286	0	-0.166	0.006	0.131	0.03
494629	-0.659	0	-0.569	0	-0.535	0
918588	0.432	0	0.496	0	0.43	0
102611	-0.42	0	-0.185	0.002	<b>-0.075</b>	<b>0.213</b>
205249	<b>0.06</b>	<b>0.319</b>	-0.342	0	-0.388	0
206887	-0.434	0	-0.384	0	-0.422	0
453332	-0.275	0	<b>-0.116</b>	<b>0.056</b>	<b>0.02</b>	<b>0.741</b>
194543	-0.314	0	-0.344	0	-0.279	0
168177	-0.332	0	-0.358	0	-0.286	0
136212	<b>-0.002</b>	<b>0.968</b>	0.295	0	0.304	0
461530	0.666	0	0.576	0	0.577	0
475056	-0.137	0.023	0.52	0	0.486	0

Table 3.4 Statistical analysis of bids and COI congestion

ResID	Hour 2		Hour 9		Hour 17	
	R	P	R	P	R	P
199871	0.186	0.002	<b>0.02</b>	<b>0.741</b>	<b>-0.074</b>	<b>0.222</b>
192115	<b>0.094</b>	<b>0.121</b>	-0.18	0.003	<b>-0.077</b>	<b>0.205</b>
282606	0.242	0	<b>0.095</b>	<b>0.116</b>	<b>0.012</b>	<b>0.844</b>
108576	<b>0.111</b>	<b>0.068</b>	<b>-0.038</b>	<b>0.526</b>	-0.125	0.038
106100	0.23	0	<b>0.03</b>	<b>0.623</b>	<b>0.053</b>	<b>0.386</b>
599841	0.146	0.015	<b>-0.021</b>	<b>0.729</b>	<b>0.014</b>	<b>0.815</b>
715337	0.134	0.027	-0.175	0.004	-0.178	0.003
104351	0.149	0.014	-0.131	0.03	-0.167	0.005
142494	0.187	0.002	<b>-0.081</b>	<b>0.183</b>	-0.145	0.017
494629	0.285	0	<b>0.053</b>	<b>0.386</b>	<b>0.076</b>	<b>0.212</b>
918588	-0.127	0.035	<b>-0.019</b>	<b>0.756</b>	<b>-0.005</b>	<b>0.935</b>
102611	0.14	0.02	-0.194	0.001	-0.092	0.13
205249	-0.127	0.036	0.182	0.002	0.246	0
206887	<b>0.107</b>	<b>0.077</b>	-0.215	0	-0.205	0.001
453332	0.125	0.039	-0.206	0.001	-0.122	0.044
194543	<b>0.049</b>	<b>0.419</b>	-0.194	0.001	-0.188	0.002
168177	<b>0.082</b>	<b>0.177</b>	-0.215	0	-0.223	0
136212	<b>-0.105</b>	<b>0.084</b>	-0.151	0.012	-0.274	0
461530	-0.23	0	-0.139	0.021	-0.156	0.009
475056	<b>-0.043</b>	<b>0.481</b>	<b>-0.072</b>	<b>0.235</b>	-0.249	0

During peak load hours, fewer generators show significant relation with the congestion. This may be due to more generators during peak tending to bid based on load. Examining the correlation coefficient more closely shows that there are more positive relations with Path 15's congestion. For example, resource 475056 shows negative significant relation with Path 15 off-peak and positive on-peak.

So far, relationships have been examined between different generator units but it also interesting to observe how a particular MP bids relative to itself at different hours. To see this, we calculate the correlation coefficient between each generator's ABP at different hours, a few such results are shown in Table 3.5 with the full results listed in Appendix C.



Table 3.5: Statistical analysis of bids at different hours

Hour\ResID	205249	136212	461530	475056	avg_delta
(2,17)	-0.06	0.72	0.50	-0.12	0.52
(2,9)	-0.02	0.73	0.70	-0.15	0.65
(9,17)	0.70	0.98	0.84	0.70	0.80

From the above table, we can see that in most of cases, the bids submitted by one generator shows significant correlation between different hours, especially among peak hours, the smallest correlation coefficient is 0.7. Still, there are a few exceptions. For example, generator 205249 shows only a small correlation between hour 2 and the other hours.

The above analysis, while far from comprehensive, has confirmed that clear patterns in bidding behavior can be observed even with very limited information. The general patterns observed were:

1. Generators tend to bid more consistently during off-peak, typically relative to marginal cost.
2. During peak hours, bids tend to depend more on load level and less on fuel price; one can assume the generators are bidding off of marginal cost in order to earn more profit at these hours.
3. Similarly with regards to congestion information during off-peak, bids tend to be directly influenced by the congestion possibility; while during peak hours, the bidding strategy is more volatile.

### **3.3.3 Bidding Prediction by Linear Regression**

Section 3.2 showed some simple general patterns found by statistical analysis but clearly such relationships may be non-linear and correlation analysis can give misleading

conclusions. In this section, more careful analysis is applied to support the previous conclusions. To simplify the work, the analysis concentrates on only three units, namely: 136212, 461530 and 475056.

Since bidding is a complex decision problem, one does not expect to capture all the factors by a simple linear approach. To see this, bids are first predicted using linear regression. The input variables include: gas price, congestion probabilities and total load. The percentage of variance ( $R^2$ ) that can be explained by the input variables is adopted to show the goodness of fit. Results are shown in Table 3.6 (see Appendix C for the complete regression results).

Table 3.6: Linear regression for example generators -  $R^2$  for bids

ResID	Hour 2	Hour 9	Hour 17
136212	19.80%	36.00%	37.90%
461530	26.00%	12.40%	12.00%
475056	4.50%	38.60%	34.10%
Overall Average	26.50%	31.90%	48.70%

These results show that even in the best case the variance explained by these variables is less than 50%, which means that the linear regression is cannot successfully capture behavior based on these variables. One possibility to improve the regression result and capture some of the non-linearity is to use a piece-wise linear function. Here, the weekday and weekend bids are separated. The Kruskal-Wallis distribution free significance test [16] was used to test if such a grouping shows significance. Results are shown in table 3.7:

Table 3.7: F-value and P-value from K-W testing for weekdays vs. weekends

ResID	Hour 2	Hour 9	Hour 17
136212	(0.4865, 0.8182)	(0.5719,0.7519)	(0.2566, 0.9563)
461530	(0.2259, 0.968)	(0.4848,0.8195)	(0.4995, 0.8085)
475056	(0.3382, 0.916)	(0.6477,0.6919)	(1.2627, 0.2749)
Overall			
Average	(0.2492, 0.959)	(3.2099,0.000)	(0.3151, 0.9288)

Results show that all p values are great than 0.05, so from a statistical point of view, the proposed grouping is not important. Another possible grouping would be according to relative load level. To do so, one must decide on the appropriate separation of load levels. It is possible to find the best separation based on explained variance but this is a difficult optimization problem and more difficult than need be for the purposes here. Here, repeated trials were used to a “good” solution while realizing a better solution might be possible. The following split was formed on the total daily load:

Total load < 599.7 GWh;	with 137 cases
599.7 GWh $\leq$ Total load $\leq$ 645 GWh;	with 73 cases
Total load $\geq$ 645.00 GWh;	with 64 cases

As previously, the Kruskal-Wallis distribution free significance tests was used to test if this grouping shows significance. The results are shown in Table 3.8. For hour 2, only generator 461530 shows significance. For hours 9 and 17, the proposed grouping does satisfy significance and suggests that at peak hour, grouping by load level helps the predictive capability. As such, the linear regression was applied for this grouping with the results shown in Table 3.9. Still, comparing these results with the previous, there is no significant improvement in explained variance.

Table 3.8: F-value and P-value from K-W testing for load levels

ResID	Hour 2	Hour 9	Hour 17
136212	(2.6693, 0.071131)	(17.8288,0)	(24.3134, 0.000)
461530	(64.9827,0.00)	(54.4399,0.000)	(49.5588,0.000)
475056	(1.5254, 0.219404)	(15.8178,0.000)	(21.2268,0.0000)
Overall			
Average	(1.6564, 0.192756)	(25.7403,0.000)	(33.8312,0.0000)

Table 3.9: Linear regression results for load grouping during peak -  $R^2$  for bids

ResID	Hour 2		Hour 9			Hour 17		
	Low	Middle\High	Low	Middle	High	Low	Middle	High
136212	20.60%	9.10%	40.30%	16.20%	10.50%	40.40%	15.50%	22.40%
461530	17.40%	15.90%	6.50%	34.40%	19.20%	6.70%	39.40%	13.50%
475056	3.60%	14.80%	37.10%	37.90%	19.70%	25.70%	43.60%	23.80%
Overall								
Average	23.40%	35.10%	30.70%	38.70%	43.20%	50.20%	52.20%	32.40%

In the preceding analysis, important factors for bid prediction may have certainly been missed but using the available input information, linear methods, i.e., both simple linear and piecewise linear method works poorly in bidding prediction. It is necessary to consider other methods for prediction.

### 3.3.4 Bidding Prediction by Neural Network

Statistical regression only considers the linear relationship between inputs and outputs. But it is unrealistic to model a complex decision problem, such as the bidding strategy problem, by linear methods. One approach to including these complexities is to use a non-linear estimator, such as, a neural network. The neural network is known as to be useful where [17]:

- precise models or algorithmic solutions are not readily available,

- extensive unanalyzed sample data is available,
- it is desired to identify structure from existing data.

These characteristics match the problem under discussion and thus, a simple feedback neural network is introduced. The input variables remain the gas price, total load and four major transmission paths with the output the average bidding price (ABP). For comparison, the same three generators and the average bidding price are chosen. The same Feb 1, 2000 to Dec. 24, 2000 CAISO real time market data used in former sections are used for training. The neural network was defined with one hidden layer using sigmoid activation functions with a linear activation in the output layer. The number of hidden neurons was chosen to be 9. Training is chosen to be a simple back propagation. Figure 3.1-3.4 shows some of the neural network simulation results for hour 17.

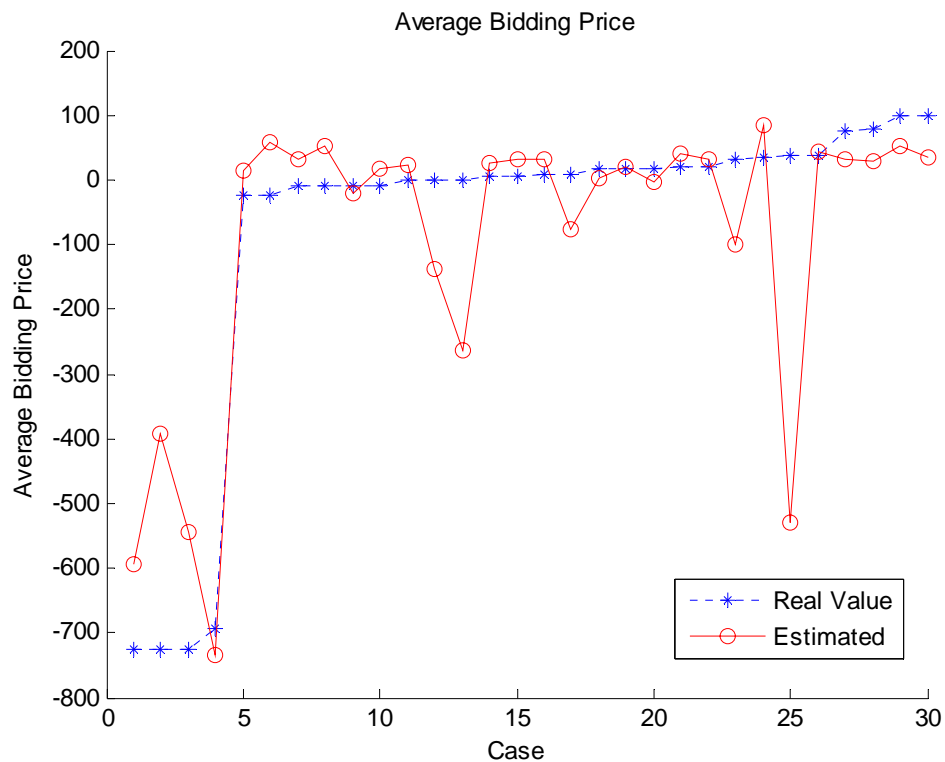


Figure 3.1: ABP forecast by neural network - hour 17, ID 136212

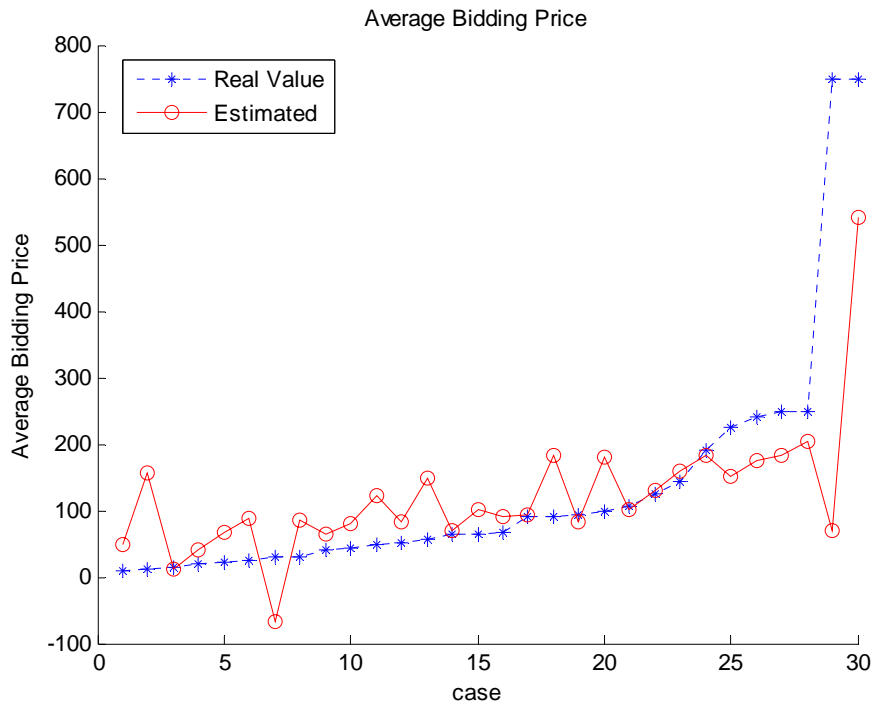


Figure 3.2: ABP forecast by neural network - hour 17, ID 461530

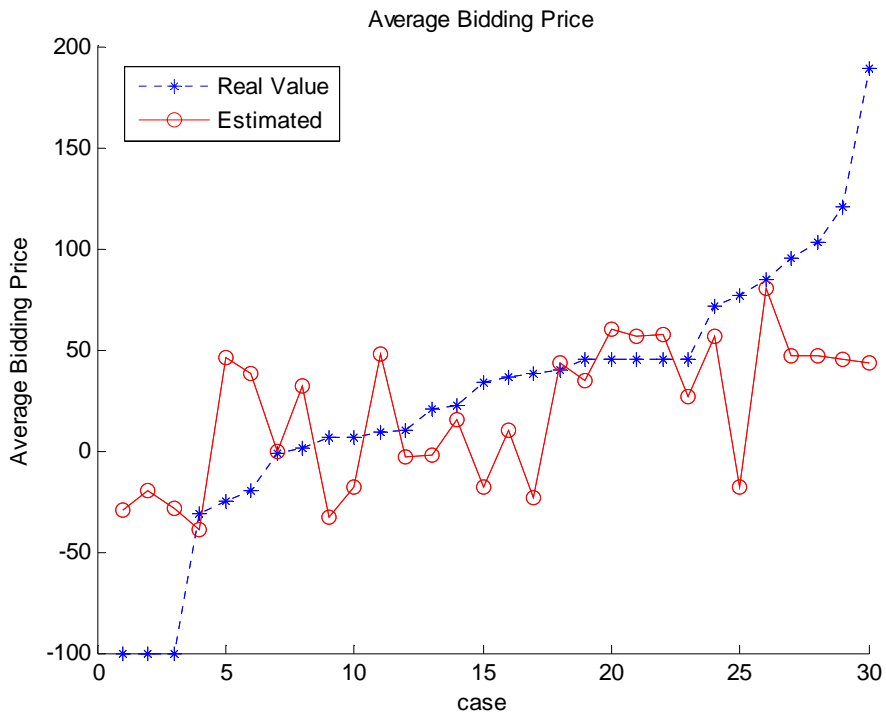


Figure 3.3: ABP forecast by neural network - hour 17, ID 475056

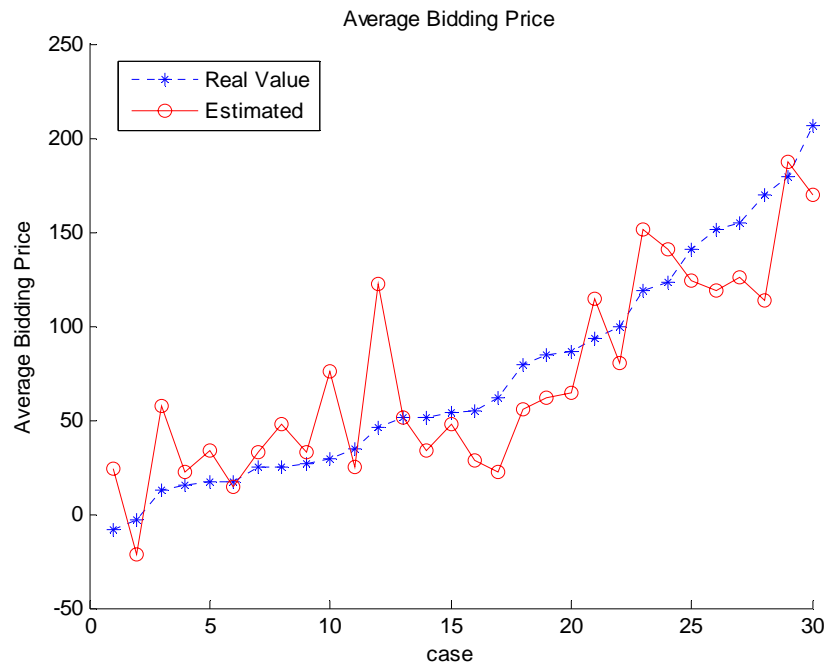


Figure 3.4: ABP forecast by neural network - hour 17, overall bid average

Generally, the results show the neural network provides some improvement but there is still significant error. Among these tests, the best match is of the overall ABP. That is, it appears far easier to predict the overall market bids than any individual MP's bidding behavior. Notice that very large errors occur at some irregular points. These salient data points are likely the key to improving prediction. Further investigation of this observation is beyond the scope of the work presented in this thesis.

In 2000, CAISO experienced a dramatic and volatile market that may not reflect behavior in a more mature market. Moreover, the input data is very limited and one expects an MP to use as much information as possible. One obvious data point to include is each MP's own historical behavior. To emulate this scenario, we include an individual MP's bids as an input variable. Results are shown in Figure 3.5-3.7 for each of three generators and the overall bidding price. Clearly, this information does lead to a significant improvement in

prediction capability. Similarly, this approach can be used for prediction of any specific MP's bids. This is shown in Figures 3.8 and 3.9 at hour 17 using information from 475056 to predict bids from 4615330 and 136212, respectively. As expected, the performance improves greatly and merely reflects that different MP's may act in a similar manner.

The above analysis shows that the neural network shows promise for forecasting bids even with extremely limited information. Again, the work here is only to show the feasibility of the proposed approach so that inputs are available for the optimization that is the main thrust of this thesis. There are certainly numerous approaches to improve the forecast results, including not only increased input information but improved neural network design.

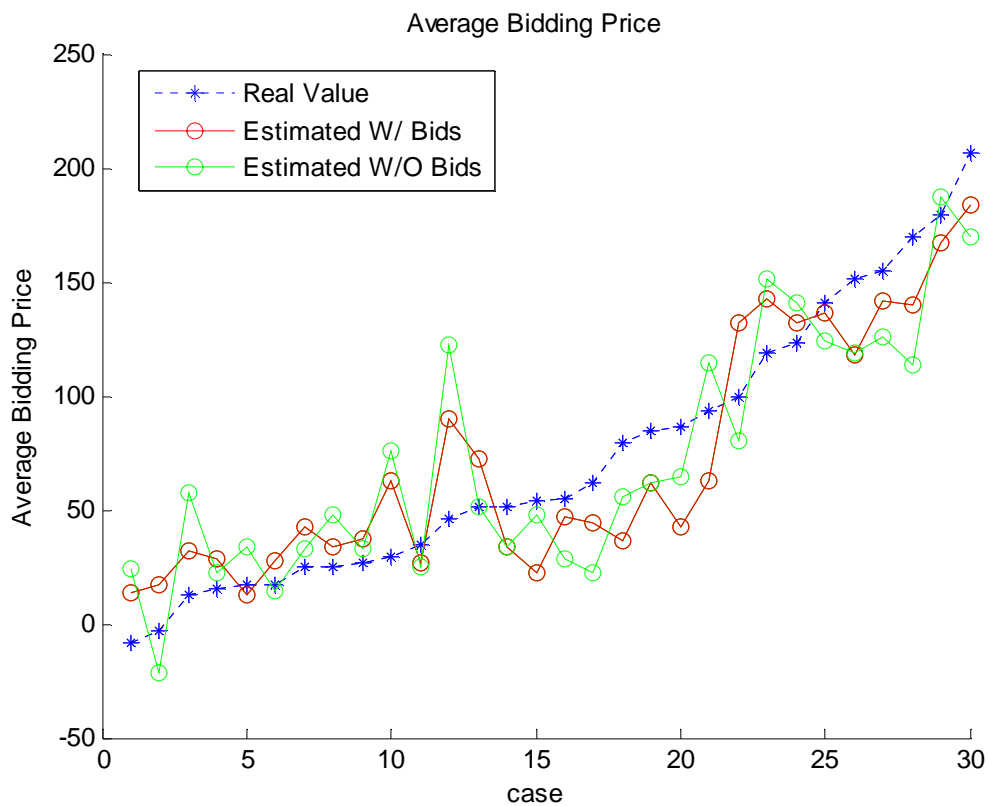


Figure 3.5 ABP forecast by neural network - hour 17 including 136212 bid information



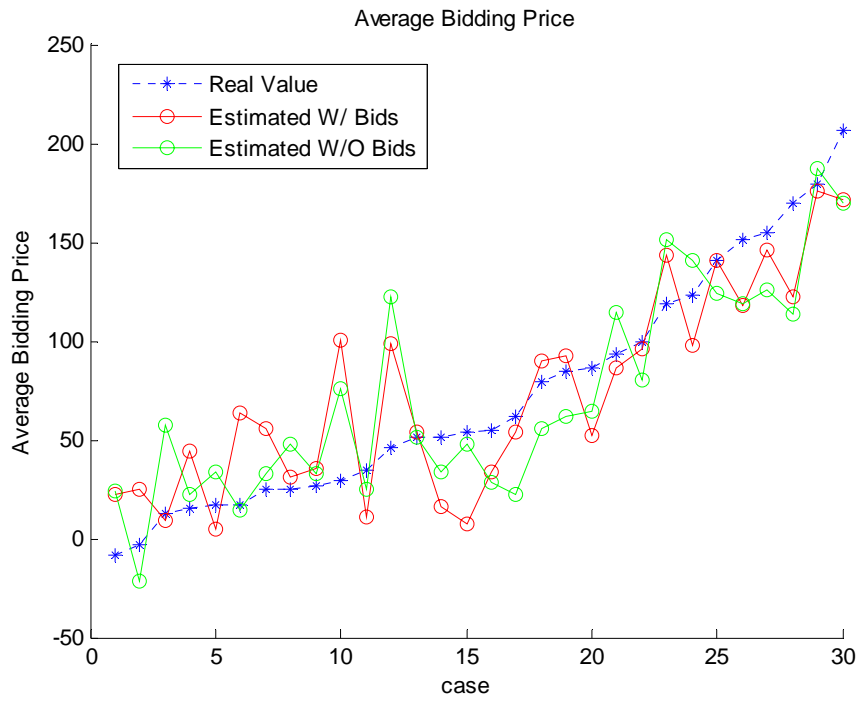


Figure 3.6 ABP forecast by neural network - hour 17 including 461530 bid information

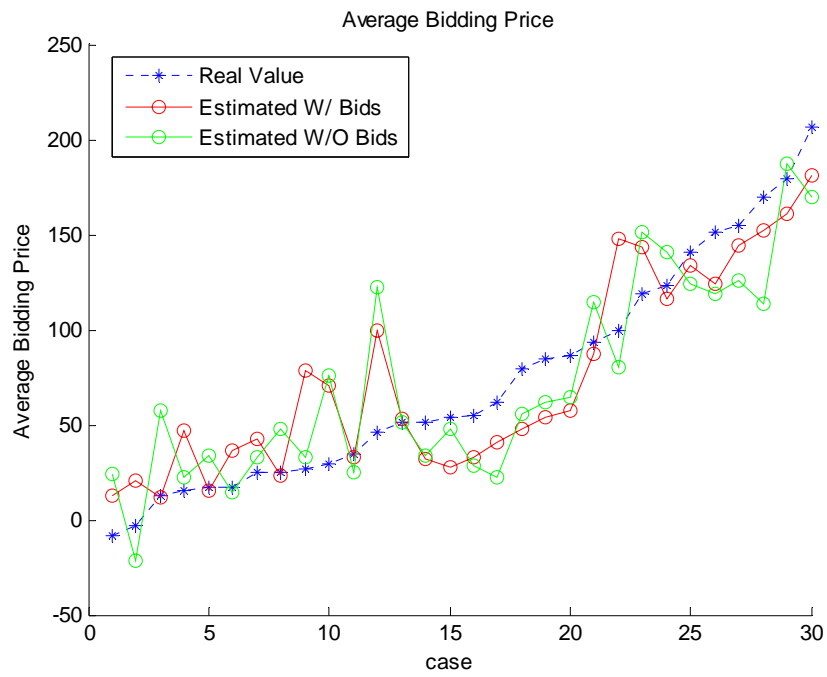


Figure 3.7 ABP forecast by neural network - hour 17 including 475056 bid information

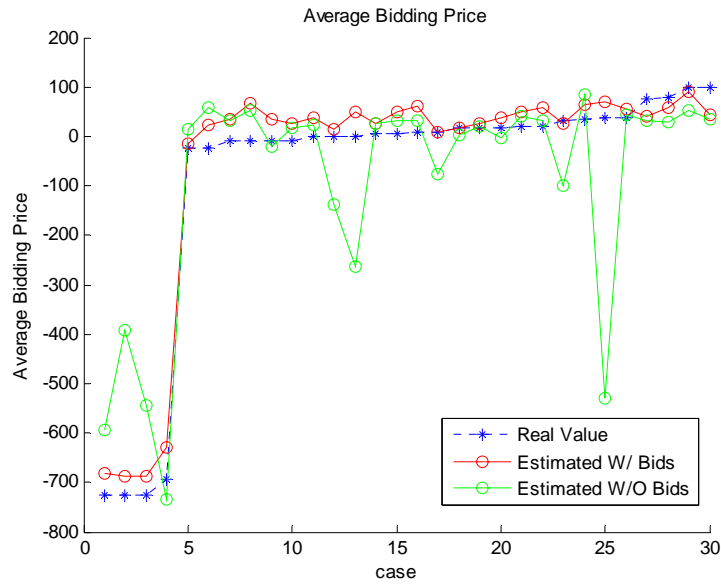


Figure 3.8: ABP forecast by neural network - hour 17, 136212 given 475606 bids

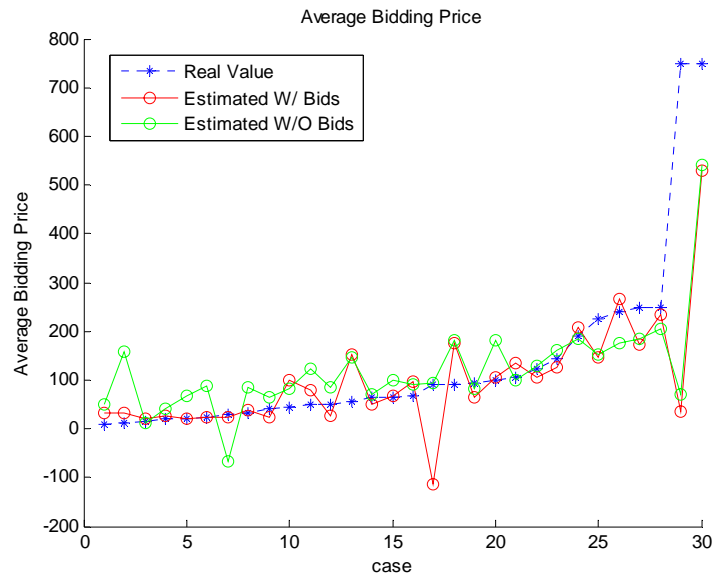


Figure 3.9: ABP forecast by neural network - hour 17, 461530 given 475606 bids

### 3.3.5 Sensitivity analysis

This chapter has mainly focused on forecasting bidding behavior given the load, gas price and congestion probability. Forecasts introduce errors and so it is important to understand the MP reactions to such uncertainties. Sensitivity analysis can indicate the factors that are most

important to the decision-making process. Three samples for hour 17 are selected as input to the last neural network trained result with the sample data information given in Table 3.10. To calculate the sensitivity, an input variable is changed by a small amount, both as an increment and a decrement, and then the overall ABP is recalculated. Results are shown in Table 3.11. Notice that the some of the decremental and incremental sensitivities show significant variation between cases. This indicates the strong non-linearity for the given variables, especially for bids. Another sensitivity of interest is relative to another MP's bids. Results are shown in Table 3.12.

Table 3.10: Sample input information for sensitivity analysis for unit 475056

	Bids(\$/MWh)	COI	PATH15	PALO	PATH26	Gas(\$/Mbtu)	Load (GW)
Case 1	27.53	0.00	0.00	100.00	94.00	3.14	23.53
Case 2	120.26	0.00	13.00	0.00	56.00	5.10	30.45
Case 3	125.00	0.00	25.00	0.00	31.00	5.08	27.60

Table 3.11: Sensitivity results for overall ABP for unit 475056

	Bids		COI		Path 15		Palo Verde		Path 26		Gas		Load	
	Inc	Dec	Inc	dec	Inc	dec	Inc	dec	Inc	dec	Inc	dec	Inc	Dec
Case 1	0.01	0.01	0.05	0.05	-0.33	-0.33	0.08	0.08	0.28	0.28	-0.02	-0.02	-0.20	-0.20
Case 2	-2.40	-2.37	-0.23	-0.22	-0.61	-0.61	-0.90	-0.89	-0.29	-0.29	0.57	0.58	-0.68	-0.67
Case 3	0.52	0.52	1.29	1.31	0.88	0.89	-0.96	-0.96	-0.24	-0.24	-2.04	-2.01	-0.04	-0.04

Table 3.12: Sensitivity results relative to 136212 and 461530

ResID	Case	Bids		COI		PATH15		PALO		PATH26		Gas		Load	
		Inc	dec	Inc	dec	Inc	dec	Inc	dec	Inc	dec	Inc	dec	Inc	dec
136212	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11	-0.09
	2	0.16	0.16	-1.75	-1.77	0.25	0.25	0.70	0.70	-0.17	-0.17	0.38	0.38	-0.02	-0.09
	3	0.00	0.00	-0.02	-0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	-0.02	-0.10
461530	1	0.13	0.14	0.10	0.10	-0.06	-0.06	-0.03	-0.03	-0.11	-0.11	-0.13	-0.13	-0.05	-0.26
	2	0.54	0.55	-0.64	-0.64	-0.16	-0.16	0.52	0.52	0.50	0.51	4.40	4.39	-0.08	-0.22
	3	-0.03	-0.04	-0.51	-0.51	0.49	0.48	0.15	0.15	0.22	0.22	2.59	2.54	-0.06	-0.24

In the bidding problem, we are mostly interested in a competitor's response to a particular operation scenario. That is, given the forecasted operational situation, i.e., forecasted load, gas price and congestion possibilities, what is the relation between system overall ABP and a particular MP's ABP. Using the data in Table 3.9, the individual bids of 475056 are varied from  $-0.5$  to  $3.0$  (from normalized values) and the forecasted system overall ABP change is found (shown in Figure 3.10). Case 3 shows strong non-linearity at higher bids. As discussed before, the relation between individual bids and overall ABP varies significantly among different operation scenarios. This result reinforces the earlier claims of difficulty in using a linear model for bid prediction. Based on the sensitivity analysis, it is easy to apply these results in the bidding problem, the next chapter extends these sensitivities to the actual bidding strategy problem.

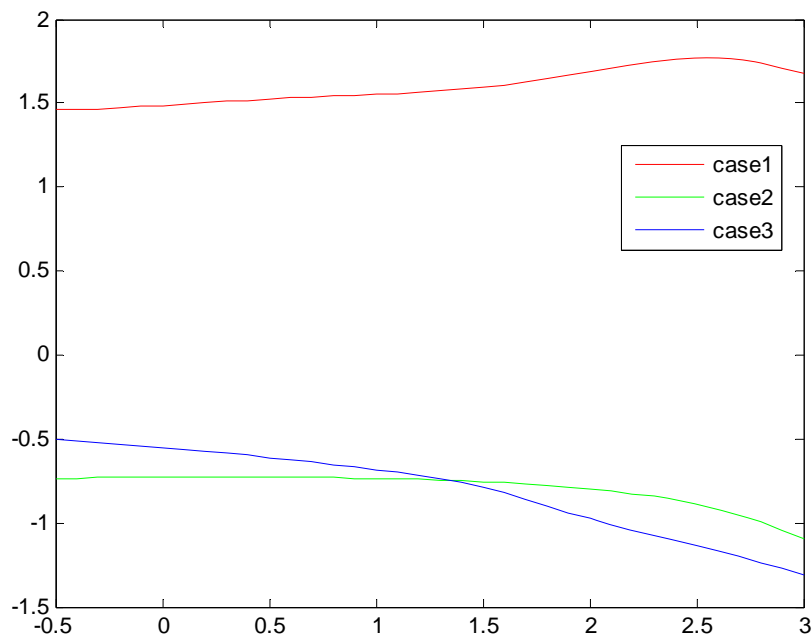


Figure 3.10: Sample sensitivity for overall ABP

### 3.4 Price Forecast

Because of the lack of storage for electricity, the demand and supply of electricity are balanced on a “knife-edge” [18]. End user demand is largely weather dependent varying significantly from day to day and moreover the reliability of the grid is paramount. Thus, electricity price is often far more volatile than that of other commodities. Volatility makes it very difficult to accurately predict prices. This section introduces a simple framework for price forecasting with a limited goal of providing input to the optimization processes developed in other chapters.

Ideally, prices under competition are equal to the marginal cost of production. Although at this stage the power market is far from a mature market, the energy price should still be strongly related to the generation cost. Accordingly, the fuel price for fossil fuel units is one of the key elements to price prediction. The two primary types of fuels are gas and coal. In the CAISO area during 2000, the generation capacity of gas units was 47.4 GW [19], or roughly 29% of total capacity (162.1 GW). Gas price tends to be more influential on price as coal prices are more stable, and moreover, coal units are base loaded and rarely dispatched on the margin. Thus in the following, only gas price is considered in the price forecast.

Intuitively, the higher the demand, the higher one expects the energy price to be. Given that energy price should be positively correlated with load, energy price should have similar weekday/weekend and seasonal patterns as the load. Thus, it is generally necessary to use different models for weekday vs. weekends, special holiday and seasons. This has been explored more fully in [20] where the seasonal factors were found to be the most important. The load forecast is very mature in power industry and includes many these salient features. Accordingly, the load forecast is used directly instead of the weather and other relevant

variables. Later, statistical analysis will show that there is no need to separate the energy price into different groups based on these variables given a load forecast.

Another major factor affecting price is the transmission system limits. To ensure power system security, it is necessary to keep the flows on major transmission paths within specified limits, which is the congestion management function of the system operator. According to the California inter-connection congestion management, the system is divided into zones, and each zone's energy price is determined by the supply and demand relation only within the zone. The factors affecting congestion status include not only load level, but also the load and generation dispatch pattern. Different locations lead to different congestion possibilities. As discussed previously for CAISO congestion management, the intra-zonal congestion could be treated as infrequent and less important as to the overall system price. Thus, while there are thousands of potentially congested transmission lines, it is reasonable to only consider the inter-zonal congestions for the zonal price forecast. In this analysis, only the four main inter-zonal paths' congestion are considered, specifically Path 15, Path 26, COI and Palo Verde. The congestion probabilities are the CAISO daily performance reports, with the percentage of congestion hours treated as congestion possibility of each day for this particular path.

Since energy prices are decided by the bids, it is impossible to ignore the bids in price prediction. Note that the CAISO publicly posts bidding information after 6 months, so that all MPs have access to bidding history information. Even with time delay, this bidding information is a useful reference. These bids implicitly contain a wide range of relevant information, including start up costs, ramp rates, and minimum up/down times that have direct influence on the unit commitment and in turn, the market clearing price. With this

observation and to simplify the analysis, only the energy price-MW is considered with all other cost and time information ignored. Finally, there are several other factors that affect energy price. Most obviously in the Western US, reservoir levels determine the amount of available hydro. This information is partially captured by the congestion information for the COI and Palo Vera, and so water levels are also not considered here even though one may expect this to affect prediction accuracy.

### 3.4.1 Empirical analysis of prices in CAISO

Similar to the previous analysis of bidding behavior, empirical analysis of energy price prediction was carried out for the CAISO real time market data from 2/1/2000 to 10/31/2000. In the absence of congestion between the ISO's active zones, one system wide energy price applies to all the system but if there is congestion, differences in zonal prices arise. Here for simplicity, only zone NP15 prices are chosen as a zonal price index and the daily average value is used in the prediction. The correlation coefficient between the energy price of NP15 and SP15 are calculated (Table 3.13) to show that the price in these two zones are highly correlated and there is little need to distinguish between these two zones during the time range of interest for the purposes of the analysis here.

Table 3.13: Correlation between NP15 and SP15:

	Hour 2	Hour 9	Hour 17
Corr_Coef	0.831119	0.869832	0.897459

Now consider the correlation between the congestion information, load level, gas price and the energy price. Correlation coefficients and corresponding t-test results are shown in Table 3.14. There is significant correlation with between congestion possibilities, gas price,

load and energy price in almost three sample hours, the only exception is the congestion for path 26, which shows insignificance for hour 9. For resource bids, not all show significance, especially for on-peak (hour 9 and 17). This suggests that during peak times, it may be more difficult to predict the energy price from MP bids.

Table 3.14: Statistical analysis results for price prediction

		Hour 2		Hour 9		Hour 17	
		Corr_Coef	P_value	Corr_Coef	P_value	Corr_Coef	P_value
ResID	199871	-0.567	0	-0.392	0	-0.006	0.917
	192115	-0.199	0.001	-0.034	0.574	0.141	0.02
	282606	-0.47	0	-0.581	0	-0.155	0.01
	108576	-0.523	0	-0.282	0	0.148	0.014
	106100	-0.616	0	-0.562	0	-0.452	0
	599841	-0.689	0	-0.622	0	-0.506	0
	715337	-0.361	0	-0.037	0.546	0.207	0.001
	104351	-0.459	0	-0.299	0	0.189	0.002
	142494	-0.287	0	-0.319	0	0.125	0.038
	494629	-0.71	0	-0.625	0	-0.522	0
	918588	0.483	0	0.464	0	0.348	0
	102611	-0.458	0	-0.151	0.012	0.055	0.368
	205249	0.093	0.124	-0.475	0	-0.528	0
	206887	-0.472	0	-0.453	0	-0.294	0
	453332	-0.263	0	-0.025	0.685	0.193	0.001
	194543	-0.293	0	-0.226	0	-0.054	0.377
	168177	-0.331	0	-0.227	0	-0.042	0.493
	136212	-0.138	0.023	0.291	0	0.459	0
	461530	0.801	0	0.716	0	0.692	0
	475056	-0.2	0.001	0.523	0	0.533	0
System Overall Bids		-0.478	0	-0.572	0	-0.559	0
COI		-0.275	0	-0.143	0.018	-0.258	0
Path15		0.794	0	0.623	0	0.559	0
PALO		0.132	0.028	-0.126	0.037	-0.244	0
Congestion PATH26		-0.092	0.13	0.073	0.225	0.196	0.001
Gas		0.842	0	0.84	0	0.775	0
Load		0.61	0	0.691	0	0.799	0



### 3.4.2 Energy Price prediction

Now the above factors will be used as input variables to predict the zonal energy price. To see the different influence on price prediction, the following five cases are considered:

- Case 1: Base case – Gas price and total Load are the only input variables
- Case 2: Base Case + congestion
- Case 3: Case 1 + all 20 resources bid information
- Case 4: Case 2 + all 20 resources bid information
- Case 5: Case 2 + individual bids + overall bidding information

The regression results are shown in Table 3.15 with detailed results in Appendix C

Table 3.15: Linear regression results  $R^2$  for price estimation

	Hour 2	Hour 9	Hour 17
Case 1	63.90%	50.70%	35.40%
Case 2	71.90%	53.30%	35.90%
Case 3	74.60%	62.10%	50.60%
Case 4	77.30%	63.60%	51.10%
Case 5	75.40%	56.60%	36.90%

From Table 3.15, observe the following:

1. Load level and gas price explain about 64% of the zone NP15 energy price variance during off-peak, while these variables only account for 35% of the variance on-peak.
2. By adding more information, the explained variance can be improved but only marginally. Comparing Cases 2, 3 and 4, explained variance increases with the number of individual bids known but is less helpful during on-peak.
3. Comparing Cases 4 and 5, it is seen that overall bidding information is nearly as explanatory as the individual bids. This is consistent with the high correlation between

different MP's bids. Thus, it is not necessary to including all 20 individual resource bids in the price prediction process and subsequent analysis will based on Case 5.

4. Finally, note that linear regression does not work well since only 77% of the variance can be explained by these factors at off-peak and only 50% on-peak.

There may certainly be other important considerations not included in the above analysis. One difficulty is insufficient information on the location of the bids since location certainly matters in determining a zonal price when congestion occurs. Another possibility for the inaccuracy of the result is non-linearities. It is possible that the including non-linearities can greatly improve the estimation. A crude attempt is made by using the piece-wise linear concept around a salient variable, e.g., dividing the data into several groups based on load level. Here, data is grouped based on weekdays and load level using the same grouping criteria as in the bid forecast. To test the significant of this separation, the Kruskal-Wallis significance tests results are calculated and shown in Table 3.16. As with the bids, there is no significant gain for the different weekdays in hour 2 and hour 17, although there is some difference at hour 9, with p-value equal to 0.02. Again, load grouping does show significance. Table 3.17 shows linear regression on these groups based on Case 5 with sample resource ID 475056 and overall ABP (detailed analysis can be found in Appendix C).

Table 3.16: Significance test results in price prediction for day and load level separation

	<b>Hour 2</b>	<b>Hour 9</b>	<b>Hour 17</b>
Load	(70.24, 0)	(98.84,0)	(177.90, 0.00)
7-days	(0.22,0.97)	(2.50,0.02)	(0.89, 0.51)

\* (test-value, p-value) pairs

Table 3.17: Linear regression results  $R^2$  for price estimation with grouping

Group	Hour 2	Hour 9	Hour 17
1	86.60%	80.40%	73.00%
2	58.50%	54.10%	26.70%
3	45.10%	30.60%	16.70%

Table 3.18: Correlation coefficient and P-value for price prediction after grouping

Grp	Hr	475056		OverAll		Gas		Load		COI		Path15		PALO		Path26	
		R2	P	R2	P	R2	P	R2	P	R2	P	R2	P	R2	P	R2	P
1	2	-0.01	0.87	-0.47	0.00	0.51	0.00	0.29	0.00	-0.17	0.04	0.54	0.00	0.19	0.02	-0.07	0.39
	9	0.37	0.00	-0.38	0.00	0.62	0.00	0.48	0.00	0.10	0.24	0.29	0.00	0.04	0.62	-0.16	0.07
	17	0.43	0.00	-0.38	0.00	0.71	0.00	0.51	0.00	0.04	0.67	0.29	0.00	-0.03	0.70	-0.18	0.03
2	2	-0.33	0.00	-0.62	0.00	0.73	0.00	0.18	0.14	-0.26	0.03	0.80	0.00	0.35	0.00	0.20	0.10
	9	0.34	0.00	-0.62	0.00	0.71	0.00	0.12	0.30	-0.12	0.31	0.56	0.00	0.14	0.22	-0.16	0.18
	17	0.51	0.00	-0.36	0.00	0.53	0.00	0.27	0.02	-0.26	0.03	0.37	0.00	-0.03	0.83	0.07	0.56
3	2	-0.43	0.00	-0.46	0.00	0.53	0.00	-0.10	0.42	N/A	1.00	0.54	0.00	0.36	0.00	0.01	0.93
	9	0.03	0.48	-0.25	0.93	0.41	0.00	-0.03	0.80	0.01	0.96	0.28	0.03	0.11	0.38	0.12	0.34
	17	<b>0.09</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>-0.01</b>	<b>0.97</b>	0.26	0.04	<b>-0.11</b>	<b>0.41</b>	<b>0.10</b>	<b>0.44</b>	<b>-0.15</b>	<b>0.23</b>	<b>0.14</b>	<b>0.27</b>

- For Hour2 group 3, all COI congestion possibility equal to 0, so no correlation coefficient to COI can be computed.

For group 3, only 16.7% of variance can be explained by the input variables. To see why this happens, the relationship between the input variables and energy price is analyzed and shown in Table 3.18. Notice that for group 3, almost all the correlation coefficients have non-significant relation with energy price, particularly at hour 17. The only variable showing significant correlation with price is total load. Thus, it is impossible to forecast energy price by a linear method using these input variables at peak hour with high loads. As with the bidding problem, there are two ways to address this problem: either increase the available information or apply non-linear methods. Since the worst performance occurs during peak, one can include not only total load but also the peak load information. This is analyzed with results shown in Table 3.19 where it is seen that peak load data is useful.

Table 3.19: Linear regression results  $R^2$  for price estimation adding peak load data

Group	Hour 2	Hour 9	Hour 17
1	86.70%	80.40%	73.70%
2	61.90%	56.10%	27.30%
3	54.90%	53.10%	35.30%

From the above analysis, we find that price prediction by a linear method works reasonably well when load is low but poor at higher loads. Again, a neural network method is introduced (the same neural network parameters as in section 3.4 are applied) combining groups 2 and 3 for all hours. Results are presented in Figures 3.11-3.13. The most significant errors occur for extremely high prices. During the time period of examination, the CAISO energy market experienced some irregular price movements caused by market power and gaming. Better results are likely given a more stable market.

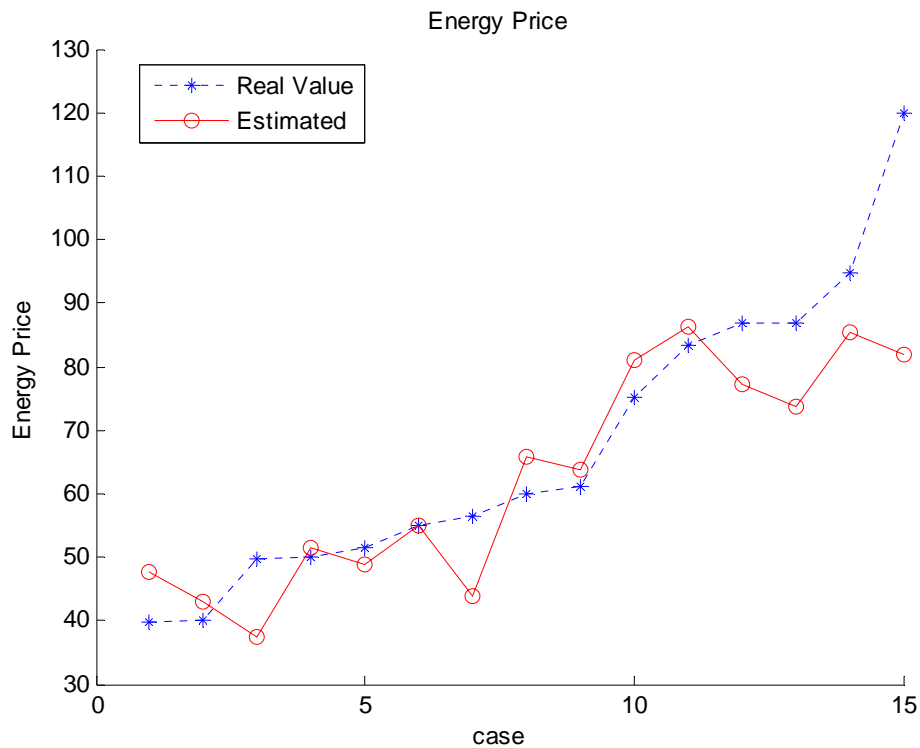


Figure 3.11: Price forecast by neural network at hour 2

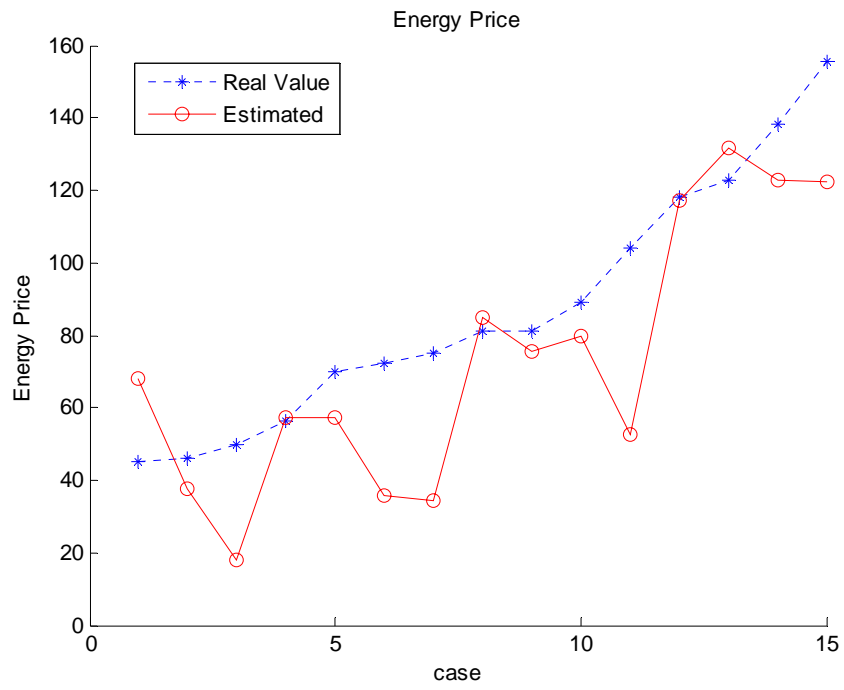


Figure 3.12: Price forecast by neural net at hour 9

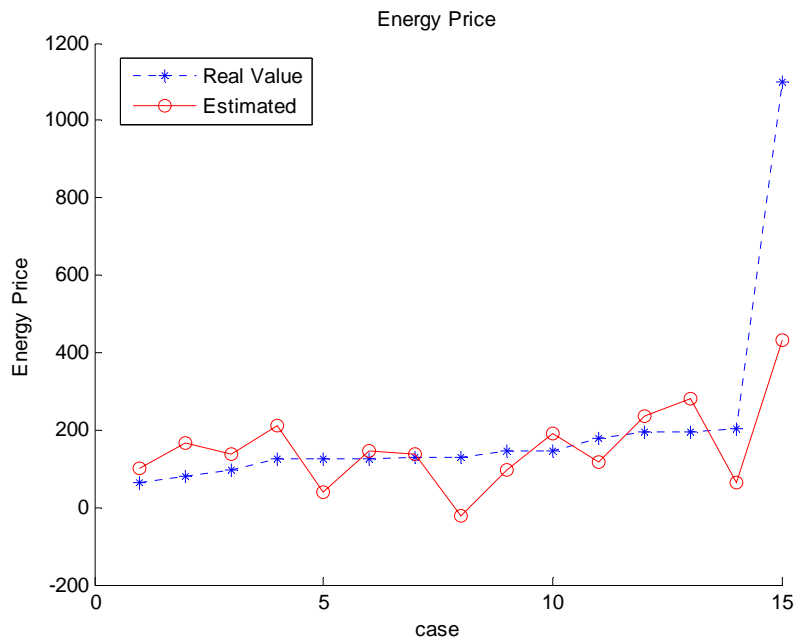


Figure 3.13: Price forecast by neural net at hour 17

### **3.5 Conclusion**

This chapter focused on an empirical analysis of supplier behavior and zonal (NP15) energy price in the 2000 CAISO real-time energy market. First, the relation between bidding behavior and the operation scenario are analyzed by a distribution free correlation and significance test method. Both linear and neural network methods are applied to predict bids. The results show that linear methods fail to adequately describe bidding behavior while the neural network shows some improvement but requires further development. Sensitivity analysis was applied based on the neural network predictor to determine the importance of different input variables. It was noted that non-linear prediction becomes increasingly important with high load.

The analysis in this chapter is rudimentary and is based on limited information. In practice, a MP has access to much more detailed information, including, for example, generator/transmission line outage data, generator location, and so on. These, of course, play an important role in the bids and final prices. Still, the function of the analysis in this chapter was to verify the overall structure of the bidding problem developed in rest of this thesis as reflective of the type of non-ideal behavior that occurs in the practical market.

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## **CHAPTER FOUR**

### **OPTIMAL BIDDING STRATEGIES: AN EMPIRICAL CONJECTURAL APPROACH**

The main objective of this chapter is to suggest one use of historical data for purposes of strategic bidding. A conjectural model based on the market clearing process is presented. In this model, participants estimate a mark-up function for their competitors in the market. Based on these estimates, an optimal bid is found. Numerical examples highlight the methodology. CAISO real time imbalanced energy market data is applied to show that this methodology is viable in practice.

#### **4.1 INTRODUCTION**

The optimal bidding strategy problem is a complex decision making problem involving numerous uncertainties. Generally, all market participants attempt to refine their strategies to earn greater profit. Success in the market requires not only successful forecasting of demand and other market conditions but also anticipating rival behaviours. As a result, many researchers have proposed a game theoretic model to address this problem [1]~[4].

There is evidence to suggest that the energy market acts mostly like an oligopoly market [5]. Game theory is often applied in oligopoly markets and certainly gaming has taken place in real markets. In an oligopoly market, a market participant's behavior will affect the market clearing price (MCP). The participant's bidding strategy problem is to determine the bids to maximize one's own benefit.

In the typical electric power market, MP's submit bids and the intersection between the aggregate supply and aggregate demand curve is the market equilibrium point that determines the MCP and the winning bids. According to economics theory, there are several ways to consider strategic market interactions, including: pure competition, Cournot strategy, Stackelberg model, conjectural model, and so on. Particularly appropriate for the bidding problem here is the Cournot model [6]

Due to the complexity of a real market with numerous participants, most of these theoretical models are too unwieldy to apply to a representative system model. Instead, numerous simplifications are needed to make application possible. For example, typically there are no pure Nash equilibrium points and instead a mixed strategy must be introduced [7]. Under mixed strategies, the optimal solution should be a probability density function. Unfortunately, this is difficult to apply as guidance to bidding activity. A deterministic decision is needed despite the underlying risks and various probabilistic outcomes.

A conjecture model can be used to consider MP interactions. The conjecture model acts as a generalized Cournot model in that each market participant attempts to guess a rival's activity corresponding to the price change [8]. Since many power markets have now been in operation for some time (albeit with frequent changes in the market rules), there exists significant historical data that can be analyzed to help participants understand likely competitor behavior. At a minimum, MPs forecast energy prices based on historical prices. Beyond this, individual market bids are made publicly available in many markets (e.g., in California, all public bids are posted online by the ISO 6 months after the day of submission). Such information provides valuable information about a rival's likely behaviors. Thus, it is feasible to apply a conjecture model to address bidding strategies.

The main objective of this chapter is to consider the best use of specific historical data. To begin, a model of the optimal bidding strategy problem is developed. Then, a statistical application method is introduced based on different price forecast techniques.

## 4.2 PROBLEM DEFINITION

In economics, there are two types of conjecture models: the general conjectural variations (GCV) and the conjectured supply function (CSF). While both models require a “guess” of rival activities beforehand, the difference between these two conjectural models lies in the focus on a rival’s price or the rival’s supply function. The CSF model is adopted in this paper as the more appropriate for assessing bidding strategies in an electricity market.

### 4.2.1 CSF Model

Assume there are  $n$  market participants, let  $k_i$  denote market participant  $i$ ’s decision. Both the market clearing price and the quantity are a function of all market participants’ decision variables denoted here by  $MCP(k_1, k_2, \dots, k_n)$  and  $p_1(k_1, k_2, \dots, k_n)$ , respectively. Assume without loss of generality the bidding decision for individual market participant 1 is desired. The conjecture variation is defined as the belief of the  $i^{th}$  market participant’s response to its rivals. Here, we are interested in the bidding strategy  $k_i$ , and the conjecture variation can be represented by  $k_i(k_1)$ . This means that market participant 1 will guess all other decision variables  $k_i$  as the function of one’s own activity, that is,

$$k_i(k_1) = f_i(k_1), \quad \forall i = 2, \dots, m \quad (4.1)$$

Since it is not possible to know a rival's actions precisely, it is necessary to represent the deviation of the  $i^{\text{th}}$  firm's behavior from this forecast. Here, the error is represented as:

$$k_i(k_1) = f_i(k_1)(1 + e_i), \forall i = 2, \dots, m \quad (4.2)$$

where  $f_i(k_1)$  are the forecast functions, which are static functions given the current market situation, and  $e_i$  are random variables representing the error in the forecast process. Since this function arises from historical data analysis, it represents the belief that these market participants will continue to bid in the same manner. Absent other information, the  $k_i(k_1)$  are assumed to normally distributed since this is the most tractable mathematically. Then

$$E(k_i(k_1)) = f_i(k_1), \quad \forall i = 2, \dots, m \quad (4.3)$$

$$\text{Var}(k_i(k_1)) = f_i(k_1)^2 \text{var}(e_i), \forall i = 2, \dots, m \quad (4.4)$$

It follows that  $e_i \sim N(0, \sigma_i^2)$ , since given a market situation  $f_i(k_1)$  is deterministic. The decision problem is to maximize profit by choosing the decision variables  $k_1$  such that:

$$\begin{aligned} \max_{k_1} \quad & \pi = MCP_0 \cdot p_1 - C_1(p_1) \\ \text{s.t.} \quad & k_2 = k_2(k_1) \\ & \vdots \\ & k_n = k_n(k_1) \\ & MCP_0 = MCP(k_1, k_2, \dots, k_n), \\ & p_1 = p_1(k_1, k_2, \dots, k_n) \end{aligned} \quad (4.5)$$

where  $C_1(p_1)$  is the generation cost function,  $p_1$  represents the MW output awarded in the auction and  $MCP_0$  is the market clear price in \$/MWh for the time period of interest.

#### 4.2.2 Decision Variable $k_i$

Let the generation cost be represented by a quadratic function as

$$C_i(p_i) = \frac{1}{2} c_i p_i^2 + b_i p_i + a_i \quad (4.6)$$

Since the constant term  $a_i$  will not affect the maximization result (assuming a unit will be committed), let  $a_i = 0$  with the marginal generation cost function of the form:

$$MC = c_i p_i + b_i \quad (4.7)$$

In an electrical power market, the market participants submit bidding curves that provide the energy price for different levels of generation output. The market operator will determine the winning bids and the MCP according to the particular set of market rules. The decision problem of interest here is the optimal bidding curve that the market participant submits to the market. Normally, a staircase bidding curve is adopted, and for simplicity here, the linearization process as depicted in Fig. 4.1 is applied. This bidding curve is represented by the following linear function:

$$IC(p_i) = \alpha_i p_i + \beta_i \quad (4.8)$$

Variables  $\alpha_i$  and  $\beta_i$  represent the bidding coefficients participant  $i$  submits, which form the decision vector  $k_i = [\alpha_i \quad \beta_i]$  for problem (4.5). We make a further assumption that both  $\alpha_i$  and  $\beta_i$  are directly related to the  $MC$  by a constant level of mark-up. Thus, (4.5) simplifies to determination of a scalar,  $k_i$ . That is let:

$$k_i = \alpha_i / c_i = \beta_i / b_i \quad (4.9)$$

Then it is easy to see that:

$$IC(p_i) = \alpha_i p_i + \beta_i = k_i (c_i p_i + b_i) \quad (4.10)$$

### 4.2.3 Conjecture Process

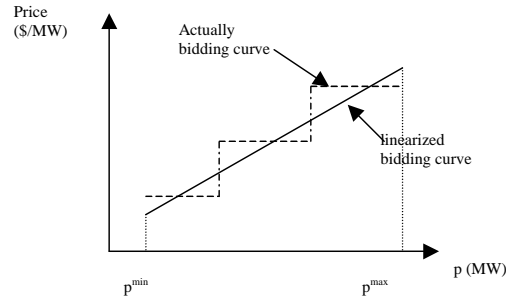


Fig. 4.1 Linearized Bidding Curve

While each market participant may have more than one bidding strategy, in the development here we consider only one bidding strategy, and then subsequently address multiple bidding strategies in the numerical examples. Under a single strategy assumption, there are  $m - 1$  forecast functions needed by each decision maker to reach a decision with  $m$  market participants. With a linearized bidding curve and market clearing process assumption, it follows that

$$p_1 = \frac{MCP(k_1, k_2(k_1), \dots, k_m(k_1))}{k_1 c_1} - \frac{b_1}{c_1} \quad (4.11)$$

Since the  $e_i$  are random variables, the  $k_i(k_1)$  and  $MCP(k_1, k_2(k_1), \dots, k_m(k_1))$  are random variables as well. Thus, the profit maximization problem is a maximum expected profit problem.

### 4.2.4 Risk

Since the rival behavior is not deterministic and knowledge of their behaviors is imperfect, there is risk in any bidding strategy. From a decision-making standpoint, it is necessary to consider the consequence of competitors' actions that deviate significantly from the forecast function. In general, each MP is willing to take a certain amount of risk to earn

more in return. This problem is commonly modeled in investment with risk represented by the standard deviation in the expected profit. Then risk is addressed by the so-called Portfolio Selection Problem (4.8), which is adopted widely in analysis of stock investments.

Portfolio management can be best characterized as obtaining the highest long-run return at the lowest risk [8]. There are two common formulations of portfolio selection: one, to minimize variance (risk) subject to achieving a specified level of return; and two, to maximize return subject to achieving a specified level of variance. Here, the former form is applied. Rewrite (4.5) as follows

$$\begin{aligned}
 \max_{k_1} E(\pi) &= E(MCP \cdot p_1 - C_1(p_1)) \\
 s.t \quad k_2 &= k_2(k_1) \\
 &\vdots \\
 k_m &= k_m(k_1) \\
 MCP &= MCP(k_1, k_2, \dots, k_m), \\
 p_1 &= p_1(k_1, k_2, k_m) \\
 \sigma_\pi^2 &\leq \sigma_*^2
 \end{aligned} \tag{4.12}$$

where  $\sigma_*^2$  represents the maximum acceptable risk determined a priori by the market participant. Those wishing to minimize the risk subject to achieving a specified level of return are essentially solving the dual of the above.

Now since the cost function is quadratic and noting that the third and fourth moments of  $e_i$  are  $E(e_i^3) = 0$   $E(e_i^4) = 3\sigma^4$ , simple algebraic manipulation yields:

$$E(\pi) = \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right) (\sigma_{mcp}^2 + \mu_{mcp}^2) \mu_{mcp} - \frac{b_1}{c_1} \mu_{mcp} + \frac{b_1^2}{2c_1} \tag{4.13}$$

$$\sigma_\pi^2 = \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right)^2 \sigma_{mcp}^2 - 2 * \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right) \cdot \frac{b_1}{c_1} (E(mcp^3) - (\sigma_{mcp}^2 + \mu_{mcp}^2) \mu_{mcp}) + \left( \frac{b_1}{c_1} \right)^2 \sigma_{mcp}^2 \tag{4.14}$$



where  $\sigma_{mcp^2}^2 = E(MCP^4) - E(MCP^2)^2$ , is the variance of  $MCP^2$ ,  $\sigma_{mcp}^2$  and  $\mu_{mcp}$  are the variance and mean of the MCP, respectively. The following section introduces a price forecasting approach to determine the necessary inputs for solving this problem. The expected profit and variance can be found by standard optimization routines given the statistics for errors in the prediction process.

### 4.3 PRICE ESTIMATION MODEL

This section places the preceding development in the context of an electricity market. Assume all other exogenous variables, including the load level, transmission limits, and fuel price, are known. The energy price is determined by the market participants' bidding strategies. Using a linear model forecast, the expected price can be represented by:

$$MCP_{\text{exp}} = \left( \sum_{i=1}^m s_i k_i + s_0 \right) (1 + e_p) \quad (4.15)$$

where  $s_i$  are the sensitivity coefficients between price and bidding strategies,  $s_0$  is a constant term corresponding to the exogenous variables, and  $e_p \sim N(0, \sigma_p^2)$  represents the error from the price estimation model. Rearranging

$$MCP_{\text{exp}} = \left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) (1 + e_i) + s_0 \right) (1 + e_{mcp}) \quad (4.16)$$

with  $e_i$  an independent random variable. Now, separating the deterministic and the random variables, and defining the deterministic term

$$p_0 = s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) + s_0 \quad (4.17)$$

and the stochastic term,

$$MCP_{\text{exp}} = \left( p_0 + \sum_{i=2}^m s_i f_i(k_1) e_i \right) (1 + e_p) \quad (4.18)$$

We have the following:

$$\mu_{mcp} = E(MCP_{\text{exp}}) = p_0 \quad (4.19)$$

and

$$\sigma_{mcp}^2 = \left( \sum_{i=2}^{m+1} s_i^2 f_i(k_1)^2 \sigma_i^2 \right) \sigma_p^2 \quad (4.20)$$

As discussed before, to consider risk, it is necessary to know the third and fourth moments of the MCP. Based on the independence Gaussian distribution assumption, it is easy to calculate the third and fourth moments for the MCP from equations (4.21) and (4.22) below

$$E[MCP^3] = E \left[ \left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) (1 + e_i) + s_0 \right)^3 \right] E[(1 + e_p)^3] \quad (4.21)$$

$$E[MCP^4] = E \left[ \left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) (1 + e_i) + s_0 \right)^4 \right] E[(1 + e_p)^4] \quad (4.22)$$

And by properties of Gaussian distribution, we know the following:

$$\left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) (1 + e_i) + s_0 \right) \sim N \left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) + s_0, \sum_{i=2}^m s_i^2 f_i(k_1)^2 \sigma_i^2 \right) \quad (4.23)$$

$$(1 + e_p) \sim N(1, \sigma_p^2) \quad (4.24)$$

Let  $X$  and  $Y$  represent the random variables in equation (4.23) and (4.24), we know the following:

$$E[X^3] = \mu_X (\mu_X^2 + 3\sigma_X^2) \quad (4.25)$$

$$E[X^4] = \mu_X^4 + 6\mu_X^2 \sigma_X^2 + 3\sigma_X^4 \quad (4.26)$$

with  $\mu_X = s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) + s_0$  and  $\sigma_X^2 = \sum_{i=2}^m s_i^2 f_i(k_1)^2 \sigma_i^2$ . Similarly, for random variable  $Y$  just replace the subscript  $X$  by  $Y$  in equation (4.25) and (4.26), and corresponding  $\mu_Y = 1$  and  $\sigma_Y^2 = \sigma_p^2$ . By applying equation (4.21), (4.22), (4.25) and (4.26) to equation (4.14), the risk corresponding to any given bidding strategy a market participant submitted can be calculated.

## 4.4 NUMERICAL EXAMPLE

### 4.4.1 Numerical Example 1

First, consider a system with four market participants. The forecast error variances and coefficients are given by:

$$\sigma_2^2 = \sigma_3^2 = \sigma_4^2 = 0.25, \sigma_{mcp}^2 = 100 \quad (4.27)$$

Assume that using statistical data analysis for a given market operation situation, participant 1 forecasts that the rivals have split strategies as given by the following bidding functions:

$$k_2(k_1) = \begin{cases} (1.0 + 0.1k_1)(1 + e_2), & k_1 \leq 2.0 \\ (0.2 + 0.5k_1)(1 + e_2), & k_1 > 2.0 \end{cases}$$

$$k_3(k_1) = \begin{cases} (1.1 + 0.2k_1)(1 + e_3), & k_1 \leq -0.5 \\ (1.2 + 0.3k_1)(1 + e_3), & k_1 > -0.5 \end{cases} \quad (4.28)$$

$$k_4(k_1) = \begin{cases} (2.0 - 0.1k_1)(1 + e_4), & k_1 \leq 1.8 \\ (2.54 - 0.4k_1)(1 + e_4), & k_1 > 1.8 \end{cases}$$

Further assume the expected price function can be represented by the following linear function:

$$MCP_{exp} = (7.973k_1 + 4.46k_2 + 82.68k_3 + 11.6k_4 + 4.014)(1 + e_{mcp}) \quad (4.29)$$

#### 4.4.2 Discussion

If there is no risk constraint, the higher the market participant 1 bids, the higher the expected profit. Thus, the optimal solution is  $k_1^* = 3.0$  (where we assume that 3.0 is the bidding cap.). Notice the risk will increase as the expected profit increases and this occurs particularly rapidly at the higher bidding prices. In the worst case, there could be significant losses. Worse from the generation company point-of-view, the expected profit changes very little beyond say  $k_1 = 1.2$ , while the risk increases rapidly. It is advantageous to sacrifice some marginal profit to reduce the risk. The optimal decision depends on the risk level a firm would like to take. For example, if the risk tolerance is about 80% of the maximum risk level (shown as the black dashed line in Fig. 4.2), then the optimal solution is  $k_1^* = 1.2$ .

Intuitively, another method to reduce risk is to employ a split bidding strategy, i.e., bid different type of generators at different strategies. For example, one may choose to bid low cost generators at a fairly low price (say, marginal cost) to ensure some profit while bidding some at a higher cost (so-called economic withholding). By doing so, risk should decrease. For simplification, here assume that market participant 1 employs such a split strategy labeled

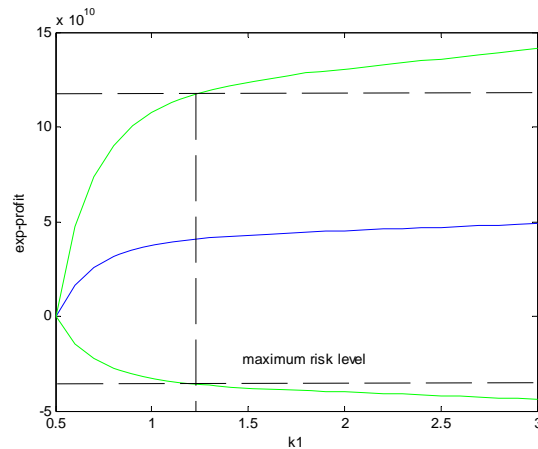


Fig. 4.2 Example 1 showing expected profit  $\pm\sigma$

for convenience as the main and minor bidding strategies. Figure 4.3 shows the results of such an approach. Since the split strategy is used, the figure is no longer a two-dimensional figure. To simplify the figure, only the main strategy is shown along the x-axis. The corresponding expected profit is the optimal value given this major bidding strategy that market participant 1 found by choosing different levels of the minor bidding strategy. In this case, if there are no risk constraints, the optimal bidding strategy is  $k_{11}=1.2$  (major) and  $k_{12}=3.0$  (minor). Again, if the maximum risk level is added as a constraint, there are two new optimal bidding strategies, shown as  $k_{11}^* = 0.75$  and  $k_{11}^* = 2.8$  with in both cases  $k_{12}=3.0$  in Fig. 3. Thus, by withholding some generation, expected profit does not increase as rapidly but risk is reduced.

#### 4.4.3 Empirical Example 2: Simplified two market participant example

In practice, it is difficult to estimate all competitors' strategies accurately and it may not be particularly profitable to do so. That is, it is unnecessary to distinguish the different market participants when they are all acting as rivals. Further in our framework, several high order moments are needed to find the optimal mark-up and these will not be readily available. Accordingly, consider a single opponent with this forecast function:

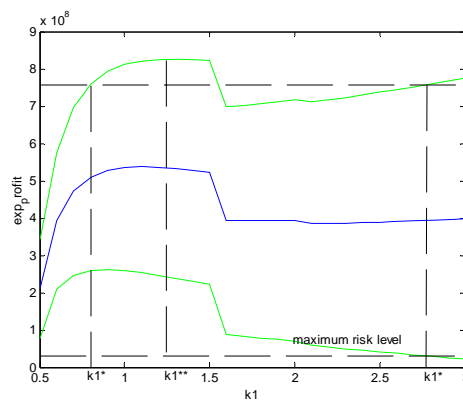


Fig. 4.3 Example 1 showing expected profit with a split strategy

$$k_2(k_1) = \begin{cases} 1.0 + 0.1k_1 + e_k, & k_1 \leq 1.5 \\ 0.55 + 0.4k_1 + e_k, & k_1 > 1.5 \end{cases} \quad (4.30)$$

All variances are as before. The profit results are shown in Fig. 4.4. Again, when there is no risk constraint, the optimal solution is  $k_1^* = 3.0$ . When an 80% risk tolerance is applied, the optimal solution is  $k_1^* = 2.4$ . For this simple case, there is a significant difference between these two models simply due to the selection of the expected rival's bidding strategy and other expected variables. In practice, it should be straightforward to obtain similar results with a two participant example.

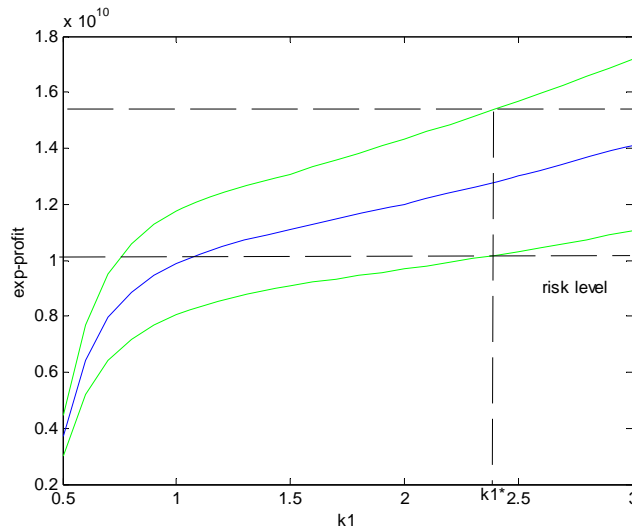


Fig. 4.4 Example 1 showing expected profit with a single competitor.

## 4.5 APPLICATION TO PRACTICAL MARKETS

To exploit the above insights in a practical application, one must address the prediction of the energy price and the rival's bidding strategy. The details of the prediction process are discussed in Chapter III. Here, the basic methodology is outlined on some representative examples in the California electricity market

#### **4.5.1 Real-time electric power market**

The California market employs a zonal pricing scheme as discussed in the introduction, which is briefly reviewed again here. There are ostensibly 14 zones observed by the CAISO. In the day/hour ahead markets, the bids are stacked in the BEEP system, and the energy price is decided by identifying the point where the aggregate supply equals the aggregate demand. The ISO then performs congestion analysis to ensure there is no security problem. If there is congestion, then the ISO initiates the congestion management procedures. Congestion management in CAISO is divided into two sub systems: inter-zonal and intra-zonal. The inter-zonal congestion management of California ISO ignores intra-zonal congestion and uses a DC optimal power flow program to dispatch between zones. The objective is to minimize the redispatch cost, as determined by the submitted adjustment bids that accompany the submitted schedules. Inter-zonal congestion management does not involve arranging or modifying trades between SCs. Further, it does not address the optimization of SC portfolios within zones. When real-time inter-zonal congestion occurs, the BEEP stack is constructed and applied separately to each zone. Thus, price differences may arise across the constrained zonal interface.

#### **4.5.2 Price forecast**

Numerous sophisticated price prediction methods that employ a large number of variables can be found in the literature [e.g., 10-11]. In this work, our interest is to consider a minimum number of factors to allow price predictions useful for bidding. Since the prices in each zone are needed to make optimal strategic bids considering congestion, it is more

informative to use the zonal price then average price as the price index for bidding. There are several interesting zones in the CAISO, the particularly important NP15 zone is considered in this chapter.

In the electric power market, the energy price is decided by the market clearing process that finds where aggregate demand equals the aggregate supply. To predict the zonal energy price, forecasted load and gas prices are the two main factors, which must be included in the energy price prediction model. In the CAISO congestion management scheme, the zone prices deviate whenever there is interzonal congestion. Thus, the possibility of congestion provides some indication for the differences in the zonal energy prices. To predict the CAISO energy prices, our input variables include the forecasted values for load, gas price and the congestion probabilities.

The CAISO real time energy market data from Feb. 2000 to Oct. 2001 is used as a sample for the proposed model. This was a particularly interesting time in the market, which helps in the analysis here. As discussed in chapter III, when load is light (in sample data, hourly average load < 25 GW), a linear model can predict prices accurately although the performance of such a linear model is poor under heavier loads. Still for simplicity, only a linear model is applied here. Using the sample data analyzed in Chapter III, the relevant input data is listed in Table 4.1. The corresponding output price at this point can be represented by equation (4.31):

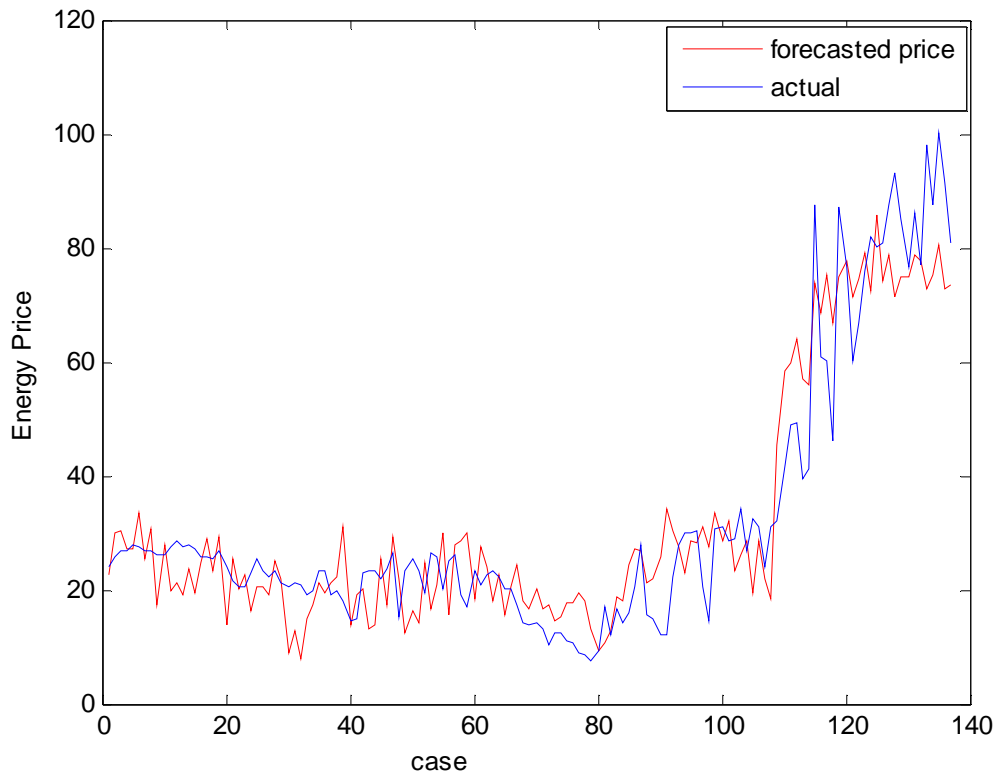
$$MCP_{\text{exp}} = (0.7554k_1 - 10.756k_2 + 30.8584)(1 + e_{mcp}) \quad (4.31)$$

where  $e_p \sim N(0, 78.04)$ . The difference between linear regression results and actual energy price can be seen in Fig. 4.5.



Table 4.1: Sample input data for price prediction

Transmission Path congestion Possibilities				Gas	Load
COI	PATH15	PALO	PATH26	(\$/Mbtu)	(MW)
0.00	0.00	100.00	94.00	3.14	23,529

*Fig. 4.5 Price prediction by linear method*

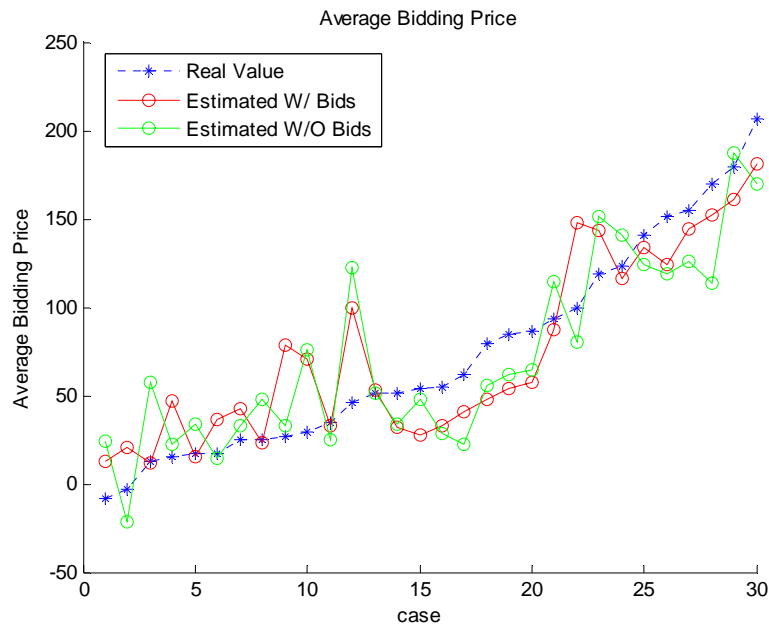
From Fig. 4.5, one can see that the linear regression matches the general trend of the energy price but fails to track the price spikes. A more complex model is necessary to investigate such price spikes and is beyond the scope of this chapter.

### 4.5.3 Forecast of bidding behavior

Predicting competitor bidding strategies ideally follows from observing their behavior over time. Unfortunately, it may not always be possible to have accurate information on recent behavior. For our application, there are two components to the bidding strategies: one's own past behavior, which must be considered when assessing competitor strategies, and competitor behavior. In California, producer bids are made public 6 months after the day in question. Simplistically, one can apply this old data assuming there have been no changes in strategy (while still considering the impact of congestion and forecasted load and prices). Still, a producer's bidding strategies may change dramatically over time. There are two possibilities for identifying these changes. One approach is to observe changes over time and project these out over the 6-month lag. Another method is to feedback the actual MCPs to adjust the bidding parameters. The competitor bidding model then outputs the average bidding price as an index of bidding behavior based on the forecasted load, forecasted gas prices, congestion possibilities and historical bidding behavior.

Bidding behavior can be extremely varied arising as it does from a complex decision-making process considering numerous objectives. The author believes it is unrealistic to use linear models to predict competitor behavior. Here, a neural network is applied to capture some of the non-linearities although certainly more sophisticated models could be employed. Detailed results are listed in chapter III Section 2, Fig. 4.6 showing such a relationship is repeated here for convenience. Note, the x-axis refers to the case number and negative values means that in this situation, a generator is better off buying energy than selling.

The sensitivities discussed in chapter III and employed in the above development represent only an individual point and it is clear that such a linear method cannot fully represent the bid forecast problem. To find a suitable relationship between individual bids and system overall bids, we follow the procedure developed in Chapter 3. Specifically:



*Fig. 4.6 Simulation Result*

- Obtain a representative training set, including all salient input and output variables as well as forecasted system operation information (load, gas price and congestion possibilities). While theoretically a market participant can do anything they desire as long as they abide by the market rules, one assumes that any market participant is rational. Based on this assumption, it is natural to find a range of possible bidding prices for a MP given the forecasted market operation scenario but this range can be quite large.
- Input this information to a trained network to find the relevant input-output mapping as shown for same example system, again as discussed in chapter III.

The detailed forecasted input information is reiterated here in table 4.2 for convenience. Similarly, assume the market participant's bidding strategy  $k_1$  will change from  $-0.5$  to  $3.0$  and the forecasted corresponding overall system response by the sensitivity analysis result is shown in Fig. 4.7.

Table 4.2: Sample Input information for Sensitivity Analysis

	Bids (\$/MWh)	Transmission Path congestion Possibilities				Gas (\$/Mbtu)	Load (MW)
		COI	PATH15	PALO	PATH26		
Case 1	27.53	0.00	0.00	100.00	94.00	3.14	23529

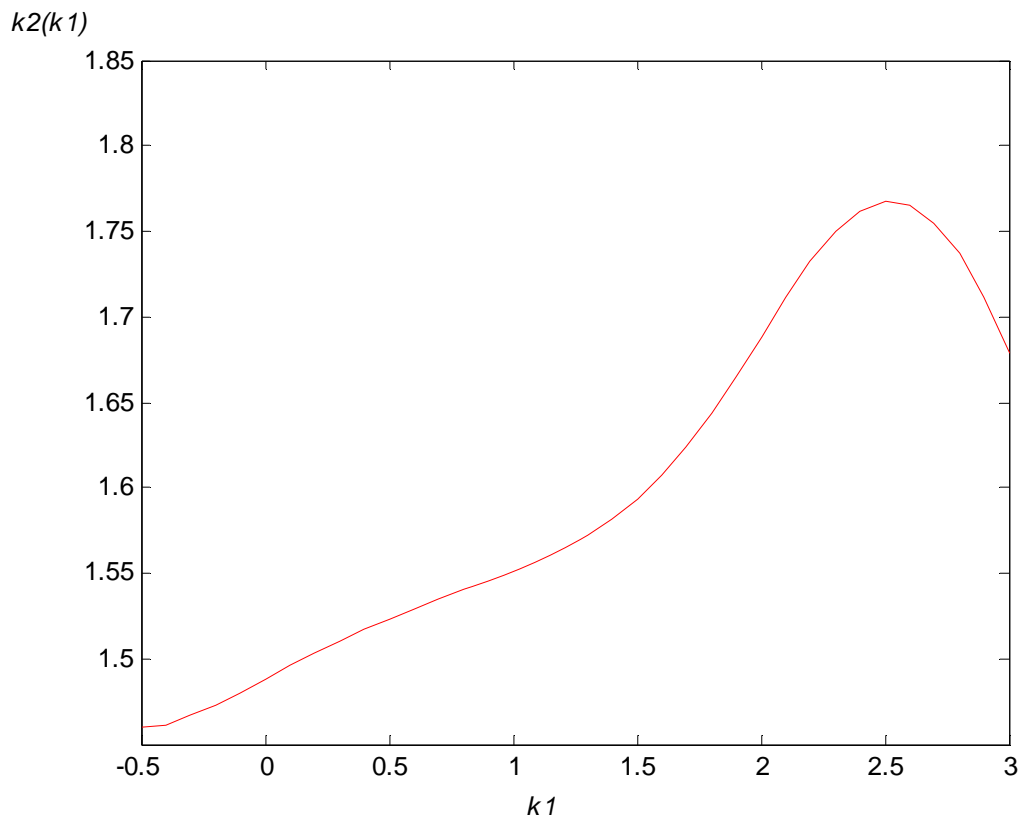


Fig. 4.7 Normalized sensitivity relationship for example 1

This relationship can now be modeled as a piece-wise linear function. For the on-going discussion this gives:

$$k_2(k_1) = \begin{cases} 0.065k_1 + 1.4825 + e_k, & k_1 \leq 1.5 \\ 0.24k_1 + 1.22 + e_k, & 1.5 \leq k_1 \leq 2.5 \\ -0.28k_1 + 2.52 + e_k, & k_1 > 2.5 \end{cases} \quad (4.32)$$

with  $e_k \sim N(0, 0.2661)$ . Assuming the aggregate generation cost function is the following:

$$Cost = \frac{1}{2} \cdot 0.0042 \cdot p^2 + 12.04 \cdot p \quad (4.33)$$

and applying 4.32 and 4.33 to the optimization in 4.12, the optimal strategy result is as shown in Fig. 4.8.

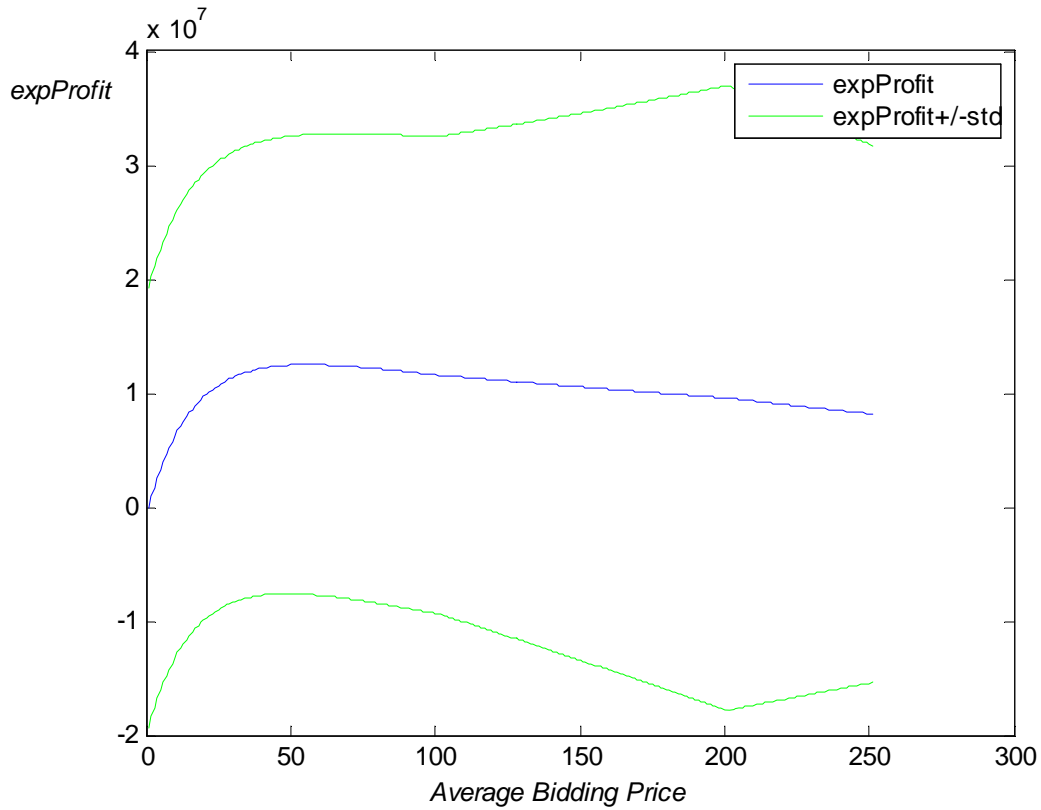


Fig. 4.8 Example 1 showing expected profit with a single competitor.

When risk is not considered, the optimal solution is  $k_i^* = 1.05$  (assuming the price cap is 3.0)

and the corresponding price and rival's bidding strategy is:

$$k_{-i}^* = 1.55, \text{ MCP} = 14.9717\$ / MWh$$

Assume an MP's risk level is the expect profit plus the standard deviation and the objective function now is not to maximize the expect profit, but to maximize the expect profit minus one standard deviation of the expect profit. In this case, the optimal solution is:

$$k_i^* = 0.99, k_{-i}^* = 1.5468 \text{ and } MCP = 14.9683\$ / MWh$$

Comparing these two optimal solutions, there is little difference. The MP's expected profit will decrease from 12.498 M\$ (millions of dollars) to 12.461 M\$, which is a mere 0.3% decrease in the expected profit. Now, let's take a look at another example, assuming the input information given in Table 4.3:

Table 4.3: Sample input data for price prediction

COI	PATH15	PALO	PATH26	Gas(\$/Mbtu)	Load (MW)
0.00	0.00	0.00	25.00	2.9977	22024

Fig. 4.9 plots the resulting sensitivities and the linearized function is now:

$$k_2(k_1) = \begin{cases} 0.268k_1 + 1.4164 + e_k, & k_1 \leq 1.3 \\ -0.48k_1 + 2.388 + e_k, & k_1 > 1.3 \end{cases} \quad (4.34)$$

with  $e_k \sim N(0, 0.2661)$ . The price model remains the same. The expect profit and standard deviation results are shown in Fig.4.10.

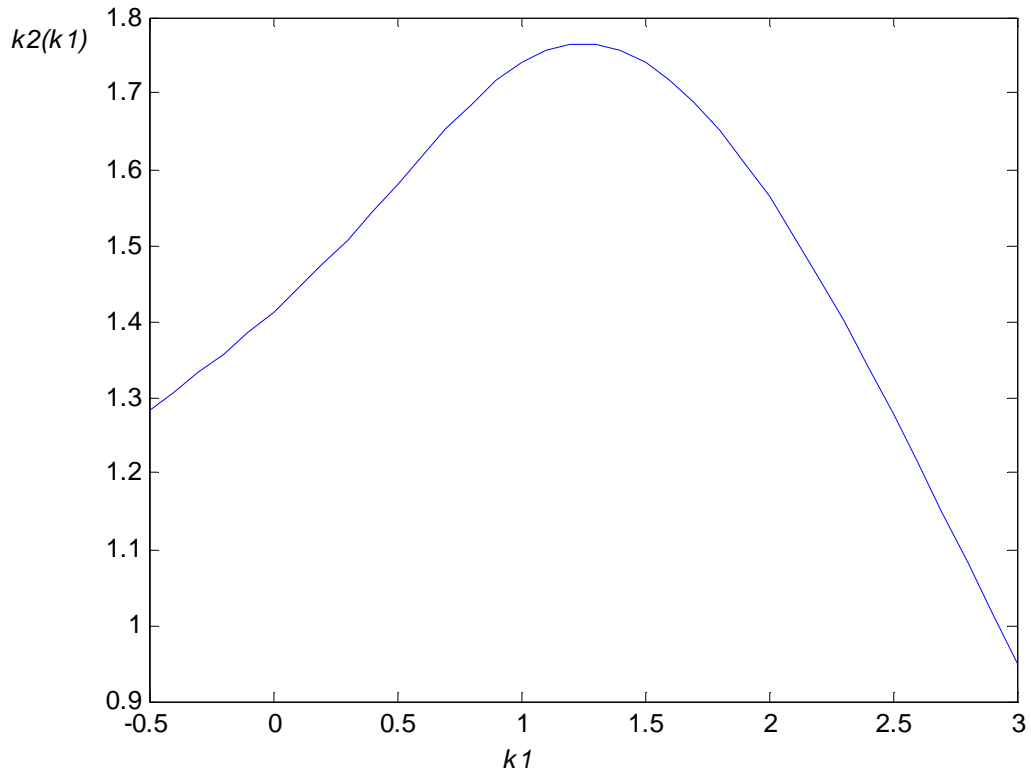
The optimal bid without considering risk is:

$$k_i^* = 1.09, k_{-i}^* = 1.71 \text{ and } MCP = 13.30\$ / MWh$$

when risk is considered, the new optimal solution is:

$$k_i^* = 3.0, k_{-i}^* = 0.95 \text{ and } MCP = 22.93\$ / MWh$$

In this case, by considering the risk, the expected profit decreases significantly from 13.27 M\$ to 4.61 M\$, a roughly 65% decrease in expected profit. Note the MCP is also significantly higher.



*Fig. 4.9 Normalized sensitivity results for example 2.*

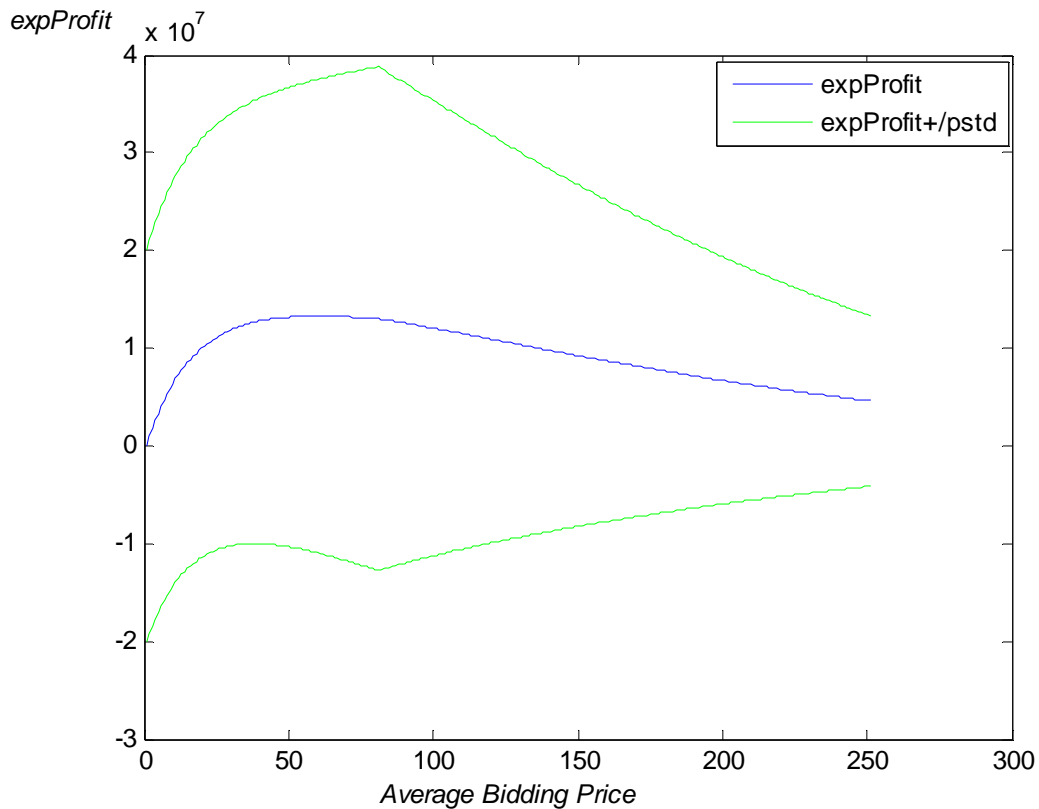


Fig. 4.10 Example 2 showing expected profit with a single competitor.

## 4.6 CONCLUSION

This chapter presents an approach for determining an optimal bid into the market as a mark-up over actual costs. It requires as input an estimate of competitors' bids in terms of price and possible variation. The solution then maximizes profit while maintaining a tolerable financial risk. Chapter III has shown the feasibility of forecasting competitor behavior, suggesting the approach in this chapter is practical. These techniques were applied to data from the California market further enforcing the feasibility of relative ease of applying this method. Results show that given the uncertainty of the market operation scenario as well as other MPs' behavior, risk will play a fundamental role in the decision making process for determining the optimal bids.



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## CHAPTER FIVE

### CONCLUSION

#### 5.1 Summary of the thesis work

Electric restructuring was started in the early 1990s as a way to increase electric power industry' efficiency and lower the energy cost. The traditional integrated system has now been separated in many parts of the country and some degree of competition has been introduced throughout the power industry. This thesis focused on how market participants (primarily generators) react under this new market operation mechanism. Specifically, this work contributed with the following three investigations:

1. Transmission system congestion influence on market clearing price and market participant bidding behavior in the framework of game theory was analyzed. The conclusion was drawn that deviation from idealized price-taker behavior is more serious when some market participants suffer disproportionately from the congestion. Due to the complexity of the calculations in the theoretical approach, this thesis suggests that a statistical analysis methodology is more appropriate. An intuitive probabilistic bidding methodology was proposed for the bidding problem to demonstrate feasibility.
2. A detailed statistical analysis has been carried out on the California real time imbalance energy market. A linear regression model was applied to a zonal energy price prediction process and a non-linear estimator based on a neural network was applied to predict bidding behavior. Sensitivity analysis was applied to understanding each factor's influence on market participant bidding behavior.

3. Statistical analysis results were applied to the optimal bidding strategy problem. The empirical conjecture approach was adopted using these results. Including risk as either an objective to be minimized or a constraint to be satisfied, a portfolio selection approach was applied. This method combines the statistical analysis technique with the optimal bidding problem. Although the results shown here are in the initial stage of development, it appears that this approach is more promising than an idealized game theoretic formulation.

## **5.2 Future work**

It is very difficult to model the bidding strategy problem in a purely theoretical mathematical framework since practical bidding behavior is not so easily captured and includes significant difficult to model human factors. Still, much statistical analysis work has already been done for market operation analysis, and it is natural for a market participant to use these analysis results to learn the best strategy. The last chapter of this thesis initiates an alternative approach to this area, although a more thorough and detailed analyses are needed. Of particular difficulty is the continually evolving market rules which render conclusions based on past behavior suspect. The industry is working hard to improve market rules that prevent market power and ensure true competition. The longer a market operates under a given set of rules, the more one can accept the validity of the bidding data, which should lead to a more reliable statistical analysis.

Market power detection and correction is the key to market monitoring and one of the major functions for ISO/RTO. So far, most of market monitors have adopted the conduct and impact testing concepts. The conduct test compares the bids with a reference level generally based on the historical bids or on cost information Independent Market Monitor (IMM)

collected beforehand. The impact test evaluates the influence of the failed bids failed on the energy price. If replacing some of the failed bids failed in conduct testing results in a significant overall pricing improvement, then such failed bids should be analyzed. For example, in the New York ISO, the AMP (Automated Mitigation Procedure) will runs the impact tests if it appears prices will exceed \$150/MWh [1]. MISO's IMM process will be triggered if there is any active binding constraint. These conduct and impact testing concepts have been criticized for failing to distinguish between resource scarcity and market power. The Edison mission objected to New York ISO's AMP proposal arguing that outside New York City the AMP could mitigate when temporary shortages, rather than market power [2]. Others have argued that reliability must be considered in these analyses. Such arguments arise as there is no widely accepted method to distinguish between market power and resource shortage. To solve this problem, more detailed studies of market operation and bidding behavior are needed. Market power mitigation remains one of the major challenges facing market analysis today.

**Reference:**

[1] 99 FERC order, Docket No. ER01-3155-002, et al.,

<http://elibrary.ferc.gov/idmws/nvcommon/NVViewer.asp?Doc=1045537:0>

[2] Edison Mission Energy, Inc. And Edison Mission Marketing & Trading Inc., “United Stated Court of Appeals for the district of Columbia Circuit”, Oct. 18, 2004

## Appendix A

Equation (17) can be found as follows. Assuming equal weightings rewrite (8) as:

$$\begin{aligned}
 & \min_{\Delta p_i} \Delta P^T \cdot \Delta P \\
 & s.t. \quad \sum_{i \in G} \Delta p_i = 0 \\
 & \quad |P_{ij}| \leq P_{ij}^{\max}
 \end{aligned} \tag{A.1}$$

With a DC power flow, the GSF is constant. So the power flow on each line can be given by:

$$P_{jk} = \sum_{\forall i} \beta_{jk,i} P_i \tag{A.2}$$

where  $\rho_{jk,i} = \frac{\Delta P_{jk}}{\Delta p_i}$  is the GSF. Then, the new flow is:

$$P_{jk}^{new} = \sum_{\forall i} \beta_{jk,i} (P_i + \Delta p_i) = P_{jk}^{old} + \sum_{\forall i} \beta_{jk,i} \Delta p_i \tag{A.3}$$

where  $\Delta p_i = 0 \quad \forall i \notin G$ . Now the flow constraint can be written as  $\left| P_{jk}^{old} + \sum_{\forall i} \beta_{jk,i} \Delta p_i \right| \leq P_{jk}^{\max}$ .

Expanding the absolute value gives

$$-P_{jk}^{\max} - P_{jk}^{old} \leq \sum_{\forall i} \beta_{jk,i} \Delta p_i \leq P_{jk}^{\max} - P_{jk}^{old} \tag{A.4}$$

Applying the Kuhn-Tucker conditions to (8), the inner solution will be:

$$2\Delta p_i + \lambda + \sum_{\forall lines} \mu_{jk}^+ \beta_{jk,i} - \sum_{\forall lines} \mu_{jk}^- \beta_{jk,i} = 0 \tag{A.5}$$

$$\mu_{jk}^+ \left( \sum_{i \in G} \beta_{jk,i} \Delta p_i - P_{jk}^{\max} + P_{jk}^{old} \right) = 0, \quad \forall jk \in lines \tag{A.6}$$

$$-\mu_{jk}^- \left( \sum_{i \in G} \beta_{jk,i} \Delta p_i + P_{jk}^{\max} + P_{jk}^{old} \right) = 0, \quad \forall jk \in lines \tag{A.7}$$

Note for line flows within limits, the  $\mu$  must equal to zero. The above can then be solved to find the  $\Delta p_i$ . Assuming a single violation in least curtailment, algebraic manipulation results in (17) with quadratic terms found by substituting the solution to (A.5) into (A.6) and (A.7).

## Appendix B

Computing mixed strategy Nash Equilibriums can be a challenging task; however, there is a particular observation about the equilibrium that can often greatly simplify this task. Note that in a mixed strategy Nash Equilibrium, the expected payoffs for any player will remain the same if he or she switches to any pure strategy that has a positive probability of being picked by the equilibrium mixed strategy. Consider a very simple example with the payoff matrix of Table B.1. There is no pure Nash Equilibrium. To calculate the mixed strategy equilibrium, player 2's probability of play L and R are  $y$  and  $1-y$ . Then player 1's expected payoff if he chooses either U or D must be equal. Let  $\sigma_2$  represents player 2's best response, so

$$E_1(U, \sigma_2) = -2y + 4(1 - y) \quad (\text{B.1})$$

and

$$E_1(D, \sigma_2) = 2y + 2(1 - y) \quad (\text{B.2})$$

Solving  $E_1(U, \sigma_2) = E_1(D, \sigma_2)$ , yields  $y = \frac{1}{3}$ . Thus, the Nash Equilibrium mixed strategy for player 2 is given by  $\sigma_2^* = (\frac{1}{3}, \frac{2}{3})$ . Similarly, player 1's is found to be  $\sigma_1^* = (\frac{3}{5}, \frac{2}{5})$ . It is easy to apply this process to the problem in this paper.

TABLE B.1  
PAYOFF MATRIX

	L	R
U	(-2,2)	(4,0)
D	(2,1)	(2,4)



## Appendix C

TABLE C.1  
Correlation coefficient and corresponding p-value between different bids

ID1	ID2	Hour17		Hour9		Hour2	
		CorrCoef	P_value	CorrCoef	P_value	CorrCoef	P_value
(199871, 192115)		0.27	0.00	0.33	0.00	0.32	0.00
(199871, 282606)		-0.06	0.30	0.20	0.00	0.37	0.00
(199871, 108576)		0.46	0.00	0.74	0.00	0.85	0.00
(199871, 106100)		0.38	0.00	0.50	0.00	0.50	0.00
(199871, 599841)		0.39	0.00	0.51	0.00	0.53	0.00
(199871, 715337)		0.19	0.00	0.25	0.00	0.17	0.00
(199871, 104351)		-0.20	0.00	0.01	0.84	0.15	0.01
(199871, 142494)		-0.14	0.02	0.03	0.65	0.07	0.22
(199871, 494629)		0.36	0.00	0.51	0.00	0.53	0.00
(199871, 918588)		-0.39	0.00	-0.36	0.00	-0.42	0.00
(199871, 102611)		0.27	0.00	0.30	0.00	0.28	0.00
(199871, 205249)		0.10	0.09	0.33	0.00	0.02	0.75
(199871, 206887)		0.43	0.00	0.44	0.00	0.46	0.00
(199871, 453332)		0.22	0.00	0.27	0.00	0.31	0.00
(199871, 194543)		0.23	0.00	0.35	0.00	0.13	0.03
(199871, 168177)		0.22	0.00	0.30	0.00	0.18	0.00
(199871, 136212)		0.04	0.53	0.06	0.35	-0.04	0.55
(199871, 461530)		-0.14	0.02	-0.30	0.00	-0.46	0.00
(199871, 475056)		-0.12	0.04	-0.30	0.00	0.15	0.01
(199871, 999999)		0.20	0.00	0.43	0.00	0.55	0.00
(192115, 282606)		-0.10	0.11	0.03	0.63	0.19	0.00
(192115, 108576)		0.25	0.00	0.42	0.00	0.29	0.00
(192115, 106100)		0.16	0.01	0.13	0.03	0.09	0.15
(192115, 599841)		0.16	0.01	0.15	0.01	0.13	0.04
(192115, 715337)		0.56	0.00	0.66	0.00	0.43	0.00
(192115, 104351)		-0.21	0.00	-0.18	0.00	-0.06	0.31
(192115, 142494)		-0.15	0.01	-0.12	0.04	0.11	0.06
(192115, 494629)		0.16	0.01	0.26	0.00	0.23	0.00
(192115, 918588)		-0.21	0.00	-0.16	0.01	-0.15	0.01
(192115, 102611)		0.72	0.00	0.71	0.00	0.58	0.00
(192115, 205249)		0.00	0.95	0.03	0.67	0.22	0.00
(192115, 206887)		0.21	0.00	0.25	0.00	0.28	0.00
(192115, 453332)		0.69	0.00	0.82	0.00	0.71	0.00
(192115, 194543)		0.30	0.00	0.44	0.00	0.20	0.00
(192115, 168177)		0.29	0.00	0.46	0.00	0.14	0.02
(192115, 136212)		-0.07	0.22	-0.02	0.81	-0.12	0.06
(192115, 461530)		0.00	0.99	-0.04	0.48	-0.11	0.08
(192115, 475056)		-0.06	0.35	-0.12	0.04	-0.10	0.09
(192115, 999999)		-0.08	0.21	0.29	0.00	0.25	0.00

ID1	ID2	Hour17		Hour9		Hour2	
		CorrCoef	P_value	CorrCoef	P_value	CorrCoef	P_value
(282606, 108576)		-0.13	0.04	0.17	0.01	0.36	0.00
282606, 106100)		0.13	0.04	0.31	0.00	0.35	0.00
(282606, 599841)		0.01	0.84	0.29	0.00	0.45	0.00
(282606, 715337)		-0.09	0.15	-0.01	0.93	0.28	0.00
(282606, 104351)		0.38	0.00	0.61	0.00	0.65	0.00
(282606, 142494)		0.52	0.00	0.66	0.00	0.67	0.00
(282606, 494629)		0.13	0.03	0.33	0.00	0.45	0.00
(282606, 918588)		0.17	0.00	-0.20	0.00	-0.14	0.02
(282606, 102611)		-0.16	0.01	0.06	0.33	0.32	0.00
(282606, 205249)		-0.02	0.77	0.23	0.00	-0.16	0.01
(282606, 206887)		0.13	0.03	0.37	0.00	0.37	0.00
(282606, 453332)		-0.03	0.65	0.02	0.71	0.21	0.00
(282606, 194543)		-0.15	0.01	0.19	0.00	0.25	0.00
(282606, 168177)		-0.15	0.02	0.19	0.00	0.26	0.00
(282606, 136212)		-0.28	0.00	-0.31	0.00	-0.18	0.00
(282606, 461530)		-0.10	0.10	-0.46	0.00	-0.44	0.00
(282606, 475056)		-0.04	0.55	-0.30	0.00	0.03	0.63
(282606, 999999)		0.01	0.93	0.18	0.00	0.15	0.01
(108576, 106100)		0.20	0.00	0.42	0.00	0.51	0.00
(108576, 599841)		0.15	0.01	0.46	0.00	0.58	0.00
(108576, 715337)		0.23	0.00	0.29	0.00	0.13	0.03
(108576, 104351)		-0.14	0.02	-0.04	0.54	0.13	0.03
(108576, 142494)		-0.10	0.10	-0.03	0.68	0.07	0.28
(108576, 494629)		0.10	0.10	0.40	0.00	0.51	0.00
(108576, 918588)		-0.30	0.00	-0.39	0.00	-0.39	0.00
(108576, 102611)		0.25	0.00	0.40	0.00	0.25	0.00
(108576, 205249)		-0.02	0.79	0.34	0.00	0.08	0.21
(108576, 206887)		0.30	0.00	0.39	0.00	0.43	0.00
(108576, 453332)		0.21	0.00	0.33	0.00	0.23	0.00
(108576, 194543)		0.12	0.06	0.37	0.00	0.09	0.12
(108576, 168177)		0.12	0.06	0.34	0.00	0.14	0.03
(108576, 136212)		0.06	0.32	0.01	0.89	-0.05	0.38
(108576, 461530)		0.08	0.21	-0.20	0.00	-0.42	0.00
(108576, 475056)		-0.06	0.33	-0.30	0.00	0.10	0.11
(108576, 999999)		-0.01	0.84	0.37	0.00	0.55	0.00
(106100, 599841)		0.71	0.00	0.70	0.00	0.75	0.00
(106100, 715337)		0.08	0.19	0.08	0.18	0.18	0.00
(106100, 104351)		-0.20	0.00	0.05	0.38	0.21	0.00
(106100, 142494)		-0.11	0.07	0.06	0.37	0.16	0.01
(106100, 494629)		0.71	0.00	0.69	0.00	0.74	0.00
(106100, 918588)		-0.48	0.00	-0.45	0.00	-0.42	0.00
(106100, 102611)		0.25	0.00	0.21	0.00	0.31	0.00
(106100, 205249)		0.33	0.00	0.29	0.00	-0.15	0.01
(106100, 206887)		0.53	0.00	0.54	0.00	0.43	0.00

ID1	ID2	Hour17		Hour9		Hour2	
		CorrCoef	P_value	CorrCoef	P_value	CorrCoef	P_value
(106100, 453332)		0.07	0.28	0.06	0.32	0.11	0.06
(106100, 194543)		0.14	0.02	0.29	0.00	0.26	0.00
(106100, 168177)		0.15	0.02	0.29	0.00	0.28	0.00
(106100, 136212)		-0.14	0.02	-0.13	0.04	0.13	0.04
(106100, 461530)		-0.44	0.00	-0.42	0.00	-0.52	0.00
(106100, 475056)		-0.41	0.00	-0.47	0.00	0.24	0.00
(106100, 999999)		0.47	0.00	0.45	0.00	0.47	0.00
(599841, 715337)		0.07	0.23	0.13	0.03	0.25	0.00
(599841, 104351)		-0.28	0.00	0.02	0.79	0.31	0.00
(599841, 142494)		-0.20	0.00	0.06	0.36	0.23	0.00
(599841, 494629)		0.84	0.00	0.84	0.00	0.86	0.00
(599841, 918588)		-0.57	0.00	-0.55	0.00	-0.47	0.00
(599841, 102611)		0.30	0.00	0.25	0.00	0.36	0.00
(599841, 205249)		0.44	0.00	0.40	0.00	-0.12	0.05
(599841, 206887)		0.44	0.00	0.43	0.00	0.37	0.00
(599841, 453332)		0.11	0.08	0.12	0.04	0.20	0.00
(599841, 194543)		0.28	0.00	0.37	0.00	0.22	0.00
(599841, 168177)		0.28	0.00	0.36	0.00	0.24	0.00
(599841, 136212)		-0.18	0.00	-0.09	0.15	0.13	0.03
(599841, 461530)		-0.53	0.00	-0.52	0.00	-0.62	0.00
(599841, 475056)		-0.41	0.00	-0.53	0.00	0.25	0.00
(599841, 999999)		0.60	0.00	0.60	0.00	0.46	0.00
(715337, 104351)		-0.12	0.05	-0.15	0.02	0.21	0.00
(715337, 142494)		-0.07	0.28	-0.10	0.12	0.19	0.00
(715337, 494629)		0.08	0.19	0.19	0.00	0.25	0.00
(715337, 918588)		-0.16	0.01	-0.12	0.04	-0.10	0.11
(715337, 102611)		0.61	0.00	0.72	0.00	0.76	0.00
(715337, 205249)		-0.04	0.51	-0.04	0.52	0.07	0.23
(715337, 206887)		0.14	0.02	0.18	0.00	0.35	0.00
(715337, 453332)		0.75	0.00	0.75	0.00	0.64	0.00
(715337, 194543)		0.36	0.00	0.37	0.00	0.32	0.00
(715337, 168177)		0.37	0.00	0.38	0.00	0.25	0.00
(715337, 136212)		0.12	0.06	0.07	0.28	0.10	0.11
(715337, 461530)		0.00	0.97	-0.04	0.49	-0.31	0.00
(715337, 475056)		0.09	0.15	-0.06	0.29	0.06	0.33
(715337, 999999)		-0.03	0.58	0.27	0.00	0.27	0.00
(104351, 142494)		0.78	0.00	0.82	0.00	0.75	0.00
(104351, 494629)		-0.23	0.00	0.06	0.32	0.35	0.00
(104351, 918588)		0.31	0.00	0.00	0.99	-0.04	0.55
(104351, 102611)		-0.22	0.00	-0.08	0.17	0.30	0.00
(104351, 205249)		-0.18	0.00	0.16	0.01	-0.24	0.00
(104351, 206887)		-0.05	0.46	0.34	0.00	0.30	0.00
(104351, 453332)		-0.12	0.06	-0.16	0.01	0.01	0.82
(104351, 194543)		-0.21	0.00	0.05	0.45	0.30	0.00

ID1	ID2	Hour17		Hour9		Hour2	
		CorrCoef	P_value	CorrCoef	P_value	CorrCoef	P_value
(104351, 168177)		-0.21	0.00	0.05	0.45	0.32	0.00
(104351, 136212)		0.10	0.10	-0.09	0.14	-0.06	0.33
(104351, 461530)		0.23	0.00	-0.21	0.00	-0.43	0.00
(104351, 475056)		0.28	0.00	0.02	0.79	-0.02	0.78
(104351, 999999)		-0.25	0.00	-0.05	0.43	0.00	0.96
(142494, 494629)		-0.13	0.04	0.10	0.10	0.29	0.00
(142494, 918588)		0.29	0.00	0.03	0.63	0.10	0.10
(142494, 102611)		-0.18	0.00	-0.07	0.26	0.23	0.00
(142494, 205249)		-0.13	0.03	0.12	0.04	-0.19	0.00
(142494, 206887)		0.09	0.14	0.35	0.00	0.35	0.00
(142494, 453332)		-0.03	0.64	-0.09	0.14	0.12	0.06
(142494, 194543)		-0.11	0.06	0.03	0.59	0.26	0.00
(142494, 168177)		-0.11	0.06	0.06	0.34	0.30	0.00
(142494, 136212)		0.06	0.34	-0.13	0.04	-0.16	0.01
(142494, 461530)		0.10	0.10	-0.20	0.00	-0.23	0.00
(142494, 475056)		0.22	0.00	-0.06	0.35	-0.08	0.20
(142494, 999999)		-0.14	0.02	-0.03	0.65	-0.06	0.34
(494629, 918588)		-0.51	0.00	-0.52	0.00	-0.43	0.00
(494629, 102611)		0.21	0.00	0.22	0.00	0.37	0.00
(494629, 205249)		0.47	0.00	0.46	0.00	-0.13	0.03
(494629, 206887)		0.48	0.00	0.46	0.00	0.37	0.00
(494629, 453332)		0.11	0.08	0.15	0.01	0.22	0.00
(494629, 194543)		0.21	0.00	0.37	0.00	0.20	0.00
(494629, 168177)		0.21	0.00	0.36	0.00	0.21	0.00
(494629, 136212)		-0.29	0.00	-0.21	0.00	0.04	0.56
(494629, 461530)		-0.53	0.00	-0.53	0.00	-0.63	0.00
(494629, 475056)		-0.48	0.00	-0.58	0.00	0.18	0.00
(494629, 999999)		0.56	0.00	0.55	0.00	0.43	0.00
(918588, 102611)		-0.33	0.00	-0.20	0.00	-0.20	0.00
(918588, 205249)		-0.41	0.00	-0.31	0.00	-0.07	0.24
(918588, 206887)		-0.41	0.00	-0.28	0.00	-0.26	0.00
(918588, 453332)		-0.15	0.02	-0.11	0.08	-0.18	0.00
(918588, 194543)		-0.30	0.00	-0.33	0.00	-0.22	0.00
(918588, 168177)		-0.30	0.00	-0.34	0.00	-0.19	0.00
(918588, 136212)		0.13	0.04	0.12	0.05	-0.25	0.00
(918588, 461530)		0.43	0.00	0.43	0.00	0.43	0.00
(918588, 475056)		0.46	0.00	0.54	0.00	-0.24	0.00
(918588, 999999)		-0.44	0.00	-0.50	0.00	-0.49	0.00
(102611, 205249)		0.03	0.63	0.00	0.96	0.04	0.51
(102611, 206887)		0.19	0.00	0.24	0.00	0.40	0.00
(102611, 453332)		0.58	0.00	0.67	0.00	0.61	0.00
(102611, 194543)		0.43	0.00	0.50	0.00	0.38	0.00
(102611, 168177)		0.42	0.00	0.49	0.00	0.31	0.00
(102611, 136212)		0.08	0.16	0.14	0.02	0.15	0.02

ID1	ID2	Hour17		Hour9		Hour2	
		CorrCoef	P_value	CorrCoef	P_value	CorrCoef	P_value
(102611, 461530)		-0.15	0.02	-0.18	0.00	-0.35	0.00
(102611, 475056)		-0.02	0.71	-0.08	0.18	0.05	0.40
(102611, 999999)		0.10	0.11	0.40	0.00	0.37	0.00
(205249, 206887)		0.18	0.00	0.21	0.00	0.03	0.62
(205249, 453332)		-0.07	0.28	-0.05	0.45	0.10	0.09
(205249, 194543)		0.14	0.02	0.11	0.06	0.14	0.02
(205249, 168177)		0.12	0.04	0.12	0.04	0.02	0.71
(205249, 136212)		-0.36	0.00	-0.21	0.00	0.14	0.02
(205249, 461530)		-0.44	0.00	-0.39	0.00	0.22	0.00
(205249, 475056)		-0.47	0.00	-0.38	0.00	-0.05	0.43
(205249, 999999)		0.39	0.00	0.36	0.00	0.25	0.00
(206887, 453332)		0.21	0.00	0.22	0.00	0.30	0.00
(206887, 194543)		0.15	0.01	0.39	0.00	0.39	0.00
(206887, 168177)		0.17	0.01	0.40	0.00	0.38	0.00
(206887, 136212)		-0.07	0.27	-0.09	0.12	0.01	0.83
(206887, 461530)		-0.39	0.00	-0.36	0.00	-0.38	0.00
(206887, 475056)		-0.23	0.00	-0.31	0.00	0.07	0.24
(206887, 999999)		0.32	0.00	0.28	0.00	0.40	0.00
(453332, 194543)		0.30	0.00	0.40	0.00	0.24	0.00
(453332, 168177)		0.32	0.00	0.42	0.00	0.22	0.00
(453332, 136212)		0.04	0.50	0.02	0.74	-0.03	0.59
(453332, 461530)		-0.02	0.81	0.00	1.00	-0.20	0.00
(453332, 475056)		0.05	0.42	-0.08	0.19	0.00	0.96
(453332, 999999)		-0.09	0.14	0.26	0.00	0.28	0.00
(194543, 168177)		0.96	0.00	0.91	0.00	0.86	0.00
(194543, 136212)		0.08	0.19	0.11	0.07	0.08	0.20
(194543, 461530)		-0.26	0.00	-0.32	0.00	-0.24	0.00
(194543, 475056)		-0.05	0.43	-0.21	0.00	0.00	0.95
(194543, 999999)		0.32	0.00	0.43	0.00	0.32	0.00
(168177, 136212)		0.10	0.10	0.09	0.13	0.00	0.98
(168177, 461530)		-0.25	0.00	-0.33	0.00	-0.29	0.00
(168177, 475056)		-0.02	0.71	-0.21	0.00	0.00	0.96
(168177, 999999)		0.32	0.00	0.43	0.00	0.25	0.00
(136212, 461530)		0.26	0.00	0.16	0.01	-0.11	0.07
(136212, 475056)		0.53	0.00	0.45	0.00	0.42	0.00
(136212, 999999)		-0.07	0.28	0.07	0.26	0.29	0.00
(461530, 475056)		0.43	0.00	0.45	0.00	-0.19	0.00
(461530, 999999)		-0.54	0.00	-0.51	0.00	-0.40	0.00
(475056, 999999)		-0.33	0.00	-0.40	0.00	0.25	0.00

TABLE C.2  
Correlation Coefficient Between Bids and PALO, PATH26

ResID	Hour2				Hour9				Hour17			
	PALO		PATH26		PALO		PATH26		PALO		PATH26	
	R	P	R	P	R	P	R	P	R	P	R	P
199871	-0.17	0.01	0.06	0.11	-0.07	0.27	0.10	0.09	-0.08	0.21	0.00	0.27
192115	-0.20	0.00	0.89	-0.01	-0.25	0.00	0.14	0.02	-0.14	0.02	0.03	0.13
282606	-0.12	0.05	0.01	0.16	0.05	0.43	0.11	0.08	-0.01	0.94	0.58	0.03
108576	-0.14	0.02	0.07	0.11	-0.12	0.04	0.06	0.34	-0.19	0.00	0.00	0.18
106100	-0.11	0.06	0.69	0.02	-0.02	0.78	0.04	0.49	-0.01	0.86	0.74	0.02
599841	-0.13	0.03	0.05	0.12	0.00	0.98	0.04	0.47	0.02	0.77	0.70	-0.02
715337	-0.15	0.01	0.22	0.07	-0.24	0.00	0.11	0.06	-0.21	0.00	0.02	0.14
104351	0.02	0.69	0.00	0.17	0.04	0.55	0.30	0.00	-0.11	0.07	0.01	0.16
142494	-0.01	0.88	0.01	0.16	0.07	0.23	0.29	0.00	-0.06	0.29	0.00	0.23
494629	-0.09	0.13	0.07	0.11	0.03	0.61	0.06	0.32	0.05	0.41	0.77	0.02
918588	0.28	0.00	0.40	-0.05	0.04	0.49	0.00	0.98	0.08	0.19	0.57	-0.03
102611	-0.14	0.02	0.49	0.04	-0.17	0.01	0.11	0.07	-0.14	0.02	0.19	0.08
205249	-0.27	0.00	0.01	-0.15	0.11	0.06	-0.04	0.54	0.09	0.15	0.11	-0.10
206887	-0.33	0.00	0.00	0.19	-0.08	0.21	0.35	0.00	-0.06	0.33	0.00	0.35
453332	-0.26	0.00	0.23	0.07	-0.30	0.00	0.11	0.06	-0.15	0.01	0.02	0.15
194543	-0.33	0.00	0.25	0.07	-0.21	0.00	0.23	0.00	-0.24	0.00	0.03	0.13
168177	-0.25	0.00	0.19	0.08	-0.20	0.00	0.23	0.00	-0.22	0.00	0.01	0.16
136212	-0.12	0.05	0.46	0.05	-0.06	0.29	0.17	0.01	-0.09	0.15	0.00	0.27
461530	0.09	0.15	0.01	-0.17	-0.12	0.04	0.03	0.69	-0.14	0.02	0.75	0.02
475056	-0.15	0.01	0.67	0.03	0.03	0.59	0.03	0.66	-0.06	0.35	0.00	0.18
<b>Overall</b>	<b>-0.29</b>	<b>0.00</b>	<b>0.33</b>	<b>0.06</b>	<b>-0.02</b>	<b>0.73</b>	<b>-0.01</b>	<b>0.94</b>	<b>0.04</b>	<b>0.55</b>	<b>0.80</b>	<b>0.02</b>

TABLE C.3  
Correlation Coefficient Between same resource ID while different hours

	Hour (2,9)		(2,17)		(9,17)	
	Corr_Coef	P-value	Corr_Coef	P-value	Corr_Coef	P-value
199871	0.79	0.00	0.54	0.00	0.70	0.00
192115	0.58	0.00	0.41	0.00	0.51	0.00
282606	0.67	0.00	0.34	0.00	0.46	0.00
108576	0.77	0.00	0.42	0.00	0.62	0.00
106100	0.89	0.00	0.84	0.00	0.94	0.00
599841	0.81	0.00	0.78	0.00	0.92	0.00
715337	0.38	0.00	0.15	0.02	0.46	0.00
104351	0.75	0.00	0.29	0.00	0.46	0.00
142494	0.86	0.00	0.41	0.00	0.49	0.00
494629	0.80	0.00	0.79	0.00	0.93	0.00
918588	0.88	0.00	0.83	0.00	0.82	0.00
102611	0.42	0.00	0.15	0.01	0.49	0.00

	Hour (2,9)		(2,17)		(9,17)		
	Corr_Coef	P-value	Corr_Coef	P-value	Corr_Coef	P-value	
205249	0.08		0.19	0.06	0.35	0.78	0.00
206887	0.96		0.00	0.94	0.00	0.96	0.00
453332	0.50		0.00	0.36	0.00	0.55	0.00
194543	0.50		0.00	0.30	0.00	0.56	0.00
168177	0.44		0.00	0.22	0.00	0.56	0.00
136212	0.27		0.00	0.30	0.00	0.79	0.00
461530	0.87		0.00	0.82	0.00	0.93	0.00
475056	-0.15		0.01	-0.10	<b>0.09</b>	0.67	0.00
<b>Overall</b>	<b>0.67</b>		<b>0.00</b>	<b>0.56</b>	<b>0.00</b>	<b>0.79</b>	<b>0.00</b>

TABLE C.4  
Bids Linear Regression Result including

	Hour	Gas	TotalLoad	COI	15 PALO	26 dummy		
136212	2	115.96	-0.01	-3.18	-0.57	-1.55	-0.14	-157.90
	9	176.76	-0.01	-1.44	-1.36	-0.80	1.09	-468.65
	17	173.47	-0.02	-1.65	-1.12	-0.77	1.50	-459.29
461530	2	44.34	0.01	-0.22	0.08	0.26	0.02	-365.76
	9	34.11	0.01	-0.34	-0.14	-0.14	-0.05	-221.84
	17	32.19	0.00	-0.45	-0.08	-0.15	-0.14	-125.17
475056	2	-9.03	0.00	-0.25	-0.02	-0.15	-0.05	38.80
	9	44.87	-0.01	0.10	0.08	0.14	0.05	-27.94
	17	32.01	0.00	-0.30	0.11	0.09	0.37	-104.04
<b>Overall</b>	<b>2</b>	<b>-30.65</b>	<b>0.02</b>	<b>0.12</b>	<b>-0.54</b>	<b>-0.49</b>	<b>-0.29</b>	<b>-51.05</b>
	<b>9</b>	<b>-33.36</b>	<b>0.00</b>	<b>-0.23</b>	<b>-0.49</b>	<b>-0.17</b>	<b>0.03</b>	<b>296.20</b>
	<b>17</b>	<b>-34.69</b>	<b>-0.01</b>	<b>-0.32</b>	<b>-0.32</b>	<b>0.10</b>	<b>0.15</b>	<b>337.22</b>

TABLE C.5  
Bids Linear Regression Result after grouping

	Hour	Gas	TotalLoad	COI	15 PALO	26 dummy			
<b>Group1</b>	136212	2	173.45	-0.06	-3.53	-1.30	-1.91	0.22	780.85
		9	263.90	-0.08	-2.15	-2.31	-0.61	3.41	588.38
		17	259.00	-0.07	-2.22	-2.04	-0.61	3.82	492.11
461530	2	44.31	0.00	-0.20	0.17	0.15	-0.25	-145.16	
	9	24.18	0.01	-0.50	-0.32	-0.04	-0.88	-212.91	
	17	27.13	0.01	-0.43	-0.34	-0.05	-0.73	-129.62	
475056	2	-1.72	0.00	-0.33	-0.06	-0.05	0.18	156.40	
	9	50.95	-0.01	0.17	0.02	0.11	0.14	73.76	
	17	38.98	-0.01	-0.04	0.03	0.10	0.13	131.67	
<b>Overall</b>	<b>2</b>	<b>-15.50</b>	<b>0.01</b>	<b>-0.02</b>	<b>-0.52</b>	<b>-0.45</b>	<b>0.40</b>	<b>128.39</b>	
	<b>9</b>	<b>-7.71</b>	<b>-0.01</b>	<b>-0.40</b>	<b>-0.65</b>	<b>-0.20</b>	<b>0.66</b>	<b>319.36</b>	
	<b>17</b>	<b>-18.25</b>	<b>-0.01</b>	<b>-0.51</b>	<b>-0.59</b>	<b>0.15</b>	<b>0.67</b>	<b>459.20</b>	
<b>Group2</b>	136212	2	59.43	0.01	1.59	-0.32	0.00	-1.62	-479.44
		9	54.39	0.02	0.63	0.05	-3.13	0.48	-709.11

	Hour	Gas	TotalLoad COI		15 PALO		26 dummy		
Group3	461530	17	48.04	0.02	0.23	0.08	-2.30	0.53	-748.39
		2	38.54	0.00	-1.71	0.79	0.21	-0.67	-102.60
		9	52.96	-0.01	-0.85	0.53	-1.84	-0.36	159.99
	475056	17	37.14	-0.01	-1.02	0.85	-8.36	-0.16	53.93
		2	-6.37	0.01	0.37	-0.05	-0.11	-0.82	-215.77
		9	39.68	0.00	-0.06	0.06	3.59	-0.28	-212.61
	Overall	17	22.40	0.02	-0.97	0.31	-8.63	0.08	-481.62
		2	<b>-31.26</b>	<b>0.02</b>	<b>2.20</b>	<b>-0.59</b>	<b>-0.28</b>	<b>-1.99</b>	<b>-255.16</b>
		9	<b>-58.01</b>	<b>0.02</b>	<b>0.44</b>	<b>-0.52</b>	<b>-4.96</b>	<b>0.33</b>	<b>-178.27</b>
Group3	136212	17	<b>-60.36</b>	<b>0.00</b>	<b>-0.15</b>	<b>-0.02</b>	<b>-3.98</b>	<b>-0.14</b>	<b>385.06</b>
		2	-41.24	0.00	0.52	-0.53	1.13	142.47	
		9	5.78	0.02	-0.04	0.09	-1.01	-0.29	-405.64
	461530	17	24.75	0.01	-1.14	0.35	-2.10	0.13	-320.01
		2	194.41	0.04	-0.88	1.16	-2.62	-1848.76	
		9	70.92	0.04	0.48	0.32	-0.33	1.52	-1352.40
	475056	17	69.16	0.02	-0.24	0.33	-1.00	0.81	-900.90
		2	-44.62	0.01	0.21	-0.86	0.28	145.15	
		9	17.90	0.00	-0.10	0.34	3.85	0.27	-128.81
Overall	17	43.06	0.01	-0.75	0.35	0.39	0.61	-443.40	
	2	<b>-200.11</b>	<b>0.03</b>	<b>-0.39</b>	<b>-0.73</b>	<b>3.44</b>	<b>422.09</b>		
	9	<b>-154.62</b>	<b>0.01</b>	<b>0.53</b>	<b>0.45</b>	<b>-0.53</b>	<b>-0.14</b>	<b>441.24</b>	
		17	<b>-73.35</b>	<b>0.00</b>	<b>1.01</b>	<b>0.34</b>	<b>0.00</b>	<b>0.08</b>	<b>470.62</b>

TABLE C.6  
Price linear regression with only gas and load

	Gas	TotalLoad	dummy	R2	
	2	28.61	0.00	-48.46	0.64
	9	27.57	0.00	-109.87	0.51
	17	12.44	0.03	-738.54	0.35

TABLE C.7  
Linear regression result of price for only with gas & load, cong

	Gas	TotalLoad COI		15 PALO		26 dummy	R2		
	2	18.09	0.00	-0.17	0.28	0.11	-0.08	-28.96	0.72
	9	22.35	0.00	-0.09	0.23	0.06	0.02	-104.40	0.53
	17	8.57	0.03	-0.20	0.17	-0.03	0.21	-707.90	0.36



TABLE C.8

Linear regression result of price only with gas &amp; load, all Bids

Hr	199871	192115	282606	108576	106100	599841	715337	104351	142494	494629	918588	102611	102611
2	16.07	0.00	-0.01	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01
9	10.91	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.02	0.01	0.00	0.02	0.02	-0.02
17	24.91	0.02	0.01	0.01	-0.01	0.01	-0.04	0.04	0.03	0.12	0.13	0.13	0.08
Hr	205249	206887	453332	194543	168177	136212	461530	475056	Overall gas	TLd	dummy		
2	0.00	0.01	-0.02	-0.02	0.01	0.00	0.00	-0.01	0.02	-0.04	-0.05	-25.67	
9	0.00	-0.01	-0.02	-0.02	-0.01	0.02	0.00	-0.01	0.05	-0.04	0.00	-71.04	
17	0.03	-0.01	-0.07	-0.10	0.00	-0.10	0.10	0.03	0.10	0.02	-0.02	-600.11	

TABLE C.9

Linear regression result of price with all information

Hr	199871	192115	282606	108576	106100	599841	715337	104351	142494	494629	918588	102611	205249	dummy
2	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	-14.37
9	0.00	0.00	-0.01	-0.01	-0.01	-0.02	0.01	0.00	0.01	-0.02	0.00	-0.02	-0.02	-70.95
17	0.01	0.02	-0.01	0.02	-0.05	0.05	0.03	0.12	0.13	0.09	0.03	-0.02	-0.07	-606.69
	2	206887	453332	194543	168177	136212	461530	475056	Overall gas	TILd	COI	Path15	PALO	Path26
9	-0.02	0.00	0.00	0.00	-0.01	0.02	-0.03	-0.04	12.06	0.00	-0.16	0.20	0.05	-0.09
17	-0.02	-0.01	0.02	0.00	-0.01	0.05	-0.04	0.01	8.10	0.01	-0.07	0.19	0.09	0.03
	-0.09	-0.01	-0.10	0.11	0.04	0.11	0.00	-0.03	17.90	0.02	-0.03	0.35	0.16	-0.05

TABLE C.10

Linear regression result of price with only 2 bids

	475056	Overall	gas	TotalLoad	COI	15	PALO	26	dummy
2	-0.04	-0.07	15.75	0.00	-0.17	0.24	0.07	-0.11	-30.87
9	-0.01	-0.06	20.76	0.00	-0.10	0.20	0.05	0.02	-87.51
17	0.01	0.10	11.63	0.03	-0.16	0.20	-0.04	0.19	-740.28

TABLE C.11

Linear regression result of price with only 2 bids by grouping

Group	Hour	475056	Overall	gas	TotalLoad	COI	15	PALO	26	dummy
	2	0.01	-0.06	18.39	0.00	-0.04	0.13	0.10	0.01	-26.74
	9	0.05	-0.05	18.51	0.00	0.04	0.05	0.08	0.01	-87.58
1	17	0.01	-0.03	24.72	0.00	-0.08	0.04	0.03	-0.04	-117.75
	2	-0.11	-0.13	14.32	0.00	-0.54	0.41	-0.14	-0.75	-90.06
	9	-0.03	-0.16	13.87	-0.01	-0.52	0.48	0.38	0.08	195.65
2	17	0.45	-0.15	4.42	0.01	-0.24	-0.21	-1.87	-0.09	-207.40
	2	-0.13	0.00	23.44	0.00	n/a	0.28	0.41	-0.40	-93.05
	9	-0.27	0.09	26.53	0.00	-0.07	0.67	1.36	0.25	-52.28
3	17	-0.66	0.76	-7.02	0.06	-1.47	1.54	-3.91	0.94	-1383.47

TABLE C.12

Linear regression result of price with only 2 bids by grouping with peak load

Group	Hr	475056	Overall	gas	TLd	PeakLoad	COI	15	PALO	26	dummy
	2	0.01	-0.06	18.35	0.00	0.00	-0.04	0.13	0.10	0.00	-26.27
	9	0.05	-0.05	18.52	0.00	0.00	0.04	0.05	0.08	0.01	-87.57
1	17	0.01	-0.03	24.74	0.00	0.01	-0.08	0.03	0.02	-0.01	-120.70
	2	-0.11	-0.13	14.71	0.01	0.00	-0.55	0.41	-0.15	-0.74	-97.87
	9	-0.09	-0.13	20.69	0.01	-0.01	-0.51	0.44	-0.64	0.02	37.69
2	17	0.39	-0.10	11.73	0.03	-0.01	-0.27	-0.26	-3.59	-0.11	-408.78
	2	-0.09	-0.03	19.20	0.02	-0.01		0.30	0.20	-0.51	-97.32
	9	-0.20	0.01	18.33	0.04	-0.04	0.16	0.52	-0.18	0.03	-65.17
3	17	-0.80	-0.08	-39.06	0.23	-0.14	0.00	1.18	-8.82	0.32	-1283.74