PHASE SPACE RECONSTRUCTION: METHODS IN APPLIED ECONOMICS AND ECONOMETRICS

By

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The members of the Committee appointed to examine the dissertation of MICHAEL PAUL MCCULLOUGH find it satisfactory and recommend that it be accepted.

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PHASE SPACE RECONSTRUCTION: METHODS IN APPLIED ECONOMICS AND ECONOMETRICS

Abstract

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Market responses to unpredictable events such as preference change, food contamination, or changes in technology and information are not always known. Phase space reconstruction, a tool designed to analyze nonlinear time series, is investigated for use as an econometric tool to detect nonlinear dynamics economic time series. It is applied to examine consumer responses to unpredictable events, changes in dynamic livestock cycles, and nonlinear structure in regression residuals. The empirical application of phase space reconstruction analyzing economic behavior demonstrates an intuitive, appealing, and straightforward demonstration as to the use of this diagnostic tool.

The first essay investigates how to reconstruct dynamic consumer reactions from market events using phase space reconstruction. This approach can provide important and unique empirical insights into consumer reactions to product recall or contaminant events. We apply phase space reconstruction analysis to U.S. meat demand, demonstrating distinct differences between intertemporal shorter run impacts from food safety incidents (e.g., E. Coli and BSE) relative to longer run health effects (e.g.,

cholesterol). Moreover, we show that consumers have reacted to food safety events differently depending on the particular food contaminate associated with that event.

In the second essay, phase space reconstruction is investigated as a diagnostic tool for determining the structure of detected nonlinear processes in regression residuals. Empirical evidence supporting this approach is provided using simulations from an Ikeda mapping and the S&P 500. Results in the form of phase portraits (e.g., scatter plots of reconstructed dynamical systems) provide qualitative information to discern structural components from apparent randomness and provide insights categorizing structural components into functional classes to enhance econometric/time series modeling efforts.

The third essay applies the technique of phase space reconstruction to investigate U.S. livestock cycles. Results are presented for both pork and cattle cycles, providing empirical evidence that the cycles themselves have slowly diminished. By comparing the two livestock cycles important insights can be made. The phase space analysis suggests that the biological constraint has become a less significant factor in livestock cycles while technology and information are relatively more significant.

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Dedication

To my beloved parents; whose support and generosity

made this possible.

And to my new wife with my deepest admiration.

INTRODUCTION

The following are three essays concerning the integration of a nonlinear tool developed in the physics literature to economic analysis. Each essay has its own set of questions addressed in a different manner but all incorporate the tool phase space reconstruction. This dissertation was influenced greatly on the need for a flexible, unassuming tool that is able to distinguish dynamic properties in a system.

The evidence presented in previous research concerning the differences between consumer reactions to health and food safety concerns was the initial question of interest. After applying a phase space analysis to that particular series in the first essay, Reconstructing Consumer Reactions to Health and Food Safety Concerns, the usefulness of phase space reconstruction became evident. The analysis confirms previous research techniques, and applies phase space reconstruction to demonstrate that health events result in more permanent consumer behavior changes than food safety concerns. In addition, the phase space analysis shows that generally the attributes of food safety contaminates are of particular importance when determining the duration and magnitude of consumer reactions to food safety events.

The second essay, Nonlinear Structure in Regression Residuals, expands on the technical questions addressed in the first essay. The realization that the tool was inefficiently estimated in the physics literature came from current work in Information and Entropy Economics where information theory is being used in a diagnostic framework. This essay investigates phase space reconstruction as a diagnostic tool for determining the structure of detected nonlinear processes in regression residuals. Outcomes of phase space reconstruction can be used to create phase portraits, providing qualitative information to discern structural components from apparent randomness in regression residuals and providing insights into categorizing structural components of regression residuals into functional classes.

As a compliment to the first essay the third, Have Livestock Cycles Diminished Over Time?, uses the technique of phase space reconstruction is applied to empirically investigate hog and cattle cycles in the U.S. The central idea is to reconstruct cattle and hog cycles to establish further evidence in evolving patterns of cycle length, magnitude, and volatility. Recent livestock and business cycle literature is then drawn on to provide plausible explanations for recent changes in livestock cycles.

This dissertation adds an innovative tool to economic analysis. The three essays address important questions individually and do so in a way that is both unique and interesting. It is the author's belief that by incorporating an interdisciplinary tool to the field of economics strengthens the discipline as a whole.

ESSAY ONE:

RECONSTRUCTING CONSUMER REACTIONS TO HEALTH AND FOOD SAFETY CONCERNS

Abstract:

We investigate how to reconstruct dynamic consumer reactions from market events using phase space reconstruction, which has been developed to analyze nonlinear dynamical systems. This approach can provide important and unique empirical insights into consumer reactions to product recall or contaminant events. We apply phase space reconstruction analysis to U.S. meat demand, demonstrating distinct differences between intertemporal shorter run impacts from food safety incidents (e.g., E. Coli and BSE) relative to longer run health effects (e.g., cholesterol). Moreover, we show that consumers have reacted to food safety events differently depending on the particular food contaminate associated with that event.

Key Words: nonlinear time series, phase space reconstruction, food safety, health effects

JEL Classification: D12, C14

Introduction

It is well known that the consumption of a food product such as beef follows a regular seasonal pattern and is sensitive to health and food safety events (Kinnucan *et al*, 1997; Piggott and Marsh, 2004; Marsh *et al*, 2004; Mazzocchi, 2006; USDA, 2006; Zhen and Wohlgenant, 2006a). However, when impacted by health and food safety concerns, characteristics of consumer reactions (i.e., the duration and magnitude of the deviations from persistent consumption patterns) are generally unknown. For instance, during the isolated Bovine Spongiform Encephalopathy (BSE) incident in the state of Washington in 2003 a rash of newspaper articles speculating on consumer reactions were published. Soon after, an article in the December 31, 2003 issue of Agriculture Online stated, "The impact of the news on consumer prices remains a giant question mark." This illustrates the uncertainty about consumer reactions to the BSE event. The uncertainty of consumer demand or confidence trickled down the supply chain from the retailer through the packing and processing sectors to the feeders and producers.

There is evidence that consumer reactions to health events, those events that deal with the dissemination of information concerning attributes of a good that have an effect on consumer health, persist for a much longer period of time than reactions to food safety events, those events that pertain to contaminate outbreaks in a particular consumption good, which affect the public's perceived quality of that good e.g. Escherichia Coli (E. coli), salmonella, and BSE. Kinnucan et al (1997) estimated the presence of a structural change in beef consumption due to the release of information in the late eighties on the link between heart disease and cholesterol in red meat. On the other hand, consumer reactions to food safety events in beef have been shown to be small and short-lived

(Piggott and Marsh, 2004; USDA, 2006). Confirming previous research techniques, this paper applies nonlinear analysis to demonstrate that health events result in more permanent consumer behavior changes. In addition, we show that care should be taken controlling for these impacts when modeling consumer reactions to subsequent food scares.

There has only been limited research as to how consumers react during food scares to different contaminate outbreaks (Piggott and Marsh, 2004; Mazzocchi, 2006). In particular, outbreaks of E. coli and isolated incidents of BSE have been studied with little evidence differentiating their impacts on consumption (Piggott and Marsh, 2004; Resende-Filho and Buhr, 2007). These studies have been limited in scope due to restrictions on economic and econometric modeling assumptions; therefore a proper overall delineation of the dynamic consumer reactions to different food safety events has yet to be performed. We show that consumer reactions to E. Coli and BSE events are in fact different. We show that generally the attributes of contaminates are of particular importance when determining the duration and magnitude of consumer reactions to food safety events.

Studies attempting to investigate the impacts of health (e.g., Kinnucan *et al*, 1997) and food safety events (e.g., Piggott and Marsh, 2004; Mazzocchi, 2006) have provided important empirical results using a wide variety of econometric tools. However, the standard techniques used have been limited in their capability to identify the differences between particular food safety and health events. Zhen and Wohlgenant (2006b) conceptualized a model based upon Becker's theory of rational addiction, providing evidence that beef consumption in the United States is persistent and that consumers may be grouped by their persistence. They argued that the more myopic the consumer the less responsive they are to food scares and health effects. Much like Kinnucan et al (1997), Zhen and Wohlgenant also show that when the incident that affects perceived quality of a good is permanent, i.e., health effects, consumption will gradually decrease to a new equilibrium at a lower level. Mazzocchi created a dynamic almost ideal demand system to determine the time-varying reactions consumers have to outbreaks. He showed that the inclusion of autoregressive parameters as consumer reactions provided decent short term forecast ability when estimating the impact of food scares (Mazzocchi, 2006). If, however, a nonlinear process generates the reaction to a food scare or health effect the autoregressive parameter will fail to correctly structure consumer behavior. Piggott and Marsh used a general almost ideal demand system to examine the impacts of public food safety information on US meat demand. They found the 1993 E. coli and non-domestic BSE food safety incidents to have small and short-lived, but statistically significant impacts on meat demand.[1] However, the impacts of food safety information about different events could not be distinguished from one another. This previous empirical evidence has also suggested that impacts from food safety outbreaks are short run and structurally different from health effects that tend to be long run. Inherently, the theoretical basis for these approaches has been static in nature that is then estimated by an empirical model augmented with dynamic components, which may or may not fully capture the dynamic impacts present in beef consumption.

In response to such pitfalls, novel techniques that are inherently dynamic provide alternative means to examine time series data. Phase space reconstruction is one such

[1] General AIDS models such as these are very useful in determining short term adjustments in behavior, as both Mazzocchi and Piggott and Marsh did, but the models are fundamentally restricted to this type of analysis. They have difficulty delineating long run dynamics from short run dynamics.

technique that has been developed to analyze nonlinear dynamical systems. Phase space reconstruction, using nonparametric nonlinear time series techniques, allows for the extraction of an underlying structural system from a single observed time series that can clearly delineate qualitative dynamic impacts of health or food safety impacts on consumer demand. It is an innovative empirical tool that provides unique insights about an economic agent by identifying dimensions of the phase space in which an attractor lies that characterize economic behavior.[2] Chavas and Holt (1991, 1993) illustrate the usefulness of using nonlinear equations of motion to model market cycles on the supply side, using time series analysis and a simple model of nonlinear dynamics. They do not use phase space reconstruction nor examine the demand side of the market process.

Indeed, our results from a phase space reconstruction analysis exhibits nonlinear dynamics in consumer meat demand. It further demonstrates distinct differences among the intertemporal responses (i.e., health versus food safety effects). Through phase space reconstruction we are able to delineate the effects of different types of contaminate outbreaks on beef consumption.

The study proceeds in the following manner; an overview of nonlinear time series methods is presented followed by the theory and method for reconstructing phase space in the Theoretical Framework. An application of phase space reconstruction to United States beef consumption follows in the section Consumer Beef Demand with results and discussion. Consumer reactions to health effects and food safety are then delineated and compared to existing research with concluding remarks for future modeling purposes.

[2] Phase space reconstruction does not necessarily lead to a final definitive model in-of-itself, but rather can provide information to specify a more complete structural model.

Theoretical Framework

Since its development phase space reconstruction has become an essential part of nonlinear dynamics (Packard *et al*, 1980; Takens, 1980). It has been incorporated into various areas of research from Schaffer and Kot's SEIR model of epidemics to Zaldívar's forecasting of Venice water levels, phase space reconstruction is the qualitative benchmark for nonlinear analysis (Schaffer and Kot, 1985; Zaldívar *et al*, 2000). It allows for the basic properties of the system to be identified and subsequent qualitative analysis to be performed without imposing prior knowledge of a system[3]. This is analogous to nonparametric regression, which allows for the relationship of variables to be determined without imposing restrictions or prior functional form.

The nonlinear time series methods discussed in this paper are motivated and based on the theory of dynamical systems in phase space (Takens, 1980).[4] Dynamical systems are usually defined by a set of first-order ordinary differential equations (or discrete time analogs). Assuming the phase space is a finite-dimensional vector space R^m and a state is defined by a vector $\mathbf{X} \in R^m$, then the continuous (or discrete) time system governing the system is $\frac{d\mathbf{X}(t)}{dt} = \mathbf{f}\left[t, \mathbf{X}(t)\right]$ $\left(\text{or } \mathbf{X}_{n+1} = \mathbf{F}\left[\mathbf{X}_n\right]\right)$. The mathematical theory of ordinary differential (difference) equations ensures the existence and uniqueness of the trajectories, if certain conditions are met (Packard *et al*, 1980; Shone, 2002). For example, Becker and Murphy (1988) and Becker, Grossman, and Murphy (1994) derived continuous and discrete time systems to model rational addiction and empirically analyze

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[3] Phase space reconstruction has been widely used as a tool to detect chaos in market behavior much like those used in Chavas and Holt (1991). We make no assumptions as to the chaotic behavior of consumption nor are we trying to test for it. Rather, the primary goal of using phase space reconstruction in this analysis is to better understand the qualitative dynamics of consumer behavior.

[4] Typically phase space is defined as the space in which some geometric structure exists. In very general terms every trajectory of the structure of question may be represented as a coordinate in its particular phase space. For qualitative analysis we will always be referring to phase spaces of two and/or three dimensions.

cigarette addiction, respectively. Moreover, Chavas and Holt (1991) illustrate the usefulness of using nonlinear equations of motion to model market cycles on the supply side. The general idea of phase space reconstruction is that a single scalar time series may have sufficient information with which to reconstruct a dynamical system, much like a single stain of DNA contains sufficient information to reproduce an entire organism.

Data are often observed as a temporal sequence of scalar values. For any event, n outcomes are observed as a subset of the total population and are denoted by the time series vector $X_t = \left[x(t), x(t-1), \ldots, x(t-n)\right]'$. For future reference the τ^{th} lag of this vector will be referred to as $X_{t-\tau} = \left[x(t-\tau), x(t-1-\tau), \ldots, x(t-n-\tau)\right]'$. The challenge is to convert the sequence of scalar observations into state vectors and reproduce dynamics in a lower dimensional phase space. Then one can study the dynamics of the system by learning the dynamics of the phase space trajectories, which is particularly useful in complex systems (see examples in Kantz and Schreiber, 1997).[5] This reproduction makes it possible to qualitatively delineate short or long run behavioral processes that evolve over time and generally better understand the nature of the underlying dynamical system. This is the essence of phase space reconstruction, which we solve using a nonparametric approach called the method of delays discussed ahead.[6]

[5] Phase space reconstruction is a diffeomorphism that reproduces a time series on a plane that mirrors the phase portrait of the underlying system. A diffeomorphism is a smooth function, Φ, that maps one differential manifold, *M*, onto another, *N*, whose inverse, Φ', is also a smooth function that maps *N* onto *M*. This mapping preserves all geometric properties of the original figure. Much like a topographical map preserves all geometric properties of the earth.

[6] The phase space reconstruction algorithms used in this paper were written in the GAUSS programming language.

Embedding

The idea of embedding attractors onto different spaces and in different dimensions is an important concept in the theory of dynamical systems.[7] It was not until Packard first proposed that this be done from measured time series that the idea of phase space reconstruction was formed (Packard *et al*, 1980). Packard proved that the embedding of the geometry of a strange attractor may be represented by a series of differential equations.[8] Takens (1980) extended this proof to encompass what is now known as the method of delays. The Method of Delays is a diffeomorphism of an attractor with dimension m onto a phase space of dimension n where $n \geq 2m+1$. For empirical application, the Method of Delays requires an optimal time lag τ be chosen followed by a minimum embedding dimension λ. Once the two parameters are estimated, the time series X_t will generate a reconstructed phase space matrix $Y_\lambda = \lfloor X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(\lambda-1)\tau} \rfloor$ with dimension $[(n-\lambda\tau)\times\lambda]$.[9]

The time lag τ is paramount to empirical applications of Takens' theorem. While the condition $n \geq 2m+1$ is sufficient, it is not necessary. By choosing a time lag that yields the highest independence between the column vectors in matrix Y_λ the geometry of the original manifold will be preserved even when the time series is contaminated with noise.

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[7] An embedding is the mapping process used to reproduce geometric figures onto different spaces. Again it is analogous to creating a two-dimensional map of the three-dimensional world. Not all embeddings are diffeomorphisms, just like not all maps contain all the properties of the area they cover. Nonetheless, even though road maps don't usually contain elevation gain they provide a great deal of information.

[8] An attractor is a subset of a space onto which a system evolves to over time. A strange attractor is an attractor that allows for a greater degree of flexibility in that the subset of the space may be fractal, i.e., the dimension of the space does not have to be a real integer.

[9] As discussed before, the process of phase space reconstruction is much like map making. The reconstruction is a map containing all geometric properties of the original system that drives the dynamics of the observed time series. Through Takens' embedding theorem it is possible to extract this map of the underlying dynamics from a single time series.

The time lag needs to be chosen optimally. If it is too small the approximation will be smooth but there will exist a high degree of correlation between components. This has the potential to force the trajectories of the attractor to lie on the diagonal in the embedding space (Broomhead and King, 1986). If, on the other hand, the time lag is chosen to be too large the dynamics of the system may unfold between components and therefore be unobserved. The optimal time lag is that which preserves the largest amount of information between components while achieving the largest degree of independence.

Time Lag for Embedding - Mutual Information

The mutual information coefficient was developed as an entropy measure of global dependencies between two random variables (Fraser and Swinney, 1986). The dependencies measured are both linear and nonlinear making it an ideal candidate for choosing an optimal time lag. In estimating the optimal time lag we are essentially asking the question: How dependent is X_t on $X_{t-\tau}$? To answer this question Fraser and Swinney defined dependence based upon conditional entropies and called it the mutual information function[10]

$$I(X_t, X_{t-\tau}) = H(X_t | X_{t-\tau}) = H(X_{t-\tau}, X_t) - H(X_{t-\tau})$$

where $H(X_t)$ is Shannon's entropy

$$H(X_t) = -\sum_t P_{x_t}(x(t)) \log P_{x_t}(x(t))$$

and

[10] When phase space reconstruction was first developed the autocorrelation function was used to find the optimal time lag. This measure of dependency is severely inferior for nonlinear analysis as it is strictly a measure of linear relation. Furthermore, the autocorrelation function hinges on estimating the sample moments of a time series. The mutual information function makes no assumptions about moments or the underlying distributions of time series; it is strictly observation dependent and therefore void of all misspecification and assumption errors.

$$H(X_{t-\tau},X_t) = -\sum_{t,t-\tau} P_{x_t x_{t-\tau}}(x(t),x(t-\tau))\log\left[P_{x_t x_{t-\tau}}(x(t),x(t-\tau))\right];$$

with $P_{x_t}(x(t))$ being the probability density of *x* occurring at time *t* (Shannon and Weaver, 1949).

The Mutual Information Function is defined as the combination of joint and marginal probabilities of the outcomes from an event in a sequence while increasing the time lag τ between components:

$$I(X_t,X_{t-\tau}) = \sum_{n-\tau}\sum_{n-\tau} P(X_t,X_{t-\tau})\log\left[P(X_t,X_{t-\tau})/P(X_t)P(X_{t-\tau})\right].$$

Based upon the standard definition of independence the $\log[\bullet]$ will equal zero, and thus the mutual information function as well, if the vectors are perfectly independent, and will tend to infinity as they become more and more dependent.

This measure is independent of the coordinates of *x* as the probability density functions are dimensionless. Since the mutual information function is based upon joint probability density functions it is a global measure of dependence and not a function of the individual time vectors (Fraser and Swinney, 1986). Choosing the time lag that yields the first local minimum of the mutual information function ensures independence of components with a maximum amount of new information (Fraser and Swinney, 1986).[11] The first minimum is chosen as the optimal time lag based on the optimality conditions

[11] There have been many entropy measures of dependence developed over the years in economics, see Granger et al (2004), and Maasoumi and Racine (2002). The main points stressed are usually to the extent which measures are a metric, so that comparisons may be made, and which are just measures of divergence. Phase space reconstruction only requires the independence of its coordinate vectors, which may be measured as either distance or divergence. It is out of the scope of this paper to compare measures of dependence so the mutual information function, with improvements, is used, as it is the norm in phase space reconstruction literature.

defined above so that it is neither too small nor too large, ensuring that the attractor unfolds correctly.[12]

Estimating the mutual information function hinges on estimating the probability density function of a time series and its lagged values. This has traditionally been done using histogram estimators that are perceived as the "most straightforward and widespread approach" (Dionísio *et al*, 2006). The histogram method of estimating density functions uniformly weights observations within a predetermined window. If the time series contains a large portion of observations located close together and some that are spread out, the histogram method will inconsistently estimate the probability density function. Algorithms have been developed that vary the window size based upon how close observations are located to each other but they are computationally intense and not easily programmed (Fraser and Swinney, 1986).

As an alternative approach we apply nonparametric estimation using kernel density approximations as a method for estimating the mutual information function.[13] In every instance the nonparametric approach took substantially less computational time. In addition to being less computationally burdensome, under appropriate conditions, the nonparametric method of estimating the mutual information function is also asymptotically efficient. By using kernel weights the possible inefficiencies encountered with the histogram method of estimating the mutual information function are minimized.

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[12] If the global minimum of the mutual information function were used as the optimal time lag the potential would be for the nonlinear system to have already completed a full cycle so that the estimate would include redundancy of the system as well. This could force the phase space reconstruction to no longer be monotonic, thus enveloping dynamical structure.

[13] Mittelhammer et al (2000) provides detailed information on nonparametric estimation and GAUSS coding examples. Simulations comparing the nonparametric and histogram methods (not reported here) demonstrated improved performance of nonparametric methods over the histogram approach in smaller sample situations. Improved performance measures included both in accuracy and precision for estimates of the time lag parameter and in computational time. Both methods converged to one another as the sample size increased as anticipated.

Embedding Dimension - False Nearest Neighbors

Given the choice of optimal time lag, the minimum embedding dimension λ can be estimated. Kennel et al (1992) developed the False Nearest Neighbors technique (discussed below) for choosing a minimum embedding dimension. Aittokallio et al (1999) suggested the embedding dimension must be chosen properly or the reconstruction may not reflect the original manifold. If λ is too small the reconstruction cannot unfold the geometry of the possible strange attractor.[14] If λ is too large procedures used to determine basic properties of the system and qualitative analysis may become unreliable (Aittokallio *et al*, 1999; Kennel *et al*, 1992).

The False Nearest Neighbors technique uses Euclidean distances to determine if the vectors of Y_λ are still "close" as the dimension of the phase space is increased. By calculating the Euclidean distance between Y_λ vectors before and after an increase in dimension, it is possible to determine if the vectors are actual nearest neighbors or "false" nearest neighbors. The test statistic developed by Kennel et al (1992) defining neighbors to be false is: $\frac{x(t+d)-x(n(t)+d)}{\|y_t-y_{n(t)}\|} > R_{tol}$ where $x(t+d)$ denotes the last coordinate in the t^{th} row of the phase space reconstruction matrix $Y_{\lambda+1}$, $n(t)$ denotes the nearest neighbor in Euclidean distance of t for each row vector y_t in matrix Y_λ, and R_{tol} is the desired tolerance level. When the percentage of false nearest neighbors is minimized or drops below a preset threshold for the entire system, the minimum embedding dimension for phase space reconstruction is found (Kennel *et al*, 1992).

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[14] An example of an embedding dimension being too small would be a 2-dimensional representation of a cube. In 2-dimensional space the cube appears to be a square. In 3-dimensional space the true geometry of the cube is clearly not a square but a much more complex figure.

The graphical false nearest neighbor test works in similar fashion; the test plots the density of false nearest neighbors in time delay space. Using the same notation as above $\left|x(t+d) - x(n(t)+d)\right|$ is plotted on the y-axis against $\frac{\|y_t - y_{n(t)}\|}{\sqrt{d}}$ on the x-axis. The first embedding dimension which yields the entire density of points contained below a 90° line is the minimum embedding dimension (Aittokallio *et al*, 1999).

Consumer Beef Demand

The empirical application is to reconstruct consumer demand for beef in the U.S. It is a useful application because it is intuitive, economically appealing, and straightforward, and at the same time interesting and important. Consumer reactions to recent health and food safety issues in beef and other meat products have received much attention in the popular press and academic literature with particular interest to policy makers and industry. Phase space reconstruction complements the available data and existing literature, as well as providing additional insights drawn out below.

Beef is a well-known staple in the American diet, and has been extensively studied (Chavas, 2000; Kinnucan *et al*, 1997; Mazzocchi, 2006; Patil *et al*, 2005; Piggott and Marsh, 2004; Zhen and Wohlgenant, 2006a; Zhen and Wohlgenant, 2006b). United States consumers eat roughly as much beef as they did forty years ago with consumption peaking in the 1970s. See Figure 1 for the time series graph of quarterly beef consumption per capita from 1960-2005. Table 1 reports descriptive statistics. Figure 1 illustrates the seasonal patterns and trends that have occurred throughout the history of beef consumption. Demand peaks in the summer months and is lowest in the winter. The average difference between the first and third quarter is 0.71, see Table 1. This difference increases slightly for the period after 1980 and then stays relatively consistent. In addition to the seasonal behavior, there appears to be an average level about which consumption has fluctuated since 1990.

We examine four events for three different quality concerns that stand out in the beef consumption literature. First is the reaction consumers had to the information regarding the negative health effects of cholesterol. The cholesterol health effect is commonly associated with the downward trend that consumption takes in the mid-eighties. The next two events can be attributed to food safety scares regarding E. coli outbreaks. The first E. coli outbreak took place in 1993 when several people became sick after consuming Jack-in-the-Box products. The second outbreak of E. coli occurred in the mid-west in 1997 and resulted in a large recall of beef. The last event is the result of BSE being detected in cattle in the Northwest in 2003. These events are also identified in Figure 1.

Empirical Issues

The empirical process progresses in several steps. First, the optimal lag is estimated and, second, the appropriate embedding dimension is determined. Third, the phase space reconstruction is completed and interpreted for U.S. beef demand. Fourth, further statistical tests are investigated to identify different health and food safety events.

The first minimum of the mutual information function determines the optimal time lag for the phase space reconstruction. The optimal time lag for beef consumption is estimated to be $\tau = 3$, as indicated in Table 2 where the mutual information function reaches its first local minimum. Interestingly, the autocorrelation function has a first minimum at lag two and is nonzero throughout the study period. At this time lag the mutual information function indicates that there is still some redundancy between the time series and its lagged vector. If used, the autocorrelation function would have resulted in a phase space reconstruction erroneously forced upon the 45° line. The difference in the mutual information and autocorrelation functions suggests evidence of nonlinear time series processes present in the data series.

The first local minimum of the mutual information function being at lag 3 demonstrates the intuitive aspect of phase space reconstruction. Beef consumption is seasonal so there would be a large amount of redundancy after thee lags. As such, the mutual information function reaches its first minimum at lag 3 and oscillates with increasing lags.

The graphical false nearest neighbors test is implemented, shown in Figure 2, to determine the minimum embedding dimension for the phase space reconstruction. Clearly the entire density of observations is contained below a line of degree less than 90 for the two-dimension case making the minimum embedding dimension $\lambda = 2$. Using the two parameter estimates we can create a graphical representation of the underlying dynamics that drive beef demand from 1960 to 2005, see Figure 3.

Before interpreting Figure 3, several general observations are in order relative to the underlying dynamical system. First, it is important to point out that linear deterministic equations of motion with constant coefficients can only have exponential or periodic solutions. Moreover, linear systems always need irregular inputs to produce bounded irregular signals (Kantz and Schreiber, 1997). Indeed, because linear models exhibit well-known analytical properties and are empirically tractable they have been applied to investigate dynamic economic behavior. Second, the reconstructed phase space in Figure 3 exhibits qualitatively different behavior relative to linear equations of motion. Nonlinear equations of motion can exhibit dynamic patterns that cannot be reconstructed from a system of linear equations. Third, the period that may most be reflective of linear equation of motions is from 1990 to 2005. In this period there appears to be periodic trajectories or trajectories that resemble a limit cycle.

Interpreting the phase space reconstruction becomes clearer by comparing the original series in Figure 1 and the reconstructed phase space in Figure 3. In Figure 3, the horizontal axis is the observed time series and the vertical axis is the time series lagged three periods. Beginning in 1960, per capita consumption was below 16 pounds per person per quarter, which appears in the lower left hand corner of Figure 3. As per capita consumption increased and peaked in the mid to late 1970s, the trajectory moved to the upper righted hand portion of the phase space. As per capita consumption began declining, the reconstructed trajectory began transitioning back towards the lower left hand part of the phase space during the 1980s. By the early 1990s the consumption series stabilized to seasonal pattern just under 17 pounds per person per quarter, reflecting a persistent cycle in the lower left part of phase space.

Seasonality of beef consumption is exhibited in the original time series and the reconstruction; people consistently eat more beef in the summer than in the winter. The reason, traditional American winter dishes consist mostly of beef's biggest substitutes poultry and pork, turkey for Thanksgiving; duck and ham for the holidays, while the traditional American summer dish is primarily barbequed beef. What is apparent from the reconstruction is that the difference between winter and summer consumption has been relatively stable over recent time, which is consistent with findings from previous studies. If, however, seasonality were the only persistent nonlinear process in beef consumption the phase space reconstruction would be contained solely on the 45° line. This indicates the complexity of consumer behavior and the need for flexible techniques when analyzing it.

The phase space reconstruction in Figure 3 shows the period from 1961-1980 was a time of transitions. In the early part of the series, the consumer increasingly incorporated beef as a part of their daily diet. Fast food restaurants such as McDonalds founded in the early fifties were beginning to takeoff. Then in the late seventies the price of beef began to increase dramatically. The consumer decreased the amount of beef she ate until a relatively stable (albeit brief) trajectory emerged in the early 1980s.

The main purpose of this application is to examine the subsequent period from 1980 to 2005, focusing on reported consumer reactions to the health concern cholesterol and food safety concerns Escherichia coli (E. coli) and bovine spongiform encephalopathy (BSE). Narrowing the analysis on the period from 1980-2005 allows for an easier delineation of the health and food safety effects on beef consumption.

Consumer Reactions to Health Effects

During the late eighties information was published on the negative effects cholesterol in beef has on health. Evidence suggested that health effects resulted in a decrease in U.S. beef consumption (Kinnucan *et al*, 1997). Figure 4 shows a subspace of the reconstruction from 1980 to 2005. The reconstruction shows a period of consumption in the early 1980s (the cyclical pattern in the upper right sector), a transition period (reflecting consumer reaction to cholesterol), and the set of more recent stable cycles at a lower level of consumption (in the lower left sector). Using a k-means clustering algorithm further delineates these patterns and transition periods.[15] The result of the cluster analysis places all the trajectory data prior to 1989 in one cluster and all the trajectory data after 1990 in another. The clustering results support the phase space reconstruction that the effects of negative information concerning health are statistically and economically significant. Consumers reacted to the information by decreasing their average level of consumption permanently while retaining a persistent seasonal pattern. The adjustment period illustrated in Figure 4 is consistent with a longer run behavioral response. This aligns with the empirical findings of Kinnucan et al (1997).

Modeling consumer reactions to health effects requires proper behavioral responses to be delineated. Using a trend to account for health information, cultural, or other variables that might affect lifestyle choices implies that consumer reactions are constantly changing. The phase space reconstruction shows that consumption has not been continually decreasing since the dissemination of the negative effects cholesterol has on health. Rather, an adjustment period occurred after which consumption returned to its regular trajectory at a permanently lower level. The two clusters in Figure 4 are almost identical in shape suggesting that consumer behavior, post health effects, is almost identical to consumer behavior before the health effect information was released. This suggests a long run shift in consumption levels that would be correctly modeled as a structural change or intercept shift opposed to a change in behavior, as a trend adjustment would imply. An incorrect specification of health effects in the 1980's with a downward trend in consumption will cause the underestimation of subsequent food safety effects.

[15] The algorithm chooses two clusters at random and then calculates the Euclidean distance from all to the cluster means. Each observation is then reclassified to the cluster that contains the mean it is closer to. The new clusters are defined and the process repeats until observations no longer change clusters (Johnson and Wichern, 2002).

Consumer Reactions to E. Coli and BSE

To examine consumer's reactions to more recent E. coli and BSE concerns the phase space trajectory is presented in Figure 5 from 1990 to 2005. As can be seen in the phase space reconstruction, the reactions from E. coli and BSE outbreaks are different for health concerns, and from one another. During an outbreak of E. coli or BSE, beef consumption was perturbed for a brief period of time until returning to the normal cyclical behavioral pattern.

The magnitude and duration of the outbreaks are delineated using Euclidean distances from the phase space mean and normal confidence ellipsoids. Observations from the phase space reconstruction are comprehensively tested under the null hypothesis of being normally distributed. First, the two coordinate vectors of the phase space reconstruction, $x(t)$ and $x(t-3)$, are tested for normality independently using the Shapiro-Wilk W test for normal data (Royston, 1983). The decision rule is to reject the null hypothesis of an underlying normal distribution if the test statistic, W, is greater than the critical level. For an $\alpha = 0.10$ confidence level in each case the null hypothesis cannot be rejected, see Table 3 for the test statistic and critical values. For the individual vectors testing normal, the next step is to test for joint normality. To do this we employ the methods described by Johnson and Wichern (2002). Constructing a Chi-Square plot of the squared generalized distances reveals that the joint probability is in fact bivariate normally distributed (the Chi-Square plot is analogous to the Normal Quantile Plot but for checking bivariate normality). Another check of bivariate normality is to check the percentage of observations that lie within the 50% confidence ellipsoid, $(x-\mu)'\Sigma^{-1}(x-\mu) \le \chi_2^2(.5)$, see Tables 4 and 5. With the normal distribution assumption verified outlying observations are those defined as lying on the largest confidence ellipsoids defined above so that the probability of that observation is very small; see Tables 4 and 5. The chi-square probability values reported in Tables 4 and 5 refer to the confidence ellipsoid that the trajectories lie on. Applying this outlier analysis to the phase space reconstruction makes it possible to differentiate the perturbation magnitude of E. coli and BSE events discussed ahead.

E. coli

Figure 6 includes the phase space trajectory from 1990 to 2005 and the 1993 E. Coli event but not the 1997 E. Coli nor the 2003 BSE event. The reconstruction of the first outbreak of E. coli in 1993, Figure 6, is noticeably larger than the second E. coli outbreak of 1997, Figure 7. This is also evident in the results reported in Tables 3 and 4 where the 1993 (1997) E. coli trajectory has a maximum chi-square probability of 0.865 (0.71). The corresponding Euclidean distance from the phase space mean is larger for 1993 than for 1997 as well. The difference between the two outbreak trajectories can be attributed to the fact that consumers may have become less sensitive to E. coli (as E. coli can be cooked out of beef, which will decrease the impact of the outbreak if the consumer takes the proper measures). Indeed, due largely to educational efforts of the United States government, consumers have become much more aware of the risks concerning food scares and the contaminate E. coli. One of the major findings of the Research Triangle Institute's evaluation of the 1996 *Pathogen Reduction; Hazard Analysis and Critical Control Point (PR/HACCP) Systems Final Rule* for the U.S. Department of Agriculture, Food Safety and Inspection Service was that consumers have become more knowledgeable of better food handling procedures and practice them often (RTI, 2002).

BSE

The 2003 BSE incident, Figure 8, resulted in a greater perturbation of consumption than those due to E. coli. The Euclidean distance from the cluster mean is larger for the 2003 BSE trajectory than the 1993 E. coli trajectory. In addition, the 2003 BSE trajectory lies outside the 95% confidence ellipse of it cluster while the 1993 E. coli trajectory does not. Unlike E. coli, BSE cannot simple be cooked out of beef. The uncertain nature of BSE appears to play an important role in consumer reactions. There are no preventions for being contaminated by BSE other than abstaining from eating contaminated beef product. As well as the lack of preventative measures, some evidence has been found for a causal link between BSE and Creutzfeldt-Jakob disease. BSE is shown to be present in the central nervous system and bone marrow of cattle, portions not normally consumed, while E. coli may be found in meat. Interestingly, the risk of becoming contaminated by BSE is much lower than E. coli. This gives evidence that people's behavior is affected more by the latent hazard of a potential longer run health impact.

Since the 2003 BSE incident in the state of Washington there have been two other confirmed cases of BSE in the United States, one in Texas in June of 2005 and another in Alabama in March of 2006. The phase space reconstruction shows that although consumers reacted in a much more sever manner during the 2003 BSE incident there is no discernable reaction to the 2005 incident. This illustrates the value of knowledge regarding contaminate attributes. In the same fashion that consumer response to the 1997 E. coli outbreak was significantly smaller than that towards the 1993 E. coli outbreak were there responses smaller to the latter BSE incidents. Consumers evidently have become less sensitive to BSE since its initial appearance in the United States. Effectively, in the same manner as E. coli, BSE has had no long-term effect on beef consumption at all.[16]

Further Discussion

The difference between the E. coli and BSE events is an important and interesting result. Piggott and Marsh (2004) estimating a demand system of beef, pork, and poultry found no significant difference between non-domestic BSE information and E. coli public food safety information as measured by the number of newspaper articles. However, the media index used by Piggott and Marsh (2004) did not include information from the 2003 BSE event in Canada or the US. Resende-Filho and Buhr (2007) modify the same model to identify the willingness to pay for a National Animal Identification System and are unclear as to the impact the 2003 BSE outbreak had on beef consumption. In addition, the marginal values tested in both these models were restricted to the individual commodity. The phase space reconstruction gives a basis for allowing variability within the marginal values of the individual contaminates. It is proof that a more flexible model needs to be developed in order to test the hypothesis of differing marginal effects from specific contaminates. In all, the nonlinear time series approach suggests different behavioral responses due to the BSE and E. Coli events, which should be taken into account when specifying theoretical models and recognized when making policy or industry recommendations.

The phase space reconstruction has provided further evidence concerning the consumption behavior of beef. The persistent phase space trajectories since 1990 qualifies previous research as to a habitual pattern of consumption (Piggott and Marsh,

[16] This is in spite of the nearly 1,000 articles found through a simple LexisNexis search on BSE printed since July 2005.

2002; Zhen and Wohlgenant, 2006b). It also provides evidence as to the importance of consumer knowledge of a good's attributes. If consumers had been more knowledgeable of the attributes of BSE contaminated beef, the 2003 BSE incident might possibly have resulted in a smaller or no perturbation from the mean phase space trajectory, as did the latter BSE incident in 2005. Quite possibly, once U.S. consumers learned that the probability of infection from BSE contaminated beef is real but very small, they quickly returned to normal behavior. Considering this possible behavioral response, and if in fact the smaller consumer response to E. coli in 1997 was due to the intervention of groups like the Partnership for Food Safety Education or USDA's risk communication strategies, it follows that consumer educational endeavors can effectively mitigate negative consumer responses to food safety concerns.

Concluding Remarks

The implementation of phase space reconstruction provides a novel approach to investigate dynamical systems that are pervasive in interesting and complex economic problems. The diagnostic tool provides a theoretically consistent approach based on the theory of dynamical systems to represent and reconstruct economic behavior. It offers additional nonparametric tools to enhance classification and diagnostics of nonlinear time series that complements existing methods for both the theoretical and empirical economist. It further opens a door for future research in nonlinear time series econometrics and provides important motivation by which to use empirical information to better specify theoretical models. Our empirical analysis provides relevant, interesting, and unique insights into the consumer demand for beef and to the impacts of health and food safety on beef demand. More generally, this approach could be used to study market reactions to recall events for alternative products.

The recent empirical evidence suggesting a different consumer response to health effects and food safety is exhibited in the phase space reconstruction. The long run health effect of cholesterol has caused consumers to shift their consumption behavior to a lower level while retaining a persistent seasonal pattern. This lower level contains some of the same behavioral dynamics present before the health information was released.

The effects of food safety information in the phase space reconstruction are shown as temporary adjustments from the consumer's phase space trajectory. These adjustments appear to be dependent on the particular contaminate. The impact of E. coli outbreaks in beef have lessened over time and may be attributed to what previous research has confirmed; consumers are more knowledgeable about the contaminate and are learning to better prepare their food. The evidence regarding the more dramatic reaction to the first BSE incident shows that consumers might be affected more by the latent hazard of a potential longer run health impact. The indiscernible reaction to the second BSE incident in 2005 confirms that the novelty of a contaminant in a society might possibly be a large determinant of consumer's reactions. For a predetermined consumption good such beef, outbreaks of contaminates have short run negative impacts. Consumers return to a steady trajectory when they believe the risk of contamination has decreased to a significant level.

This analysis has also provided a framework for better understanding consumer behavior in general. As shown, if knowledge of the consumption good, contaminate, or factor that affects perceived quality is relatively small, consumers will have a larger reaction to changing attributes. This suggests the need for further research into the effects that increased consumer knowledge of a particular contaminate (or positive attribute) pre-incident has on decreasing (increasing) the consumer response during the incident.

Tables

Descriptive Statistics	
Mean	18.62
Standard Deviation	2.14
Minimum	14.92
Maximum	24.38
Mean post Health Effects	16.61
Average Seasonal Adjustment	0.71
Average Seasonal Adjustment 1980 - 1990	1.01
Average Seasonal Adjustment 1990 - Present	0.96

Table 1: Descriptive statistics for United States beef consumption.

Lag	Mutual Information Function	Autocorrelation Function
1	12.832321	0.93398562
2	13.16868	0.88915847
3	10.675077	0.89276679
4	15.64411	0.89938676
5	9.3894612	0.84142258
6	11.614153	0.7980301
7	8.3321991	0.80056196
8	10.42145	0.79877481
9	7.0329578	0.73660526
10	9.6901346	0.68506057
11	5.9306914	0.68480521
12	8.5038975	0.67389864
13	6.0765665	0.61758272
14	7.9425017	0.57791756
15	4.7249772	0.59158978
16	6.7885208	0.59141034
17	4.987119	0.54004567
18	7.0196091	0.50942787
19	4.3248576	0.51685105
20	6.0548875	0.52364019
21	4.3643202	0.4736557
22	5.7147465	0.4465524
23	3.7226371	0.45503268
24	5.4606001	0.46283112
25	3.9729883	0.41268237
26	4.6753803	0.39274703
27	3.5112068	0.39660277
28	4.6453013	0.39379443
29	3.495097	0.34909124
30	4.5347639	0.32961898

Table 2: The Mutual Information Function and Autocorrelation Function for the United States Beef time series.

Shapiro-Wilk W test for normal data				
Variable	**W**	**V**	**z**	**Prob>z**
x(t)	0.96543	1.879	1.359	0.087
x(t-3)	0.96085	2.128	1.628	0.05175

Table 3: Shapiro-Wilk test for normality.

Year	Quarter	X(t)	X(t-3)	EDM[a]	Z-Score	Chi-Square Prob.
1991	**:1**	15.94566	17.36501	0.997686	3.1795338	0.796026848
1991	**:2**	16.99344	17.40165	0.884672	2.3553583	0.692007285
1991	**:3**	17.49375	16.40234	0.919458	2.7350217	0.745259742
1991	**:4**	16.14206	15.94566	0.805991	1.9608056	0.624840038
1992	**:1**	16.3169	16.99344	0.475318	0.71812132	0.301668011
1992	**:2**	16.86232	17.49375	0.922186	2.5603657	0.722013534
1992	**:3**	17.12036	16.14206	0.701380	1.5818642	0.546578023
1992	**:4**	15.80377	16.3169	0.846760	2.2471471	0.674884102
1993	**:1**	15.7764	16.86232	0.859502	2.3930159	0.697752164
1993	**:2**	16.07941	17.12036	0.727514	1.6981674	0.572193246
1993	**:3**	16.89766	15.80377	0.860295	2.3061907	0.684341817
1993	**:4**	15.80742	15.7764	1.149105	4.011897	0.865467369
1994	**:1**	16.09441	16.07941	0.731758	1.6269465	0.556684356
1994	**:2**	16.71562	16.89766	0.310530	0.29008778	0.135015672
1994	**:3**	17.18974	15.80742	0.997263	3.1616041	0.794190038
1994	**:4**	16.28458	16.09441	0.603624	1.0981133	0.42250566
1995	**:1**	16.14104	16.71562	0.469055	0.7117537	0.299441111
1995	**:2**	16.89846	17.18974	0.652727	1.2824296	0.47334774
1995	**:3**	17.40735	16.28458	0.872313	2.4664355	0.708646435
1995	**:4**	16.11935	16.14104	0.670327	1.367895	0.495378933
1996	**:1**	16.82493	16.89846	0.366784	0.40701662	0.184136575
1996	**:2**	17.40494	17.40735	1.134194	3.9152339	0.858805506
1996	**:3**	16.88762	16.11935	0.569895	1.0253005	0.401093773
1996	**:4**	16.00658	16.82493	0.629281	1.2833884	0.473600157
1997	**:1**	16.01267	17.40494	0.987066	3.0972393	0.787458847
1997	**:2**	16.99463	16.88762	0.483924	0.72030949	0.302431627
1997	**:3**	16.74025	16.00658	0.620157	1.1875078	0.447749703
1997	**:4**	15.9315	16.01267	0.895226	2.4459972	0.705653784
1998	**:1**	16.3621	16.99463	0.451069	0.64311621	0.274981496
1998	**:2**	16.85652	16.74025	0.289286	0.26009426	0.121945953
1998	**:3**	17.13265	15.9315	0.863951	2.3776729	0.695424553
1998	**:4**	16.32551	16.3621	0.368609	0.41450085	0.187183925

[a]Euclidean distance from the cluster mean

Table 4: Chi-Square test based on the Confidence Ellipsoid for the phase space reconstruction 1991-1998. The Chi-Square probability indicates which confidence ellipsoid each trajectory lies on.

Year	Quarter	X(t)	X(t-3)	EDM[a]	Z-Score	Chi-Square Prob.
1999	**:1**	16.33342	16.85652	0.361548	0.41996993	0.189403567
1999	**:2**	17.36981	17.13265	0.931913	2.674087	0.737379039
1999	**:3**	17.42001	16.32551	0.869826	2.451809	0.706507883
1999	**:4**	16.38007	16.33342	0.352362	0.37564564	0.171238464
2000	**:1**	16.69684	17.36981	0.765975	1.7742395	0.588159748
2000	**:2**	17.1003	17.42001	0.952866	2.7369851	0.745509698
2000	**:3**	17.54635	16.38007	0.975781	3.0810532	0.785731762
2000	**:4**	16.41529	16.69684	0.202291	0.13261837	0.064158528
2001	**:1**	16.07483	17.1003	0.716928	1.6510895	0.56200367
2001	**:2**	16.79001	17.54635	0.955593	2.754619	0.747743663
2001	**:3**	17.03063	16.41529	0.474427	0.72951923	0.305636454
2001	**:4**	16.39706	16.07483	0.571897	0.98399534	0.388596207
2002	**:1**	16.19647	16.79001	0.440034	0.62758717	0.269330164
2002	**:2**	17.50916	17.03063	1.003378	3.1368781	0.791629816
2002	**:3**	17.36452	16.39706	0.795511	2.0491237	0.641046293
2002	**:4**	16.63321	16.19647	0.415243	0.52716766	0.231706786
2003	**:1**	16.18452	17.50916	0.989511	3.0682214	0.784352619
2003	**:2**	16.90426	17.36452	0.814059	1.9935851	0.6309387
2003	**:3**	16.87853	16.63321	0.281369	0.25277519	0.118726796
2003	**:4**	14.95477	16.18452	1.697575	9.097842	0.989421387
2004	**:1**	15.96204	16.90426	0.700735	1.5914138	0.548737861
2004	**:2**	16.88647	16.87853	0.393883	0.47298947	0.210609961
2004	**:3**	16.9217	14.95477	1.686787	8.7615424	0.987484297
2004	**:4**	16.28722	15.96204	0.718883	1.5550338	0.540454295
2005	**:1**	15.63747	16.88647	0.999561	3.2356196	0.801667388
2005	**:2**	16.91096	16.9217	0.441471	0.5930573	0.25660567
2005	**:3**	17.00067	16.28722	0.516134	0.85859924	0.349035141
2005	**:4**	16.52987	15.63747	0.975146	2.8828018	0.763403921

[a]Euclidean distance from the cluster mean

Table 5: Chi-Square test based on the Confidence Ellipsoid for the phase space reconstruction 1999-2005. The Chi-Square probability indicates which confidence ellipsoid each trajectory lies on.

Figures

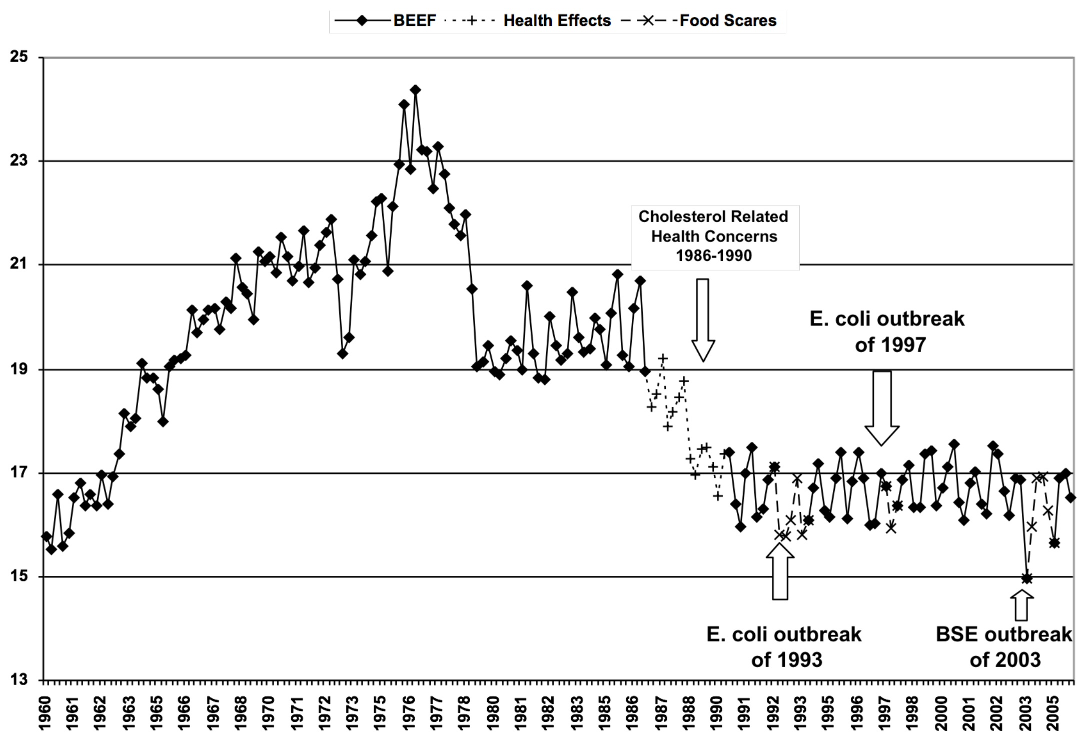

Figure 1: United States consumption of beef with indicated Health Effects and Food Scares; 1960-2005.

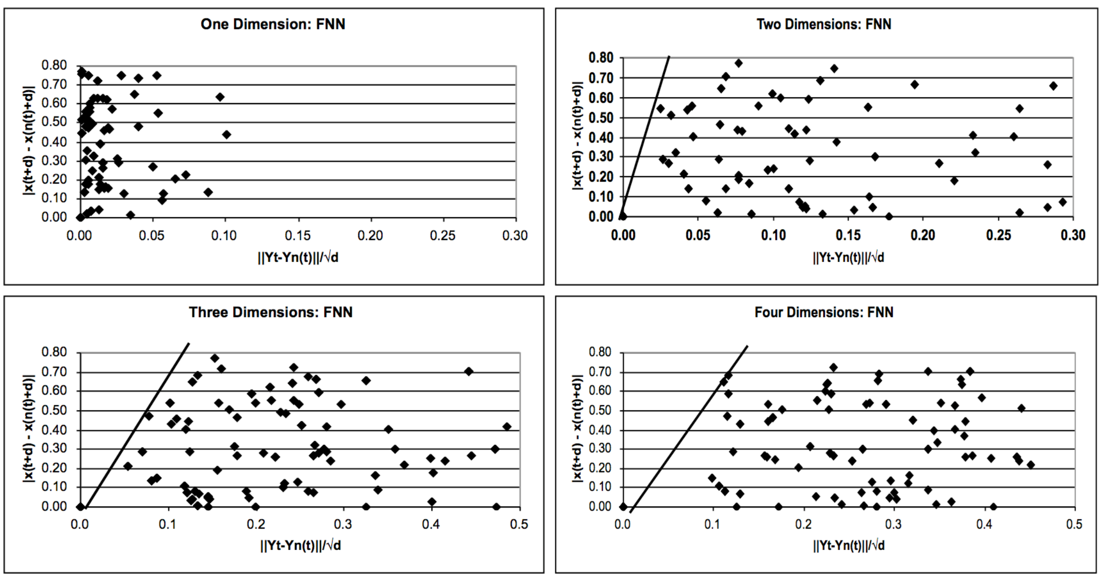

Figure 2: Graphical false nearest neighbors test for minimum embedding dimension indication a dimension of $\lambda = 2$.

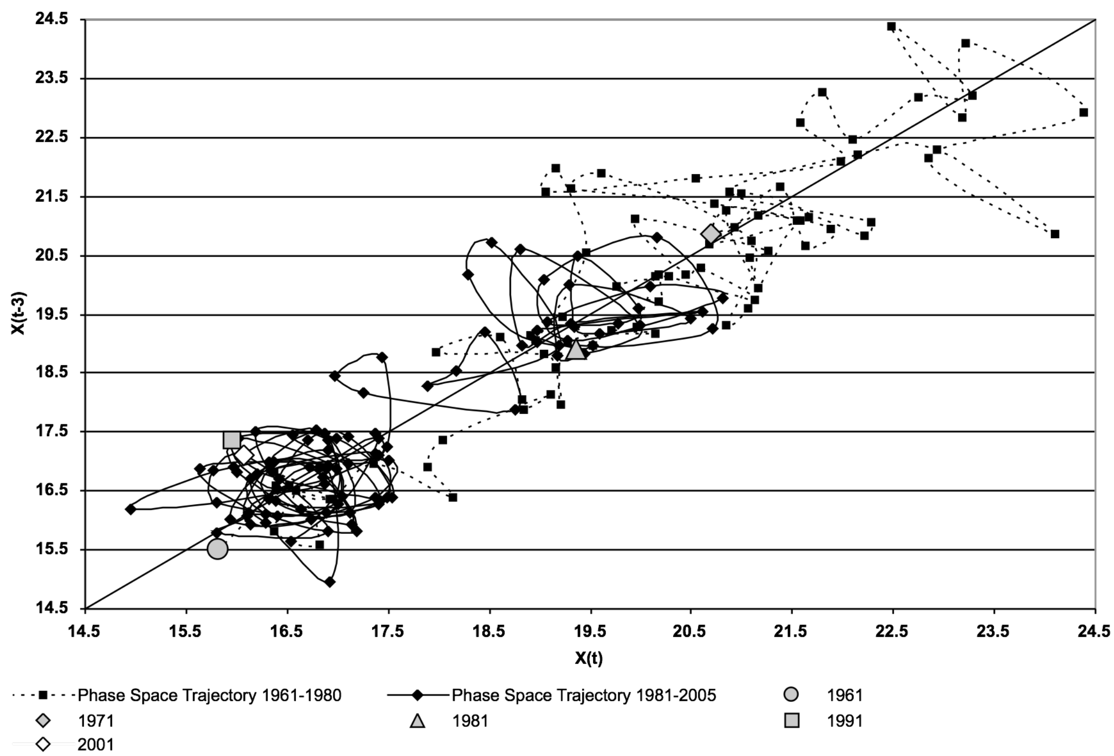

Figure 3: Reconstructed phase space for U.S. beef consumption; 1960-2005.

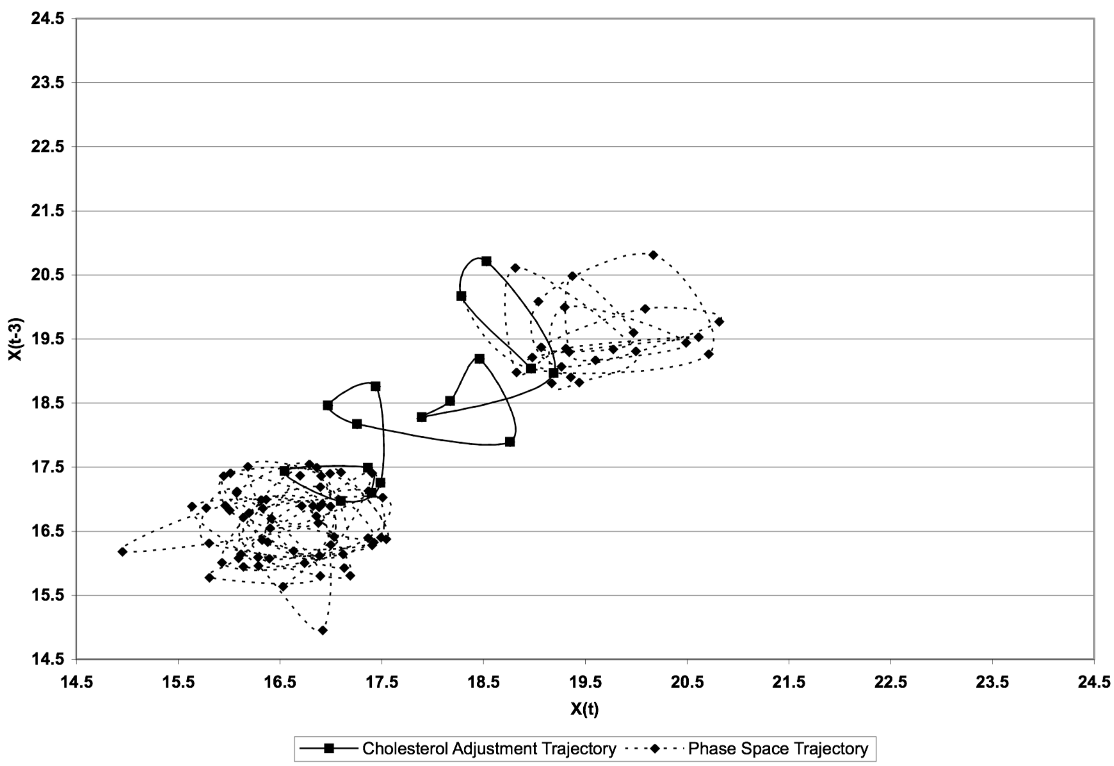

Figure 4: The effects of cholesterol on U.S. Beef consumption; 1980-2005.

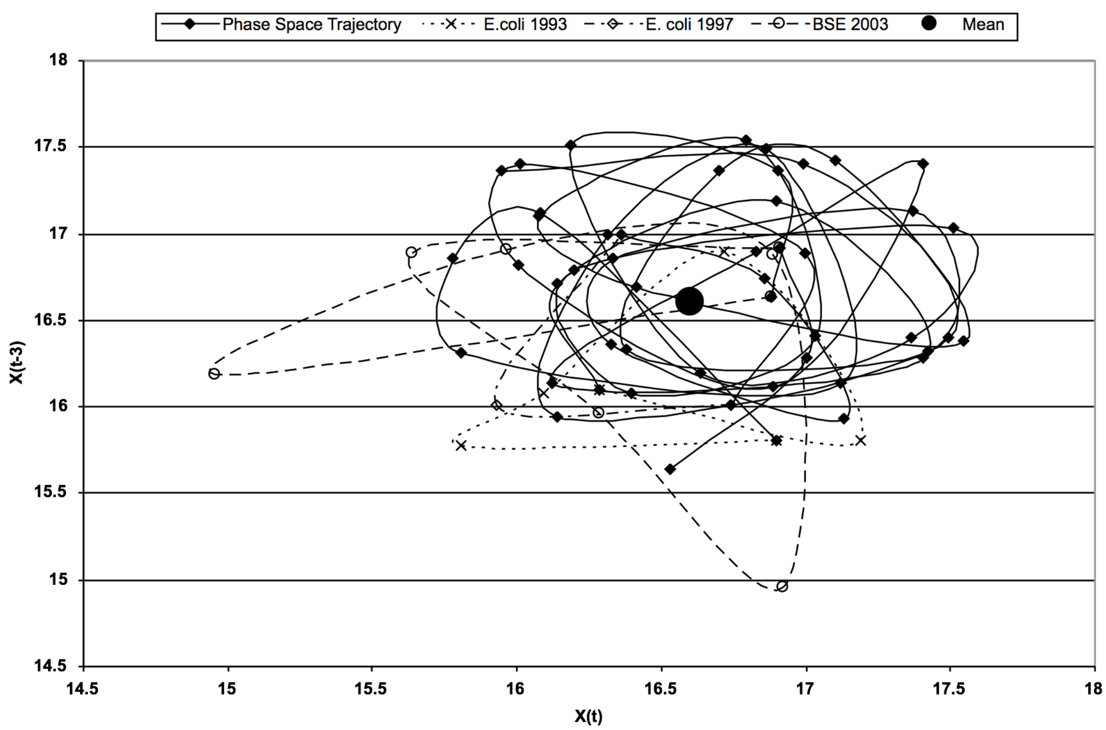

Figure 5: Reconstructed phase space with the effects of cholesterol controlled for; 1990-2005.

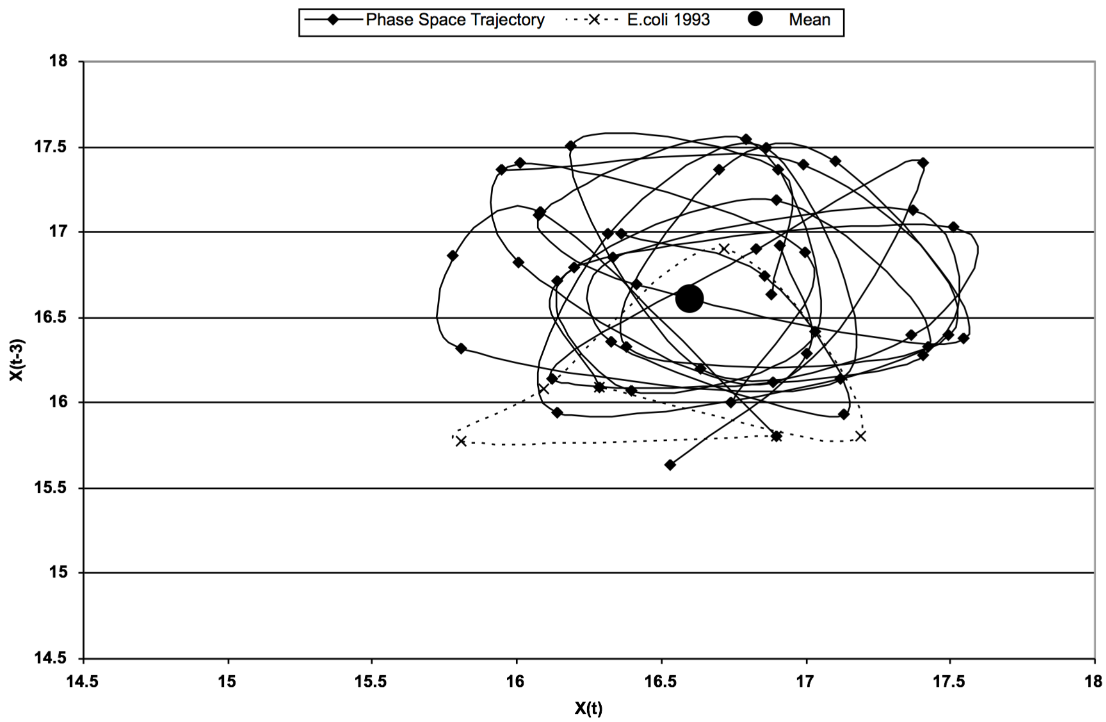

Figure 6: Reconstructed phase space isolating the effects of the Jack-in-the-Box E. coli outbreak of 1993; 1990-2005 (without other E. coli and BSE food scares).

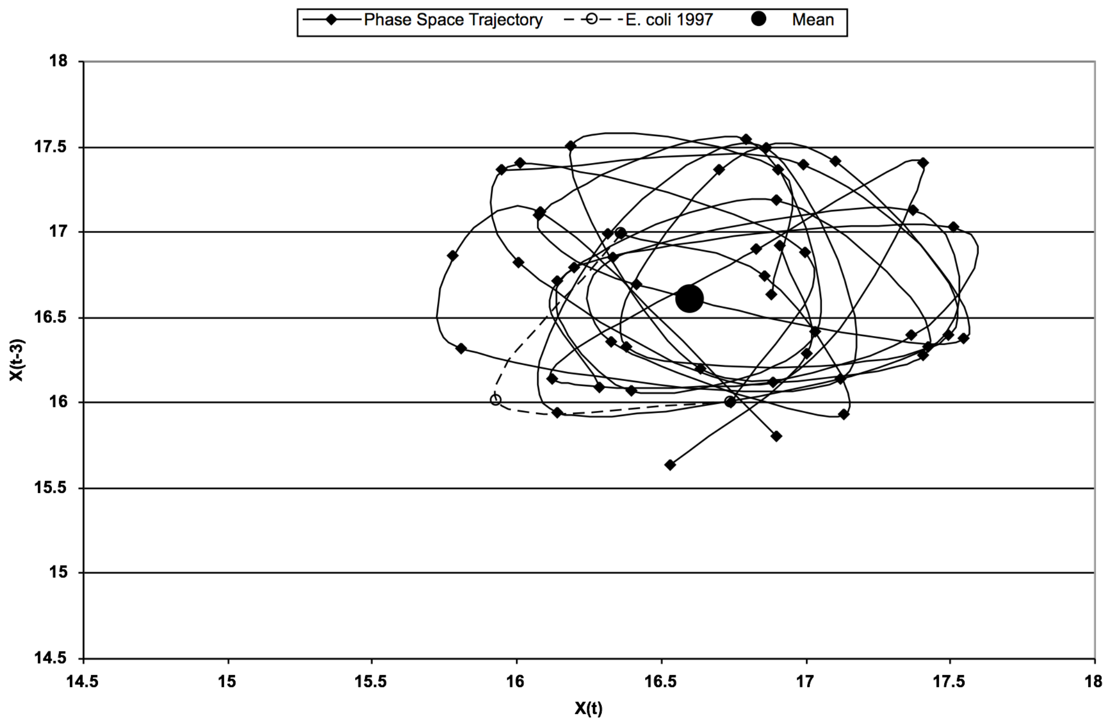

Figure 7: Reconstructed phase space isolating the effects of the Hudson Beef E. coli outbreak of 1997; 1990-2005 (without other E. coli and BSE food scares).

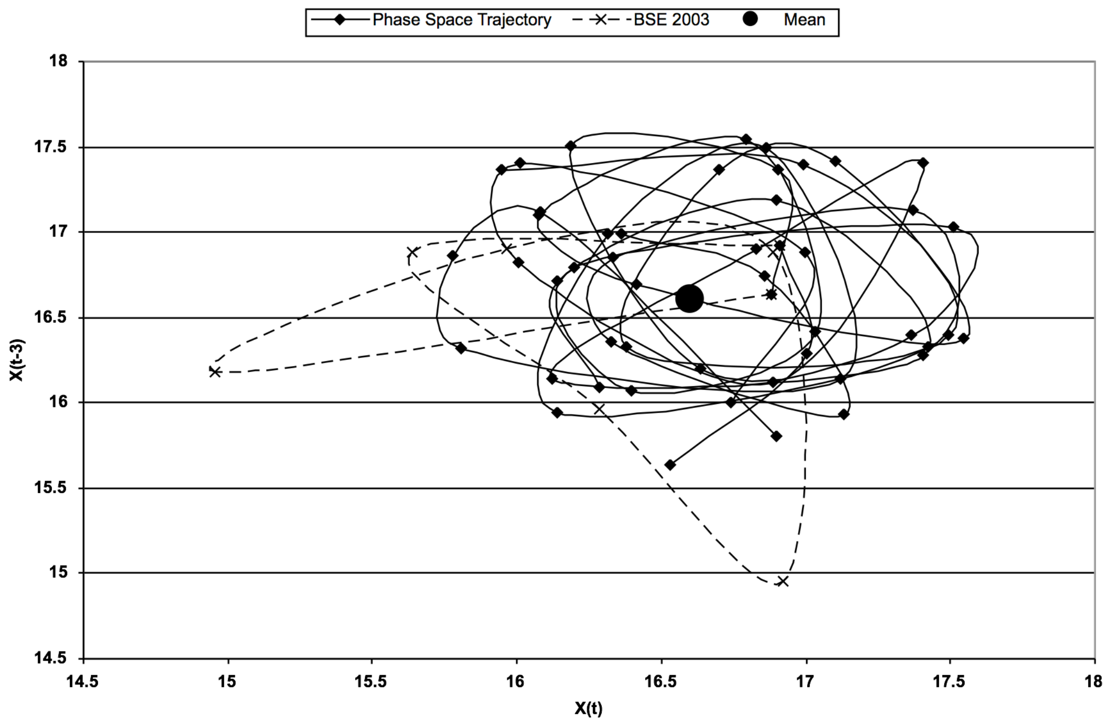

Figure 8: Reconstructed phase space isolating the effects of the BSE outbreak in the state of Washington in 2003; 1990-2005 (without E. coli food scares).

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ESSAY TWO:

NONLINEAR STRUCTURE IN REGRESSION RESIDUALS

Abstract:

Phase space reconstruction is investigated as a diagnostic tool for determining the structure of detected nonlinear processes in regression residuals. Empirical evidence supporting this approach is provided using simulations from an Ikeda mapping and the S&P 500. Results in the form of phase portraits (e.g., scatter plots of reconstructed dynamical systems) provide qualitative information to discern structural components from apparent randomness and provide insights categorizing structural components into functional classes to enhance econometric/time series modeling efforts.

Key Words: phase space reconstruction, mutual information, information and entropy economics, residual diagnostics

JEL Classification: C52

Introduction

This paper investigates phase space reconstruction as a diagnostic tool for determining the structure of detected nonlinear processes in regression residuals. Outcomes of phase space reconstruction can be used to create *phase portraits*, providing qualitative information to discern structural components from apparent randomness in regression residuals and providing insights into categorizing structural components of regression residuals into functional classes. In effect, this approach can be thought of as an alternative to simple scatter plots in linear or nonlinear models that are standard techniques in the practice of econometrics and statistics.

Although novel to analyzing regression residuals, the use of phase space reconstruction to analyze the structural nature of nonlinear processes has been applied in a variety of fields. The medical and engineering research used phase space reconstruction to separate different processes observed in the same time series (Richter and Schreiber, 1998). Physics and econometrics research have used phase space reconstruction as a tool to detect chaos (Takens, 1980; Chon *et al*, 1997; and Barnett and Chen, 2004). Epidemiological studies have used phase space reconstruction as a qualitative "check" for dynamic system of (Schaffer and Kot, 1985).

At the same time econometrics research has addressed the issue of detecting dependence consistent with nonlinear processes (Maasoumi and Racine, 2002; Granger, Maasoumi, and Racine, 2004; and Dionísio, Menezes, and Mendes, 2006). While it is important that the presence of nonlinear processes be detected, being able to make qualitative inferences about the structure of the nonlinear process may improve model fit. These three papers show that in the event dependence between random variables is a result of some process which is either nonlinear or of some complicated nature, i.e. non-Gaussian, traditional time series routines that are functions of the correlation coefficient fail. They provide a measure of dependence based upon entropies that does well in detecting relationships between random variables be they discrete, non-Gaussian, or driven by nonlinear functions. If dependence were detected in regression residuals, inference on the structure of the dependence would provide help in determining the appropriate modeling routine. If one knew the specific form of the dependence of two random variables, or model residuals, it would be possible to fit a more specific functional form opposed to a blanket nonlinear approach such as GARCH models.

Through a series of simulated illustrations and a real world example, we demonstrate how nonlinear structure in time series data may be analyzed using phase space reconstruction. The application follows along the lines of detection of nonlinear processes as demonstrated by Granger, Maasoumi, and Racine (2004) in that it employs the use of mutual information, an entropy measure of dependence, as a fundamental part of identifying parameters of the embedding process in phase space reconstruction. The current analysis extends Granger, Maasoumi, and Racine (2004) by reconstructing phase planes and interpreting phase portrait.

The paper proceeds in the following manner. First, residual reconstruction is motivated using phase space reconstruction and Takens' embedding theorem. Second, methods to apply phase space reconstruction are discussed. Third, the Ikeda system is simulated to demonstrate the use of phase space reconstruction in the presence of nonlinear processes and their performance in the presence of noise. Then, an application to the S&P 500 is performed. Finally, concluding remarks are provided. Appendix A presents a step procedure for empirical application and a software package written in the Gauss programming language.

Residual Reconstruction

The central idea is to construct a phase portrait of regression residuals using outcomes from phase space reconstruction that provides qualitative or quantitative information in the econometric or statistical modeling effort. A single time series of residuals is analyzed applying the process of phase space reconstruction, which extracts existing nonlinear dynamics inherent in the dynamical system from which the data series arises. For example the reconstruction could provide a foundation for model building and prediction, as in Schaffer and Kot (1985), where causal relationship is inferred based on a dynamic model replicating the phase space reconstruction. The dimension of the phase portrait and the appropriate lag length are determined in the phase space reconstruction, which we now discuss.

Initially developed to analyze fluvial dynamics, Takens (1981) proved an embedding theorem for a time series (void of noise), demonstrating that it was possible to approximate the systems state space through reconstruction. Takens' embedding theorem, based upon Whitney's definition of embedding (Whitney, 1936), has been used widely in practice to motivate all types of nonlinear analysis.

To motivate phase space reconstruction, the theory of dynamical systems in phase space needs to be addressed. Dynamical systems are usually defined by a set of first-order ordinary differential equations (or discrete time analogs). Assuming the phase space is a finite-dimensional vector space R^m and a state is defined by a vector $\mathbf{X} \in R^m$, then the continuous (or discrete) time system governing the system is $\frac{d\mathbf{X}(t)}{dt} = \mathbf{f}\left[t, \mathbf{X}(t)\right]$ $\left(\text{or } \mathbf{X}_{n+1} = \mathbf{F}\left[\mathbf{X}_n\right]\right)$. The mathematical theory of ordinary differential (difference) equations ensures the existence and uniqueness of the trajectories, if certain conditions are met (Packard *et al*, 1980; Shone, 2002; Takens, 1981).

Phase space reconstruction is a diffeomorphism that reproduces a time series on a plane that mirrors the phase portrait of the underlying system. A diffeomorphism is a smooth function, Φ, that maps one differential manifold, M, onto another, N, whose inverse, Φ', is also a smooth function that maps N onto M. Given that the phase space reconstruction is a diffeomorphism, it has the desirable property of differentiable equivalence. This ensures the qualitative dynamic representation of the vector field $F[X_n]$ that is the underlying system structure, in the reconstructed vector field $\Phi(F[X_n])$.

There have been two methods used to reconstruct phase space: Takens' (1981) method of delays and Broomhead and King's (1986) singular value decomposition. Each method has its positive and negative attributes in empirical applications. We focus on the method of delays as it has the more easily interpretable results.

Method of Delays

The method of delays concentrates on time valued vectors of observed time series. When reconstructing a system from the time series $X_t = \left[x(t), x(t-1), \ldots, x(t-n)\right]'$, the method of delays requires an optimal time lag τ be chosen followed by a minimum embedding dimension λ. Once the two parameters are estimated, the time series X_t will generate a reconstructed phase space matrix $Y_{\lambda} = \lfloor X_t, X_{t-\tau}, X_{t-2\tau}, ..., X_{t-(\lambda-1)\tau} \rfloor$ with dimension $[(n - \lambda\tau) \times \lambda]$.

The time lag τ is paramount to empirical applications of Takens' theorem. While the Whitney's embedding condition $n \geq 2m + 1$ is sufficient, it is not necessary. By choosing a time lag that yields the first minimum of redundancy between the column vectors in matrix Y_{λ} the geometry of the original manifold will be preserved even when the time series is contaminated with noise. However, if the estimated time lag τ is too small the approximation will be smooth but there will exist a high degree of correlation between components. This has the potential to force the trajectories of the attractor to lie on the diagonal in the embedding space (Broomhead and King, 1986). If, on the other hand, the time lag is chosen to be too large the dynamics of the system may unfold between components and therefore be unobserved. The optimal time lag is that which preserves the largest amount of information between components while achieving the smallest degree of redundancy. The incorporation of advances in information, entropy, and nonparametric economics make estimating the optimal time lag asymptotically efficient as well as computationally less burdensome as discussed below.

Mutual Information

There have been many entropy measures of dependence proposed for diagnostic modeling (see Granger, Maasoumi, and Racine (2004) and references therein). Analogous to the autocorrelation function, these measures have been used widely to test for peculiarities in error terms of econometric and time series models. However, unlike the autocorrelation function entropy dependence is both linear and nonlinear making it a much more suitable candidate for nonlinear analysis. Furthermore, the autocorrelation function hinges on estimating the sample moments of a time series. Entropy dependence measures generally make no assumptions about moments or underlying distributions of a time series; they are only limited in the probability density estimation technique used.

The method of delays only requires the independence of its reconstructed coordinate vectors, which may be measured as either distance or divergence. For this reason we shall only focus on the conditional entropy measure of global dependence between two random variables called the mutual information function developed by Fraser and Swinney (1986). In estimating the optimal time lag we are essentially asking the question: How dependent is X_t on $X_{t-\tau}$? To answer this question Fraser and Swinney defined dependence based upon conditional entropies and called it the mutual information function

$$I(X_t, X_{t-\tau}) = H(X_t | X_{t-\tau}) = H(X_{t-\tau}, X_t) - H(X_{t-\tau})$$

where $H(X_t)$ is Shannon's entropy

$$H(X_t) = -\sum_t P_{x_t}(x(t)) \log P_{x_t}(x(t))$$

and

$$H(X_{t-\tau}, X_t) = -\sum_{t,t-\tau} P_{x_t x_{t-\tau}}(x(t), x(t-\tau)) \log\left[P_{x_t x_{t-\tau}}(x(t), x(t-\tau))\right];$$

with $P_{x_t}(x(t))$ being the probability density of *x* occurring at time *t* (Shannon and Weaver, 1949).

The mutual information function is defined as the combination of joint and marginal probabilities of the outcomes from an event in a sequence while increasing the time lag τ between components:

$$I(X_t, X_{t-\tau}) = \sum_{n-\tau} \sum_{n-\tau} P(X_t, X_{t-\tau}) \log\left[P(X_t, X_{t-\tau}) / P(X_t) P(X_{t-\tau})\right].$$

Based upon the standard definition of independence the argument $\log[\bullet]$ will equal zero, and thus the mutual information function as well, if the vectors are perfectly independent and will tend to infinity as they become more dependent.

This measure of dependence is independent of the coordinates of x as the probability density functions are dimensionless. Since the mutual information function is based upon joint probability density functions it is a *global* measure of dependence and not a function of the individual time vectors (Fraser and Swinney, 1986). Choosing the time lag that yields the first local minimum of the mutual information function ensures independence of components with a maximum amount of new information (Fraser and Swinney, 1986). The first minimum is chosen as the optimal time lag based on the optimality conditions defined above so that it is neither too small nor too large, ensuring that the attractor unfolds correctly.[17]

Estimating the mutual information function hinges on estimating the probability density function of a time series and its lagged values. This has traditionally been done in the phase space reconstruction literature by using histogram estimators that are perceived as the "most straightforward and widespread approach" (Dionísio *et al*, 2006). The histogram method of estimating density functions uniformly weights observations within a predetermined window. If the time series contains a large portion of observations located close together and some that are spread out, the histogram method will inconsistently estimate the probability density function. Algorithms have been developed that vary the window size based upon how close observations are located to

[17] If the global minimum of the mutual information function were used as the optimal time lag the potential would be for the nonlinear system to have already completed a full cycle so that the estimate would include redundancy of the system as well. This could force the phase space reconstruction to no longer be monotonic, thus enveloping dynamical structure.

each other but they are computationally intense and not easily programmed (Fraser and Swinney, 1986).

Following the information and entropy literature we apply nonparametric estimation using kernel density approximations as a method for estimating the mutual information function. In addition to being less computationally burdensome, under appropriate conditions, the nonparametric method of estimating the mutual information function is also asymptotically efficient. By using kernel weights the possible inefficiencies encountered with the histogram method of estimating the mutual information function are minimized (Pagan and Ullah, 1999).

False Nearest Neighbors

Given the choice of optimal time lag, the minimum embedding dimension λ can be estimated. Kennel and Brown (1992) developed the False Nearest Neighbors technique (discussed below) for choosing a minimum embedding dimension. Aittokallio (1999) suggested the embedding dimension must be chosen properly or the reconstruction may not reflect the original manifold. If λ is too small the reconstruction cannot unfold the geometry of the strange attractor. If λ is too large artificial symmetry is created and procedures used to determine basic properties of the system and qualitative analysis may become unreliable (Aittokallio, et al., 1999, Kennel, et al., 1992).

The false nearest neighbors technique uses Euclidean distances to determine if the vectors of Y_λ are still "close" as the dimension of the phase space is increased. By calculating the Euclidean distance between Y_λ vectors before and after an increase in dimension, it is possible to determine if the vectors are actual nearest neighbors or "false" nearest neighbors. The test statistic developed by Kennel and Brown defining neighbors to be false is:

$$\frac{x(t+d)-x(n(t)+d)}{\left\|y_t - y_{n(t)}\right\|} > R_{tol}$$

Where $x(t+d)$ denotes the last coordinate in the t^{th} row of the phase space reconstruction matrix $Y_{\lambda+1}$, $n(t)$ denotes the nearest neighbor in Euclidean distance of t for each row vector y_t in matrix Y_λ, and R_{tol} is the desired tolerance level. When the percentage of false nearest neighbors is minimized or drops below a preset threshold, the minimum embedding dimension for phase space reconstruction is found (Kennel, et al., 1992).

Given the efficient estimation of the embedding and time lag parameters required in phase space reconstruction the tool may be used to embed residuals driven by a nonlinear process to make qualitative inference on the structure of the process. As with any statistical estimation technique, phase space reconstruction is affected in the presence of noise. The simulations below are designed to demonstrate that *if* nonlinear dependence is *detected* it can be qualitatively analyzed under a threshold level of noise.

Simulations: The Ikeda Map

Prior to examining phase space reconstruction on actual regression residuals, we apply it to a set of data simulations generated from a variation of a known nonlinear process, the Ikeda Map, and contaminated versions of the process. Investigating the effect of noise contamination is important because real world data and dynamics will likely be due to both deterministic and noise components. Further, this will illustrate the process of drawing qualitative information from the structure of existing nonlinear processes in the form of a phase portrait with and without data contamination.

The simulations use a numerically iterated deterministic time series, X_n, from the Ikeda Map, discussed below, added to randomly generated time series, $\varepsilon_{n,i}$, as the full data generating process. The simulation follows that in Chon *et al* (1997), except that we add noise post numerical iteration of the map and focus on the *structural* analysis of dependence whereas Chon *et al* (1997) included a noise term within the numerical iteration of the Ikeda map and focused on the *detection* of chaos. This allows us to separate the two components more distinctly, in particular X_{n+1} is independent of ε_n.

The isolation of X_n in the data generating process allows for a comparison of the degree of structure that can be detected with various levels of noise. This type of additive noise is analogous to measurement error in an economic time series, as measurement error is defined by the independence of the error and the unobserved process. In economics, measurement error can severely decrease the ability to determine relationships between variables, and the opportunity cost of observing a naturally occurring time series without measurement error is arguably very high. By controlling the volatility of the random component, this simulation illustrates the general usefulness of phase space reconstruction as a residual diagnostic tool. The variance of the randomly generated component is determined as a fraction, 10%, 25%, 50% and 100%, of the variability of the deterministic component. Each series is tested for structure with conventional methods discussed below and then embedded using the method of delays.[18]

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[18] For demonstrations of the application of phase space reconstruction as a diagnostic modeling tool refer to McCullough (2008), Schaffer and Kot (1985), and Zaldívar, *et al* (2000).

Ikeda Map

The Ikeda Map is a system of discrete-time difference equations that produces chaotic-like behavior for different values of the parameter u.

$$X_{n+1} = 1 + u(X_n \cos t_n - Y_n \sin t_n)$$

$$Y_{n+1} = u(X_n \sin t_n + Y_n \cos t_n)$$

and

$$t_{n+1} = 0.4 - \frac{6}{1 + X_n^2 + Y_n^2}.$$

For initial conditions $(X_0, Y_0, t_0) = (0,.1,.4)$, and parameter value $u = .85$ (Figure 1 shows a plot of X_n for $n = 500$ and a plot of randomly generated data from a normal distribution. A casual inspection of the time series X_n in Figure 1 might suggest a noisy process. We use this type of structure for our simulations in order to demonstrate the simple nature of phase space reconstruction that allows for complex qualitative analysis. If a nonlinear process was detected in the residuals of a model it could be analyzed in the fashion we employ here.

To contaminate the Ikeda data series, a randomly generated time series, ε_n, is drawn from four different Gaussian distributions each with increased standard deviations. By increasing the standard deviation of the random component it is possible to observe the point at which phase space reconstruction can no longer differentiate structure from noise. Five hundred observations are numerically iterated and used in each of the simulations to illustrate the full dynamic structure of the Ikeda map.

More explicitly the simulations are defined as $Z_i(t)$ such that

Model 1: $Z_1(t) = X_n$

Model 2: $Z_2(t) = X_n + \varepsilon_{n,1}$

Model 3: $Z_3(t) = X_n + \varepsilon_{n,2}$

Model 4: $Z_4(t) = X_n + \varepsilon_{n,3}$

Model 5: $Z_5(t) = X_n + \varepsilon_{n,4}$

Model 6: $Z_6(t) = \varepsilon_{n,4}$.

where X_n is defined in the Ikeda map above and $\varepsilon_{n,i}$ are independently identically distributed as:

$\varepsilon_{n,1} \sim N(0,0.05)$, $\varepsilon_{n,2} \sim N(0,0.125)$, $\varepsilon_{n,3} \sim N(0,0.25)$, and $\varepsilon_{n,4} \sim N(0,0.5)$.

In the analysis below, Model 1 or $Z_1(t) = X_n$, the uncontaminated deterministic time series, is used as the benchmark to compare the remaining models. The similarity of the phase space reconstructions of $Z_2(t),...,Z_5(t)$ to $Z_1(t)$ is what is referred to as the degree at which nonlinear structure can be detected. Model 6 or $Z_6(t) = \varepsilon_{n,4}$, has no underlying structure but a pure random noise process.

Results

The six time series, $Z_i(t), i = 1,...,6$, are tested in a manner similar to diagnostic residual analysis in a typical econometric setting. The Skewness-Kurtosis and Shapiro-Wilks test for normality are used to assess the distribution of the time series. These tests are based upon estimated moment conditions of the observed time series (see Kutner *et al*, 2005; or Mittelhammer, 1996). The Portmanteau test is a function of the autocorrelation coefficients of a time series and is used to determine if it is white noise (Tsay, 2002). Table 1 contains the outcomes of each test and, as expected, after applying the conventional tests each time series appears to have some sort of deviation from the tests null hypotheses.

The highly deterministic time series, $Z_1(t),...,Z_4(t)$, reject normality and white noise hypotheses. Autocorrelation and mutual information functions, reported in Figure 2 for $Z_1(t)$, are plotted and have significant lag components. The misgiving is that these functions cannot contribute to the structure of the determinism; they only report the presence of dependence. It is especially critical to be able to analyze the nature of dependence in a case such as this because both linear and nonlinear measures detect dependence. When both types of measures detect dependence it is natural to assume linear dependence, as it is the simpler of the two. Given the nonlinear process of the Ikeda Map, measures to correct for linear dependence will fail. As demonstrated below, by applying phase space reconstruction, it becomes apparent that the nature of the dependence is not linear and more complicated measures are needed to capture the true effect.

Using the parameter estimation techniques described above, Figure 3 provides the phase portraits of the reconstructed phase space for the Ikeda map with increasing randomness. For example, it is clear in the first phase space reconstruction of $Z_1(t)$, the top left graph, that the process is of a nonlinear structure.

Model 1: The first phase space reconstruction simulation, see Figure 3a, is that of the uncontaminated time series, $Z_1(t)$. The traditional statistical tests reported in Table 1 all reject the null hypothesis of normal a distribution and white noise. Both mutual information and autocorrelation functions shown in Figure 2 have significant lags. All the tests of dependence confirm the presence of a nonlinear process. The phase portrait, demonstrates the structure of the nonlinear process that drives the time series. The structure of the process is well defined and appears to follow a stable trajectory. The interpolating lines show the triangular flow of the phase space trajectory while the observations appear to lie on a very distinct attractor. It would be up to the analyst in how they would control for this type of nonlinear process. For instance, if the purpose of the regression analysis were to isolate the impact that the specific variables in the design matrix have on a dependent variable, modeling the nonlinear process structurally in the error term would be sufficient. If the purpose were to detail all the possible variables that impact the dependent variable, the phase space reconstruction could help identify an omitted variable.

Model 2: The second phase space simulation contains the same deterministic component as the first with the addition of the noise term, $\varepsilon_{n,1} \sim N(0,0.05)$. It is clear in Figure 3b that the contamination increases the spread of the phase space trajectories about their respective paths. The difference between the phase space reconstructions of $Z_1(t)$ to $Z_2(t)$ is relative level of clarity. For nonlinear processes, being able to distinguish the paths that trajectories lie on is the most important part in analyzing the structure. With a small level of contamination, as Figure 3b shows, it is possible to delineate the structure of the paths, i.e. direction and intertemporal differences between observations, from the variability about those paths.

Model 3: For the third simulation, the noise component has a variance roughly equal to 25% of the deterministic process, $\varepsilon_{n,2} \sim N(0,0.125)$. Takens' embedding theorem states that for a process void of noise reconstruction is always possible. However, as we see in Figure 3c when the noise component of the time series increases, the ability to distinguish structure becomes harder. While the general direction of the trajectory paths remain the tightness, or clarity, of the paths has greatly diminished. The noise component decreases the ability to discern intertemporal differences between observations on the phase space trajectory through time. In Model 1, accurate predictions of future observations conditional on known observations could be made; it is not the case in Model 3. The added noise stretches the phase space trajectories towards the telltale "ball" that is the phase space reconstruction of Gaussian white noise seen in the sixth simulation of $Z_6(t)$, Figure 3f.

Model 4: When the additive noise variability is 50% of the deterministic process, the phase space reconstruction in Figure 3d shows signs of a nonlinear process that is only slightly distinguishable from noise. The phase portrait appears to fall within a triangular pattern. While the ability to discern the exact structural nature of the nonlinear process has been lost, it is still possible to follow the general direction of the phase space trajectories. We see that when an error component such as measurement error has a variance that is half that of the nonlinear component it becomes nearly impossible to determine the exact structure of the nonlinear process, however general observations of the nature of the process can still be made.

Model 5: For simulation five, Figure 3e, the standard deviation of the randomly generated time series is equal to that of the deterministic time series. The phase portrait is nearly identical to that of pure Gaussian randomly generated data. The traditional methods of determining independence can no longer detect suspect behavior. In fact,

$Z_5(t)$ tests as being both normally distributed by the skewness/kurtosis and Shapiro-Wilk tests, and white noise by the Portmanteau test. The volatility of the random component has overcome the deterministic component so that phase space analysis can no longer distinguish the structure of the nonlinear process.

Model 6: The final simulation of variable $Z_6(t)$, Figure 3f, is that of the same noise component in the fifth simulation with no component from the Ikeda map. When comparing the phase portrait of the sixth simulation to that of $Z_5(t)$, it appears that $Z_5(t)$ is randomly drawn from a Gaussian distribution with a larger variance than $Z_6(t)$.

The simulations above illustrate the ability to analyze the structure of detected dependence. It was found that after the variance of the additive error component increased above 25% of the variance of the deterministic component only general structure could be inferred. Below this level more detailed descriptions of the nonlinear structure may be made. After the variance of the error component increased passed 50% of the deterministic component the ability to distinguish structure from noise decreased rapidly.

If the first four time series of the simulation happened to be the residuals of an econometric model, the conclusion of model misspecification would be made and the analyst would be left to ad hoc or blanket measures for correction. Having that qualitative representation of the residual structure in the phase space reconstruction will undoubtedly make the necessary additional modeling easier. This representation can be related to basic regression diagnostic analysis. For example, if two variables had an unknown quadratic relationship, simple linear regression analysis would result in a high

R^2 value. However, if a plot of the residuals vs. predicted values were performed it would be clear that the model was misspecified and further analysis would dictate regressing the one variable on the other, squared.

In the case that nonlinear dependence in regression residuals is found and structural analysis is performed with phase space reconstruction, numerous model correction techniques may be employed. The technique used would depend on the scope of the regression analysis and the nature of the detected dependence. For cases in which the dependence followed a simple nonlinear form, as in the simple linear regression example above, polynomial functions of autoregressive parameters could be fit. In the same case, independent variables may be used to extract causal explanation for the structure. When more complex structures are found and no causal explanations can be made, it may be appropriate to employ blanket nonlinear time series methods such as ARCH and GARCH processes or heteroskedastic residual correction techniques like Robust Regression. There is also the possibility that a dynamic system of equations may be created that exhibit structure in their simulated phase portraits that mirror the structure of the detected dependence. For example, Schaffer and Kot (1985) use phase space reconstruction on measles data to confirm the SEIR model of epidemics. Phase space reconstruction may be used on regression residuals in the same framework.

Application: The S&P 500

An empirical demonstration of phase space reconstruction as a residual diagnostic tool is applied to regression residuals from a nonlinear time series model of the weekly opening value of the Standards and Poor (S&P) 500 from January 3, 1996 to December 22, 1997.

For those models in which nonlinear processes are detected a phase space reconstruction can be used to add insight as to the nature of the nonlinear structure. For residual series in which no dependency is detected, phase space reconstruction can be used to confirm the absence of nonlinear structure, as we do here.[19]

An Integrated Auto Regressive model with ARCH and GARCH structured residuals has been used in the past to estimate the S&P 500 (see Granger, Maasoumi, and Racine (2004); Dionísio, Menezes, and Mendes, 2006; and references therein) so we do the same. Table 2 contains the model results, and Figure 4 presents the residual time series from the estimated model. The lagged variables were chosen to minimize the Akaike's information criterion and maximize R^2 as demonstrated in Tsay (2002). This estimation technique follows the previously indicated literature on entropy in which the residuals of the nonlinear time series model are tested for nonlinear dependence.

To demonstrate the absence of nonlinear structure the residuals, the residuals are reconstructed using the method of delays. The optimal time lag is estimated with the mutual information function to be $\tau = 2$; see Table 3. This is the lag at which the mutual information function reaches its first minimum. Next, the minimum embedding dimension is estimated using the false nearest neighbors technique to be $\lambda = 3$. Both the graphical and traditional methods confirm this as the minimum embedding dimension; see Figure 5 and Table 4.

The S&P 500 reconstructed phase portrait, Figure 6, resembles the 'bird's nest' that appears in the phase space reconstruction of Model 6 in the above simulation,

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[19] Barnett and Chen (2004) suggest using phase space reconstruction, as well as other nonlinear measures, as a test for detecting chaos in a time series. While the purpose of this paper is not to demonstrate that which was done in Barnett and Chen, the S&P 500 application helps to define the range of phase space reconstruction as a residual diagnostic tool.

randomly generated data from a Gaussian distribution. For further comparison, a three-dimensional reconstructed phase portrait of white noise is presented in Figure 7. It is clear that Figure 6 and Figure 7 share very similar qualitative structure confirming the speculation of random noise. Figure 7 has a larger spread than Figure 6 but the apparent cluster or 'bird's nest' is similar for the two. For the S&P 500, since the phase space reconstruction of the residual time series resembles that of the reconstruction for a randomly generated time series, we can conclude that the time series is random error or speculate that if there are any underlying nonlinear processes present their influence is outweighed by noise as was the case in Model 5 of the previous simulation.

Like Barnett and Chen (2004) we illustrated the usefulness of mutual information function in detecting dependence. In addition to the entropy measures of dependence discussed above, embedding residuals allows for further distinction between random and nonlinear processes. Unlike previous research we reconstruct residuals for a qualitative analysis of the structure of the underlying nonlinear process. In cases when it is not sufficient to just detect dependence, phase space reconstruction can add valuable insight on the nature of the dependence.

Conclusions

This paper investigates phase space reconstruction as a diagnostic tool for determining the structure of detected nonlinear processes in regression residuals. Outcomes of phase space reconstruction are used to create phase portraits, providing qualitative information to discern structural components from apparent randomness in regression residuals and providing insights into categorizing structural components of regression residuals into functional classes. Empirical evidence supporting this approach is provided using simulations from an Ikeda mapping and the S&P 500. Results provide qualitative information to discern structural components from apparent randomness and provide insights categorizing structural components into functional classes to enhance econometric/time series modeling efforts.

Specifically, the set of simulations provided evidence that if nonlinear structure persists in the residuals of an econometric/time series model, they may be embedded in a manner to be qualitatively analyzed with phase space reconstruction. This approach is different from using phase space reconstruction in the detection of chaos or providing evidence of dependence using entropy metrics, which previous research has done. Rather, the current approach may be thought of as an alternative to simple scatter plot analysis on linear or regression models. An important product of this approach is that it supplies qualitative evidence of nonlinear structure potentially providing the foundation for causal explanations (as in the Ikeda mapping). The S&P example shows the general use of phase space reconstruction and phase portraits for analysis of regression residuals.

Moreover, since phase space reconstruction is an extension of Takens' embedding theorem, it is relatively free of a priori assumptions and restrictive constraints. The approach does not require specification of functional forms and can be estimated using nonparametric econometric methods. It is not a "data mining" technique and is not affected by discrete or non-Gaussian processes. In all, phase space reconstruction can be thought of as a scatter plot for dynamical systems. It is a tool with a relatively low marginal cost of implementation and potential high marginal benefit, making it an appealing addition to the regression toolbox.

Tables

Shapiro-Wilk W test for normal data					
Variable	Obs	W	V	z	Prob > z
Z_1(t)	498	0.96125	12.989	6.163	0
Z_2(t)	498	0.96603	11.387	5.846	0
Z_3(t)	498	0.97928	6.946	4.658	0
Z_4(t)	498	0.99226	2.594	2.291	0.01098
Z_5(t)	498	0.99607	1.318	0.664	0.2534
Z_6(t)	498	0.99677	1.083	0.191	0.4241

Skewness/Kurtosis tests for Normality				
Variable	Pr(Skewness)	Pr(Kurtosis)	adj. $X^2(2)$	Prob > $X^2(2)$
Z_1(t)	0.912	0	22.3	0
Z_2(t)	0.933	0	21.1	0
Z_3(t)	0.928	0	15.5	0.0004
Z_4(t)	0.915	0.006	7.26	0.0266
Z_5(t)	0.905	0.186	1.77	0.4123
Z_6(t)	0.095	0.863	2.82	0.2439

Portmanteau test for white noise		
Variable	Q-statistic	Prob > $X^2(40)$
Z_1(t)	2078.2247	0.0000
Z_2(t)	2053.8117	0.0001
Z_3(t)	1844.4558	0.0003
Z_4(t)	1273.9056	0.0104
Z_5(t)	445.4445	0.4378
Z_6(t)	42.6208	0.359

Table 1: Conventional econometric residual tests applied to $Z_i(t)$.

Model Section	Variable	Coefficient	Standard Error	Z-Value	P>|z|	[95% Confidence Interval]	
Exogenous	Constant	-0.9504154	0.4428517	-2.15	0.032	-1.818389	-0.082442
ARIMA	ARI(1,1) at lag 10	0.1132148	0.0413289	2.74	0.006	0.0322117	0.1942179
ARCH/GARCH	ARCH(1)	0.0679655	0.0190663	3.56	0	0.0305962	0.1053348
	GARCH(1)	0.9083263	0.0266032	34.14	0	0.8561851	0.9604676
	Constant	2.212102	1.129958	1.96	0.05	-0.0025753	4.426779

Table 2: Nonlinear time series model results for the S&P 500.

Lag	Mutual Information
1	0.081168171
2	0.065175734
3	0.0753686
4	0.080049195
5	0.064124407
6	0.09124448
7	0.064965585
8	0.069914514
9	0.065449536
10	0.081261469

Table 3: The average mutual information function for S&P 500 residual series.

Dimension	Percentage of False Nearest Neighbors
1	1.189066059
2	0.797266515
3	0.400911162
4	0.348519362
5	0.355353075
6	0.410022779
7	0.464692483
8	0.574031891
9	0.649202733
10	0.776765376

Table 4: The percentage of false nearest neighbors for the S&P 500 residual series.

Figures

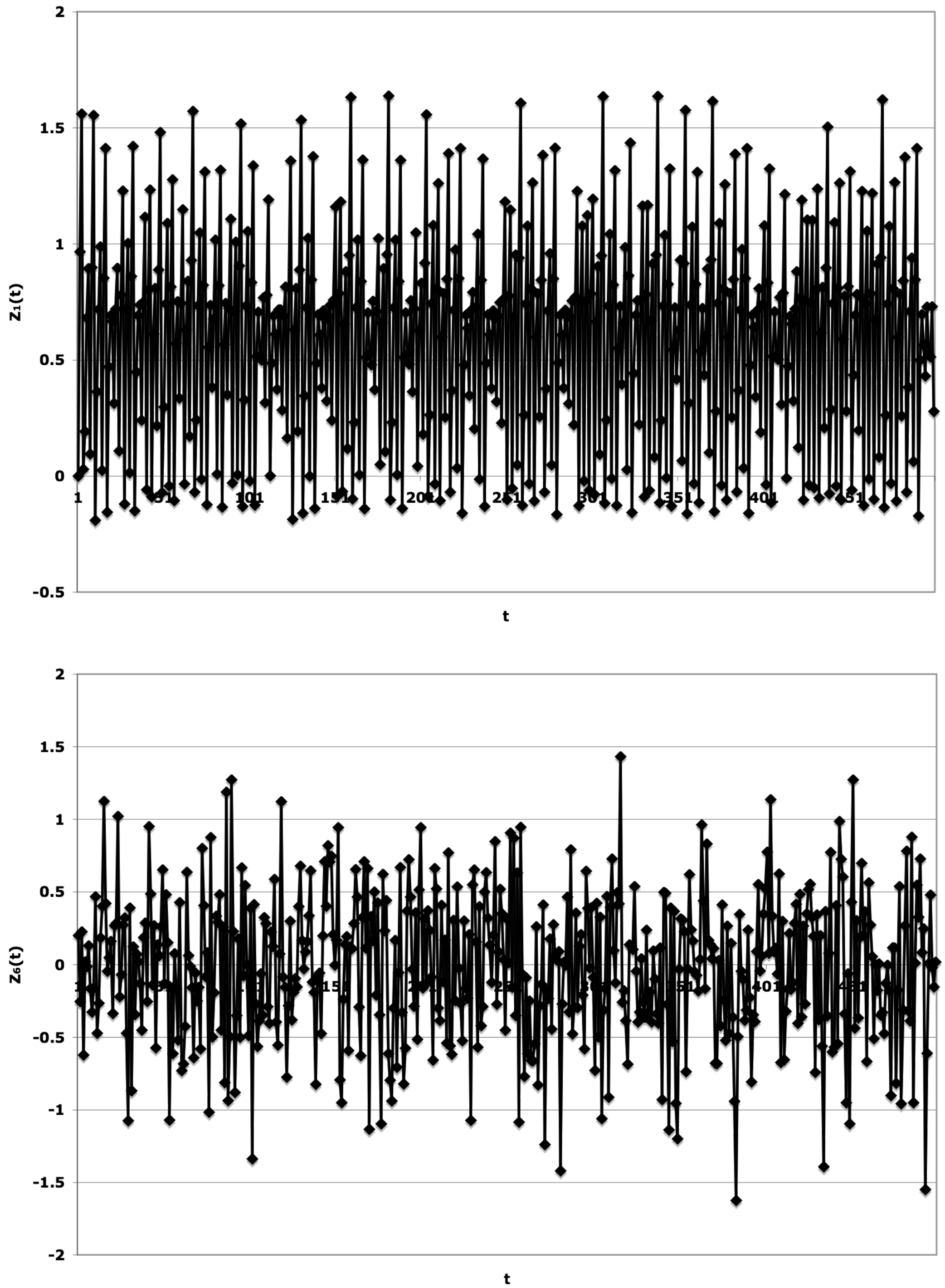


Figure 1: Time series for $Z_1(t)$ generated by the Ikeda Map and $Z_6(t)$ randomly generated data from a $N(0,0.5)$ distribution.

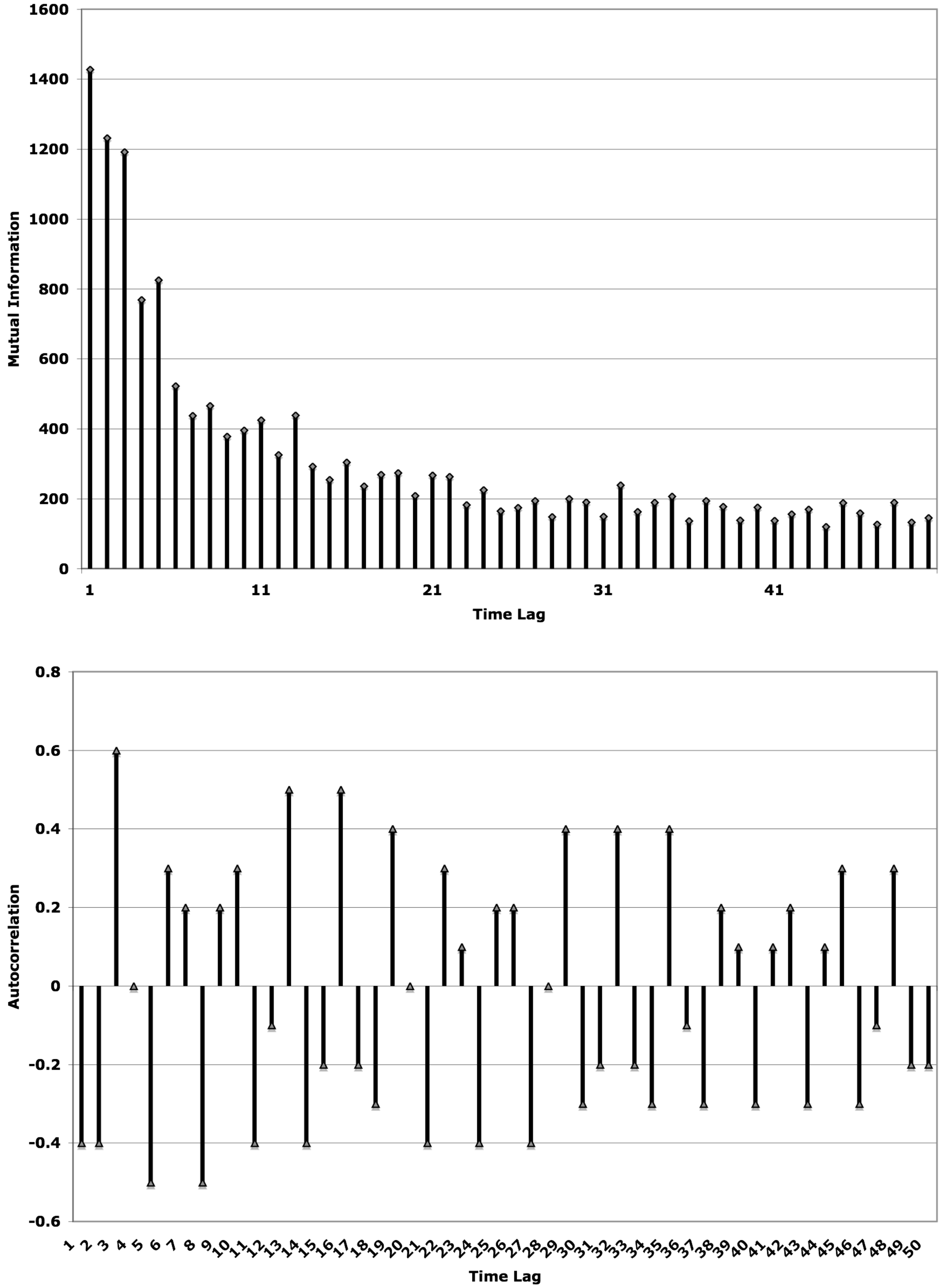
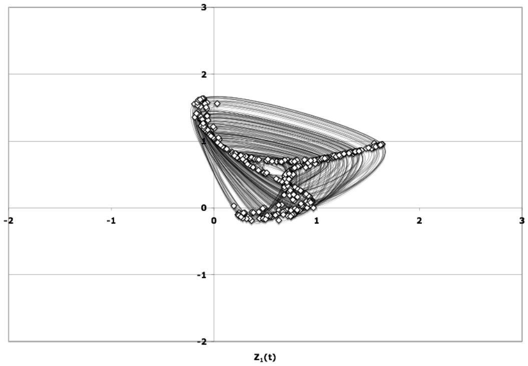
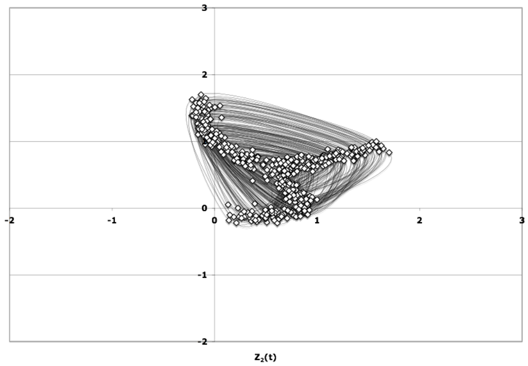


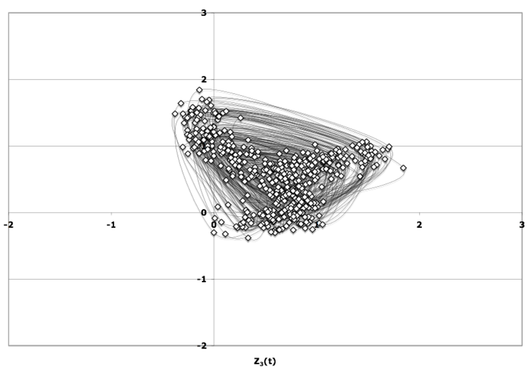
Figure 2: The autocorrelation and mutual information functions for $Z_1(t)$.



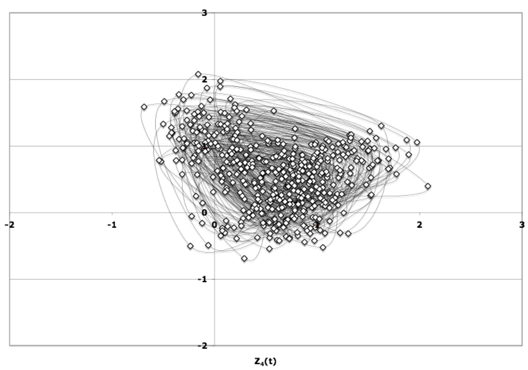
a) Z_1(t)



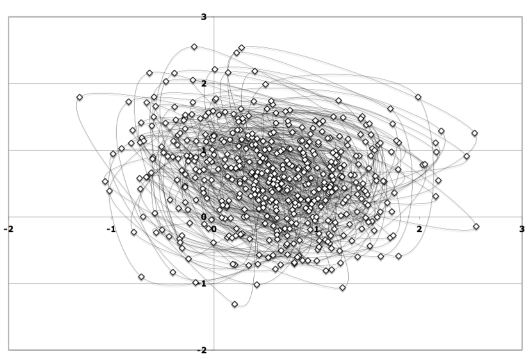
b) Z_2(t)



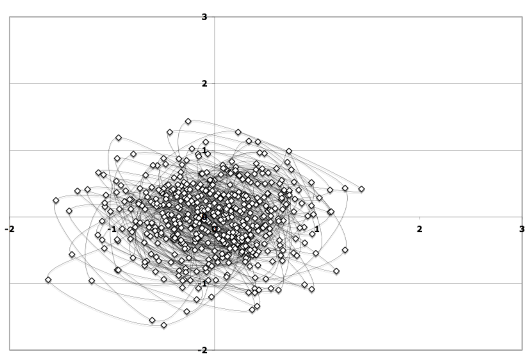
c) Z_3(t)



d) Z_4(t)



e) Z_5(t)



f) Z_6(t)

Figure 3: The phase space reconstructions of $Z_i(t)$ for $i = 1,...,6$.

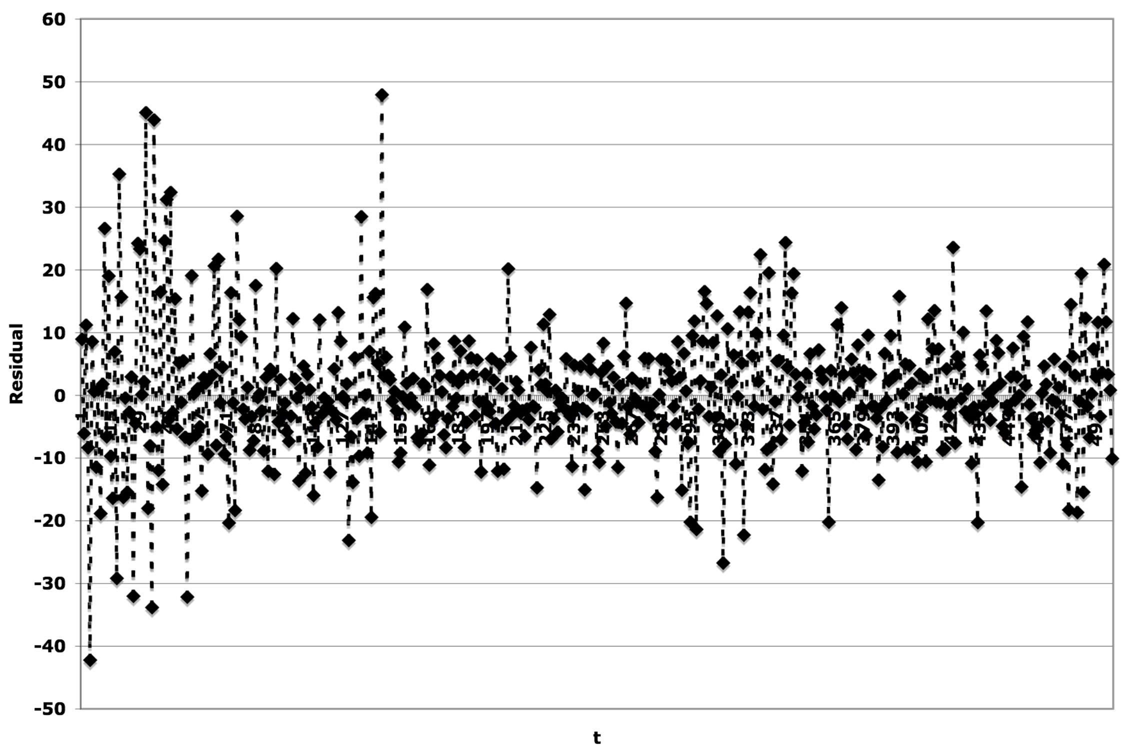


Figure 4: Residuals from the nonlinear time series model of the S&P 500.

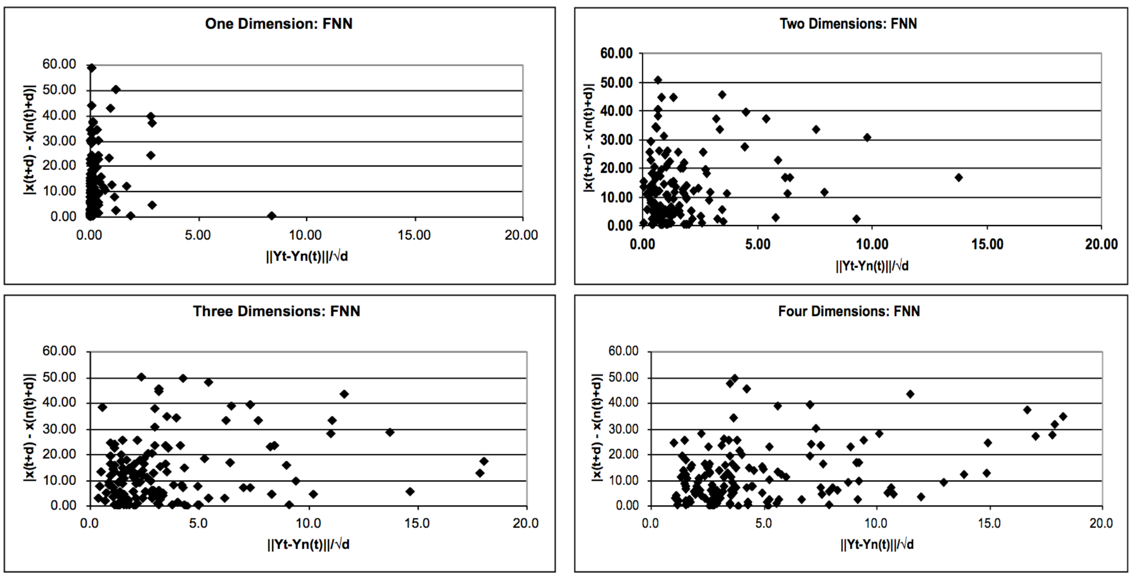


Figure 5: Graphical false nearest neighbors method for determining the minimum embedding dimension for the S&P 500.

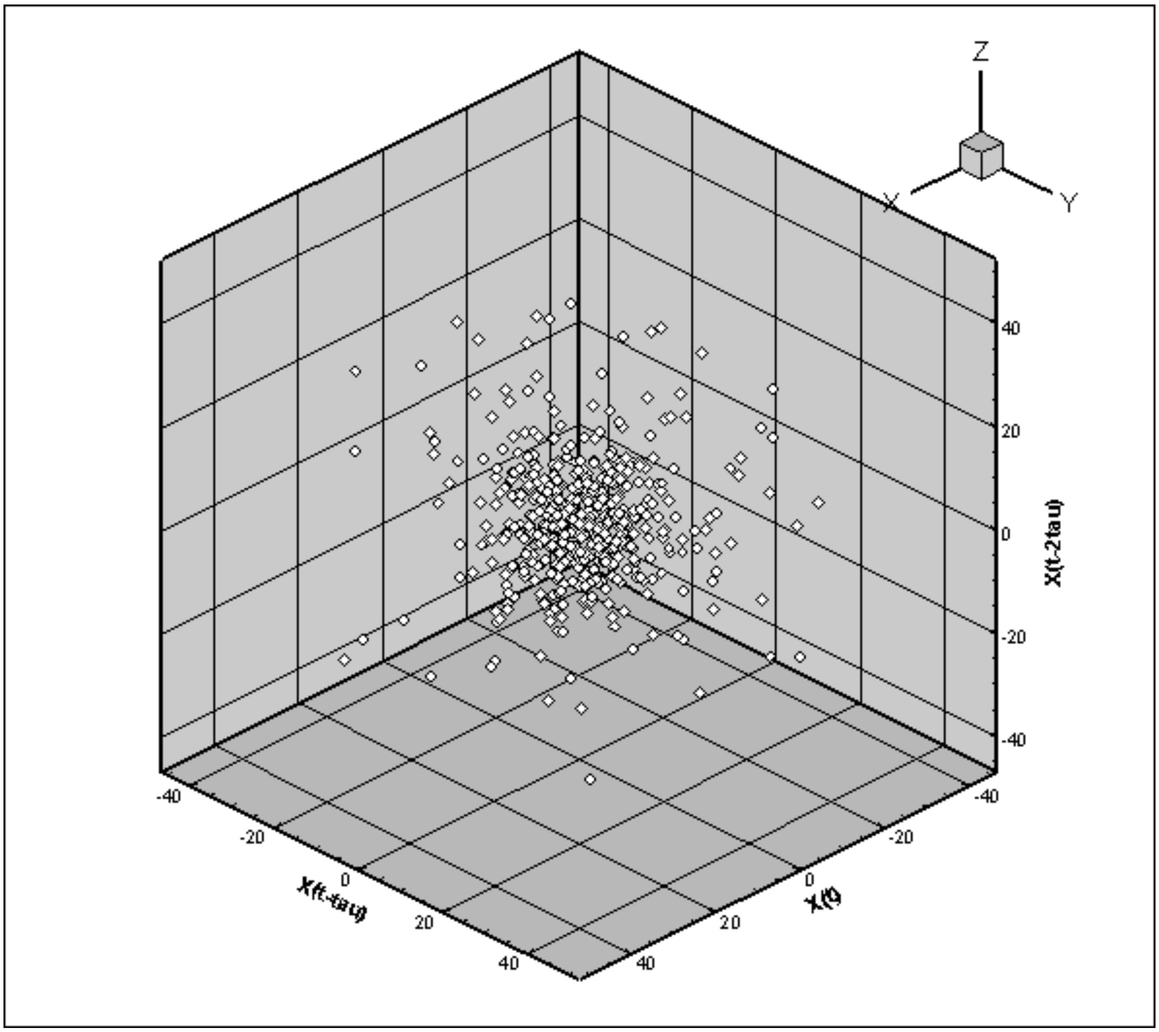


Figure 6: Reconstructed phase space of the residuals of the S&P 500 model.

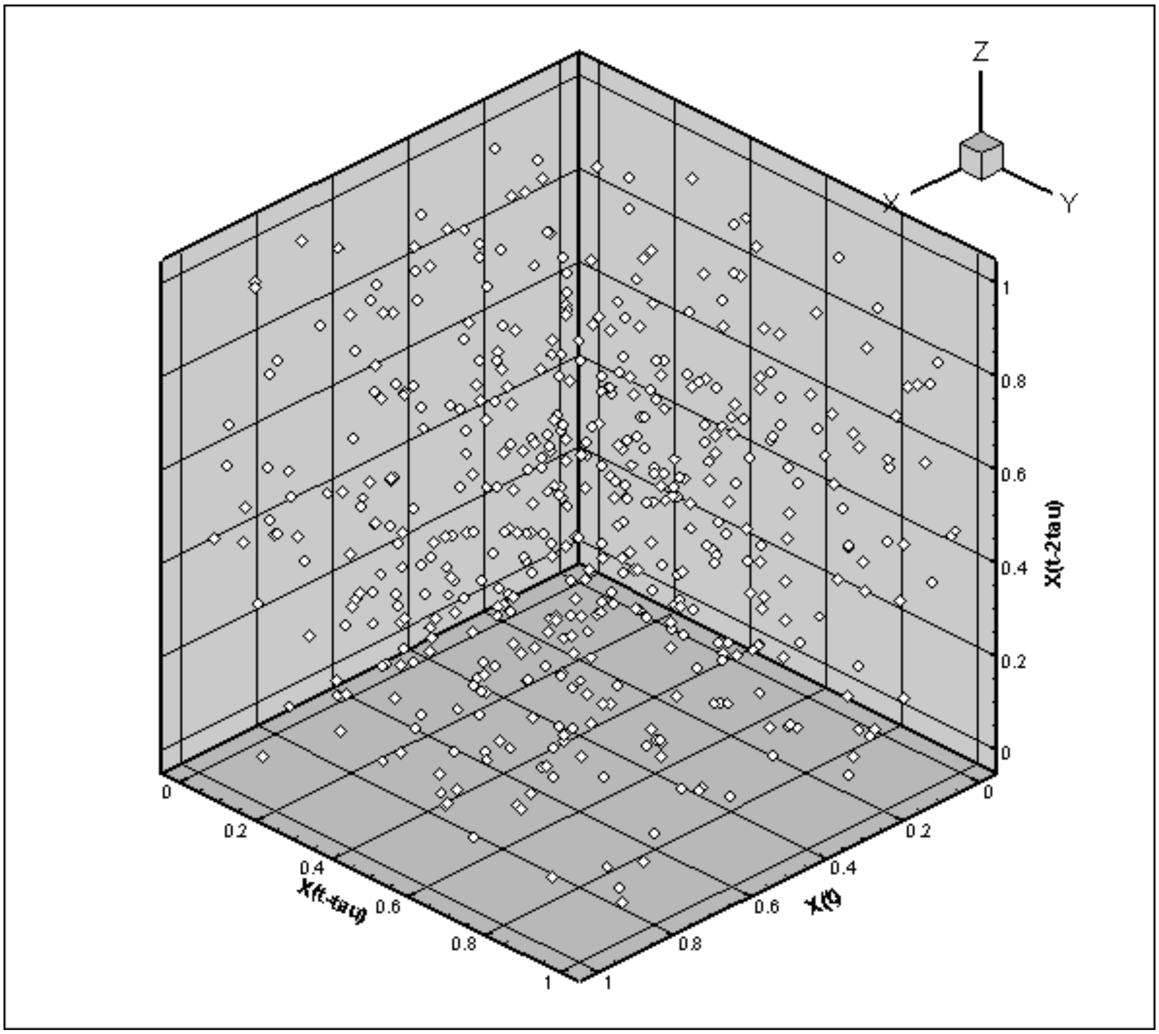


Figure 7: Phase space reconstruction for a randomly generated Gaussian time series.

Appendix A:

Step-by-Step Procedure for Phase Space Reconstruction

Phase space reconstruction of economic or soft science time series is starting to break through. It provides a useful benchmark for dynamical model building and insights into previously unknown properties of the system. A proposed method of approach for phase space reconstruction was developed and follows as such, keeping in mind that the reconstructee has thorough knowledge of the time series being analyzed:

Phase Space Reconstruction:

1. Check the time series a for potential unit root. If present, proceed directly to the implementation of singular value decomposition. If not present, proceed to the method of delays.

Method of Delays:

2. Choose an optimal time lag τ by means of the mutual information function and/or the 45º linear dependency graphical method depending on time series length.
3. Using the time lag τ, estimate the minimum embedding dimension using the false nearest neighbors technique. Again, if the time series is too short for this method, (300 observations has appeared to be a useful cutoff amount), use the graphical false nearest neighbors technique to approximate the minimum embedding dimension.
4. If possible create a graphical representation of the reconstructed phase space for qualitative analysis and proceed with the desired use of the reconstructed space.

If the minimum embedding dimension is estimated to be larger than 3 and a graphical representation is desired, proceed to singular value decomposition for a possible reduction in dimensions.

Singular Value Decomposition:

5. Using a time lag of $\tau = 1$, estimate the minimum embedding dimension using the false nearest neighbors technique.
6. Perform principal component analysis on the reconstructed phase space vectors' covariance matrix.
7. Choose the number of principal components to use based on the proportion of variance explained. Rotate the reconstructed phase space vectors by the principal components.
8. If singular value decomposition was successful in reducing the number of dimensions to 3 or less, create a graphical representation of the reconstructed phase space. As this reconstruction is related to that in step 4 of the method of delays by a diffeomorphism, it contains all the same qualitative and quantitative properties and may be used in the same desired fashion.

Gauss code for implementing phase space reconstruction

```
new;
cls;
gausset;
library gaussplot pgraph;
// Set up library and include file with structure
definitions
#include gp.sdf
// Create gpData structure
struct gpData gdat;
struct gpPlotControl gp;


pqgwin many;


print "Phase Space Reconstruction ... written by Michael
McCullough";
print "";
print "The purpose of this program is to allow users to
implement";
print "phase space reconstruction on a given time series.
Phase space";
print "reconstruction may be implemented in a number of
ways.  This";
print "program utilizes the standard Method of Delays and
Singular Value";
print "Decomposition.  The Method of Delays is the primary
method for";
print "phase space reconstruction with the most up-to-date
parameter";
print "estimation techniques employed.  Singular Value
Decomposition";
print "is implemented in the instance where the Method of
Delays does not";
print "yield a phase space reconstruction embedding under
four dimensions.";
print "This program is included in Appendix A of the
dissertation";
print "PHASE SPACE RECONSTRUCTION:”;
print "METHODS IN APPLIED ECONOMICS AND ECONOMETRICS";
print "by Michael McCullough (contact author)";
print "The program is usable for academic purposes only
with explicit permission";
print "from the contact author.";
print "";
print "Press any key to continue...";
```

```
waitc;
cls;

print "The Method of Delays uses a two step procedure:";
print "";
print "Step 1  An optimal time lag is chosen so that the";
print "coordinates of the reconstructed phase space
contain";
print "the largest amount of new information without
redundancy.";
print "";
print "Step 2  A minimum embedding dimension is chosen.";
print "The embedding dimension must be large enough for
the";
print "strange attractor to completely unfold and yet not
so";
print "large that the attractor becomes indiscernible.";
print "";
print "Press any key to continue...";
waitc;
cls;

print "Let's get Started";
print "First the data set needs to be loaded";
print "Remember Gauss doesn't like spaces in the load
statement";
print "so make sure the file path doesn't have any.  Also,
the data should be";
print "organized as a single column vector without headings
saved as a tab";
print "delineated 'txt' or 'dat' file.";
print "";
print "Please type the file path below:";
print "For example c: gauss7.0 example.txt";
print "";
filepath = cons;
print "";
print "Please enter the number of observations for your
time series;";
nobs = con(1,1);
print "";
load x[nobs,1] = ^filepath;
cls;


print "The following is the graph of your time series.";

tim = seqa(1,1,nobs);
```

```
ylabel ("Your time series");
xlabel("Observation Number");
xy(tim,x);


print "Now the first Step of the Method of Delays: Choosing
the optimal time lag.";
print "";
print "A few different methods have been proposed for
choosing the optimal time lag.";
print "The first and most efficient is the Mutual
Information Method.  First introduced";
print "by Fraser and Swinney in 1986 the accompanying paper
provides an asymptotically efficient";
print "nonparametric method of estimation.  The Mutual
Information Function plots the";
print "average amount of predictable information
(redundancy) between time lagged vectors";
print "when increasing the time lag.  The optimal time lag
is chosen when the MIF reaches its";
print "first minimum.  This maximizes global information
between lagged time series vectors.";

print "";
print "For comparison and reference illustration two other
methods used in the past have been included.";
print "By plotting the time series against itself lagged to
the mth period a reference";
print "is made as to the optimal time lag.  Chose the time
lag where one of the following occur:";
print "The scatter plot losses form about the 45degree
line.  If the scatter plot exhibits";
print "a strong nonlinear pattern choose the time lag after
which the scatter plot either";
print "losses form or no new form is added.";
print "";
print "The plot of the autocorrelation function is also
supplied.  Previous research has";
print "suggested choosing the time lag where the ACF
reaches the first minimum or decreases";
print "below the tolerance 1/e.  This estimation technique
generally is no longer in use";
print "but the plot may add valuable insight into the
linear behavior of the time series.";
print "";
print "Press any key to continue...";

waitc;
```

```
cls;

maxlag:
print "Please input the maximum number of lags to include
in the analysis below:";
lag = con(1,1);

if lag > 100;
     print "It takes some time to compute the Mutual
Information Function so maybe a";
    print "smaller maximum time lag should be used.";
    print "";
    print "Do you really want to use this lag?  Please
enter Yes or No ...";
ans = cons;
if(ans $== "No") or (ans $== "N") or (ans $== "n") or (ans
$== "no") or (ans $== "NO");
     cls;
     goto maxlag;
endif;
elseif lag > rows(x);
    print "Woops, you can't test more lags than there are
observations! Try a smaller max lag.";
    goto maxlag;
endif;
//print "Press any key to find the Optimal Time Lag for
Phase Space Reconstruction...";
//waitc;
cls;

n=rows(x);
{akMI} = mike(x,lag,n);
{mifmin} = firstmin(akMI);
{auto} = OTL(x,lag);
{acfmin} = firstmin(auto);
cls;
print "The Mutual Information estimate of the Optimal time
lag is...";
mifmin;
print "";
print "";
print "The first minimum of the ACF is...";
acfmin;
print "";
print "";
print "Press any key to continue on to the Second part of
the Method of Delays:";
print "Finding the minimum embedding dimension.";
```

```
waitc;
print "";
print "";
print "For the minimum embedding dimension two estimation techniques will be utilized:";
print "The False Nearest Neighbors test and the Graphical False Nearest Neighbors Test.";
print "The False Nearest Neighbors test was introduced by Kennel et al. in 1992 with the";
print "the later Graphical test introduced by Aittokallio et al. in 2003.  Combining the";
print "two techniques decreases the chances of a conservative bias in the minimum embedding";
print "dimension.  The Minimum Embedding Dimension should be chosen where either the False";
print "Nearest Neighbors falls below some specified minimum percentage (Usually specified";
print "between 10% - 20%) or the density of observations in the Graphical False Nearest";
print "Neighbors test may be plotted below a line having an angle less than 90 degrees.";
print "";
print "Press any key to continue...";

waitc;
cls;

print "Both tests for a minimum embedding dimension require the previously estimated";
print "optimal time lag.  Please indicate below the chosen optimal time lag from the";
print "previous estimation techniques.";
print "The first minimum of the Mutual Information function was.";
mifmin;


tau = con(1,1);


fnntoll:
print "Please input the desired tolerance level for the False Nearest Neighbors test below:";
print "Remember the normal tolerance level is between 0.1 - 0.2.";
tol = con(1,1);

if tol < 0;
```

```
     cls;
     print "The tolerance level must be non-negative.";
     goto fnntoll;
endif;


maxdim:
print "Please input the maximum number of dimensions to
include in the analysis below:";
print "The normal amount of maximum dimensions is usually
under 100 for computational time";

mdim = rows(x)/tau;
print "";
print "";
mdimm = mdim - 5;
print "The maximum amount of dimensions you should test
with your time series is " mdimm;
print "The minimum shouldn't be below 5";
FNNDIM = con(1,1);

if FNNDIM > 100;
     print "It takes some time to compute the False Nearest
Neighbors test so maybe a";
    print "smaller maximum dimension should be used.";
    print "";
    print "Do you really want to use this dimension?
Please enter Yes or No ...";
ans = cons;
if(ans $== "No") or (ans $== "N") or (ans $== "n") or (ans
$== "no") or (ans $== "NO");
     cls;
     goto maxdim;
endif;
elseif FNNDIM*tau > rows(x);
    print "Woops, you can't test this many dimensions with
the length of your time series.";
    print "Try a smaller maximum embedding dimension.";
    goto maxdim;
endif;
//print "Press any key to find the Minimum Embedding
Dimension for Phase Space Reconstruction...";
//waitc;
cls;

{FalseNN,d} = FNN(FNNDIM,tau,x,tol);
cls;
```

```
print "The False Nearest Neighbors drops below the
tolerance level at the following dimension";
d;

print "";
print "";
print "If the Graphical test yields a dimension containing
the density of observations";
print "below a line with angle less than 45 degrees use
that as the minimum embedding";
print "dimension.";
print "";
print "Please indicate the chosen minimum embedding
dimension below based on the False";
print "Nearest Neighbors and Graphical tests";

d = con(1,1);
print "";
print "";
print "Now we're ready to reconstruct phase space.  The
following graph is the";
print "reconstructed phase space for your time series.  If
the time series is greater";
print "than three dimensions then obviously it cannot be
graphed but quantitative analysis";
print "may still be performed.  If a visualization is still
desired a prompt will ask if";
print "singular value decomposition should be implemented";
print "";
print "The phase space reconstruction matrix may be
accessed by";
print "clicking on the symbols tab and then on the Matrices
tab.  The matrix labeled 'PSR'";
print "is the phase space reconstruction matrix.";
print "";
print "Press any key to continue...";
waitc;
print "";
print "";
if d > 3;
cls;
print "It appears that the minimum embedding dimension
estimated using the Method of Delays";
print "is too large to graph.  Would you like to implement
Singular value decomposition?";
ans = cons;
if(ans $== "Yes") or (ans $== "Y") or (ans $== "y") or (ans
$== "yes") or (ans $== "YES");
```

```
     cls;
     goto singular;
endif;
endif;


mod:
cls;
print "The following graph is the phase space
reconstruction of your time series.";
print "Use it to perform qualitative and/or quantitative
analysis on the time series.";
print "If nonlinear dynamics drive your time series they
will have been embedded one-to-one";
print "onto the phase space reconstruction.";
print "";
//print "Press any key to continue...";
//waitc;

/*----------------------------Plotting the Phase Space
Reconstruction------------------------*/


if d == 2;
goto twodim;
elseif d == 3;
goto threedim;
elseif d > 3;
goto maxdimm;
endif;


twodim:

n = rows(x);
y = zeros(n-tau,2);

i=1;
do until i > n-tau;
y[i,1]=x[i];
y[i,2]=x[i+tau];
i=i+1;
endo;
graphset;
ylabel ("X(t)");
xlabel("X(t-tau)");
xy(y[.,1],y[.,2]);
goto finish;
```

```
threedim:

gp = gp3DCartesianPlotCreate;


n=rows(x);
y = zeros(n-2*tau,3);

i=1;
do until i > n-2*tau;
y[i,1]=x[i];
y[i,2]=x[i+tau];
y[i,3]=x[i+2*tau];
i=i+1;
endo;


// Create gpPlotControl structure and set members to
default values
struct gpPlotControl gp;
gp = gp3DCartesianPlotCreate;
// Put data for each zone into a different array
a = aconcat(y[.,1],y[.,2],3);
a = aconcat(a,y[.,3],3);

// List variable names
string vnames3 = { "X(t)", "X(t-tau)", "X(t-2tau)" };

// Set data in gpData structure
gdat = gpInitPlotData(vnames3);
ret = gpAddZone(&gdat,"Phase Space Reconstruction",a); //
field zone one

// Write data file
ret = gpWritePlotData(&gdat,"psr.plt");

// Specify names for macro file to be created and data file
to use
gpSetMacroFile(&gp,"psr.mcr");
ret = gpSetDataFile(&gp,0,"psr.plt");

// Set axes titles
ret = gpSetXAxisTitle(&gp,1,1,"X(t)");
ret = gpSetYAxisTitle(&gp,1,1,"X(t-tau)");
ret = gpSetZAxisTitle(&gp,1,"X(t-2tau)");

// Set the offset of the Z-axis title from the axis line
```

```
ret = gpSetZAxisTitleOffset(&gp,1,12);

// Change plot fit type so that you can see the entire grid
in the frame
ret = gpSetPlotFit(&gp,1,2);

// Add field zone legend
ret = gpAddFieldZoneLegendFrame(&gp,1);
ret = gpMoveFieldZoneLegend(&gp,1,99,9);
ret = gpSetFieldZoneLegendFont(&gp,1,"helv",14,2);
ret = gpSetFieldZoneLegendLineSpacing(&gp,1,1.25);

// Plot the graph
ret = gpPlot(&gp);

goto finish;


maxdimm:
n=rows(x);
p=D*tau;
y = zeros(n-p,D);

for h (1,n-p,1);
    for u (0,D-1,1);
    y[h,u+1] = x[h+u*tau];
    endfor;
endfor;
goto finish;


finish:
PSR = y;

goto done;



singular:
cls;
print "The following graphs are the phase space
reconstruction using singular value";
print "decomposition for one and two dimensions.  Choose
which one to use based on";
print "proportion of variance explained by each eigenvalue
displayed in the third graph.";
print "";
print "Press any key to continue...";
```

```
waitc;
{PSR} = SVD(X,d);

goto done;

done:
end;


/*---------------------------Procedures Used in PSR---------
-----------------*/

/*   MIKE() Procedure {akMI} = mike(X,lag,n);
**
**   This procedure calculates the mutual information
function I(x) for increasing
**  time lags for use in phase space reconstruction (Fraser
and Swinney, 1986).
**  The procedure uses the Epanechnikov kernel density
estimator to calculate
**  the joint and marginal density functions (Mittelhammer
et al., 2000).
**
**
**   The required inputs to the procedure are
**
**       X              an (n x 1) column vector time
series.
**      lag         the number of lags to include in the
analysis.
**      n           number of observations in time series
X.
**
**   The output returned by the Average Mutual Information
Procedure is
**
**        AKMI      the average mutual information function
from kernel density estimator.
**
*/



/*============================= Procedures used in MIKE
==============================*/

/*   Procedure for the two-dimensional kernel function */
```

```
proc Mkernel(xo, x, h);
     local kernvals, u, points;
     u = (xo - x)/h;
     points = sumc((u.*u)');
     kernvals = (2/pi)*(1 - points).*(points .<= 1);
     retp(sumc(kernvals)/(rows(x)*h*h));
endp;


/*   Procedure for the one-dimensional kernel function */

proc kernel(xo,x,h);
     local kernvals,u;
     u= (xo-x)/h;
          kernvals = .75*(1-u^2).*(abs(u).<=1);
     retp(sumc(kernvals/(rows(x)*h)));
     endp;


/*   Procedure for the convolution of the Epanechnikov
kernel function  */

proc EpanConv(u);
     local epn, a, signs;

     signs = (u .>= 0) - (u .< 0);
     a = u - signs;
     epn = (u/2)-((a^5)/5)+(u.*(a^4)/2)+(2*(a^3)/3)-
((a^3).*(u^2)/3)-(u.*a.*a)-a+(a.*(u^2));
     epn = signs.*epn + (8/15) - (2*(u^2)/3);

retp(0.5625*(abs(u) .< 2).*epn);
endp;



/*   Procedure to calculate Mutual Information using Kernel
density Estimation */


proc mike(x,lag,n);
local hx,hy,hz,akmi,m,i,p,y,k,fxi,fxj,fxij,j,a,laga;

/*   Calculate the Mutual Information function using the
Epanechnikov Kernel
**  density Estimation function
*/
```

```
akmi = zeros(lag,1);
m = rows(x);
{hx} = hopt(x,m);
i=1;
do until i > lag;



format /mat /rd 5,1;
     locate(5,1);
     print /flush "  Evaluating the mutual information
functions at the time lag " i " out of " lag;

p = m-i;

y = zeros(p,i+1);
for h (1,p,1);
    for u (0,i,1);
    y[h,u+1] = x[h+u];
    endfor;
endfor;

k=rows(y);

//{hy} = hopt(y[.,i+1],k);
// hz  = (hx+hy)/2;


fxi = zeros(k,1);

for jj (1, k, 1);
     j = jj;
     fxi[j] = kernel(y[j,1], y[.,1], hx);
endfor;

fxj = zeros(k,1);
for jj (1, k, 1);
     j = jj;
     fxj[j] = kernel(y[j,i+1], y[.,i+1], hx);
endfor;

fxij = zeros(k,1);
for jj (1, k, 1);
     j = jj;
     fxij[j] = mkernel(y[j,1 i+1], y[.,1 i+1], hx);
```

```
endfor;

AkMI[i] = sumc(fxij.*ln(fxij./(fxi.*fxj)));
i = i+1;
endo;


laga = seqa(1,1,lag);

cls;

ylabel ("Mutual Information Function");
xlabel("Lag");
bar(laga,akmi);

retp(akMI);
endp;


/*   Procedure to calculate the optimal h using the
Epanechnikov kernel function  */

proc hopt(x,n);
local
hgrid,ngrid,mh,u,h,holdit,hstar,hstarind,sumpart,xMINUSi;
hgrid = seqa(0.1, 0.1, 50);
ngrid = rows(hgrid);
mh = zeros(ngrid,1);
u = zeros(n,n);

for k (1,ngrid,1);
     h = hgrid[k];

for i (1,n,1);
for j (1,n,1);
    u[i,j] = (x[i] - x[j])/h;
endfor;
endfor;

     mh[k] = mh[k] + sumc(sumc(EpanConv(u)));
     mh[k] = mh[k]/(h*n*n);
     sumpart = 0;
     for i (1, n, 1);
          xMINUSi = setdif(x, x[i], 1);
          sumpart = sumpart + kernel(x[i], xMINUSi, h);
     endfor;
     sumpart = 2*sumpart/n;
     mh[k] = mh[k] - sumpart;
```

```
endfor;
holdit = hgrid~mh;
hstarind = minindc(holdit);
hstarind=hstarind[2,1];
hstar = holdit[hstarind,1];

retp(hstar);
endp;

/*   Procedure to calculate the first minimum of a vector
     */

proc firstmin(x);
local diff,fmin,i,n;
diff = zeros(rows(x)-1,1);
fmin = 1;
n = rows(x);
for i (1,n-1,1);
diff[i] = x[i+1]-x[i];
endfor;

if maxc(diff) < 0;
fmin = maxindc(diff) + 1;
goto fin3;
endif;

i=1;
do until diff[i] >= 0;
if diff[i] <= 0;
    fmin = fmin + 1;
endif;
i=i+1;
endo;
fin3:
retp(fmin);
endp;

/*=====================        Optimal Time Lag Graphical
Plots     =====================*/

proc OTL(x,lag);
local m,p,y,a,auto,pauto;


m = rows(x);
p = m-lag;
y = zeros(p,lag+1);
for h (1,p,1);
```

```
    for u (0,lag,1);
    y[h,u+1] = x[h+u];
    endfor;
endfor;

a = lag;
auto = acf(x,a,1);
lag = seqa(1,1,a);

graphset;
_plctrl = {-1};
begwind;
window(3,3,0);
setwind(1);
xlabel ("X(t)");
ylabel("X(t-1)");
xy(y[.,1],y[.,2]);

nextwind;
ylabel("X(t-2)");
xy(y[.,1],y[.,3]);

nextwind;
ylabel("X(t-3)");
xy(y[.,1],y[.,4]);

nextwind;
ylabel("X(t-4)");
xy(y[.,1],y[.,5]);

nextwind;
ylabel("X(t-5)");
xy(y[.,1],y[.,6]);

nextwind;
ylabel("X(t-6)");
xy(y[.,1],y[.,7]);

nextwind;
ylabel("X(t-7)");
xy(y[.,1],y[.,8]);

nextwind;
ylabel("X(t-8)");
xy(y[.,1],y[.,9]);

_plctrl = {0};
```

```
_pline = {1 1 0 0 50 0 1 5 0};
nextwind;
ylabel ("Autocorrelation Function");
xlabel("Lag");
bar(lag,auto);

endwind;

retp(auto);
endp;
```

```
/*-----------------------False Nearest Neighbors Tests----
--------------------*/
/*   FNN() Procedure {FalseNN,d} = FNN(FNNDIM,tau,x,tol);
**
**   This procedure is designed to perform the false
nearest neighbors test
**  on a given time series.  The procedure calculates the
percent of false
**  nearest neighbors for a given optimal time lag and
tolerance level as proposed
**  by Kennel et al. (1992).  The procedure also implements
the Graphical method
**  for a minimum embedding dimension proposed by
Aittokallio et al. (2003).
**
**
**   The required inputs to the procedure are
**
**   X         an (n x 1) column vector time series.
**   tau       the optimal time lag.
**      FNNDIM      the maximum number of dimensions.
**      tol              tolerance level of false nearest
neighbors.
**
**   The outputs returned by the False Nearest Neighbors
procedure are
**
**   D         the minimum embedding dimension.
**   FalseNN        the percent of false nearest neighbors
for each dimension.
**      Plot             Plot of percent of false nearest
neighbors by dimension
**
```

```
*/


proc (2) = FNN(FNNDIM,tau,x,tol);
local
m,p,y,n,D1,k,FalseNN,atol,rtol,rd,rdelta,DEN,nei,NUM,dim1,d
im,j,i,d,dims,ra;

m = rows(x);
p = m-FNNDIM*tau;
y = zeros(p,FNNDIM+1);
for h (1,p,1);
    for u (0,FNNDIM,1);
    y[h,u+1] = x[h+u*tau];
    endfor;
endfor;

n = rows(y);
D1 = zeros(n,1);
k=1;
FalseNN = zeros(FNNDIM,1);
rd = zeros(n,FNNDIM);
rdelta = zeros(n,FNNDIM);
do until k > FNNDIM;

format /mat /rd 5,1;
     locate(5,1);
     print /flush "  Evaluating the False Nearest Neighbors
for dimension " k " out of " FNNDIM;

dim = k;
atol = 2;
     /*False Nearest Neighbor Tolerance Level*/
rtol = 10;
ra = (stdc(x));



for i (1,n,1);
    j = 1;
    do until j > n;
    D1[j] = sqrt((y[i,1:k]-y[j,1:k])*(y[i,1:k]-y[j,1:k])');
     /*Euclidean Distance*/
    j = j+1;
    endo;
        D1[i] = 10000000;
     /*Delete Distance from self*/
```

```
    DEN = minc(D1);
     /*Nearest Neighbor Distance*/
     nei = minindc(D1);
        dim1 = dim + 1;

    NUM = abs(y[i,dim1] - y[nei,dim1]);
     /*Coordinate Distance of Nearest Neighbor increased
Dimension*/

        if (NUM/DEN) > rtol;
            FalseNN[k] = FalseNN[k] + 1/n;
        endif;
        if (DEN^2+NUM^2)/(ra^2) > atol;
            FalseNN[k] = FalseNN[k] + 1/n;
        endif;

    rdelta[i,k] = NUM;
    rd[i,k] = DEN/sqrt(dim);
endfor;
k=k+1;


endo;
if minc(FalseNN) > tol;
d=0;
goto fin2;
endif;

d = 1;
i=1;
do until FalseNN[i] < tol;
if FalseNN[i] > tol;
    d = d+1;
endif;
i=i+1;
endo;

cls;

fin2:

cls;

begwind;
_plctrl = {-1};
window(3,2,0);
setwind(1);
ylabel ("");
```

```
xlabel("Dimension 1");
xy(rd[.,1],rdelta[.,1]);

nextwind;
xlabel("Dimension 2");
xy(rd[.,2],rdelta[.,2]);

nextwind;
xlabel("Dimension 3");
xy(rd[.,3],rdelta[.,3]);

nextwind;
xlabel("Dimension 4");
xy(rd[.,4],rdelta[.,4]);

nextwind;
xlabel("Dimension 5");
xy(rd[.,5],rdelta[.,5]);
endwind;

/*Plot of False Nearest Neighbors with increasing
Dimensions*/
_pline = {1 1 0 .2 50 .2 1 5 0};
_plctrl = {0};
dims = seqa(1,1,rows(FalseNN));
ylabel("Percent of False Nearest Neighbors");
xlabel("Dimension");
xy(dims,FalseNN);

cls;
retp(FalseNN,d);
endp;


/*------------------------Singular Value Decomposition-----
--------------------*/

proc SVD(X,FNN);
local
SVDdim,m,p,y,n,vary,psr,e1,e2,e3,evalues,evectors,var,por,d
ims;

SVDdim=FNN;
m = rows(x);
p = m-SVDdim;
y = zeros(p,SVDdim);
for h (1,p,1);
    for u (0,SVDdim-1,1);
```

```
    y[h,u+1] = x[h+u];
    endfor;
endfor;
n = rows(y);
vary = vcx(y);
{ evalues,evectors } = eigv(vary);
PSR = zeros(n,3);

e1 = evectors[.,1];
PSR[.,1] = y*e1;
e2 = evectors[.,2];
PSR[.,2] = y*e2;
e3 = evectors[.,3];
PSR[.,3] = y*e3;
psr = real(psr);

evalues = real(evalues);
var=sumc(evalues);
por=evalues/var;
dims = seqa(1,1,rows(por));
por;
/*Plot of Eigenvalues*/
begwind;
window(3,1,0);
setwind(1);

graphset;
ylabel("Eigenvalues");
xlabel("Number");
xy(dims,por);

nextwind;
ylabel("e1");
xlabel("e2");
xy(psr[.,1],psr[.,2]);

nextwind;
ylabel("");
xlabel("");
xyz(psr[.,1],psr[.,2],psr[.,3]);
endwind;
retp(psr);
endp;
```

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ESSAY THREE:

HAVE LIVESTOCK CYCLES DIMINISHED OVER TIME?

Abstract:

This paper applies the technique of phase space reconstruction to investigate U.S. livestock cycles. Results are presented for both pork and cattle cycles, providing empirical evidence that the cycles themselves have slowly diminished. By comparing the two livestock cycles important insights can be made. The phase space analysis suggests that the biological constraint has become a less significant factor in livestock cycles while technology and information are relatively more significant.

Key Words: cattle cycles, pork cycles, nonlinear time series, phase space reconstruction, technological change

JEL Classification: C14, Q11

Introduction

The technique of phase space reconstruction is applied to empirically investigate hog and cattle cycles in the U.S. Current research on hog and cattle cycles employs a variety of modeling techniques to analyze the dynamic properties found in the cycles. In particular, the focus of this literature, discussed in detail below, has been to identify the primary factors driving these cycles. Concurrently, innovations modeling macroeconomic business cycles suggest that the business cycle in-and-of-itself has been changing over time. The central idea of the current paper is to reconstruct cattle and hog cycles to establish further evidence in evolving patterns of cycle length, magnitude, and volatility. We then draw on recent livestock and business cycle literature to provide plausible explanations for recent changes in livestock cycles.

It is well known in business cycle literature that variability in output growth across the G-7 nations has greatly decreased. Recent contributions to this literature: Stock and Watson (2005); Doyle and Faust (2005); and Canova, Ciccarelli, and Ortega (2007) estimate the converging properties of these cycles. The debate therein lies in the cause of volatility change; whether or not it is due to good monetary policy, inventory management, or simply reduced external shocks. McConnell and Perez-Quiros (2000); Kahn, McConnell, and Perez-Quiros (2002); and Stock and Watson (2003) effectively show that for the United States the reduction of volatility in output growth can be attributed to changes in technology and information.

Through phase space reconstruction, we are able to identify short and long run dynamics and other qualitative features of livestock cycles. Phase space reconstruction, using nonparametric nonlinear time series techniques, allows for the extraction of an underlying structural system from a single observed time series. In the spirit of Chavas and Holt (1991) the dynamic properties of the cattle and hog cycles are illustrated using Takens' (1981) embedding theorem.

Results from the phase space reconstruction exhibit nonlinear dynamics consistent with previous findings, while also suggesting a longer run change in the characteristics of the livestock cycle much like that observed in business cycles. We show through a combination of descriptive statistics, phase space analysis, and previous literature that livestock cycles have been diminishing in recent years.

The study proceeds in the following manner; previous livestock cycle research is discussed, followed by the theory and application of phase space reconstruction to United States cattle and pork inventories. The next section makes specific comparisons between the two livestock cycles and draws upon past research and economic reasoning for plausible explanations for recent patterns. Final remarks and suggestions for future research conclude the paper.

Previous Livestock Cycle Research

Both cattle and pork cycles have been researched extensively using a wide variety of methods. The observation that cycles are prevalent in livestock markets is far from new. From Ezekiel's (1938) treatment of the pork and milk industry with the cobweb theorem to recent contributions involving nonlinear time series analysis the previous assessments have genuinely shown the depth of ingenuity in the field of economics. After John Muth's (1961) theory of rational expectations the majority of this research has focused on explaining why cycles might exist in competitive markets such as these. Five observations become apparent after a review of the livestock cycle literature:

1) The biological constraints inherent in livestock accumulation have played a large role in the pervasive cyclical effect.
2) Model results are driven by how ranchers are assumed to form expectations about markets.
3) Nonlinear methods such as dynamic and nonlinear time series models describe the cycles more accurately than linear methods.
4) Since most of the research was published before 2003, data sets have been limited to the 19^{th} and 20^{th} centuries.
5) The livestock cycle literature can be loosely split into two groups, dynamic models and time series models.

Dynamic Livestock Models

The dynamic model literature follows Jarvis (1974) in applying basic microeconomic principles to the cattle market. Jarvis illustrated that by correctly modeling the cattle herd as being comprised of different capital goods, i.e. calves, cows, steers, etc., a better explanation of producer responses to prices can be found. He also found the supply of beef to be backward bending. By behaving rationally, producers react to price increases by decreasing slaughter rates in the short run to increase herd size thus increasing supply in the long run.

The next set of dynamic models came from Rosen in 1987 and Rosen, Murphy, and Scheinkman in 1994. Rosen simplified the analysis and focused on the specific interaction of rational ranchers market decisions with the population dynamics of a herd.

The analysis concluded that the backward bending supply curve was a result of a permanent demand shock, and that transitory demand shocks result in what economists would view as "normal" behavior. The second adaptation of Rosen, Murphy, and Scheinkman incorporated a more accurate adaptation of the biological constraints faced by ranchers in a rational expectation framework. Through this framework they show that the cyclical process in cattle inventories is due to exogenous demand shocks.

In response to these models, Nerlove and Fornari (1998) created a model that tested the existence of quasi-rational expectations over fully rational expectations. They find that in a dynamic framework their model follows the results of the previous research while allowing for a greater degree of flexibility.

In 2000, two papers were published independently that offered somewhat similar insights into what might be causing the cyclical effects. Chavas estimated, using a novel dynamic model, the existence of heterogeneous expectations among ranchers. The evidence presented confirms rational expectations in the cattle market while maintaining the persistence of the cattle cycle as a result of not all ranchers having these forward-looking expectations. Similarly, Hamilton and Kastens use the dynamic model incorporating what they call a "market timing effect." They proposed that if some ranchers behaved in a counter-cyclical fashion, i.e. had rational expectations, the cycle would eventually die out. However, if there existed a mix of ranchers whom had different decision-making processes, countercyclical, constant-inventory, and representative of the aggregate, the cycle will persist.

The last set of dynamic models come from David Aadland and DeeVon Bailey. Aadland and Bailey (2001) incorporate the most sophisticated biological and market constraints to date. Unlike the previous two articles, they assume producers to be the traditional identical forward-looking profit maximizers. They separate the fed and unfed cattle sector and allow producers to make adjustment decisions between these two capital goods which allows for the possibility of Jarvis' backward bending supply curve. In Aadland (2004) the developments of Chavas (2000) are combined with the previously mentioned paper so that for the first time extensive biological and market constraints are combined with heterogeneous expectations. The model made it possible for the observed decade long cattle cycle to be endogenously propagated. He then related the periodicity of the cattle cycle to four large macroeconomic shocks.

Time Series Livestock Models

The time series models have experienced a similar development as the dynamic models. They are found more readily in the analysis of the pork cycle and are comprised of a rich array of econometric and time series techniques. The first, Rucker, Burt, and LaFrance (1984) uses the flexibility of their estimation technique to estimate Jarvis' model. They find that by analyzing both fed beef and corn prices a superior model is built. They also make the concession that aggregate models could be severely understating the differing regional production technologies and suggest this as potential future research.

In 1991 Chavas and Holt published their seminal paper on pork dynamics. They expressed the importance of nonlinear state equations in dynamic models. This type of setup allows for an exceptionally large scope of analysis as linear state equations, while useful, cannot fully capture nonlinear processes. They use nonlinear time series analysis to estimate the pork cycle and show that given rational expectations the existence of a pork cycle must be due to some unknown randomness; the cycle cannot be perfectly predictable for it to persist.[20]

Mundlak and Huang (1996) use autocorrelation and cross-correlation functions as well as spectral decomposition to show that, ceteris paribus, countries with differing production technologies exhibit similar production cycles and different price cycles. While their treatment does isolate interesting idiosyncrasies between countries, the use of linear time series techniques, as will be shown below, did not fully capture the cyclical effects.

The last paper published addressing livestock issues follows Chavas and Holt (1991). Holt and Craig (2006) use sophisticated nonlinear time series techniques to analyze the hog-corn cycle from 1910-2004. The variant STAR model allowed for the explicit modeling of structural change in addition to the inherent nonlinear behavior. They found that the cycle's response to shocks has fundamentally changed over the years, and that by the end of their sample period has all but disappeared. They hypothesize that the pork cycle has been evolving over time as a function of institutional and technological change.

Reconstructing Livestock Cycles

Since its development phase space reconstruction has become an essential part of nonlinear dynamics (Packard et al., 1980, Takens, 1981). It has been incorporated into various areas of research from Schaffer and Kot's SEIR model of epidemics to Zaldívar's

[20] It is interesting to note that the embedding theorem that drives Chavas and Holt (1991) and Chavas and Holt (1993) is the same that motivates this paper. While this application of the embedding theorem is novel in economics, through Chavas and Holt the theorem itself has laid the foundation for other important economic research.

forecasting of Venice water levels, phase space reconstruction is the qualitative benchmark for nonlinear analysis (Schaffer and Kot, 1985, Zaldívar et al., 2000). It allows for the basic properties of the system to be determined and subsequent qualitative analysis to be performed without any prior knowledge of a system. This is analogous to nonparametric regression, which allows for the relationship of variables to be determined without imposing restrictions or prior functional form.

The nonlinear time series methods used in this paper are motivated and based on the theory of dynamical systems in phase space (Takens, 1981).[21] The general idea of phase space reconstruction is that a single scalar time series may have sufficient information with which to reconstruct a dynamical system, much like a single stain of DNA contains sufficient information to reproduce an entire organism.

Data are often observed as a temporal sequence of scalar values. For any event, n outcomes are observed as a subset of the total population and are denoted by the time series vector $X_t = \left[x(t), x(t-1), \ldots, x(t-n)\right]'$. For future reference the τ^{th} lag of this vector will be referred to as $X_{t-\tau} = \left[x(t-\tau), x(t-1-\tau), \ldots, x(t-n-\tau)\right]'$. The challenge is to convert the sequence of scalar observations into state vectors and reproduce dynamics in phase space. Then one can study the dynamics of the system by learning the dynamics of the phase space trajectories, which is particularly useful in complex systems (see examples in Kantz and Schreiber, 1997).[22] This reproduction makes it possible to

---

[21] Typically phase space is defined as the space in which some geometric structure exists. In very general terms every trajectory of the structure of question may be represented as a coordinate in its particular phase space. For qualitative analysis we will always be referring to phase spaces of two and/or three dimensions.

[22] Phase space reconstruction is a diffeomorphism that reproduces a time series on a plane that mirrors the phase portrait of the underlying system. A diffeomorphism is a smooth function, Φ, that maps one differential manifold, M, onto another, N, whose inverse, Φ', is also a smooth function that maps N onto M.

qualitatively delineate short or long run processes that evolve over time and generally better understand the nature of the underlying dynamical system; this is the essence of phase space reconstruction.

Embedding

The idea of embedding attractors onto different spaces and in different dimensions is an important concept in the theory of dynamical systems.[23] It was not until Packard first proposed that this be done from measured time series that the idea of phase space reconstruction was formed (Packard et al., 1980). Packard proved that the embedding of the geometry of a strange attractor may be represented by a series of differential equations.[24] Takens extended this proof to encompass what is now known as the method of delays (Takens, 1981). The Method of Delays is a diffeomorphism of an attractor with dimension m onto a phase space of dimension n where $n \geq 2m + 1$. For empirical application, the Method of Delays requires an optimal time lag τ be chosen followed by a minimum embedding dimension λ. Once the two parameters are estimated, the time series X_t will generate a reconstructed phase space matrix $Y_\lambda = \lfloor X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(\lambda-1)\tau} \rfloor$ with dimension $[(n - \lambda\tau) \times \lambda]$.[25]

This mapping preserves all geometric properties of the original figure. Much like a topographical map preserves all geometric properties of the earth.

[23] An embedding is the mapping process used to reproduce geometric figures onto different spaces. Again it is analogous to creating a two-dimensional map of the three-dimensional world. Not all embeddings are diffeomorphisms, just like not all maps contain all the properties of the area they cover. Nonetheless, even though road maps don't usually contain elevation gain they provide a great deal of information.

[24] An attractor is a subset of a space onto which a system evolves to over time. A strange attractor is an attractor that allows for a greater degree of flexibility in that the subset of the space may be fractal, i.e., the dimension of the space does not have to be a real integer.

[25] As discussed before, the process of phase space reconstruction is much like map making. The reconstruction is a map containing all geometric properties of the original system that drives the dynamics of the observed time series. Through Takens' embedding theorem it is possible to extract this map of the underlying dynamics from a single time series.

The time lag τ is paramount to empirical applications of Takens' theorem. While the condition $n \geq 2m + 1$ is sufficient, it is not necessary. By choosing a time lag that first minimizes the redundancy between the column vectors in matrix Y_λ the geometry of the original manifold will be preserved even when the time series is contaminated with noise (Fraser and Swinney, 1986).

Given the choice of optimal time lag, the minimum embedding dimension λ can be estimated. Kennel, Brown, and Abarbanel (1992) developed the False Nearest Neighbors technique for choosing a minimum embedding dimension. If λ is too small the reconstruction cannot unfold the geometry of the possible strange attractor.[26] If λ is too large procedures used to determine basic properties of the system and qualitative analysis may become unreliable (Aittokallio, et al., 1999, Kennel, et al., 1992).

Cattle Cycles

Cattle cycle inventory data come from the National Agricultural Statistics Service (NASS) of the USDA. The time series consists of yearly quantities of cattle and calves measured in January from 1887-2007. We have provided descriptive statistics, Table 1, portioned every twenty years and refer the reader to the standard deviations as measures of unadjusted volatility. The table provides some basic insights into the changing behavior of the cattle cycle over the sample period. The mean is greater than the median for seven of the eight periods indicating that generally the time series has been increasing, hence the unit root. The standard deviation decreased substantially from 1867-1950 after which a large increase in volatility occurred. The increase in volatility can be attributed to producer response in large fluctuations in retail beef consumption.

[26] An example of an embedding dimension being too small would be a 2-dimensional representation of a cube. In 2-dimensional space the cube appears to be a square. In 3-dimensional space the true geometry of the cube is clearly not a square but a much more complex figure.

The last observation period contains the smallest standard deviation of the entire sample; meaning that volatility has decreased to its minimum over the sample period in the last twenty years.

The quantity of beef supplied to a market in a year by the rancher depends on the size of her stock of cattle. The stock of cattle at the rancher's disposal depends on the amount of cattle retained from the previous year. This time dependent process is indicative of the statistical process known as a unit root and can be seen clearly in the supply time series in Figure 1. The traditional methods for estimating the optimal time lag, the mutual information function, exhibits a long memory process that complicates the estimation of the optimal time lag. This long memory process is presented in Table 2, the mutual information and autocorrelation functions for the supply of beef. However, since the mutual information function is relatively small we know that the optimal time lag must also be small.

Traditional methods of analysis like those presented in the section above are impeded in the presence of a unit root. This is the motivation for the majority of the literature to analyze detrended livestock data. However, Broomhead, Huke, and Muldoon (1992) show that certain filters will eliminate nonlinear processes. As phase space reconstruction was designed to analyze nonlinear processes, unit roots have no effect on the embedding theorem itself. They do disrupt the nonparametric parameter estimation techniques so that they must be supplemented with the geometric methods discussed in McCullough (2008). In doing so this analysis is free of restrictive filters and intertemporal dynamics may be delineated whereas in previous research they could not. Using the geometric method the optimal time lag is estimated to be $\tau = 3$.

To estimate the minimum embedding dimension we use the false nearest neighbors method described in Kennel et al (1992). Table 3, contains the percentage of false nearest neighbors found for each dimension. It is clear that the structure of the system may be embedded on a dimension as small as two, certainly no more dynamics will unfold by increasing the embedding dimension larger than three. The phase space reconstruction is presented in Figure 2 and will be that which the qualitative analysis will be based upon.

It is no surprise that the phase space reconstruction of cattle inventories exhibits an overlapping cyclical pattern from the beginning of the trajectory in the lower left corner of Figure 2. The cluster of overlapping cycles in the phase space trajectory show that cattle inventories, while increasing slightly, followed the same pattern over the first three cycles. What is not necessarily apparent in the original time series, Figure 1, is that these cycles have been steadily diminishing with time.

Observing the middle portion of the phase space reconstruction indicated with a solid line, the overlapping cycles are distinctly decreasing so that by the time when the last overlap should occur only a slight bump appears. This is an indication that the severity of the cycles has decreased. In fact, it can be seen in the cycles of the original time series quit easily. Table 4 contains descriptive statistics that differentiate the cattle cycles. Amplitude is the difference between the initial cycle inventory and its lowest inventory and can be thought of as a type of severity measure, or magnitude, for each cycle. Table 4 also contains the wavelength of the cycles measured from peak to peak. Form the late 19th century a downward trend in amplitude is present with a large increase during the 1975-1981 cycle.

The next few cycles of the phase space trajectory follow a recursive trend as the result of permanent demand shocks as well as other external factors. After 1990, however, per capita consumption of beef reached a steady state in which only temporary adjustments about the mean have been observed. McCullough (2008) shows that the steady pattern of beef consumption is permanent and should be expected baring any unforeseen large macroeconomic shocks or unlikely fundamental changes in behavior. Figure 3 overlays per capita consumption with total inventories to illustrate the point that since 1960, inventories have nearly mirrored consumption. The phase space reconstruction during the final period shows a small cycle nearly half the size of its early counterparts, suggesting a mitigation of the cyclical process. During the same period, the final two cycle amplitudes in Table 4 decrease substantially from the previous cycle. This is also the trend that is seen in the standard deviation reported in Table 1. That which is called the "Great Moderation" in the business cycle (see Stock and Watson, 2003) appears to be present in the cattle cycle as well. Combining the change of hog cycles and general business cycles helps to explain the cause of this mitigation.

Hog Cycles

Yearly total inventory of hog and pigs in the United States from 1866-2006 are analyzed. Taken from NASS, inventory is measured on December 1 and descriptive statistics for every twenty years are provided in Table 5. What was seen as slight moderation in cattle inventories is more than apparent in pork inventories. The standard deviation has decreased ten-fold from the 1929-1949 cycle while the mean and median have slightly increased. This is direct evidence of decreased volatility in pork inventories, which the following phase space analysis confirms.

As with the total inventory of cattle the time series of total inventory of hogs and pigs requires the use of methods unaffected by unit roots. The optimal time lag is estimated to be $\tau = 3$. The minimum embedding dimension is estimated using the false nearest neighbors test, see Table 6. The percentage of false nearest neighbors decreases from 30% to 19.75% when increasing the embedding dimension from $\lambda = 2$ to $\lambda = 3$. For an application such as this, a percentage of false nearest neighbors between 20% and 30% is quite acceptable. For this reason, as well as for purposes of clarity, we present and draw inferences from a phase space in two dimensions.

The phase space reconstruction of pig and hog inventories requires a slightly more detailed analysis. In Figure 4, the time series of total pig and hog inventories, the well-documented hog cycle can be seen with a fluctuation between peaks and troughs between four and six years. The first six or seven cycles are just as apparent in the phase space trajectory, Figure 5. After this point the phase space trajectory enters what appears to be a birds nest type state, indeed Table 5 shows an increase in standard deviation from 1908-1970. This period was one of large macroeconomic shocks and technological change, after which volatility and amplitude, see Table 7, decrease to levels comparable to those before refrigeration. Comparing volatility, wavelength, and amplitude post 1980 to that before 1910 raises the question whether or not the *structure* of pork cycles during these periods are the same as well. To answer this question, Figure 6 shows the phase space reconstruction of total hogs and pigs between 1886-1910 and 1980-2006. From this phase space reconstruction we can draw two general observations.

1. The volatility of pork inventories has decreased substantially from previous decades to a level comparable to the time when pork markets were entirely local.

2. The cyclical nature of the pork cycle has changed. The type of cyclical present pre-1910 is fundamentally different from that post-1980.

Figure 7 plots retail pork consumption overlaid with total hog and pig inventories and the same general effect seen in the overlay of cattle inventory and consumption can be seen, but with much more accuracy. Total inventories follow retail consumption quarterly since 1990 nearly identically. Seasonal patterns are met without fail, as are the slightly longer-run adjustments in consumption. As it is with cattle cycles, the amplitude of the pork cycle has decreased significantly in the last twenty-five years. The volatility of the pork cycle is plotted in Figure 8 and as expected has decreased to a level that has not been seen since the time before refrigeration (see Table 7 for statistics).

Comparing Livestock Cycles

It is important to note that the phase space reconstruction was performed on *inventories* of both cattle and swine not slaughter rates or the commonly studied hog-corn price ratio. The amount of generality obtained from studying inventories as opposed to prices or slaughter rates increases due to the fact that it is not an analysis of the results of market equilibrium but one of the forces that drive to obtain market equilibrium. In markets such as these where the product has short shelf lives it is up to the rancher to maintain the inventory to supply demand.

The existence of the livestock cycle has been attributed to biological constraints, weather, grain and beef trade, commodity programs, structural effects, exogenous demand shocks, and market-timing effects (Mathews et al, 1999). Simple economic analysis agrees with these as determinants of market equilibrium. However, since the markets for the two species were almost entirely local in the 19th century, we can negate the effects of producer responses to timing and larger market shocks, and attribute the cycles to the biological constraints in stock accumulation and weather. It was not until the markets began to grow nationally that these other determinants began to play a significant role, and it is at this time that the cycles appear to begin to change. This trend is apparent in the volatility measure of standard deviation, Tables 1 and 5, and the amplitude measure of both livestock cycles, Tables 4 and 7.

As technology increased and operations became more efficient and confined, especially in the swine industry, the ability to meet demand increased dramatically. Regardless of how expectations are formed if biological effects restrict producers there will be a lag between increased demand and increased quantity supplied. The decrease in the variability of the pork phase space reconstruction is a testament to the producers' ability to match demand. What is qualitatively apparent in the phase space reconstruction is also apparent in the decrease in standard deviation and volatility presented in Tables 1, 4, 5, and 7. This is parallel to the factors that affect beef producers. The gestation period for swine is approximately four months compared to the approximate nine for cattle. Combined with the shorter time lag for births swine have litters comprised of around 10 piglets. The biological constraints for pig farmers a much less restrictive than those for cattle ranchers. With consistent demand, they should therefore be able to meet demand with greater ease. This is exactly what the phase space reconstructions, Figures 2 and 6, the demand-supply overlays, Figures 3 and 7, the measures of cycle amplitude, Tables 4 and 7 and Figure 8, and the overall standard deviations, Tables 1 and 5, illustrate.

In addition to decreased biological constraints the hog and pig industry has experienced a drastic change in the last twenty years. The supply chain has become increasingly vertically integrated. Having vertically integrated supply chains decreases the impact of restrictions such as lack of consumer information. It is not uncommon now in the hog industry to have direct relations between retail suppliers and operations that breed and slaughter.

Mathews et al (1999) demonstrate this consolidation effect with their estimated effects of increasing concentration of the beef-packing sector. Their estimate of the Herfindahl-Hirshman Index (HHI) for the packing industry suggests the ability for market power, however they reported that they industry does not appear to be exercising this power and that farm prices are actually slightly higher because of the concentration. They suggest that factors not included in their analysis associated with size and concentration to be more important than monopolistic pricing effects. Two of the main factors associated with improved efficiency with economies of scale are better information and improved use of technology. While the cattle industry appears to evolving in the same way the hog industry has, it is not nearly as vertically integrated. This difference between supply chain linkages is most likely one of the largest reasons for seeing a more severely diminished cycle in the hog industry.

The reduction of cycle volatility because of better adjustment of inventories due to technological advancement and vertical integration is not a new theory. It is this argument that drives the business cycle research of McConnell and Perez-Quiros (2000) and Kahn, McConnell, and Perez-Quiros (2002). They explicitly show that the diminishing of the business cycle over the previous two decades was greatly influenced by:

1) "the reduction in the volatility of durable goods"..."coincident with a break in the proportion of durables output accounted for by inventories"

and

2) "changes in inventory behavior stemming from improvements in information technology (IT)..."

Improvements in information technology through supply chain management can be seen in livestock industries as well, indeed much more so for pork than beef. This causal representation of changes in the United States business cycle can effectively be applied to the general change in the livestock cycle. For example, the direct link of farm-gate supply in the pork industry to retail demand allows for an increase in consumer information and a more efficient supply response, resulting in decreased volatility of long-run cyclical processes.

The final remark by Holt and Craig (2006) suggests that which the phase space analysis has confirmed: "the hog-corn cycle" (in fact the inventory cycles of both hogs and cattle) "itself is not a stationary process, but rather a feature of these markets that has, itself, evolved through time as dictated by institutional and technological change." The research detailed above suggested many different causes of the livestock cycle that would eventually lead to a steady state of inventories given a consistent demand. Over time producers have been able to adjust herd size at an increasing rate. It is only because cattle inventories contain such a restrictive biological constraint that cattle producers cannot match demand as efficiently as hog producers.

Given that beef consumption has settled on a steady state level, producers that cull an average percent of total inventory, especially with increasing prices, will receive consistent profits. The profit maximizing decision, as detailed in the dynamic literature discussed above, of agents whom generate expectations *in any form* will converge to the same decision. This is the response found in the swine industry where the biological constraints have been all but completely overcome.

Final Remarks and Future Research

As patterns of United States consumption stabilize over time while production technologies advance, and supply chains become more vertically integrated, the ability of the producer to match demand has inevitably increased. Reductions in volatility and amplitude suggest the more efficient supply responses, and the phase space reconstructions of the hog cycle and the cattle cycle confirm this converging behavior. What has been called the "Great Moderation" in international business cycles is apparent in United States livestock cycles as well. The persistence of the livestock cycle has diminished in the swine industry and has slowed considerably in the beef industry.

The observations made above on the livestock literature pertaining to biological constraints, producer expectations, nonlinear methods, and data limitations parallel the recent contributions to business cycle analysis. The dynamic livestock models have shown that the early large cyclical effects are largely due to endogenously propagated exogenous shocks. This is apparent in the phase space analysis, as the livestock cycles have diminished over time with an increased link from to farm-gate supply to retail consumption. The time series livestock models suggest the use of nonlinear tools to effectively analyze cycles and through phase space reconstruction, this analysis was able to circumvent the typical assumption problems that traditional models face and expose the similarities and differences between the swine and beef production decision.

The phase space reconstructions, demand-supply overlays, measures of cycle amplitude, and overall standard deviations have shown that whether or not all producers are forming fully rational expectations they have been able to greatly reduce the volatility of livestock cycles. In particular, the level of vertical integration is the main difference between the cattle and hog industries. The diminishing pork cycle is a testament to the greater ability of hog farmers to adjust inventories to match demand, and the diminished cattle cycle is indicative of the more binding biological constraint and smaller tendency to consolidate.

It seems prudent that future testing of the evidence of diminishing cyclical effects in a more traditional manner be done. Methods similar to those used in the business cycle literature presented should be able to capture the effects hypothesized in this research. What the phase space reconstruction has been able to provide is a clear idea on the direction that the livestock markets have taken. The 19th century production cycles depended greatly on the biological constraints faced by producers, and after the advent of refrigeration, hormonally induced growth rates, stable demands, and other institutional, informational, and technological changes the biological constraints have lessened greatly. Conceptually, if the rate these cycles have decreased and whether or not they have been consistently decreasing over time can be estimated, as it has been done in the business cycle literature, it could be possible to determine a causal link to the particular institutional and technological factors driving these changes.

The level of supply chain integration is certainly not consistent throughout the United States. In testing the determinants of livestock cycle reduction, the use of disaggregated data may prove informative. Areas where environmental regulation prohibits the development of large closed hog operations may have more predominant inventory cycles than areas in which regulation is nonexistent.

Tables

Total Inventory of Cattle and Calves (1000 Head)			
Period	Mean	Median	Standard Deviation
1867-2007	77164	70979	25657.07
1867-1887	39867	37333	8692.03
1888-1908	58891	59739	5667.33
1909-1929	63614	63373	5702.66
1930-1950	72929	71755	7462.26
1951-1971	100457	97700	8958.55
1972-1992	111898	113360	11055.62
1993-2007	98623	98199	2678.38

Table 1: Descriptive statistics for the total inventory of cattle and calves: 1867-2007.

Lag	Mutual Information Function	Autocorrelation Function
1	0.128989959	0.72276174
2	0.128807415	0.353590386
3	0.128623553	0.009057123
4	0.128438354	-0.263429289
5	0.128251798	-0.39530035
6	0.128063865	-0.335062452
7	0.127874535	-0.174605927
8	0.127683786	-0.005688347
9	0.127491598	0.152744535
10	0.127297949	0.270365454

Table 2: The mutual information and autocorrelation functions for the United States total inventory of cattle and calves. The long memory process indicative of a nonstationary time series is clearly present.

Dimension	False Nearest Neighbors
1	0.604938272
2	0.222222222
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

Table 3: The percentage of false nearest neighbors found when increasing from the indicated dimension.

Cycle	Wavelength	Amplitude
1890-1903	14	10809
1904-1917	14	10767
1918-1933	16	15718
1934-1944	11	9120
1945-1954	10	8743
1955-1964	10	5416
1965-1974	10	217
1975-1981	7	21164
1982-1995	14	19628
1995-2007	12	8660

Table 4: Descriptive statistics differentiating the cattle cycles. Wavelength refers to the number of years between cycle peaks; Amplitude is the difference between the initial cycle inventory and its lowest inventory.

Total Inventory of Hogs and Pigs (1000 Head)			
Period	Mean	Median	Standard Deviation
1866-2006	53756	55366	8526.07
1866-1886	40050	39794	4566.91
1887-1907	49593	49154	4149.28
1908-1928	58052	57578	5160.26
1929-1949	56727	56810	10067.80
1950-1970	57023	57125	5384.00
1971-1991	56792	55466	4419.90
1992-2006	59776	59722	1736.27

Table 5: Descriptive statistics for the total inventory of hogs and pigs: 1866-2006.

Dimension	False Nearest Neighbors
1	0.851851852
2	0.308641975
3	0.197530864
4	0.135802469
5	0.271604938
6	0.296296296
7	0.283950617
8	0.395061728
9	0.469135802
10	0.567901235

Table 6: The percentage of false nearest neighbors for embedding total hog and pig inventories.

Cycle	Wavelength	Amplitude
1867-1871	5	734
1872-1878	7	4079
1879-1883	5	1761
1884-1888	5	5196
1889-1896	8	4478
1897-1906	10	5424
1907-1910	4	10316
1911-1914	4	2541
1915-1917	3	3018
1918-1921	4	5384
1922-1926	5	17199
1927-1931	5	7038
1932-1938	7	23061
1939-1942	4	6812
1943-1944	2	24368
1945-1949	5	6716
1950-1954	5	17155
1955-1958	4	3837
1959-1961	3	3466
1962-1967	6	12207
1968-1969	2	3783
1970-1972	3	8268
1973-1978	6	11347
1979-1982	4	12785
1983-1987	5	5692
1988-1991	4	1678
1992-1993	2	262
1994-1997	4	3614
1998-2000	3	2868
2001-2006	6	167

Table 7: Descriptive statistics differentiating the hog cycles. Wavelength refers to the number of years between cycle peaks; Amplitude is the difference between the initial cycle inventory and its lowest inventory.

Figures

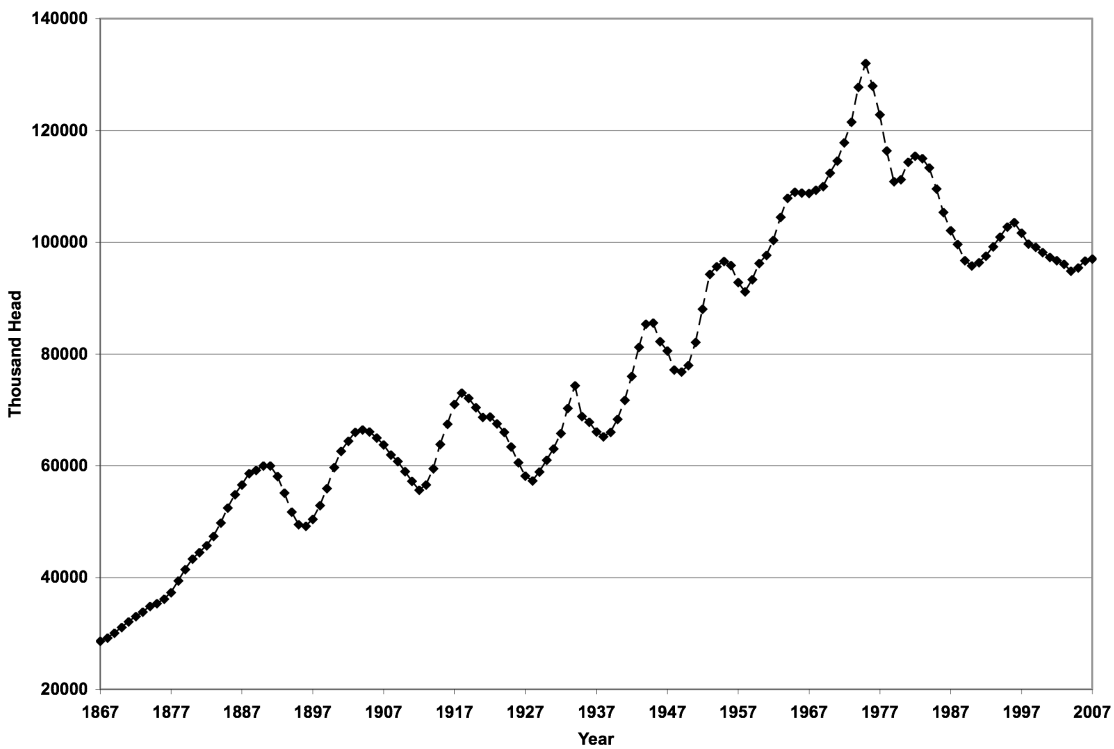

Figure 1: Total inventory of cattle and calves (1000’s): 1867-2007.

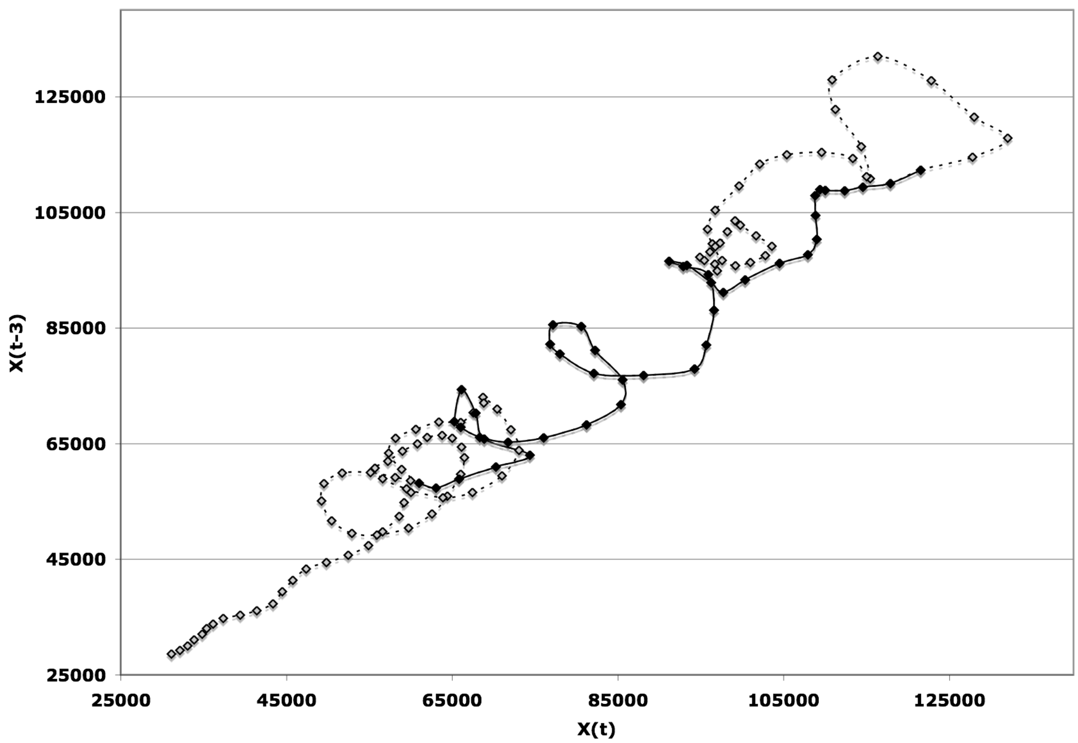

Figure 2: Phase space reconstruction of the total inventory of cattle and calves in the United States: 1867-2007.

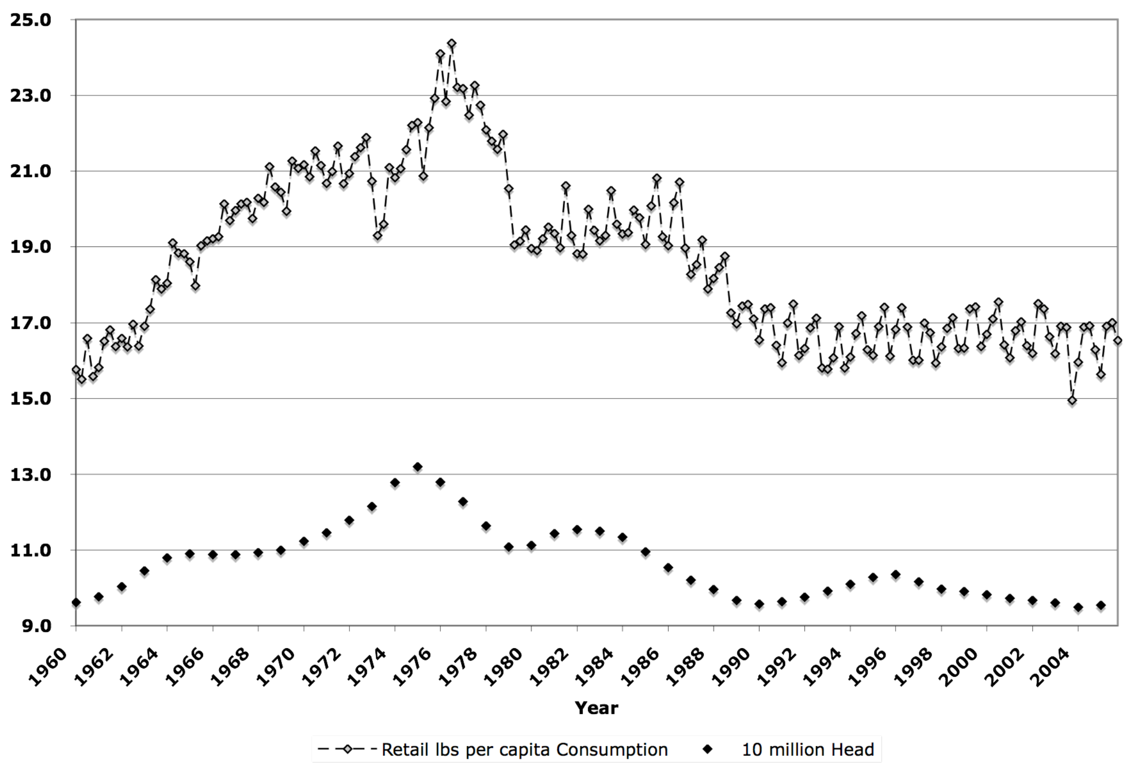

Figure 3: Retail lbs per capita beef consumption and the total inventory of cattle and calves: 1960-2005.

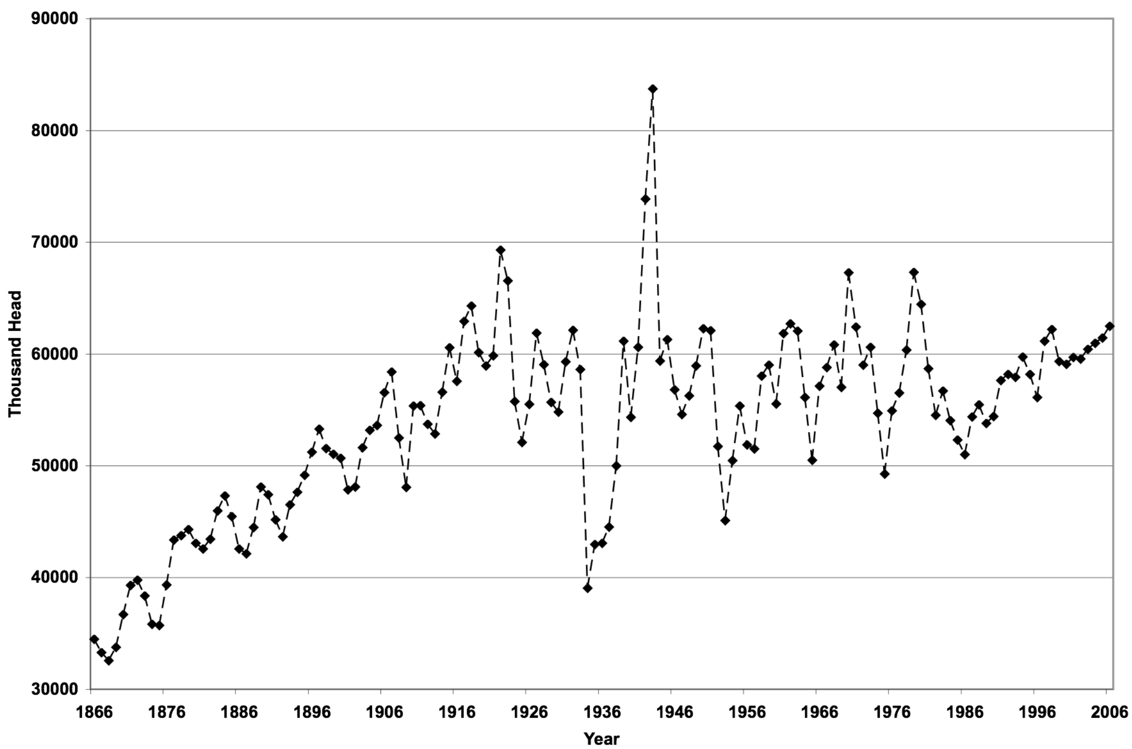

Figure 4: Total inventory of hogs and pigs (1000's): 1866-2006.

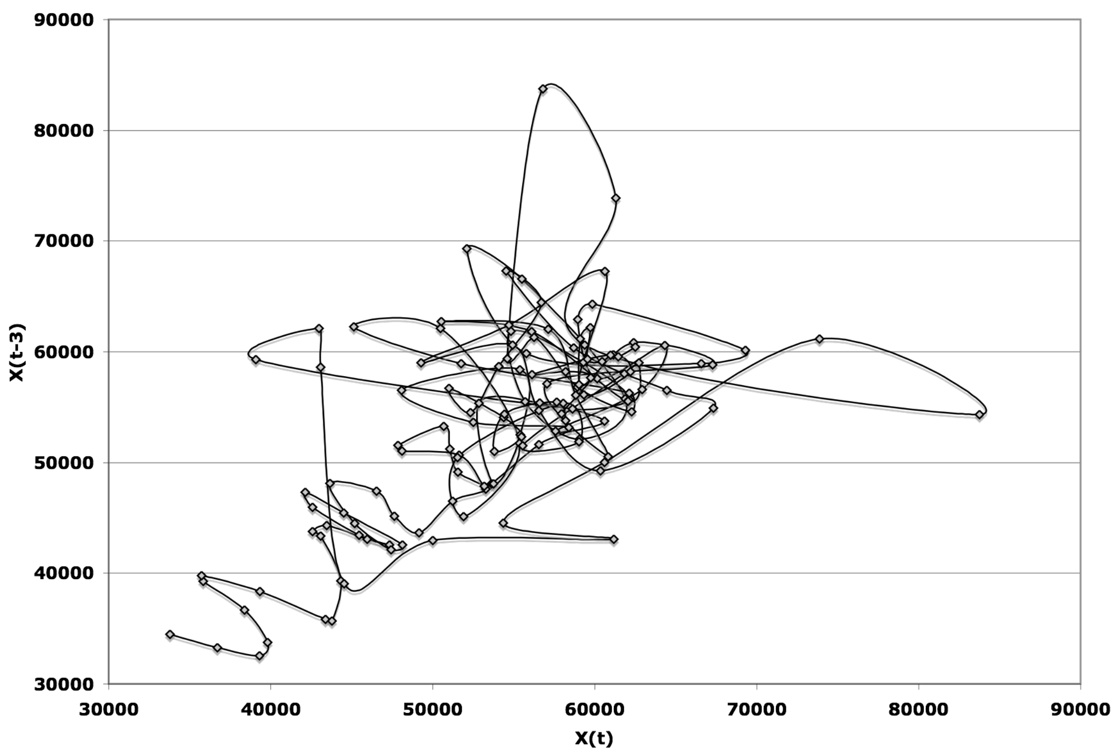

Figure 5: Phase space reconstruction of the total inventory of hogs and pigs in the United States: 1866-2006.

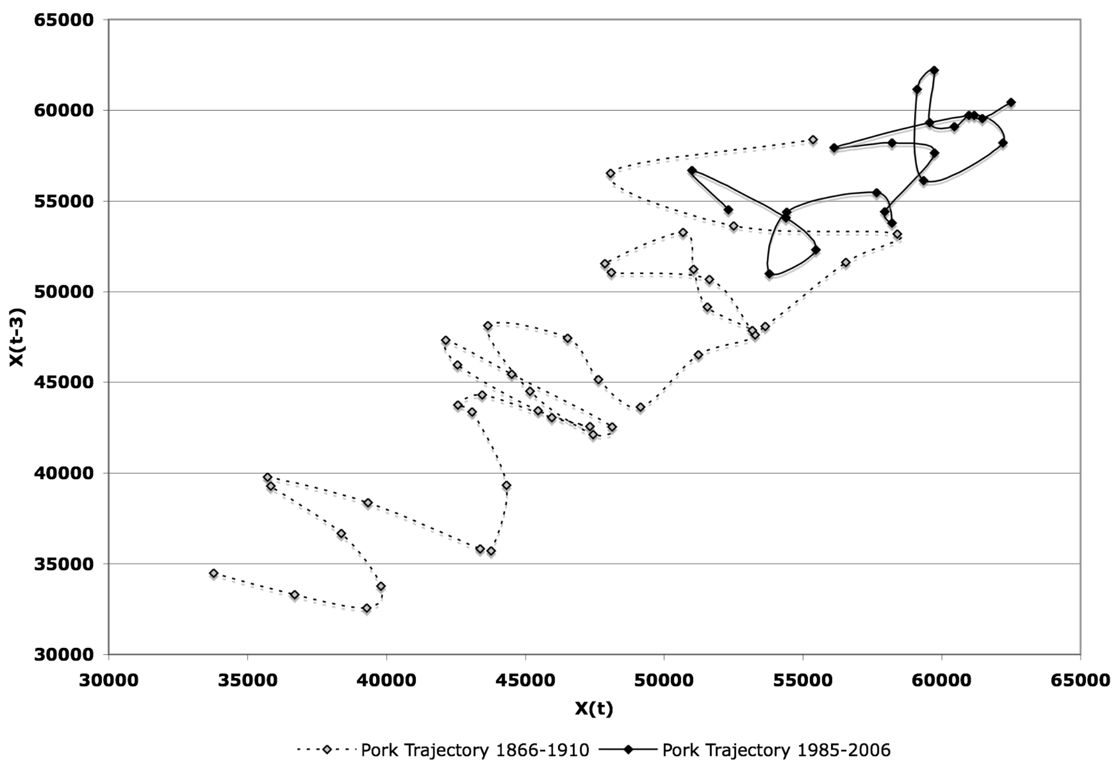

Figure 6: Phase space reconstruction of the total inventory of hogs and pigs in the United States: 1866-1910 and 1985-2006.

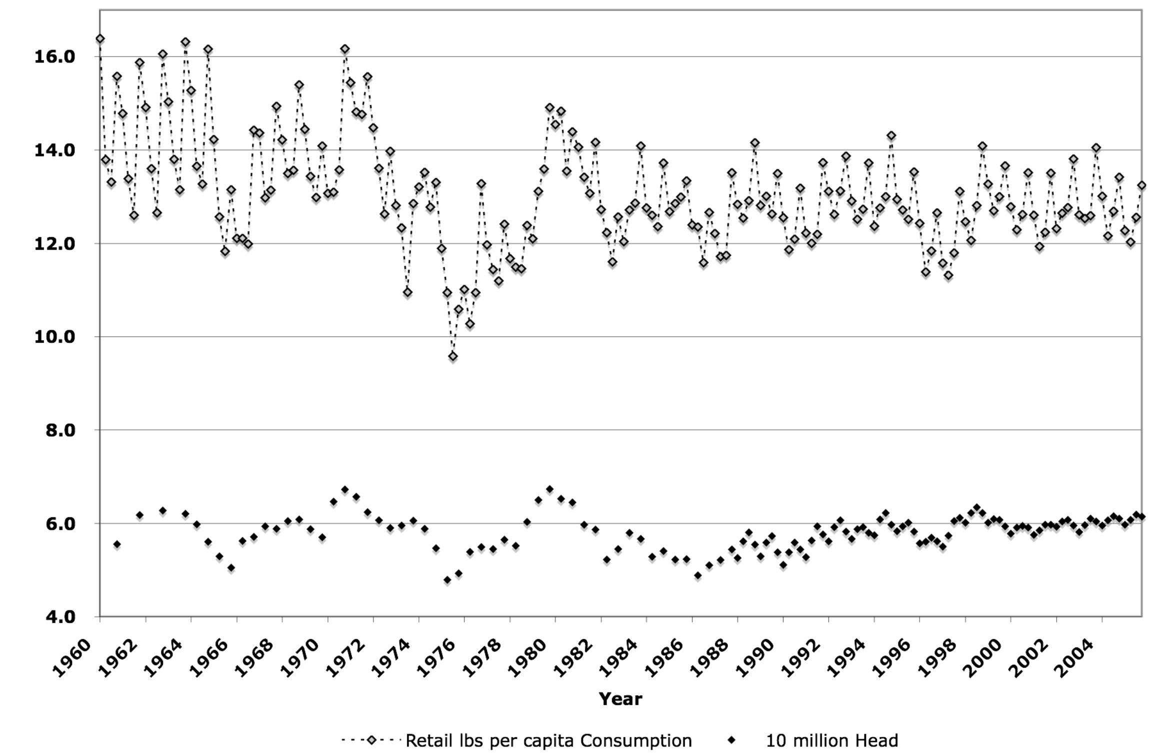

Figure 7: Retail lbs per capita pork consumption and the total inventory of hogs and pigs: 1960-2005.

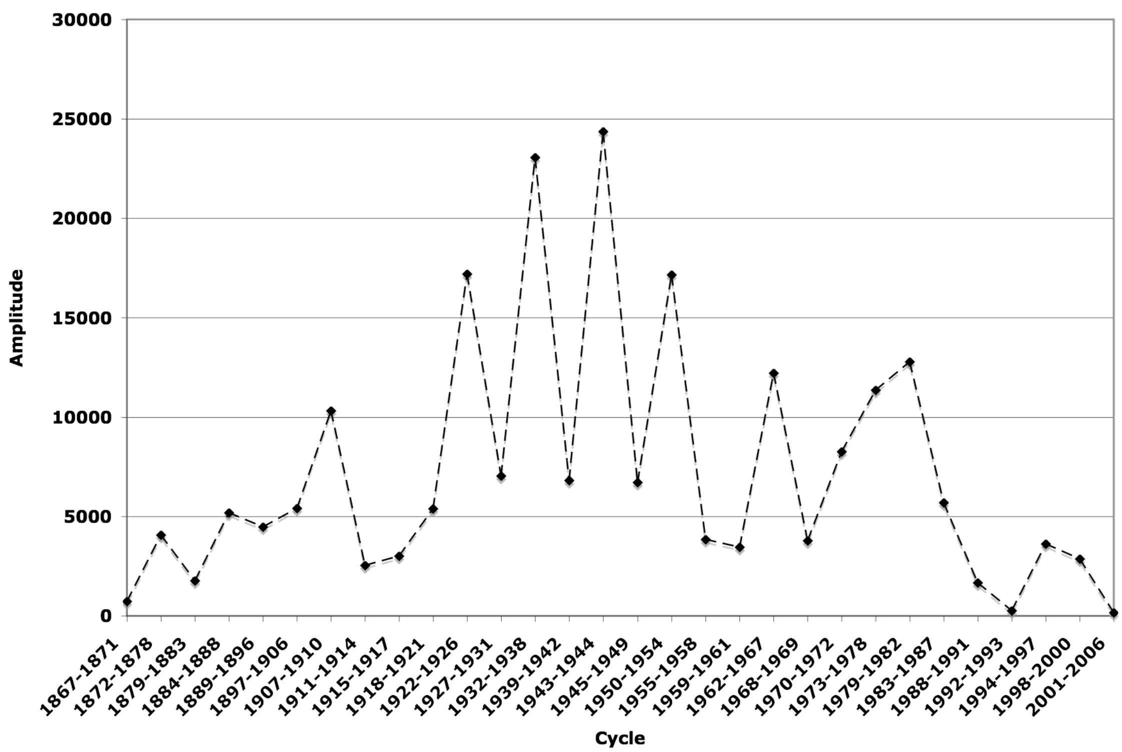

Figure 8: The amplitude of the pork cycle defined by the difference between initial inventory and the minimum inventory for each cycle.

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