NEWSVENDORS AND SUPPLY CHAIN COORDINATION

UNDER SATISFICING OBJECTIVES AND MULTIPLE OBJECTIVES

By

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To the Faculty of Washington State University:

The members of the Committee appointed to examine the dissertation of CHUNMING SHI find it satisfactory and recommend that it be accepted.

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Chair

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Abstract

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The vast majority the research in Operations Management and Supply Chain Management assume the maximizing objectives, particularly the objective of expected profit (cost) maximization (minimization). In this dissertation, we study the optimal behaviors of individual firms and supply chains under the satisficing objectives and multiple objectives. A satisficing objective is to maximize the probabilities to attain some required target performance levels.

This dissertation consists of five chapters. In Chapter 1, we first provide the background and literature review.

In Chapter 2, we study individual firm(s) under the satisficing objectives. We first derive the optimal order quantity and/or retail price for a newsvendor under three possible objectives. Next we study multiple newsvendors under inventory competition.

We obtain analytic results which are quite different from those obtained under the objective of expected profit maximization. Finally, we briefly study the problem of target setting in the framework of newsvendors.

In Chapter 3, we study contract design and supply chain coordination under the satisficing objectives. We first focus on the profit satisficing objective, under which we derive the Pareto-optimal contracts and provide a necessary and sufficient condition for supply chain coordination. Our results provide an important justification of the wide of wholesale price contracts besides their lower implementation cost. We also propose the use of slotting fee contracts to coordinate such a supply chain. Finally in Chapter 3, we extend our study to the case where each agent adopts the profit satisficing and the revenue satisficing objectives simultaneously.

In Chapter 4, we study the supply chain under multiple objectives. We first study quantity flexibility contracts under both the profit satisficing objective and the objective of expected profit maximization. We show that more degrees of freedom benefit the supply chain under multiple objectives. Next we make an initial attempt to study the supply chain where different agents adopt different types of objectives.

Finally in Chapter 5, we summarize our contributions in this study. We also suggest a number of future research projects.

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Chapter 1

Introduction

This dissertation is a study on newsvendors and supply chains under the satisficing objectives and multiple objectives. In business practice, firms often adopt satisficing objectives, i.e., to maximize the chances to achieve some required target performance levels.

This chapter is to introduce the background and motivation of this research. Supply Chain and Supply Chain Management are first briefly described in Section 1.1. In Section 1.2, we introduce satisficing objectives, including the profit satisficing objective and the revenue satisficing objective. We lay out the structure of this dissertation in Section 1.3.

1.1 Supply Chain Management

A supply chain is a chain of individuals and/or firms who are involved in providing some products and/or services to end customers. The individuals and firms include suppliers, manufacturers, transporters, warehouses, retailers, and finally customers themselves. The functions in supply chain management include, but not limited to, procurement, new product development, production, logistics, inventory, marketing, and customer services. From another perspective, a supply chain could be traditionally characterized by the flow of materials, finances and information.

The purpose of supply chain management is to satisfy customers' needs and wants as quickly as possible at a cost as low as possible. In the increasingly competitive and global market nowadays, supply chain management becomes increasingly critical. Use the function of logistics as an example. In the United States alone, annual cost on non-military logistics is estimated to be $670 billion, which is more than 11% of the gross national product. For manufacturers in the United States, it is common that logistics costs account for 30% of cost of goods sold (Douglas and Griffin 1996). Furthermore, according to Paul H. Zipkin, as of March 1999, businesses in the United States held about $1.1 trillion worth of inventories. This is about 1.35 times their total sales for the month. Therefore, even a small percent decrease in supply chain cost amounts to a huge saving. A number of firms have achieved phenomenal growth partially due to successful supply chain management, including Wal-Mart and Dell. In this dissertation, we focus on the aspect of inventory management in supply chain management.

1.2 Satisficing Objectives

The vast majority of the research in Operations Management, including Supply Chain Management, studies the maximizing behavior of individual firms and/or supply chains. The concept of maximizing behavior has its roots in traditional economic theory, which postulates an economic person. This person is assumed to possess (nearly) perfect information. Furthermore, this person is assumed to have unlimited capacity to process all the relevant information. Under such assumptions, the rational behavior of an

economic person can be generally modeled by the maximization of an appropriate utility function of personal preference (von Neuman and Morgenstern, 1944).

However, such an economic person does not exist in reality. In the words of March and Simon (1958): "Most human decision-making, whether individual or organizational, is concerned with the discovery and selection of satisfactory alternatives; only in exceptional cases is it concerned with the discovery and selection of optimal alternatives". In one of his classical papers (1959), Simon summarizes the five most important attacks on the crucial assumption of profit maximization in the theory of the firm. One of them is due to the fact that maximizing profit is an ambiguous goal when there is imperfect competition among firms. Simon then argues that most firms' goals are to attain a certain level or rate of profit, a certain share of the market, or a certain level of sales.

There have been empirical studies on the objectives of firms and organizations. In one of the earliest studies examining the objectives of managers, Lanzillotti (1958) interviews the officials of 20 large companies (including General Electric, General Foods and Goodyear) and verifies empirically that the most typical goal cited by the managers was a target return on investment. Shipley (1981) studies the pricing and profit objectives of 728 British manufacturing firms. It is found out that most firms specify a required profit target or rate of return on capital employed as an important performance measure. Furthermore, a required profit target or rate of return is the principle performance measure for two-thirds of the 728 firms. In a recent paper, Brown and Tang (2006)

survey 250 MBA students and 6 professional buyers and find that their important performance metrics including meeting a profit target and meeting targets on both sales and gross margin.

In reality, very often it is critical for firms to achieve targets on some performance measures. Such performance measures include cost, profit, revenue, market share and so on. When public companies deliver their quarterly results, they want to make sure that their results beat the consensus estimates from Wall Street analysts. Otherwise, their stocks are very likely to fumble. This is clearly demonstrated by what happened to the online auction house eBay in the 4th quarter of 2004. Its profit of 33 cents a share missed the expectation of 34 cents from Wall Street analysts by just ONE cent. However, its stock price fell by 12% right after the report. In the 3rd quarter of 2005, Yahoo! reported $875 million in revenue. Though it represents a 44% gain over the same period last year, the consensus estimate was for $881 million. As a result, the stock was down 10 percent in after-hours trading. These two examples clearly demonstrate the importance of achieving profit and/or revenue targets.

One related problem of both theoretical and practical importance is target setting. It is said that setting performance targets and managing to achieve them is fundamental to business success. Targets provide explicit directions to an organization and motivate management to strive for even higher levels of performance. However, target setting is both an art and a science. Too high a target will provoke frustration and cynicism, whereas too low a target will engender apathy and risk the firm's survival. Different

methods of target setting have been practiced by many decision makers. In the approximate order of most to least used, they include plucking it out of thin air, a percentage improvement on last period, and benchmarking (Barr, 2003). For example, Jack Welch describes the setting of stretch targets as one of General Electric's three main operating principles (McTaggart and Gillis, 1998). Lovell *et al* (1997) evaluate the target setting procedure employed by a large financial institution, whose management annually sets performance targets for each of its branch offices based on local, regional and national economic conditions.

Suppose a firm faces one or more targets on some performance measures. The reality is that if the firm fails to achieve the target(s), there will be some serious consequence, such as a fumble on its stock price and loss of investors' confidence. Facing with such a reality, will the firm endeavor to maximize the performance measures? Very likely, the answer is no. Instead, the firm will adopt satisficing objectives, i.e., to maximize the chances to achieve the pre-specified targets. Note that the word "satisficing", an amalgam of 'satisfy' and 'suffice', is coined by the Nobel economics laureate Herbert Simon.

The most common objective adopted in the majority of Operations Management research is the objective of expected profit (cost) maximization (minimization). For simplicity, the following discussion will be based on the objective of expected profit maximization. Similar arguments apply to the objective of expected cost minimization. Obviously, the objective of expected profit maximization is one type of maximizing objectives. However, maximizing expected profit means the average profit would be

maximized if a firm repeats the same decision many times. There is no guarantee that the realized profit at some specific time period will be as high as the maximal expected profit. In another word, the objective of expected profit maximization is an objective for a relatively long term. If a firm has to take into account shorter-term results, satisficing objectives are at least as important as maximizing objectives.

On the other hand, the objective of expected profit maximization is a risk-neutral objective. It implies a firm is only concerned with expected profit but not the risk associated with it. In reality, firms often time are risk-averse. The rational behavior of a risk-averse agent can be generally modeled by the maximization of the expected value of an appropriate utility function of a performance measure (von Neuman and Morgenstern, 1944). An important approach to operationalize risk-aversion is the mean-variance analysis due to Markowitz (1959). This approach works the best when the random variable under consideration is close to be symmetrically distributed, which, however, may not be the case for many inventory and supply chain management problems. Without a relatively symmetrical distribution, it is better to operationalize risk-aversion using different measures of downside risk. These measures include semi-variance and critical probability. With semi-variance, one is concerned with the volatility of the outcomes below the mean. With critical probability, one is concerned with the probability of the outcomes in some critical region.

Satisficing objectives operationalize risk-aversion using one measure of downside risk: the critical probability that the target performance levels will be obtained. Given a

specified target, the associated satisficing objective is highly risk-averse. If a firm adopts the satisficing objective alone, it implies that the firm is only concerned with the risk of not achieving the target; the firm is not motivated to over-achieve the targets. Intuitively, such a satisficing objective applies when under-achievement is very undesirable and over-achievement is not very rewarding. However, whether the satisficing objective is ultimately risk averse or not depends on the magnitude of the target. The higher a target, the more risk-taking behavior the target will induce.

So far, it has been shown that satisficing objectives are supported by both theory development, empirical studies and examples of real companies. However, satisficing objectives do have some drawbacks. First, they are more short-term oriented. Second, they are very risk-averse given pre-specified targets. Therefore, satisficing objectives, although important, are not sufficient for firms. The ultimate goal for most firms is to be successful in the long term, particularly in terms of profitability. Furthermore, firms are certainly not very risk-averse all the time. They will not only be penalized by under-achievement, but also be rewarded by over-achievement. Hence, it is very likely that firms will adopt multiple objectives, including maximizing and satisficing objectives. Note that both maximizing and satisficing objectives can be in terms of profit, revenue, market share, return on investment, and so on. If a firm is to maximize profit, we call such an objective the profit maximizing objective. If a firm is to maximize the chance to achieve a profit target level, we call such an objective the profit satisficing objective. if a firm is to maximize the chances to achieve a profit target and a revenue target

simultaneously, we call such an objective the profit and revenue satisficing objective. Similar arguments apply to other performance measures.

1.3 Dissertation Structure

The focus of this dissertation is to study newsvendors and supply chains under satisficing objectives and multiple objectives. In Chapter 2, we first study a single newsvendor under three possible objectives: profit satisficing objective, revenue satisficing objective, and profit and revenue satisficing objective. Then we study a single price-setting newsvendor under those three possible objectives. Next, we study competitive newsvendors under the profit satisficing objective. Finally, we briefly study the problem of target setting in newsvendors. To be more specific, a manger has two newsvendors working for him and he has a target profit to achieve. The research question is how to allocate profit targets to both newsvendors who will also adopt the profit satisficing objective.

In Chapter 3, we study contract design and supply chain coordination under satisficing objectives. We first consider a basic supply chain where each agent adopts profit satisficing objective. The research question is how to design contracts such that the supply chain's performance can not be improved further. The idea is to design Pareto-optimal contract. A contract is said to be Pareto-optimal when there is no other contract within the same contractual form such that at least one agent will strictly increase his payoff without making any other agent worse off. The contractual types we consider

includes linear tariff contract, including wholesale price contract as a special case, buy back contract, quantity flexibility contract. Once we obtain the Pareto-optimal contract(s) within each particular contractual form, we ask the following question: are those Pareto-optimal contracts capable of supply chain coordination? We propose a theorem specifying necessary and sufficient conditions for supply chain coordination under profit satisficing objective. Next, we consider the same supply chain where each agent adopts the profit and revenue satisficing objective. As a first step, we consider wholesale price contract only in this chapter.

As argued Section 1.2, it is more likely that a firm will adopt both maximizing objectives and satisficing objectives simultaneously. Therefore, in Chapter 4, we first study contract design and supply chain coordination under multiple objectives. To be more specific, each agent adopts two important objectives simultaneously, i.e., the profit satisficing objective and the objective of expected profit maximization. The contractual form we consider is quantity flexibility contract partially due to its three degrees of freedom. One of our intentions is to see if more degrees of freedom benefit a supply chain under multiple objectives. Next, we study the same supply chain where the supplier adopts the profit satisficing objective and the retailer adopts the objective of expected profit maximization. As a first step, we only consider wholesale price contract.

Finally in the last chapter, Chapter 5, we summarize our research contributions in this dissertation. We also propose some potential future research directions.

Chapter 2

Newsvendors under Satisficing Objectives

2.1 Introduction and Literature Review

The classical newsvendor model, also know as the newsboy model or the single-period model, is to decide a product's order quantity that maximizes (minimizes) the expected profit (cost). There is only one selling season. The newsvendor has to make the order quantity decision before the selling season knowing only a stochastic demand of the product. The newsvendor purchases the product at a unit constant cost and sells at a unit constant selling price. If she does not order enough, she forgoes some potential sales during the selling season. If she orders more than the soon-to-be-realized demand, she has to salvage the products at the end of the selling season at a salvage price, which is (much) lower than the purchase cost. Note that the salvage price could be negative, which indicates a disposal cost.

The newsvendor model has a long history as a decision tool to a variety of real-life applications, where the product or service has a short life time cycle. In fact, as early as nineteen century, Edgeworth (1888) applied a variant of the model to a bank cash-flow problem. More recently, the model has also been used to aid decision making in industries such as fashion, sporting (Gallego and Moon, 1993) and service (Weatherford

and Pfeifer 1994). Furthermore, more and more products and services subscribes to short life cycle nowadays. This is particularly true due to the fact that customers are increasing demanding due to global competition and exploding information and communication technologies.

There have been extensive studies on the newsvendor model. For comprehensive reviews, readers are referred to Porteus (1990), Silver et al (1998) and Khouja (1999). In particular, there have been two relatively new research streams on newsvendors, namely price-setting newsvendors and competitive newsvendors.

Traditionally, research on newsvendors has focused a single newsvendor given exogenous retailing price and customer demand. However, the reality is that firms can exert a number of marketing efforts to affect demand, including price, manufacturer rebates and retailer rebates. Whitin (1955) is the first to study a price-setting newsvendor. He established a sequential procedure to determine optimal stocking quantity and price. Petruzzi and Dada (1999) review the history of the problem, generalize existing results and provide a more integrated framework for two alternative demand formulations, i.e., additive and multiplicative demand models. Monahan *et al* (2004) study the dynamic pricing problem from a newsvendor's perspective. Assuming a multiplicative demand form, they find the dynamic problem can be re-interpreted as a price-setting newsvendor with recourse.

The second research stream has been on competitive newsvendors. There are two basic forms of competitions, namely the inventory competition and price competition. Inventory competition occurs because newsvendors sell (partially) substitutable and/or complementary products. Substitutable effect occurs when newsvendors sell the same or similar products. If a customer can not get what she wants from a newsvendor, it is likely that she will continue and buy the same or similar product from other newsvendors. This is called stock-out-based substitution, the study of which could be complicated. To make the analysis simple, different assumptions may be imposed. A common assumption in stock-out-based substitution is that each newsvendor has an exogenous independent demand and if a newsvendor is out of stock, a fixed proportion of her customers will go to other newsvendors (McGillivray and Silver 1978, Wang and Palar 1994, Netessine and Rudi 2003). One major finding is that most of the times, but not always, newsvendors selling substitutable products tend to overstocking as compared with centralized inventory management for all newsvendors (Netessine and Rudi 2003). However, it should be noted that substitutable effect may also due to the perceived service level/or and the displayed inventory levels. A mass display of some products in a store can serve as "physic stock" to stimulate sales (Larson and DeMarais, 1990).

Inventory competition also occurs when newsvendors are selling complementary products. For example, if one newsvendor is selling DVD movies and another newsvendor is selling DVD players, they are selling complementary products. Netessine and Zhang (2005) are the first to study this kind of inventory competition. They show that

competition with complementary products ALWAYS leads to inventory under-stocking by newsvendors.

Besides inventory competition, newsvendors may also engage in price competition. Obviously, if a newsvendor charges a lower price for the same or similar product, she may attract customers from other newsvendors. Bernstein and Federgruen (2004) model the situation where each retailer decides on both stocking quantity and price, but her demand depends on the prices of all retailers only. Chen *et al* (2004) considers a horizontal market of multiple firms that face stochastic price-dependent only demand. Each firm decides on stocking quantity but only use pricing to compete for demand. With general demand model they prove the existence and uniqueness of pure-strategy Nash equilibrium. They also show that the equilibrium exhibits an under-pricing bias due to competition.

Obviously it is more realistic to assume that both inventory levels and price will affect customer demand. As a result, newsvendors will under simultaneous inventory and pricing competitions. Zhao (2003) consider four types of market: no competition, inventory competition, price competition, both inventory and price competitions. It is shown that under either inventory competition or price competition, Nash equilibrium strategy exists. Under both inventory and price competition, it is shown that pure Nash equilibrium exists, and is unique under some conditions. However, the results are based on a specific demand model as a linear function of inventory and price. Therefore, it is unknown whether such results hold under general demand models.

In this chapter, we study newsvendors with satisficing objectives. Kabak and Schiff (1978) are the first to study newsvendors with zero shortage cost under profit satisficing objective. Lau (1980) and Sankarasubramanian and Kumaraswamy (1983) generalize the problem to non-zero shortage cost under profit satisficing objective. Lau and Lau (1988) further extend the results to a newsvendor with two products. A special case when the demand is exponentially distributed is studied by Li (1991). More recently, Parlar and Weng (2003) study the problem of balancing two desirable but conflicting objectives, i.e., maximizing the expected profit and maximizing the probability of achieving the expected profit. Finally, Brown and Tang (2006) identify various performance metrics for the newsvendor problem and analyze their impacts on the order quantity. The performance metrics include: meeting a profit target, meeting targets on both sales and gross margin, and minimizing excess inventory. A common finding in the literature is that if a newsvendor endeavors to maximize the probability to achieve her target profit t, the optimal order quantity will lead to an expected profit less than t.

However, to our best knowledge, there are a few research gaps on newsvendors under satisficing objectives. First, only the profit satisficing objective is considered. Second, the price and customer demand are assumed to be exogenous. Third, there is no research on inventory competition and price competition. Finally, there is no research on target setting in the newsvendor framework. This chapter attempts to fill these gaps.

In Section 2.2, we first review the newsvendor model under profit satisficing objective, i.e., to maximize the probability to achieve a target profit. In Section 2.3, we consider the

newsvendor model under revenue satisficing objective. We then study the newsvendor under the profit and revenue satisficing objective in Section 2.4. In Section 2.5, we study price-setting newsvendors under profit satisficing objective. In Section 2.6, we study price-setting newsvendors under both profit and revenue satisficing objectives simultaneously. One of our particular focuses is on two common demand models, namely the additive demand model and the multiplicative demand model. In Section 2.7, we study competing newsvendors under the profit satisficing objective. In Section 2.8, we briefly study the target-setting problem in newsvendor setting. Finally we summarize our findings in Section 2.9.

2.2 Newsvendor under Profit Satisficing Objective

In this section, we briefly review and extend the results for the newsvendor model under profit satisficing objective. The purpose is to provide a basis for the subsequent analysis.

Consider a standard newsvendor with a procurement cost c and sell to her customers at a fixed unit price r. The customer demand, a random variable D, follows a probability density function (PDF) $f(\cdot)$ and a cumulative density function (CDF) $F(\cdot)$. The newsvendor has only one order opportunity and determines her order quantity before the actual demand is realized. If the newsvendor under-orders, he will suffer lost sales. For simplicity, we assume zero shortage cost first. If he over-orders, he will dispose the unsold inventory at salvage price v, which leads to a loss as well. Moreover, the newsvendor has a preset target profit of t. Her objective is to choose an order quantity q

to maximize the probability of achieving the target profit, i.e., to maximize $P(q) = P\{\Pi(q) \geq t\}$, where $\Pi(q)$ denotes the random profit function.

The random profit of the newsvendor for a given order quantity q is given by:

$$\Pi(q) = (r-c)q - (r-v)\,(q-D)^{+}$$

$$= \begin{cases} (r-v)D - (c-v)q & if\ D < q \\ (r-c)q & if\ D \geq q \end{cases} \tag{2.1}$$

It is well know that under the objective of expected profit maximization (PM), the optimal order quantity q^{*}_{PM} is given by:

$$q^{*}_{PM} = F^{-1}\left(\frac{r-c}{r-v}\right) \tag{2.2}$$

Figure 2.1 shows the random profit as a function of demand D given a fixed order quantity.

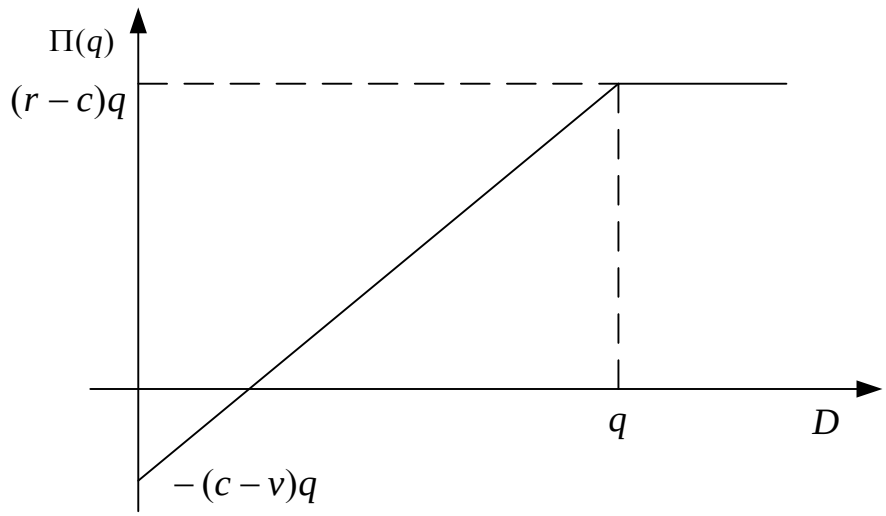

Figure 2.1: The profit function of a newsvendor for a given order quantity.

In view of Figure 2.1, the probability of achieving the target profit t for the newsvendor is given by:

$$P(q) = \begin{cases} 0 & if\ q < \dfrac{t}{r-c} \\ 1 - F\left(\dfrac{(c-v)q+t}{r-v}\right) & if\ q \geq \dfrac{t}{r-c} \end{cases} \tag{2.3}$$

For simplicity, we call this probability "profit probability". Figure 2.2 shows the profit probability as a function of order quantity q.

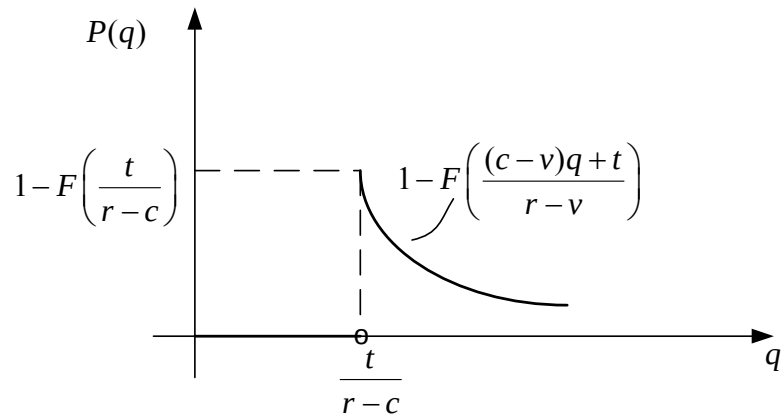

Figure 2.2: The profit probability for the newsvendor as a function of order quantity.

Clearly the participation constraint for the newsvendor is $q \geq \frac{t}{r-c}$. Once the participation constraint is satisfied, the profit probability $P(q)$ decreases in q. Therefore, under profit satisficing objective, the optimal order quantity and the associated maximal probability are given by $q^* = \frac{t}{r-c}$ and $P^* = 1 - F\left(\frac{t}{r-c}\right)$, respectively.

Notice that the optimal order quantity takes a surprising simple form and is independent of the demand distribution. Furthermore, with the optimal order quantity q^*, the newsvendor's maximal possible profit is the preset target profit t, which occurs only when the realized demand D is larger than the optimal order quantity.

It is interesting to compare the optimal quantity q^* above with the optimal quantity q^*_{PM} that maximizes the expected profit. If the newsvendor orders q^*, her expected profit is given by:

$$E\Pi(D,q^*) = t - \int_0^{q^*} F(x)dx \tag{2.4}$$

Interestingly, although the probability of achieving t is maximized, the associated expected profit will be less than t. On the other hand, if the newsvendor sets the target profit at $t = E\Pi(D,q_{PM}^*)$, the associated optimal order quantity to maximize the probability of achieving the target is given by:

$$q^* = q_{PM}^* - \frac{r-v}{r-c}\int_0^{q_{PM}^*} F(x)dx \tag{2.5}$$

which is obviously less than q_{PM}^*. This is consistent with the finding that a risk-averse newsvendor tends to order less in comparison with a risk-neutral newsvendor.

The results above are based on the assumption that there is no zero shortage cost. However, it is more likely that shortage cost exists in practice. When a customer comes to a store which is out of stock, the stock of course will lose the potential sale opportunity. Furthermore, there is a good-will cost associated with out-stock. We call such a cost the shortage cost, denoted by s. Shortage cost is more critical from a long term perspective. For example, a customer may be less likely to repeat the purchasing if stock-out occurred before. The firm may risk her reputation in potential customer base because of stock-out.

Under non-zero shortage cost s, if the newsvendor orders q, her random profit as a function of demand D is given by:

$$\Pi(D) = (r-c)q - (r-v)(q-D)^{+} - s(D-q)^{+}$$

$$= \begin{cases} (r-v)D-(c-v)q & if\ D<q \\ -sD+(r+s-c)q & if\ D \geq q \end{cases} \tag{2.6}$$

Figure 2.3 illustrates the profit function graphically.

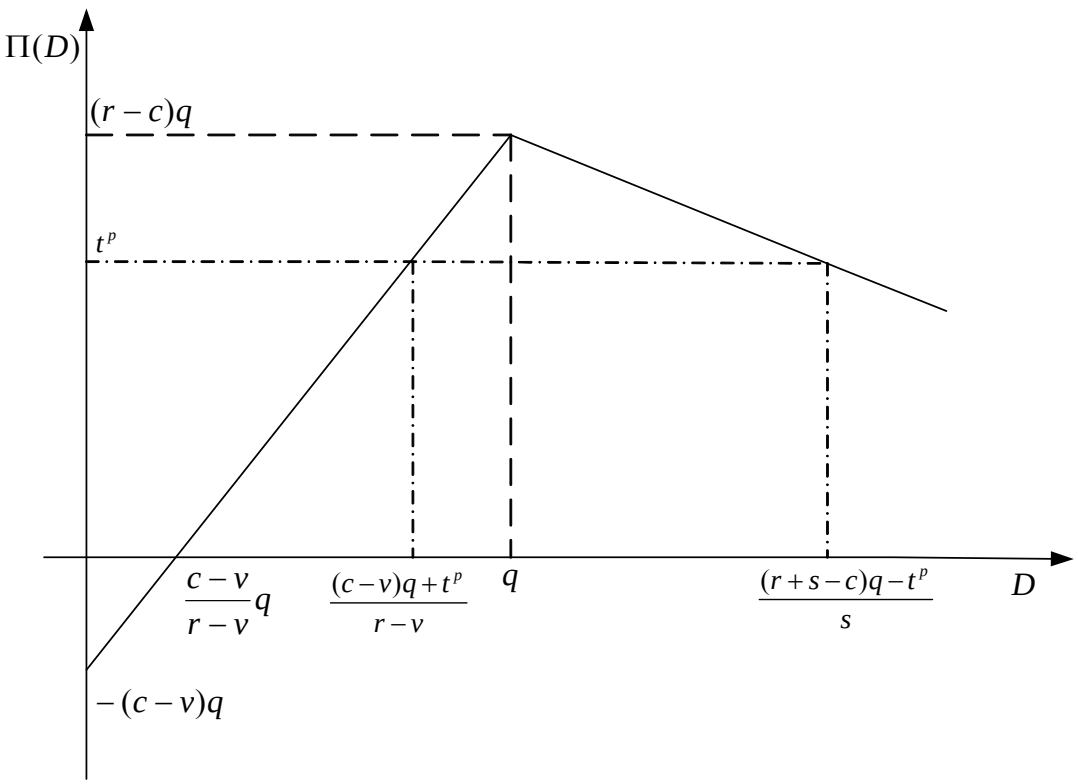

Figure 2.3: The profit function of a newsvendor with non-zero shortage cost.

The following theorem formulate the optimization problem of maximizing the probability of achieve her target profit when shortage cost is present (Lau 1980, Sankarasubramanian and Kumaraswamy (1983).

Theorem 2.1:

If a newsvendor with a non-zero shortage cost adopts profit satisficing objective and her target profit is set at t, her optimization problem is:

$$\max_{q} \; P(q) = F\left(\frac{(r+s-c)q-t}{s}\right) - F\left(\frac{(c-v)q+t}{r-v}\right)$$

$$\text{s.t. } q > \frac{t}{r-c} \tag{2.7}$$

Proof.

If $q \le \frac{t}{r-c}$, or $t \ge (r\text{-}c)\, q$, we have $P(q) = 0$. This is because $(r\text{-}c)q$ is the maximal possible profit given an order quantity q. Note that the probability of achieving the maximal possible profit is 0 since we assume a continuous distribution for the random demand.

If $q > \frac{t}{r-c}$, or $t < (r\text{-}c)\,q$, such a target is achievable. Such a target profit t intersects with the newsvendor's profit function twice, first at order quantity $\frac{(c-v)q+t}{r-v}$, then at order quantity $\frac{(r+s-c)q-t}{s}$. Furthermore, under $t \geq (r\text{-}c)\,q$, it can be verified that:

$$\frac{(r+s-c)q-t}{s} \geq \frac{(c-v)q+t}{r-v} \tag{2.8}$$

Therefore, the newsvendor's profit probability is given by:

$$P(q) = F\left(\frac{(r+s-c)q-t}{s}\right) - F\left(\frac{(c-v)q+t}{r-v}\right) \tag{2.9}$$

In summary, the newsvendor's optimization problem is given by the proposition. □

To solve the optimization problem Theorem 2.1 (See equation (2.7)), we look at the first and second order derivatives:

$$\frac{\partial P(q)}{\partial q} = \frac{r+s-c}{s} f\left(\frac{(r+s-c)q-t}{s}\right) - \frac{c-v}{r-v} f\left(\frac{(c-v)q+t}{r-v}\right) \tag{2.10}$$

$$\frac{\partial P^2(q)}{\partial q^2} = \left(\frac{r+s-c}{s}\right)^2 f'\left(\frac{(r+s-c)q-t}{s}\right) - \left(\frac{c-v}{r-v}\right)^2 f'\left(\frac{(c-v)q+t}{r-v}\right) \tag{2.11}$$

However, an analytic solution to the optimization problem does not exist for general distribution. The general approach would be through numerical optimization given a particular demand distribution.

2.3 Newsvendor under Revenue Satisficing Objective

We considered a newsvendor under the profit satisficing objective in Section 2.2. However, there are other types of satisficing objectives in firms. For example, a firm may have a revenue target to achieve. If that is the case, the firm has the objective to maximize the probability to achieve her revenue target t^e. We use the superscript "*e*" to denote revenue-related parameters and variables.

Given an order quantity, the newsvendor's revenue function is:

$$\Pi^e(q) = rq - (r-v)(q-D)^+ \tag{2.12}$$

The probability for the newsvendor to achieve her revenue target can be written as:

$$P^e(q) = \begin{cases} 0 & if\ q < \dfrac{t^e}{r} \\ 1 - F\left(\dfrac{-vq + t^e}{r-v}\right) & if\ \dfrac{t^e}{r} \le q < \dfrac{t^e}{v} \\ 1 & if\ q \ge \dfrac{t^e}{v} \end{cases} \tag{2.13}$$

For simplicity, we call it the revenue probability, which is plotted in Figure 2.4.

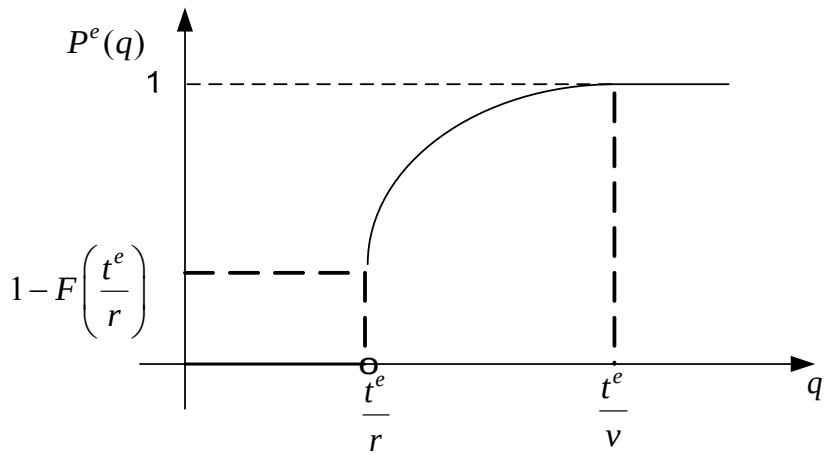

Figure 2.4: The revenue probability for a newsvendor as a function of order quantity.

The implication is clear. The newsvendor has to order large enough such that her revenue target is achievable. Once the newsvendor orders $q = \frac{t^e}{v}$, her revenue will be t^e under the worst scenario: even if there is no regular demand, she will achieve her revenue target simply by salvaging the stock.

2.4 Newsvendor under Profit and Revenue Satisficing Objective

It is likely that a firm has both profit and revenue satisficing objectives. In this section, we study a newsvendor under both satisficing objectives. The newsvendor has a target profit and a target revenue at t^p and t^e, respectively. We use the superscript "p" to denote profit-target-related parameters and variables when both profit and revenue targets are considered. For notational simplicity, we will drop the superscript "p" when only the profit-target-related parameters and variables are present.

The goal of the newsvendor is to maximize the joint probability to achieve both targets, i.e., to maximize $P^{pe}(q) = P\{\Pi^p(q) \geq t^p \ \& \ \Pi^e(q) \geq t^e\}$. It is worthwhile noticing that by this formulation, we treat profit target and revenue target equally important.

Before we proceed to study the newsvendor's joint probability to achieve both targets, it would be interesting to see when the profit probability or the revenue probability is larger. To this end, we first assume the participation constraint is satisfied: both the profit and revenue targets have to be attainable, i.e.,

$$q \geq \max\left\{\frac{t^p}{r-c}, \frac{t^e}{r}\right\} \tag{2.14}$$

We consider two cases separately. In the first case of $\frac{t^e}{t^p} \leq \frac{r}{r-c}$, the participation constraint becomes q $\geq \frac{t^e}{r-c}$ and

$$1\text{-}F\left(\frac{t^e}{r}\right) = P^e(q) \geq P^p(q) = 1\text{-}F\left(\frac{t^p}{r-c}\right) \tag{2.15}$$

It is worthwhile noticing that *r* and *r-c* are the revenue margin and profit margin, respectively. Therefore, if the ratio between the revenue target and the profit target is no larger than the ratio between the revenue margin and the profit margin, the revenue probability is automatically larger than the profit probability. In another word, if the

revenue target is relatively lower than the profit target, the newsvendor needs concern about only the profit target.

In the second case of $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, the participation constraint becomes $q \geq \frac{t^e}{r}$. Since the profit probability decreases and the revenue probability increases in the order quantity, respectively, they intersect at order quantity such that:

$$1-F\left(\frac{(c-v)q+t^p}{r-v}\right)=1-F\left(\frac{-vq+t^e}{r-v}\right) \tag{2.16}$$

which gives $q = \frac{t^e - t^p}{c}$. The associated profit and revenue probability is $1-F\left(\frac{(c-v)t^e+vt^p}{c(r-v)}\right)$. We plot both the profit and revenue probabilities in Figure 2.5 as below.

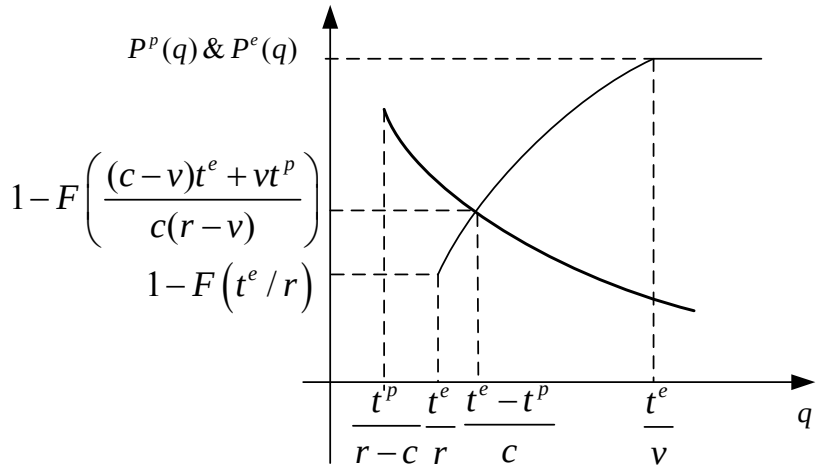

Figure 2.5: The profit (bold line) and revenue probabilities as functions of order quantity when $\frac{t^e}{t^p} \geq \frac{r}{r-c}$.

Therefore, if the revenue target is large enough, i.e., $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, the revenue probability will be less than the profit probability in the interval $\left[\frac{t^e}{r}, \frac{t^e-t^p}{c}\right]$. When the order quantity becomes larger than $\frac{t^e-t^p}{c}$, the revenue probability will be larger than the profit probability. This is due to the fact that the revenue always increases in order quantity while profit first increases and then decreases in order quantity.

Now we maximize the joint probability to achieve both profit and revenue targets, i.e., to maximize $P^{pe}(q) = P\{\Pi^p(q) \geq t^p \ \& \ \Pi^e(q) \geq t^e\}$. We have the following theorem.

Theorem 2.2:

Suppose a newsvendor is to maximize the joint probability to achieve profit and revenue targets. If $\frac{t^e}{t^p} < \frac{r}{r-c}$, her optimal order quantity and the associated maximal joint probability are given by:

$$q^* = \frac{t^p}{r-c} \tag{2.17}$$

$$P^{pe*} = 1 - F\left(\frac{t^p}{r-c}\right) \tag{2.18}$$

On the other hand, if $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, her optimal order quantity and the associated maximal joint probability are given by:

$$q^* = \frac{t^e - t^p}{c} \tag{2.19}$$

$$P^{pe*} = 1 - F\left(\frac{(c-v)t^e + vt^p}{c(r-v)}\right) \tag{2.20}$$

Proof.

We first derive the joint probability as a function of order quantity. The newsvendor's goal is to maximize $P^{pe}(q) = P\{\Pi^p(q) \geq t^p \ and \ \Pi^e(q) \geq t^e\}$, which can be rewritten as:

$$P^{pe}(q) = P\ \{\ (q-D)^{+} \le \frac{(r-c)q-t^{p}}{r-v}\ \&\ (q-D)^{+} \le \frac{rq-t^{e}}{r-v}\ \}$$

We consider two cases separately. In the first case where $q \le \frac{t^e - t^p}{c}$, we have:

$$\frac{rq-t^{e}}{r-v} \le \frac{(r-c)q-t^{p}}{r-v} \tag{2.21}$$

which means $P^{pe}(q) = P^{e}(q)$. Recall the revenue probability as in Figure 2.4, we need determine whether $\frac{t^e - t^p}{c}$ or $\frac{t^e}{r}$ is larger. Note that $\frac{t^e - t^p}{c} < \frac{t^e}{v}$ is always true. If $\frac{t^e - t^p}{c} < \frac{t^e}{r}$, i.e., $\frac{t^e}{t^p} < \frac{r}{r-c}$, we have $P^{pe}(q) = P^{e}(q) =$ 0. If $\frac{t^e}{t^p} \ge \frac{r}{r-c}$, we have the following based on Figure 2.4:

$$P^{pe}(q) = 0 \text{ if } q < \frac{t^e}{r}$$

$$P^{pe}(q) = 1 - F\left(\frac{-vq+t^{e}}{r-v}\right) \text{ if } \frac{t^e}{r} \le q \le \frac{t^e - t^p}{c} \tag{2.22}$$

Now we consider the second case where $q > \frac{t^e - t^p}{c}$, which implies $P^{pe}(q) = P^p(q)$.

Based on Figure 2.2, we need to determine the relative magnitude of $\frac{t^e - t^p}{c}$ and $\frac{t^p}{r-c}$. If $\frac{t^e}{t^p} \geq \frac{\text{r}}{\text{r-c}}$, we have $\frac{t^e - t^p}{c} > \frac{t^p}{r-c}$ and the following:

$$P^{pe}(q) = 1 - F\left(\frac{(c-v)q + t^p}{r-v}\right) \text{ if } q \geq \frac{t^e - t^p}{c} \tag{2.23}$$

On the other hand, if $\frac{t^e}{t^p} < \frac{\text{r}}{\text{r-c}}$, we have $\frac{t^e - t^p}{c} < \frac{t^p}{r-c}$ and the following:

$$P^{pe}(q) = 0 \text{ if } \frac{t^e - t^p}{c} \leq q < \frac{t^p}{r-c} \tag{2.24}$$

$$P^{pe}(q) = 1 - F\left(\frac{(c-v)q + t^p}{r-v}\right) \text{ if } q \geq \frac{t^p}{r-c} \tag{2.25}$$

Now we summarize the results of the two cases. If $\frac{t^e}{t^p} < \frac{r}{r-c}$, we have:

$$P^{pe}(q) = 0 \text{ if } q < \frac{t^p}{r-c} \tag{2.26}$$

$$P^{pe}(q) = 1 - F\left(\frac{(c-v)q + t_p}{r-v}\right) \text{ if } q \geq \frac{t^p}{r-c} \tag{2.27}$$

which is plotted in Figure 2.6. Therefore, $P^{pe}(q)$ decreases in order quantity when $q \geq \frac{t^p}{r-c}$. The joint probability is maximized at $\frac{t^p}{r-c}$ and the associated maximal joint probability is given by (2.18). If $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, we have:

$$P^{pe}(q) = 0 \text{ if } q < \frac{t^e}{r}$$

$$P^{pe}(q) = 1 - F\left(\frac{-vq + t^e}{r-v}\right) \text{ if } \frac{t^e}{r} \leq q \leq \frac{t^e - t^p}{c}$$

$$P^{pe}(q) = 1 - F\left(\frac{(c-v)q + t^p}{r-v}\right) \text{ if } q \geq \frac{t^e - t^p}{c} \tag{2.28}$$

which is plotted in Figure 2.8. Therefore, $P^{pe}(q)$ first increases and then decreases in order quantity. The joint probability is maximized at $\frac{t^e - t^p}{c}$ and the associated maximal joint probability is given by (2.20). □

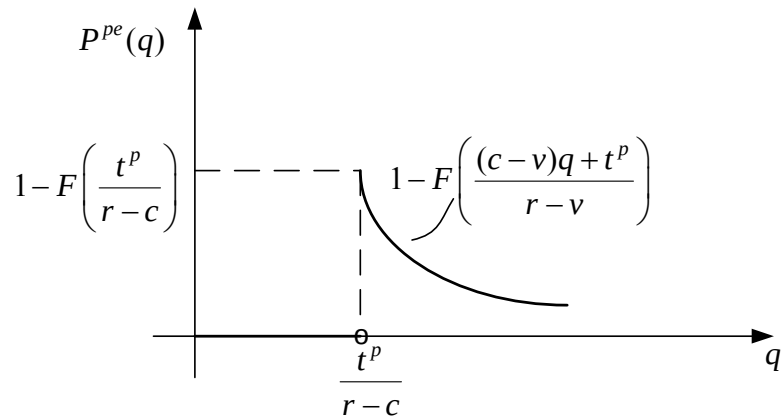

Figure 2.6: The joint probability for a newsvendor to achieve her profit and revenue targets when $\frac{t^e}{t^p} < \frac{r}{r-c}$.

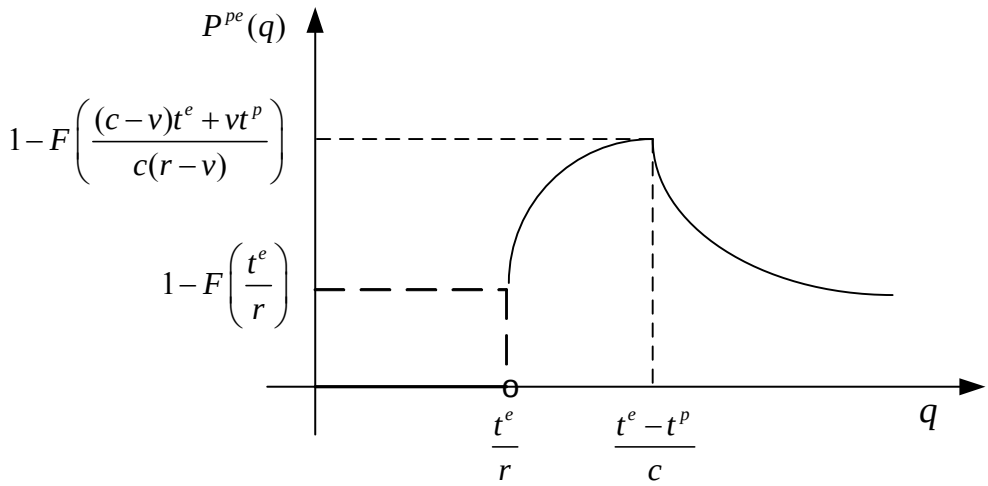

Figure 2.7: The joint probability for a newsvendor to achieve her profit and revenue targets when $\frac{t^e}{t^p} > \frac{r}{r-c}$.

Now we have a few interesting observations. We have two important ratios. One is the ratio between the revenue target and profit target, for which we call it target ratio. The

other one is the ratio between the revenue margin and profit margin, for which we call it margin ratio. If the target ratio is lower than the margin ratio, the revenue target is "redundant". The newsvendor only needs to take care of her profit target. Her joint probability to achieve both targets is just the profit probability. Therefore, the discussions in Section 2.2 apply.

On the other hand, if the target ratio is higher than the margin ratio, the revenue target matters for the newsvendor. When the joint probability increases in order quantity first, it is because a larger order quantity helps the newsvendor to achieve her revenue target. When the order quantity becomes sufficiently large, the joint probability decreases because an order quantity of this high will risk the profit probability. As a result, the joint probability decreases first and then increases in order quantity. The maximal joint probability is achieved when the revenue probability and profit probability are equal, i.e., $q^* = \frac{t^e - t^p}{c}$. Note that this formulation is in a surprisingly simple form. It only depends on procurement cost and the difference between the revenue and profit targets. The associated maximal joint probability is $P^{pe*} = 1 - F\left(\frac{(c-v)t^e + vt^p}{c(r-v)}\right)$, which depends on both targets, demand distribution and all parameters of a newsvendor. It is obvious that the maximal profit and revenue probability P^{pe*} decreases with regard to both profit target and revenue target. To see how P^{pe*} changes with regard to the procurement cost c we define $G(c) = \frac{(c-v)t^e + vt^p}{c} = t^e - \frac{v}{c}\left(t^e - t^p\right)$. Therefore $G(c)$ increases with regard to c, which means that P^{pe*} decreases with regard to c.

To see how the probability changes with regard to the salvage price, we take a derivative:

$$\frac{\partial P^{pe*}}{\partial v} = -f\left(\frac{(c-v)t^e + vt^p}{c(r-v)}\right) * \frac{ct^e - r(t^e - t^p)}{c^2(r-v)^2} > 0 \tag{2.29}$$

The inequality above holds due to $\frac{t^e}{t^p} > \frac{r}{r-c}$. It makes intuitive sense: a larger salvage price will help the newsvendor to achieve her profit and revenue targets.

2.5 Price-setting Newsvendor under Profit Satisficing Objective

Now we consider the situation where the newsvendor also decides on retail price *r*. The critical modeling assumption here is how to incorporate randomness in demand as a function of price. There are two types of demand models, namely the additive and multiplicative demand models (Petruzzi and Dada 1999). In this section, we will consider the additive case first and the multiplicative case next.

2.5.1 Additive Demand Model

In the additive case, demand is models as $D(r,\varepsilon) = y(r) + \varepsilon$. The random part ε is a random variable independent of retail price *r* with cumulative distribution function (CDF) $F(x)$. The deterministic part $y(r)$ is decreasing in *r*, i.e., $y'(r) < 0$. This model implies that the retail price only affect the location of the demand but not the shape. The

demand variance is a constant. Furthermore, if we use $F_D(x)$ to denote the CDF of the random demand, we then have:

$$F_D(x) = F(x - y(r)) \tag{2.30}$$

The next theorem determines the optimal retail price for the additive demand model.

Theorem 2.3:

Suppose a price-setting newsvendor faces with an additive demand $D(r,\varepsilon) = y(r) + \varepsilon$. where $y(r)$ is a deterministic and ε is the stochastic. If $y(r)$ is concave with regard to *r*, the optimal price r^* is obtained by solving:

$$-y'(r) = \frac{t}{(r-c)^2} \tag{2.31}$$

Proof.

Given *r*, the maximal profit probability is given by:

$$\begin{aligned} P^*(r) &= 1 - F_D\left(\frac{t}{r-c}\right) \\ &= 1 - F\left(\frac{t}{r-c} - y(r)\right) \end{aligned} \tag{2.32}$$

To maximize $P^*(r)$, we need to minimize

$$A(r) = \frac{t}{r-c} - y(r) \tag{2.33}$$

We take the first and the second derivatives with regard to *r*,

$$\frac{\partial A(r)}{\partial r} = -\frac{t}{(r-c)^2} - y'(r) \tag{2.34}$$

$$\frac{\partial^2 A(r)}{\partial r^2} = \frac{2t}{(r-c)^3} - y''(r) \tag{2.35}$$

If $y(r)$ is concave, then we have $y''(r) < 0$ and $\frac{\partial^2 A(r)}{\partial r^2} > 0$. This means $A(r)$ is convex with regard to the selling price *r*. Therefore, the optimal r^* is obtained by setting $\frac{\partial A(r)}{\partial r} = 0$. □

As an example, we consider a specific additive demand model where $y(r) = a - b\,r$ and $D(r,\varepsilon) = a - br + \varepsilon$. This is a model commonly adopted in economics literature. Obviously $y(r) = a - br$ is concave with regard to *r*. By solving $-y'(r) = b = \frac{t}{(r-c)^2}$, we have:

$$r^* = \sqrt{\frac{t}{b}} + c \tag{2.36}$$

Therefore, the optimal retail price is to mark up the production cost by $\sqrt{\frac{t}{b}}$. The higher the profit target, the higher the optimal retail price. The higher price elasticity, the lower the optimal retail price. Furthermore, the optimal retail price is robust to the target profit and price elasticity. This is certainly good news because it is not always easy to figure out the target profit and price elasticity.

Substituting r^* into $q^*(r) = \frac{t}{r-c}$ and $P^*(r) = 1 - F_D\left(\frac{t}{r-c}\right) = 1 - F\left(\frac{t}{r-c} - a + br\right)$, we have:

$$q^* = \sqrt{bt} \tag{2.37}$$

$$P^* = 1 - F\left(2\sqrt{bt} + bc - a\right) \tag{2.38}$$

Once again, both optimal order quantity and the associated profit probability are robust in the target profit.

2.5.2 Multiplicative Demand Model

Now we consider the multiplicative demand model $D(r,\varepsilon) = y(r)\ \varepsilon$. The random part ε is independent of selling price r and has a CDF of $F(x)$. The deterministic part $y(r)$ is

again decreasing with regard to r. It can be seen that this formulation implies that the retail price will affect the shape of the demand but not the location. Furthermore, we have:

$$F_D(x) = F\left(\frac{x}{y(r)}\right) \tag{2.39}$$

Before we solve the optimal price, we introduce the concept of Increasing Pricing Elasticity (IPE). Price elasticity is defined as:

$$e(r) = -\frac{ry'(r)}{y(r)} \tag{2.40}$$

It measures the percent decrease in demand as a result of a percent increase in price. IPE then means $\frac{de(r)}{dr} \geq 0$. If a demand has the property of IPE, then price elasticity is larger when price is setting a higher level, which indicates that it is less desirable to raise price further.

The following theorem determines the optimal retail price for the multiplicative demand model.

Theorem 2.4:

Suppose a price-setting newsvendor faces with a multiplicative demand $D(r,\varepsilon)=y(r)\ \varepsilon$. where $y(r)$ is a deterministic and ε is the stochastic. If $y(r)$ has the IPE property, then the optimal selling price r^* is obtained by solving:

$$-\ y'(r)=\frac{y(r)}{r-c} \tag{2.41}$$

Proof.

Given *r*, the maximal profit probability is given by:

$$P^*(r)=1\text{-}F_D\left(\frac{t}{r-c}\right)=1\text{-}F\left(\frac{t}{(r-c)y(r)}\right) \tag{2.42}$$

To maximize $P^*(r)$, we need maximize $B(r)=(r-c)y(r)$. We have:

$$\frac{\partial B(r)}{\partial r}=y(r)+(r-c)y'(r) \tag{2.43}$$

$$\frac{\partial^2 B(r)}{\partial r^2}=(r-c)y''(r)+2\,y'(r) \tag{2.44}$$

For simplicity, suppose the maximal allowed price is positive infinity. We have:

$$\frac{\partial B(r)}{\partial r}\Big|_{r=c} = y(c) > 0 \tag{2.45}$$

$$\frac{\partial B(r)}{\partial r}\Big|_{r=\infty} = (\infty - c)\, y'(\infty) < 0 \tag{2.46}$$

Therefore, $\frac{\partial B(r)}{\partial r}$ =0 has at least one solution. We have the following based on the assumption that $y(r)$ has the IPE property:

$$y''(r) \le \left[\frac{ry'(r)}{y(r)} - 1 \right] \frac{y'(r)}{r} \tag{2.47}$$

Hence, we have:

$$\frac{\partial^2 B(r)}{\partial r^2} \le (r\text{-}c)\, [\, \frac{ry'(r)}{y(r)} - 1\,] \frac{y'(r)}{r} + 2\, y'(r) = y'(r)\ C(r) \tag{2.48}$$

where $C(r) = \frac{r-c}{r} [\, \frac{ry'(r)}{y(r)} - 1\,] + 2.$

At $\frac{\partial B(r)}{\partial r} = 0$, we have:

$$(r-c)\, y'(r) = -\, y(r) \tag{2.49}$$

Therefore, we have:

$$C(r)\Big|_{\frac{\partial B(r)}{\partial r}=0} = \frac{r-c}{r}[-\frac{r}{r-c}-1]+2$$

$$=\frac{c}{r} > 0 \tag{2.50}$$

which indicates:

$$\frac{\partial^2 B(r)}{\partial r^2}\Big|_{\frac{\partial B(r)}{\partial r}=0} < 0 \tag{2.51}$$

So we conclude that $B(r)=(r-c)y(r)$ is quasi-concave in retail price r. The optimal retail price r^* is then obtained by setting $B'(r)$=0, which gives the equation (2.41). □

As an example, we consider the special case where $y(r)=r^{-b}$, which is called iso-elastic demand curve in economics literature. The parameter b is the demand elasticity and b > 1. Therefore, the demand is modeled as $D(r,\varepsilon)=r^{-b}\ \varepsilon$. This is the most frequently used demand model in econometrics and marketing empiricists. Besides it analytic appeal, Monahan *et al* (2004) summarizes four reasons for the model's popularity.

Based on Theorem 2.4, we have the optimal retail price by solving $-\ y'(r)=\frac{y(r)}{r-c}$:

$$r^* = \frac{b}{b-1}c \tag{2.52}$$

So the optimal price only depends on two parameters, namely the price elasticity and the procurement cost. Therefore, the optimal price takes a surprising simple form: it is just a multiple of the procurement cost. Surprisingly, the optimal price is independent of the profit target.

The corresponding optimal order quantity and the maximal profit probability are given by:

$$q^* = \frac{(b-1)t}{c} \tag{2.53}$$

$$P^* = 1 - F\left(\left(\frac{c}{b-1}\right)^{b-1} b^b t\right) \tag{2.54}$$

Therefore, the optimal order quantity is proportional to the target profit. The associated maximal profit probability is decreasing in the target profit.

2.6 Price-setting Newsvendor under Profit and Revenue Satisficing Objective

We study price-setting newsvendor under profit satisficing objective only in Section 2.5. What if the newsvendor adopts the profit and revenue satisficing objective? In such a

situation, the newsvendor has both a profit target and a revenue target to achieve. Suppose her profit and revenue targets are set exogenously at t^p and t^e, respectively. Her objective is then to maximize the joint probability $P^{pe}(q,r)$ = $P\{\Pi^p(q,r) \geq t^p \ \& \ \Pi^e(q,r) \geq t^e\}$. Note that the newsvendor now has two decision variables, the retail price and the order quantity.

In Section 2.3, we assume a traditional newsvendor with fixed retail price *r*. We obtain analytic solutions to the optimal order quantity and the associated maximal joint probability to achieve both profit and revenue targets. In this section, we assume the newsvendor will decide on both order quantity and retail price. To gain more managerial insights, we adopt the two special models used in Section 2.5, one belongs to additive demand model and the other belongs to multiplicative demand model. The detailed analysis follows.

2.6.1 Additive Demand Model

The demand is models as $D(r,\varepsilon) = a - br + \varepsilon$, where ε follows a CDF of $F(x)$. Then the demand follows CDF $F_D(x) = F(x - a + br)$. For simplicity, we define two constants:

$$b_1 = \frac{(t^e - t^p)^2}{\mathrm{c}[(\mathrm{c}-\mathrm{v})t^e + vt^p]} \tag{2.55}$$

$$b_2 = \frac{(t^e - t^p)^2}{ct^p} \tag{2.56}$$

It can easily verified that $b_1 < b_2$. We have the following theorem on the newsvendor's optimality.

Theorem 2.5.

Suppose a price-setting newsvendor adopts the profit and revenue satisficing objective. If the customer demand is specified by the additive demand model $D(r,\varepsilon) = a - br + \varepsilon$, the optimal retail price and order quantity and the associated maximal joint probability are given by Table 2.1:

Table 2.1: Price-setting newsvendor under the additive demand model.

Price Elasticity b	*Optimal Retail Price* r^*	*Optimal Order Quantity* q^*	*Maximal joint probability* P^{pe*}
$b < b_1$	$\sqrt{\frac{(c-v)t^e + vt^p}{bc}} + v$	$\frac{t^e - t^p}{c}$	$1 - F\left(2\sqrt{b}\sqrt{\left(1-\frac{v}{c}\right)t^e + \frac{v}{c}t^p} + bv - a\right)$
$b_1 < b < b_2$	$\frac{t^e}{t^e - t^p}c$	$\frac{t^e - t^p}{c}$	$1 - F\left(\frac{t^e - t^p}{c} + \frac{t^e}{t^e - t^p}bc - a\right)$
$b > b_2$	$\sqrt{\frac{t^p}{b}} + c$	$\sqrt{bt^p}$	$1 - F\left(2\sqrt{bt^p} + bc - a\right)$

Proof.

From Section 2.3, it can be seen that we need consider two separate cases.

Case 1: $\frac{t^e}{t^p} < \frac{r}{r-c}$, i.e., $r < \frac{t^e}{t^e - t^p} c$

Under Case 1, we have:

$$q^*(r) = \frac{t^p}{r-c} \tag{2.57}$$

$$P^{pe*}(r) = 1 - F\left(\frac{t^p}{r-c} - a + br\right) \tag{2.58}$$

To maximize $P^{pe*}(r)$, we need to minimize $\frac{t^p}{r-c} - a + br$, which is concave in r. From the first order condition, we have:

$$r^* = \sqrt{\frac{t^p}{b}} + c \tag{2.59}$$

Combining the constraint for Case 1, we have:

$$r^* = \min\left\{\sqrt{\frac{t^p}{b}} + c,\ \frac{t^e}{t^e - t^p} c\right\} \tag{2.60}$$

It can verified easily that if $b \geq b_2$, $r^* = \sqrt{\frac{t^p}{b}} + c$; otherwise, $r^* = \frac{t^e}{t^e - t^p} c$.

Case 2: $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, i.e., $r \geq \frac{t^e}{t^e - t^p} c$

Under Case 2, we have:

$$q^*(r) = \frac{t^e - t^p}{c} \tag{2.61}$$

$$P^{pe*}(r) = 1 - F\left(\frac{(c-v)t^e + vt^p}{c(r-v)} - a + br\right) \tag{2.62}$$

To maximize $P^{pe*}(r)$, we need to minimize $\frac{(c-v)t^e + vt^p}{c(r-v)} - a + br$, which again is concave. By setting the first order condition to zero, we have:

$$r^* = \sqrt{\frac{(c-v)t^e + vt^p}{bc}} + v \tag{2.63}$$

Combining with the constraint for Case 2, we have:

$$r^* = \max\ \{\sqrt{\frac{(c-v)t^e + vt^p}{bc}} + v,\ \frac{t^e}{t^e - t^p} c\} \tag{2.64}$$

So it depends on the relative magnitudes of these two terms. If b $< b_1$, it can be verified $r^* = \sqrt{\frac{(c-v)t^e + vt^p}{bc}} + v$. Otherwise, $r^* = \frac{t^e}{t^e - t^p} c$.

Summarizing the above-mentioned two cases, we have the optimal retail prices in the Theorem 2.5. By substituting the optimal retail prices appropriately, we can have the corresponding optimal order quantity and maximal join probability as in Theorem 2.5. □

2.6.2 Multiplicative Demand Model

The demand is modeled as $D(r,\varepsilon) = r^{-b}\,\varepsilon$, where $b > 1$ is the demand elasticity and random variable ε follows a CDF of $F(x)$. Then the demand follows CDF $F_D(x) = F\left(\frac{x}{y(r)}\right)$. For simplicity, we define two constants:

$$b_1 = \frac{t^e}{(c-v)t^e + vt^p} c \tag{2.65}$$

$$b_2 = \frac{t^e}{t^p} \tag{2.66}$$

It can be easily verified that $b_1 < b_2$. Now we determine the optimal retail price, optimal order quantity, and the maximal joint probability for the newsvendor, which is stated in Theorem 2.6.

Theorem 2.6:

Suppose a price-setting newsvendor adopts the profit and revenue satisficing objective. If the customer demand is specified by the multiplicative demand model $D(r,\varepsilon)=r^{-b}\,\varepsilon$, the optimal retail price and order quantity and the associated maximal joint probability are given by Table 2.2:

Table 2.2: Price-setting newsvendor under the multiplicative demand model.

Price Elasticity b	*Optimal Retail Price* r^*	*Optimal Order Quantity* q^*	*Maximal joint probability* P^{pe*}
$b<b_1$	$\frac{b}{b-1}v$	$\frac{t^e-t^p}{c}$	$1\text{-}F\left(\frac{(c-v)t^e+vt^p}{cb^{-b}(b-1)^{-(b-1)}v^{-(b-1)}}\right)$
$b_1<$ $b<b_2$	$\frac{t^e}{t^e-t^p}c$	$\frac{t^e-t^p}{c}$	$1\text{-}F\left(\left(t^e\right)^b\left(\frac{t^e-t^p}{c}\right)^{-(b-1)}\right)$
$b>b_2$	$\frac{b}{b-1}c$	$\frac{b-1}{c}t^p$	$1\text{-}F\left(\left(\frac{c}{b-1}\right)^{b-1}b^b t^p\right)$

Proof.

Similar to the proof in the previous theorem, we consider two separate cases first.

Case 1: $\frac{t^e}{t^p} < \frac{r}{r-c}$ or $r < \frac{t^e}{t^e - t^p} c$

$$P^{pe*}(r) = 1 - F\left(\frac{t^p}{(r-c)r^{-b}}\right) \tag{2.67}$$

To maximize $P^{pe*}(r)$, it is enough to maximize $(r-c)r^{-b}$, which is convex in r. By setting the first order condition to zero, we have:

$$r^* = \frac{b}{b-1} c \tag{2.68}$$

Combining with the constraint in Case 1, we have the optimal retail price as:

$$r^* = \max\{\frac{b}{b-1} c, \frac{t^e}{t^e - t^p} c\} \tag{2.69}$$

It can be verified that if $b > b_2$, $r^* = \frac{b}{b-1} c$; Otherwise $r^* = \frac{t^e}{t^e - t^p} c$.

Case 2: $\frac{t^e}{t^p} \geq \frac{r}{r-c}$, or $r \geq \frac{t^e}{t^e - t^p} c$

$$P^{pe*}(r) = 1 - F\left(\frac{(c-v)t^e + vt^p}{c(r-v)r^{-b}}\right) \tag{2.70}$$

Similarly, $P^{pe*}(r)$ is maximized at $\frac{b}{b-1}v$. Combining with the constraint $r \geq \frac{t^e}{t^e - t^p} c$ in Case 2, we have:

$$r^* = \max\{\frac{b}{b-1}v, \frac{t^e}{t^e - t^p} c\} \tag{2.71}$$

If $b > b_1$, $r^* = \frac{t^e}{t^e - t^p} c$; Otherwise $r^* = \frac{b}{b-1}v$.

Summarizing the results obtained above, we have the optimal retail prices as in Theorem 2.6. Substituting the optimal prices appropriately, we can have the corresponding optimal order quantity and maximal joint probability as in Theorem 2.6. □

2.7 Competitive Newsvendors under Profit Satisficing Objective

So far we have focused on a single newsvendor, either a classical newsvendor or a price-setting newsvendor. In this section, we study multiple newsvendors under inventory competition. There has been extensive research literature on competitive newsvendors

(Lippman and McCardle 1997, Netessine and Rudi 2003, and references therein). In this research stream, one critical assumption is that how individual stocking level will affect everyone's customer demand as well as the aggregate demand of the whole industry. In the first group of this research stream, a newsvendor's stocking level decision will affect the demands of all newsvendors, including her own. One typical assumption in the literature is that the aggregate industry demand is fixed and a newsvendor's demand will be proportional to her stocking level (Lippman and McCardle 1997). In this way, the newsvendors will compete by choosing their stocking levels. For clarity, we call the demand model in the first group Model I.

In the second group of this research, a newsvendor's stocking level decision will affect all other newsvendors' demands, but not her own. One typical assumption in the literature is that each newsvendor has an "initial" independent customer demand. Then the "initial" demand, or more likely the sale, of a newsvendor will affect the demand of all other newsvendors. The similar or the same product sold by the newsvendors can either be substitutable and/or complementary (Netessine and Zhang 2005). If the newsvendors are selling substitutable products, inventory competition occurs: if a customer goes to a newsvendor who is out of stock, the customer will make a second attempt and try another newsvendor. If the newsvendors are selling complementary products, a customer buying a product from one newsvendor will buy some complementary product from another newsvendor. Either way, the stocking level of a newsvendor will affect the demand of other newsvendors, but not her own. It is also

worthwhile noticing that the aggregate demand here is variable. Again for clarity, we call the demand model in the second group the Model II.

In this section, we study n newsvendors competing in stocking level. Newsvendor i has a target profit t_i to achieve and she adopts a profit satisficing objective. We use the following notation: q_i denotes newsvendor i's order quantityand q_{-i} = $[q_1, \ldots q_{i-1}, q_{i+1}, \ldots q_n]$ denotes the order quantity vector of all but newsvendor i's order quantity.

2.7.1 Model I

In this subsection, we study the situation where newsvendors engage in inventory competition under fixed total retail demand. The total demand D is a random variable with a CDF of $F(x)$. A fixed total retail demand makes sense for relatively mature products. In such a situation, a critical assumption to make is how to allocation demand among competing retailers. We adopt a common allocation model in the literature, namely, the proportional allocation model. (Wang and Gerchak 2001, Cachon 2003). As the result, the newsvendor i's demand $D_i = \frac{q_i}{q} D$. If the CDF of D_i is denoted by $F_i(x)$, then the following relationship holds:

$$F_i(x) = F\left(\frac{q}{q_i} x\right) \tag{2.72}$$

From Section 2.2, we know that the newsvendor has a participation constraint of $q_i \geq \frac{t_i}{r_i - c_i}$. Once the constraint is satisfied, her profit probability is given by:

$$P_i(q_i) = 1 - F_i\left(\frac{(c_i - v_i)q_i + t_i}{r_i - v_i}\right)$$

$$= 1 - F\left(\frac{q}{q_i}\frac{(c_i - v_i)q_i + t_i}{r_i - v_i}\right) \tag{2.73}$$

The following theorem gives the optimal order quantity.

Theorem 2.7:

Suppose there are *n* competing newsvendors with fixed total demand. If the individual demand follows the proportional allocation model, then the newsvendor *i*'s optimal order quantity is given by: $q_i^* = \max\{\frac{t_i}{r_i - c_i}, q_i^0\}$ where q_i^0 is obtained by solving:

$$\frac{q_i^2}{q - q_i} = \frac{t_i}{c_i - v_i} \tag{2.74}$$

Proof.

From equation (2.73), it can be seen that to maximize $P_i(q_i)$, it is sufficient to minimize the following:

$$A(q_i) = (c_i - v_i)q + t_i \frac{q}{q_i} \tag{2.75}$$

because $P_i(q_i) = 1 - F\left(\frac{A(q_i)}{r_i - v_i}\right)$. The first order and second order optimality conditions are given by:

$$\frac{\partial A(q_i)}{\partial q_i} = (c_i - v_i) - t_i \frac{q - q_i}{q_i^2} \tag{2.76}$$

$$\frac{\partial^2 A(q_i)}{\partial q_i^2} = \frac{2t_i(q - q_i)}{q_i^3} > 0 \tag{2.77}$$

Therefore, $A(q_i)$ is convex in q_i. By setting the first order condition to zero, we can obtain q_i^o. Combining with the participation constraint, we proved the results in this Theorem 2.7. □

To gain more managerial insights, we consider the following two special cases.

Case 1: Symmetrical Newsvendors

In this case, we assume symmetrical newsvendors, i.e., they have the same cost structure and the same target profit t. Therefore, equation (2.74) becomes:

$$\frac{q_i^2}{q-q_i}=\frac{t}{c-v} \tag{2.78}$$

If the solution to newsvendor i is q_i^0, then the total will be $q^0 = n\,q_i^0$ due to the symmetry, which leads to:

$$q_i^0=\frac{(n-1)t}{c-v} \tag{2.79}$$

Therefore, the newsvendor i's optimal order quantity is given by:

$$q_i^* = \max\ \{\frac{(n-1)t}{c-v},\frac{t}{r-c}\ \} \tag{2.80}$$

So the optimal order quantity depends on n, the number of the newsvendors. If $n \geq \frac{r-v}{r-c}$, $q_i^*=\frac{(n-1)t}{c-v}$, $q^*=\frac{n(n-1)t}{c-v}$, and the associated maximal profit probability is given by:

$$P_i^* = 1 - F\left(\frac{n^2 t}{r-v}\right) \tag{2.81}$$

On the other hand, if $n \le \frac{r-v}{r-c}$, $q_i^* = \frac{t}{r-c}$, $q^* = \frac{nt}{r-c}$, and the maximal profit probability is:

$$P_i^* = 1 - F\left(\frac{nt}{r-c}\right) \tag{2.82}$$

For comparison purpose, let us consider the centralized version. Since each newsvendor has a profit target of *t*, the total profit target for the system would be nt. Then the optimal order quantity and maximal profit probability is given by:

$$q^* = \frac{nt}{r-c} \tag{2.83}$$

Therefore, if the number of newsvendor is not high, i.e., $n \le \frac{r-v}{r-c}$, the total inventory will be the same for the decentralized and centralized system. On the other hand, if the number of newsvendors is sufficiently high, i.e., $n \ge \frac{r-v}{r-c}$, the total inventory will be higher for the decentralized system than the centralized system. This is due to the fact that:

$$\frac{n(n-1)t}{c-v} \ge \frac{nt}{r-c} \tag{2.84}$$

The results above make great sense. When the competition is not as severe, it will not affect the behavior of the newsvendors. However, when the competition gets more severe, the newsvendors will tend to order more.

Case 2: Asymmetrical Newsvendors

Now we consider asymmetrical newsvendors. Asymmetrical newsvendor general means they have different cost structure and different target profit. For simplicity, we consider a special situation where two newsvendors have the same cost structure but different target profits. Under such a situation, equation (2.74) becomes:

$$\frac{q_1^2}{q_2} = \frac{t_1}{c-v} \tag{2.85}$$

$$\frac{q_2^2}{q_1} = \frac{t_2}{c-v} \tag{2.86}$$

We have the following solutions:

$$q_1^0 = \frac{\left(t_1^2 t_2\right)^{1/3}}{c-v} \tag{2.87}$$

$$q_2^0 = \frac{\left(t_2^2 t_1\right)^{1/3}}{c-v} \tag{2.88}$$

The newsvendor's optimal order quantity is then given by:

$$q_1^* = \max\ \{\, q_1^0 , \frac{t_1}{r-c}\ \} \tag{2.89}$$

$$q_2^* = \max\ \{\, q_2^0 , \frac{t_2}{r-c}\ \} \tag{2.90}$$

It can be verified that if the product is relatively more profitable, i.e., $r-c \geq c-v$, we have the following Table 2.3:

Table 2.3: Optimal order quantities for the newsvendors under a more profitable product.

	q_1^*	q_2^*
$\left(\frac{t_1}{t_2}\right)^{1/3} \leq \frac{c-v}{r-c}$	$\frac{\left(t_1^2 t_2\right)^{1/3}}{c-v}$	$\frac{t_2}{r-c}$
$\frac{c-v}{r-c} \leq \left(\frac{t_1}{t_2}\right)^{1/3} \leq \frac{r-c}{c-v}$	$\frac{\left(t_1^2 t_2\right)^{1/3}}{c-v}$	$\frac{\left(t_2^2 t_1\right)^{1/3}}{c-v}$
$\left(\frac{t_1}{t_2}\right)^{1/3} \geq \frac{r-c}{c-v}$	$\frac{t_1}{r-c}$	$\frac{\left(t_2^2 t_1\right)^{1/3}}{c-v}$

On the other hand, if the product is relatively less profitable, i.e., $r-c \leq c-v$, we have the following Table 2.4:

Table 2.4: Optimal order quantities for the newsvendors under a less profitable product.

	q_1^*	q_2^*
$\left(\frac{t_1}{t_2}\right)^{1/3} \leq \frac{r-c}{c-v}$	$\frac{\left(t_1^2 t_2\right)^{1/3}}{c-v}$	$\frac{t_2}{r-c}$
$\frac{r-c}{c-v} \leq \left(\frac{t_1}{t_2}\right)^{1/3} \leq \frac{c-v}{r-c}$	$\frac{t_1}{r-c}$	$\frac{t_2}{r-c}$
$\left(\frac{t_1}{t_2}\right)^{1/3} \geq \frac{c-v}{r-c}$	$\frac{t_1}{r-c}$	$\frac{\left(t_2^2 t_1\right)^{1/3}}{c-v}$

It can be seen that the total inventory is strictly larger except under the situation where the product is relatively less profitable and $\frac{r-c}{c-v} \le \left(\frac{t_1}{t_2}\right)^{1/3} \le \frac{c-v}{r-c}$. The latter implies that the ratio between the two newsvendor's target profits are relatively in the same level.

2.7.2 Model II

In this section, we consider the situation where the total retail demand is variable and is dependent of individual newsvendor's stocking level. Suppose the newsvendors now sell substitutable or complementary products. For newsvendor *i*, he has an initial demand D_i^0, which is independent of each other. After the substitution or complementary effects, her effective demand is given by D_i.

Adopting from (Netessine and Zhang, 2005), the following formally defines complementarity and substitutability.

Definition. If for all *i*, $q_{-i}^{'} \ge q_{-i}$ implies $D_i\left(q_{-i}^{'}\right) \ge D_i\left(q_{-i}\right)$ stochastically, then the newsvendors are selling complementary products. If for all *i*, $q_{-i}^{'} \ge q_{-i}$ implies $D_i\left(q_{-i}^{'}\right) \le D_i\left(q_{-i}\right)$ stochastically, then the newsvendors are selling substitutable products.

Let's first give an example of substitution. If an "original" customer of newsvendor *i* comes but newsvendor *i* is out of stock, s/he will go to go to newsvendor *j* at a deterministic fraction a_{ij}. Then the newsvendor *i*'s effective demand is given by:

$$D_i = D_i^0 + \sum_{j \neq i} a_{ji} \left(D_j^0 - q_j \right)^+ \tag{2.91}$$

Considering the complementary effect. If a customer make an "initial" purchase from newsvendor *i*, s/he will also make a purchase from newsvendor *j* at a deterministic fraction a_{ij}. Then the newsvendor *i*'s effective demand is given by:

$$D_i = D_i^0 + \sum_{j \neq i} a_{ji} \min\left(D_j^0, q_j \right) \tag{2.92}$$

where $\min\left(D_j^0, q_j \right)$ represents the initial sales of newsvendor *j*.

We have the following theorem on inventory competition under substitutable or complementary effect.

Theorem 2.8:

If competitive newsvendors sell substitutable or complementary products, each newsvendor will have the same stocking levels as the situation without the substitutable

or complementary effect. However, the profit probability of each newsvendor will be higher.

Proof.

Each newsvendor's profit probability is given by:

$$P_i^*(q_i) = 1 - F_{D_i}\left(\frac{(c_i - v_i)q_i + t_i}{r_i - v_i}\right) \tag{2.93}$$

Since the effective demand D_i is independent of order quantity, the optimal order quantity will not be affected and the maximal profit probability is given as:

$$P_i^* = 1 - F_{D_i}\left(\frac{t_i}{r_i - c_i}\right) \tag{2.94}$$

However, since $D_i \geq D_i^0$ stochastically, we have:

$$P_i^* \geq 1 - F_{D_i^0}\left(\frac{t_i}{r_i - c_i}\right) \tag{2.95}$$

which indicates the maximal profit probability is higher comparing with the situation where there is no substitutable or complementary effect. □

In summary, the presence of substitution and/or complementary effects will not affect the stocking quantity decisions for newsvendors. It does, however, decrease the chance for them to achieve their goals. Furthermore, our results are very different from those under the objective of expected profit maximization (Netessine and Zhang, 2005).

2.8 Target Setting for Newsvendors

So far we have studies satisficing objectives assuming that the profit and/or revenue targets are exogenous. Obviously, target need to be set properly to be useful. Too high a target will provoke frustration and cynicism, whereas too low a target will engender apathy and risk the firm's survival. Therefore, it is an important topic to study target setting in Operations Management and Supply Chain Management.

In this section, we study the following scenario: A manager employs two newsvendors and the manager adopts profit satisficing objective with a target profit T. The manager's problem is to assign targets to the two newsvendors such that the manager's profit probability is maximized. Given a target, each newsvendor is assumed to adopt profit satisficing objective as well.

Suppose newsvendor i is assigned to have a target profit of t_i. As a result, newsvendor i will order $q_i^* = \frac{t_i}{r_i - c_i}$ to maximize her profit probability. Then the manager's total profit, the sum of the two newsvendors' profits, is given by:

$$\Pi_c(t_1,t_2) = (r_1 - c_1)\, q_1^* - (r_1 - v_1)(q_1^* - D_1)^+ + (r_2 - c_2)\, q_2^* - (r_2 - v_2)(q_2^* - D_2)^+$$

$$= t_1 + t_2 - (r_1 - v_1)\left(\frac{t_1}{r_1 - c_1} - D_1\right)^+ - (r_2 - v_2)\left(\frac{t_2}{r_2 - c_2} - D_2\right)^+ \qquad (2.96)$$

The manager wants to maximize his profit probability:

$$P_c(t_1,t_2) = \mathrm{P}\{\, t_1 + t_2 - (r_1 - v_1)\left(\frac{t_1}{r_1 - c_1} - D_1\right)^+ - (r_2 - v_2)\left(\frac{t_2}{r_2 - c_2} - D_2\right)^+ \geq T \,\} \quad (2.97)$$

and his optimization problem is:

$$\max_{t_1,\, t_2} \mathrm{P}\{\, (r_1 - v_1)\left(\frac{t_1}{r_1 - c_1} - D_1\right)^+ + (r_2 - v_2)\left(\frac{t_2}{r_2 - c_2} - D_2\right)^+ \leq t_1 + t_2 - T \,\} \qquad (2.98)$$

Obviously, it is required that $t_1 + t_2 \geq T$. Otherwise, the manager's profit probability will be zero, i.e., $P_c(t_1,t_2) = 0$. Unfortunately, the optimization problem generally does not have analytic solution. As the first step, we consider two special cases here. In the first case, suppose it is desired that $t_1 + t_2 = T$. In the second case, we consider identical newsvendors.

Case 1: $t_1 + t_2 = T$ and i.i.d. demand

If $t_1 + t_2 = T$, the manager's optimization problem is given by:

$$\max_{t_1, t_2} \mathrm{P}\{ (r_1 - v_1)\left(\frac{t_1}{r_1 - c_1} - D_1\right)^+ + (r_2 - v_2)\left(\frac{t_2}{r_2 - c_2} - D_2\right)^+ \leq 0 \} \tag{2.99}$$

Since the function $(\bullet)^+ \geq 0$, the optimization problem reduces to:

$$\max_{t_1, t_2} \mathrm{P}\{ D_1 \geq \frac{t_1}{r_1 - c_1} \text{ AND } D_2 \geq \frac{t_2}{r_2 - c_2} \} \tag{2.100}$$

which further reduces to:

$$\max_{t_1 \geq 0} \bar{F}_1\left(\frac{t_1}{r_1 - c_1}\right) \bar{F}_2\left(\frac{T - t_1}{r_2 - c_2}\right) \tag{2.101}$$

It can be seen that up to now, the optimization problem is easy to solve. Unfortunately, analytic solution is not available generally. As an example, suppose the i.i.d. demand follows a uniform distribution with a lower bound of *a* and an upper bound of *b*. Given such a demand distribution, it is naturally to assume the profit target satisfies: $(r_1 - c_1)a \leq t_1 \leq (r_1 - c_1)b$ and $(r_2 - c_2)a \leq t_2 \leq (r_2 - c_2)b$.

For the uniform distribution, we have:

$$\bar{F}(x) = \frac{b-x}{b-a},\ a \le x \le b \tag{2.102}$$

Then the optimization problem is equivalent to:

$$\max_{t_1 \ge 0} Z(t_1) = [b(r_1 - c_1) - t_1][b(r_2 - c_2) - T + t_1] \tag{2.103}$$

The first order and the second order conditions are given by:

$$\frac{\partial Z(t_1)}{\partial t_1} = T + b[(r_1 - c_1) - (r_2 - c_2)] - 2t_1 \tag{2.104}$$

$$\frac{\partial^2 Z(t_1)}{\partial t_1^2} = -2 \tag{2.105}$$

Therefore, $Z(t_1)$ is concave function in t_1. Setting $\frac{\partial Z(t_1)}{\partial t_1} = 0$, we have:

$$t_1^* = \frac{T + b[(r_1 - c_1) - (r_2 - c_2)]}{2} \tag{2.106}$$

$$t_2^* = \frac{T + b[(r_2 - c_2) - (r_1 - c_1)]}{2} \tag{2.107}$$

and this concludes the proof. □

The results are surprisingly simple and intuitive. Given a total target of T, the manager will assign $T/2$ to each newsvendor and adjust it further based on difference in their profit margins. If a newsvendor has a relatively larger profit margin, she will be assigned a relatively higher profit target.

Case 2: Identical Newsvendors with i.i.d. Demand

For identical newsvendors, it is natural to assign an identical target to each newsvendor, say t. The manager's optimization problem becomes:

$$\max_{t \geq 0} P_c(t) = \mathrm{P}\{ \left(\frac{t}{r-c} - D_1 \right)^+ + \left(\frac{t}{r-c} - D_2 \right)^+ \leq \frac{2t-T}{r-v} \} \tag{2.108}$$

Again, the inequality of $t \geq T/2$ is necessary to guarantee $P_c(t) \geq 0$. Now we try to solve the optimization problem. First we define:

$$A(t) = \frac{t}{r-c} \tag{2.109}$$

$$B(t) = \frac{2t-T}{r-v} \tag{2.110}$$

It can be verified that $A(t) > B(t)$. The optimization problem now becomes:

$$\max_{t \geq 0} P_c(t) = \mathrm{P}\{ \left(A(t) - D_1\right)^+ + \left(A(t) - D_2\right)^+ \leq B(t) \} \tag{2.111}$$

So it depends on the following inequality:

$$\left(A(t) - D_1\right)^+ + \left(A(t) - D_2\right)^+ \leq B(t) \tag{2.112}$$

and we have the following possible scenarios:

Scenario 1: $D_1 < A(t)$ and $D_2 < A(t)$

Equation (2.112) becomes $D_1 + D_2 \geq 2\,A(t) - B(t)$. it can be verified that the profit probability of scenario 1 is given by:

$$P_1(t) = \int_{A(t)-B(t)}^{A(t)} f(x_2)dx_2 \int_{2A(t)-B(t)-x_2}^{A(t)} f(x_1)dx_1 \tag{2.113}$$

Scenario 2: $D_1 < A(t)$ and $D_2 > A(t)$.

Equation (2.112) becomes $D_1 \geq A(t) - B(t)$. The profit probability is given by:

$$P_2(t) = \left[F\left(A(t)\right) - F\left(A(t) - B(t)\right) \right] \bar{F}\left(A(t)\right) \tag{2.114}$$

Scenario 3: $D_1 > A(t)$ and $D_2 < A(t)$

Due to the symmetry between the newsvendors, we have $P_3(t) = P_2(t)$ immediately.

Scenario 4: $D_1 > A(t)$ and $D_2 > A(t)$

Equation (2.112) becomes $\frac{2t - T}{r} \geq 0$, which is always true given the constraint. Therefore, the profit probability is given by $P_4(t) = \bar{F}^2\left(A(t)\right)$.

Summarizing the four scenarios, the manager's profit probability is given by:

$$P_c(t) = P_1(t) + 2\,P_2(t) + P_4(t) \tag{2.115}$$

and the manager's optimization problem is then:

$$\max_{t \geq 0} \; P_c(t) \tag{2.116}$$

Again, analytic solution is not available generally. So we give a numerical example here. Suppose the two identical newsvendors have the following parameters: retail price r

=10, procurement cost c =5, salvage price v =2, and uniform demand with lower bound 0 and upper bound 30. The manager has a profit target T =100 and he assign identical target profit t for the two newsvendors. Figure 2.8 illustrate the manager's profit probability as a function of t. The simulation is implemented using Matlab. The probabilities are obtained based on the interpretation of probability as a relative frequency. The experiment was repeated by one million times.

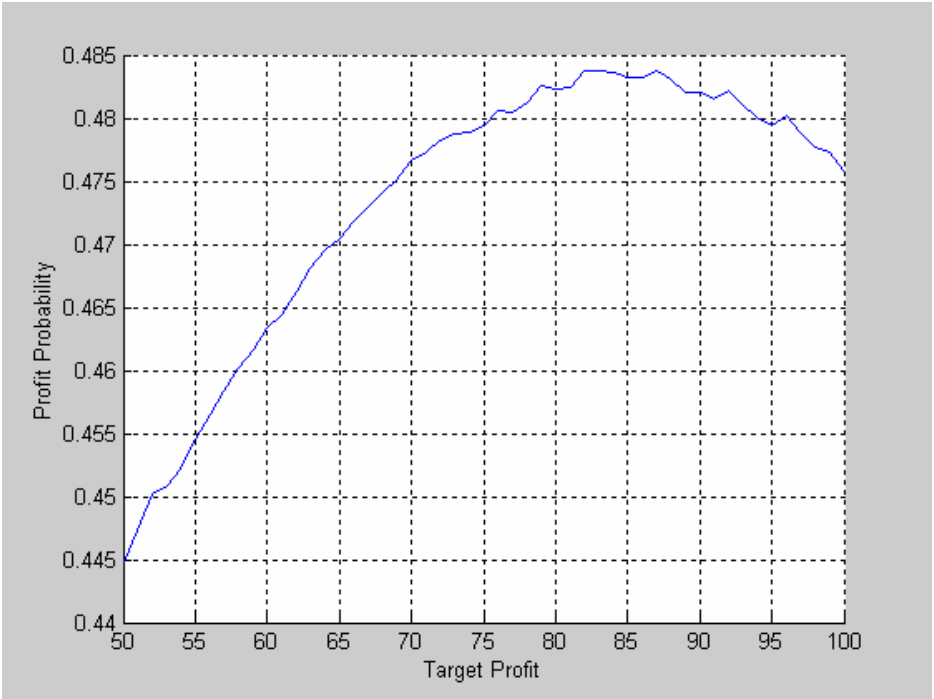

Figure 2.8: The profit probability for the manager as a function of profit target.

It can be seen that the manager's profit probability is approximately concave in the target t. The optimal target is about 85 and the manager's maximal profit probability is about 49%.

2.9 Conclusions

Setting performance targets and managing to achieve them is fundamental to business success. As a result, it is a common practice that firms adopt satisficing objectives, i.e., to maximize the chances to achieve some preset target performance levels, including profit, revenue, market share, sales and so on. In this chapter, we study a single newsvendor and multiple competitive newsvendors under both profit and revenue satisficing objectives.

After a brief review on a single newsvendor under the profit satisficing objective, Our first contribution is the study on a single newsvendor under a variety of scenarios: a newsvendor under the revenue satisficing objectives, a newsvendor under both the profit satisficing and the revenue satisficing objectives, a price-setting newsvendor under the profit satisficing objective, and a price-setting newsvendor under both the profit satisficing and the revenue satisficing objectives. In the scenarios of a price-setting newsvendor, we consider both the additive and multiplicative demand models.

Our results under the satisficing objectives are very different from those under the objective of expected profit maximization. For a newsvendor under both the profit and revenue satisficing objective, it turns out that if the revenue target is relatively lower, the newsvendor only needs to concern about the profit target. The joint probability of achieving both targets is then the same as the profit probability. The newsvendor needs to concern about the revenue target only when it is relatively larger than the profit target. As

a result, the optimal order quantity is higher and the maximal joint probability is lower. This sheds some light on the setting of the profit target and revenue target. For example, when the manager of a newsvendor is actually risk-neutral but he knows that the newsvendor adopts the profit satisficing objective, the manager needs to set the revenue target relatively higher than the profit target. The reasoning is as follows. Given a profit and/or revenue target, if a newsvendor adopts the satisficing objective, she is actually risk-averse: a profit and/or revenue higher than the target doesn't benefit. A general conclusion is that risk-aversion, comparing with risk-neutrality, leads to ordering less. Therefore, if the manager is risk-neutral, he will try to encourage or "force" the newsvendor to order more. One way to achieve that is to assign a relatively larger revenue target. To obtain such a revenue target, the newsvendor tends to order more.

For a price-setting newsvendor, we obtain analytic solutions to the optimal order quantity, optimal retail price and the associated maximal profit or joint probability. It turns out the optimality depends on the value of the price elasticity. Again, our results are very different from those under the objective of expected profit maximization, where analytic solutions are generally not available.

The second contribution of this chapter is the study of competitive newsvendors. As an initial step, we limit ourselves to inventory competition for newsvendors under the profit satisficing objective. We consider two scenarios: fixed total demand and variable total demand. In the scenario of fixed total demand, we consider the proportional allocation model, where each newsvendor’s demand is proportional to her stocking level. We show

that whether the newsvendor's ordering behavior will change due to competition depends on how competitive the market is. If the number of newsvendors is sufficiently small, the newsvendor will order the same quantity as that without competition. Otherwise, the newsvendor will order more, which leads to higher inventory in the whole market. In the scenario of variable total demand, we consider both the substitutable and complementary effects. Our results are very interesting: Under both effects, each newsvendor will not change her optimal order quantity. However, their profit probabilities will be higher.

Our last contribution is the study of target setting in the newsvendor setting. To our best knowledge, there is little research on target setting although it is of great practical importance. We study a simple scenario: a manager manages two newsvendors and all of them adopt the profit satisficing objective. Given a target profit level, the problem is that how the manager should assign target profit for the newsvendors. It turns out that closed-form analytic solutions are not available. To gain more managerial insight, we consider two special cases. In the first special case, it is assumed that the sum of the newsvendor's targets will be equal to the manager's target. Furthermore, if the demand is uniform and i.i.d., we obtain the analytic expressions for the newsvendors' target profits. Each newsvendor is assigned half of the manger's profit target and is further adjusted by the difference of profit margins between the newsvendors. The higher the profit margin, the higher target profit the newsvendor will be assigned to. Once again, our results are very different from those under the objective of expected profit maximization, where the manager will simply ask each newsvendor to maximize her expected profit.

In summary, in this chapter, we extend the research on newsvendor(s) under satisficing objectives by a great deal. However, a number of practical problems remain, which will be potential research projects. For example, we can consider both price competition and inventory competition for competitive newsvendors. We also feel that lots of more research needs to be done on target setting in Operations Management and Supply Chain Management. For example, if a manager manages multiple newsvendors, how should the manager assign profit and/or revenue targets? The expected research results will provide guidelines for important everyday business decisions.

Chapter 3

Contract Design and Supply Chain Coordination under Satisficing Objectives

3.1 Introduction and Literature Review

Supply chain has been an important topic in academic research and industrial practice. The reason can be best summarized by the quote "today's competition is not about company versus company any more, but about supply chain versus supply chain". Due to a number of factors including global competition and outsourcing, nowadays it is usually a good idea for a firm to focus on its core competency. As the result, most supply chains are decentralized and different agents owns different stages. If there is no coordination mechanism, each agent will try to optimize his/her own objective, which results in sub-optimization or the so-called double marginalization phenomenon (Spengler, 1950).

Therefore, it is vital to align the incentives of all the agents such that global optimization and supply chain coordination is achieved. The general idea of supply chain coordination is to manage the supply chain to have the best performance possible. Of course, what we mean by "best possible" depends on what kinds of objective each involved agent adopts.

The vast majority of supply chain coordination research assumes that each agent adopts the objective of expected profit (cost) maximization (minimization) (Cachon, 2003).

Under such an objective, best supply chain performance is equivalent to say that the expected value of the total profit is maximized. Furthermore, supply chain contracts, as popular coordination mechanisms, have been studied extensively. For clarity, we will use the objective of expected profit maximization as an example and similar arguments apply to the objective of expected cost minimization.

Under the objective of expected profit maximization, the best possible supply chain performance dictates that the total expected profit is maximized. Various contractual forms prove to be able to achieve supply chain coordination under such an objective. This includes: (two-part) linear tariff contracts (including wholesale price contracts as special cases) (Lariviere and Porteus, 2001 and Corbett *et al*, 2004), buy back contracts (Pasternack, 1985), revenue sharing contracts (Cachon and Lariviere, 2005), quantity flexibility contracts (Tsay, 1999), sales rebate contracts (Taylor, 2002) and quantity discount contracts (Weng, 1995).

The objective of expected profit maximization is a risks-neutral objective. For a supply chain where each agent adopts this objective, the objective function of the supply chain is obviously the objective of expected profit maximization as well. When the agents involved in a supply chain adopt satisficing objectives, it is not obvious as to what the objective function of the supply chain entity should be. In our research, we adopt the general definition of supply chain coordination with contracts proposed by Gan *et al* (2004). A supply chain is said to be coordinated with a contract if the optimizing actions of the agents under the contract lead to Pareto optimality, i.e., no agent in the chain can

be better off without making any other agent worse off. The associated contract is said to be a Pareto-optimal contract. By definition, only Pareto-optimal contracts should be selected. This is due to the fact that if a contract is not Pareto optimal, it is open to counteroffers that make no one worse off and at least one of the other agents strictly better off. In another word, Pareto-optimal contracts are self-enforcing. Furthermore, a contract has to be Pareto optimal first to coordinate a supply chain. There have been limited studies on supply chain contracts utilizing Pareto optimality criterion. Cachon (2004) considers the Pareto-optimal push (same as wholesale price), pull, and advance-purchase discount contracts in the traditional framework of expected profit maximization.

To the best of our knowledge, there have been few studies that deal with supply chain coordination where all agents involved adopt the profit satisficing objective. This chapter attempts to fill this gap. The contractual forms we consider include liner tariff (including wholesale price as a special case), buy back and quantity flexibility contracts. We focus these three contractual forms for three reasons. Firstly, they are popular contracts in business practice (Tayur *et al*, 1999). Secondly, they have been extensively studied in the supply chain literature, mostly in the framework of expected profit maximization (Cachon, 2003). Thirdly, wholesale price, buy back, and quantity flexibility contracts have one, two, and three contract parameters, respectively. Hence, our research will answer the question if increaseing degrees of freedom in contractual forms will be beneficial to coordination of supply chains under the satisficing objectives.

In this chapter, we will restrict ourselves to the feasible Pareto-optimal contracts. A contract is feasible when it satisfies the participation constraint of each agent. There are various forms of participation constraint. For example, an agent may enter a contract only if he will get his reservation profit level. In this chapter, we operationalize participation constraint in terms of probability of achieving a target profit: the probability must be larger than a threshold for each agent. For simplicity, the threshold probability is assumed to be zero, i.e., each agent will enter a contract only if his target is attainable. Under this assumption, each agent then tries to maximize the probability of attaining that target.

We consider a supply chain where a supplier sells to a retailer facing a random demand from customers. The target profit levels for the supplier and the retailer are set externally at t_s and t_r, respectively. The retail price is fixed at r. The supplier procures or produces the good at a constant marginal cost c. Any unsold unit in the supply chain can be salvaged at a price v per unit. It is assumed $v < c < r$ to avoid trivial situations.

The business transaction and profit allocation between the agents are determined by a contract, which in turn is specified by its parameter set θ. The contractual forms we consider include (two-part) linear tariff, buy back, and quantity flexibility contracts. Both the supplier and the retailer in the supply chain adopt the profit satisficing objective. The supplier wants to maximize the probability of achieving the predetermined target profit t_s, i.e., to maximize $P_s(q,\theta) = P\{\Pi_s(q,\theta) \geq t_s\}$. Similarly, the objective of the retailer is to maximize $P_r(q,\theta) = P\{\Pi_r(q,\theta) \geq t_r\}$. We assume that the target profits t_s and t_r, once

endogenously determined, become a common knowledge. For convenience, let $\beta = \frac{t_s}{t_r}$ denote the target ratio. A larger β thus indicates a higher relative target profit level of the supplier.

In the reminder of this chapter, we first design Pareto-optimal linear tariff contracts (Section 3.2), buy back contracts (Section 3.3), and quantity flexibility contracts (Secton 3.4) for a supply chain where all agents adopt the profit satisficing objective. These Pareto-optimal contracts are then evaluated based on whether they can coordinate the supply chain or not in Section 3.6. In Section 3.7, we proceed to consider supply chain coordination under multiple satisficing objectives, including the profit and the revenue satisficing objectives. Then we study a situation where the supplier adopts the profit satisficing objective and the retailer adopts the objective of expected profit maximization in Section 3.8. Finally we summarize the obtained results in the last section, Section 3.9.

3.2 Pareto-optimal Linear Tariff Contracts under Profit Satisficing Objective

In this section, we consider (two-part) linear tariff contracts. With a linear tariff contract, the supplier charges a per unit wholesale price w but offers a fixed lump sum side payment l to the retailer. Hence, a linear tariff contract is characterized by its parameter set $\theta = [w, l]$. When $l = 0$, linear tariff contracts reduce to wholesale prices contracts. When $l < 0$ and $w = c$, linear tariff contracts reduce to franchisee fee contracts. Furthermore, it is reasonable to assume that $-t_s \le l \le t_r$, i.e., the side payment is no more than either of the target profits. For simplicity of exposure, we consider the case of $-t_s < l < t_r$ first.

Under a linear tariff contract with parameter set θ, the supplier's profit is given by:

$$\Pi_s(q,\theta) = (w-c)q - l \tag{3.1}$$

which is deterministic. Hence, the supplier's participation constraint is $q \geq \frac{t_s + l}{w-c}$ for given a target profit t_s. Once the participation constraint is satisfied, the supplier's probability of achieving his target is always 1.

The retailer is similar to a newsvendor with a fixed side payment l. Therefore, the participation constraint for the retailer is $q \geq \frac{t_r - l}{r-w}$ and the probability of achieving her target is given by

$$P_r(q,\theta) = \begin{cases} 0 & if\ q < \frac{t_r - l}{r-w} \\ 1 - F\left(\frac{(w-v)q + t_r - l}{r-v}\right) & if\ q \geq \frac{t_r - l}{r-w} \end{cases} \tag{3.2}$$

Figure 3.1 shows the probabilities of achieving their target profits for the supplier and the retailer when $\frac{t_r - l}{r-w} \geq \frac{t_s + l}{w-c}$, i.e., $w \leq \frac{ct_r + rt_s + (r-c)l}{t_r + t_s}$. The figure corresponds to the other case is similar.

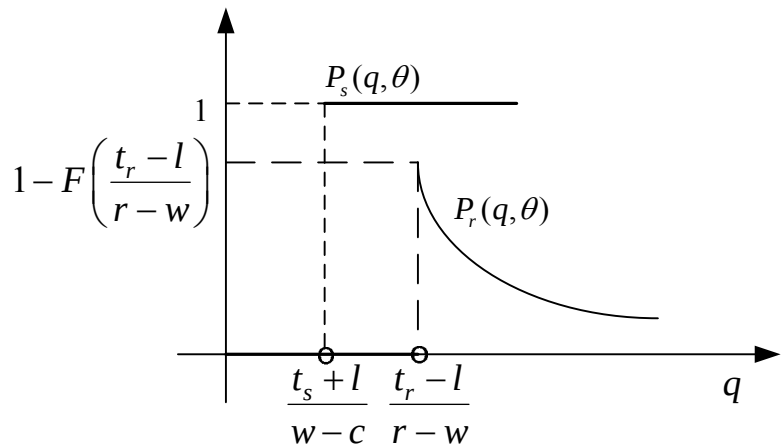

Figure 3.1: The probabilities of achieving the target profits for the supplier (bold line) and the retailer under a two-part linear tariff contract with $w \le \frac{ct_r + rt_s + (r-c)l}{t_r + t_s}$.

Theorem 3.1: A linear tariff contract is Pareto optimal if its contract parameters satisfy:

$$w = \frac{ct_r + rt_s + (r-c)l}{t_r + t_s} \tag{3.3}$$

The associated Pareto-optimal order quantity and maximal probabilities of achieving the targets are given by:

$$q^* = \frac{t_r + t_s}{r-c} \tag{3.4}$$

$$P_s^* = 1 \text{ and } P_r^* = 1 - F\left(\frac{t_r + t_s}{r-c}\right) \tag{3.5}$$

Proof.

It can be seen from Figure 3.1 that, given a linear tariff contract with parameter set $\theta = [w, l]$, the unique feasible Pareto optimal order quantity is given by:

$$q^*(\theta) = \max\left\{\frac{t_s + l}{w - c}, \frac{t_r - l}{r - w}\right\} \tag{3.6}$$

The corresponding probabilities of achieving the target profits for the supplier and the retailer are, respectively,

$$P_s\left(q^*(\theta)\right) = 1 \tag{3.7}$$

$$P_r\left(q^*(\theta)\right) = 1 - F\left(\frac{(w - v)q^*(\theta) + t_r - l}{r - v}\right) \tag{3.8}$$

Since $P_s\left(q^*(\theta)\right) = 1$, for linear tariff contracts to be Pareto optimal, it suffices to maximize $P_r\left(q^*(\theta)\right)$, or equivalently, to minimize

$$(w - v)\max\left\{\frac{t_s + l}{w - c}, \frac{t_r - l}{r - w}\right\} - l \tag{3.9}$$

Note that (3.9) first decreases and then increases with respect to w and the minimum is attained when $\frac{t_s + l}{w - c} = \frac{t_r - l}{r - w}$, which is equivalent to equation (3.3). Once the contract parameter set $\theta = [w, l]$ satisfies (3.3), it can be verified that (3.9) reduces to $\frac{(r - v)t_S + (c - v)t_r}{r - c}$, which is independent of l. Hence, the Pareto optimal linear tariff contract parameters satisfy (3.3). Equations (3.4) and (3.5) are obtained by substituting (3.3) into (3.7) and (3.8). □

It can be verified that the results in Theorem 3.1 also hold when the side payment $l = -t_s$ or $l = t_r$. When $l = -t_s$, the Pareto-optimal linear tariff contract reduces to the Pareto-optimal franchisee fee contract. When $l = 0$, the Pareto optimal linear tariff

contract reduces to the Pareto-optimal wholesale price contract with the wholesale price $\hat{w}$ given by:

$$\hat{w} = \frac{ct_r + rt_s}{t_r + t_s} \tag{3.10}$$

Notice that $\hat{w}$ is a linear combination of procurement cost c and selling price r, with the target profit levels of the retailer and the supplier as the weights, respectively.

Example:

Consider a toy supply chain with a manufacturer (the supplier) and a retailer who faces a Normal demand with mean of 300 units and standard deviation of 70 units. There is only one selling season for this particular type of toys. Due to the long lead time, the retails has to commit an order quantity before the selling season so that the manufacturer can plan the manufacturing accordingly. The manufacturer produces the toys at a unit cost c = \$18 and the retailer sells at a unit price r = \$25. Any unsold toy has a unit salvage cost v = \$8. The target profits for the manufacturer and the retailer are set at t_s = \$800 and t_r = \$600, respectively. If the profit allocation is implemented through a wholesale price contract, what would be the Pareto-optimal contract parameter w and how sensitive is it?

The answer to the above example is summarized in Table 3.1. Under a wholesale price contract with a wholesale price w, $q^*(w)$ and $P_r^*(w)$ denote, respectively, the associated Pareto-optimal order quantity and the retailer's maximal probability of achieving her profit target. The Pareto-optimal wholesale price is given by $\hat{w}$=22. Note that under a

feasible wholesale price contract with a wholesale price w, the supplier's profit is deterministic and $P_s^*(w) = 1$ always holds.

Table 3.1: The Pareto-optimal order quantity and the retailer's maximal probability of achieving her target profit under a wholesale price contract.

w	21	21.6	21.8	$\hat{w}$=22	22.2	22.4	23
$q^*(w)$	267	222	211	200	214	231	300
$P_r^*(w)$	0.81	0.89	0.91	0.92	0.89	0.84	0.50

It can be seen from Table 3.1 that the retailer's probability of achieving her profit target is quite sensitive near the Pareto-optimal wholesale price $\hat{w}$. As the wholesale price decreases from 22 to 21 (4.5%), $P_r^*(w)$ decreases from 0.92 to 0.80 (13%). As the wholesale price increases from 22 to 23 (4.5%), $P_r^*(w)$ decreases from 0.92 to 0.50 (45.7%).

The probabilities for the supplier and the retailer to achieve their respective target profits depend on not only the wholesale price but also the retailer's order quantity q. This is demonstrated by the Table 3.2. The first and the second values in each cell, if not infeasible, represent the probabilities of achieving target profits for the supplier and the retailer, respectively. Any combination is infeasible if at least one of the participation constraints is not satisfied.

Table 3.2: The probabilities of achieving the target profits for the supplier and the retailer with different combinations of wholesale price and order quantity under wholesale price contracts.

q \ w	21	21.6	21.8	$\hat{w}$=22	22.2	22.4	23
170	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
200 (q^*)	Infeasible	Infeasible	Infeasible	**(1,0.92)**	Infeasible	Infeasible	Infeasible
230	Infeasible	(1,0.88)	(1,0.87)	(1,0.86)	(1,0.85)	Infeasible	Infeasible
284 (q_e^*)	(1,0.75)	(1,0.70)	(1,0.69)	(1,0.67)	(1,0.65)	(1,0.63)	Infeasible

In Table 3.2, q^* is the corresponding Pareto-optimal order quantity for the satisficing objectives, and q_e^* is the optimal order quantity for the objective of expected profit maximization. Notice that the performance of the supply chain under the satisficing objectives is very sensitive to the choice of both w and q. Even though most combinations of w and q we choose are close to the optimal one ($\hat{w}$=22, q^*=200), more than half of the combinations are infeasible.

3.3 Pareto-optimal Buy Back Contracts under Profit Satisficing Objective

With wholesale price contracts, the supplier takes no risk from the demand uncertainty. With buy back contracts, the supplier charges a unit wholesale price w but offers the retailer a unit buy back price b for all of the unsold units. Therefore, with buy back

contracts, the supplier takes part of the risk from demand uncertainty. Each buy back contract can be characterized by a parameter set $\theta = [w, b]$. To avoid uninteresting scenarios, we assume that $v \le b \le w$.

We will study the supplier first. Under a buy back contract with parameter set $\theta = [w, b]$, if the retailer orders q products, the supplier's profit is given by:

$$\Pi_s(q, \theta) = (w - c)q - (b - v)(q - D)^+ \tag{3.11}$$

It is worthwhile noticing that that the contract parameters (w, b) need to be chosen such that $w - (c - v) \le b \le w$. The requirement of the second inequality is to prevent the retailer from profiting through the buy back process. Similarly, since the supplier's marginal revenue and the marginal cost are $w - b$ and $c - v$, respectively, the first inequality prevents the supplier from profiting through the buy back process.

It follows that the supplier's probability of achieving his target profit t_s is given by:

$$P_s(q, \theta) = \begin{cases} 0 & \text{if } q < \dfrac{t_s}{w - c} \\ 1 - F\left(\dfrac{(b + c - w - v)q + t_s}{b - v}\right) & \text{if } q \ge \dfrac{t_s}{w - c} \end{cases} \tag{3.12}$$

Therefore, the participation constraint for the supplier is $q \geq \frac{t_s}{w-c}$, which only depends on the wholesale price *w*.

Now we consider the retailer. The retailer is a newsvendor with procurement cost of w and "salvage price" b. In view of equation (2.3), the retailer's probability of achieving her target t_r is given by:

$$P_r(q,\theta) = \begin{cases} 0 & if\ q < \frac{t_r}{r-w} \\ 1 - F\left(\frac{(w-b)q + t_r}{r-b}\right) & if\ q \geq \frac{t_r}{r-w} \end{cases} \tag{3.13}$$

Hence, the participation constraint for the retailer is $q \geq \frac{t_r}{r-w}$. The probability functions $P_s(q,\theta)$ and $P_r(q,\theta)$ when $w < \hat{w}$ are plotted in Figure 3.2.

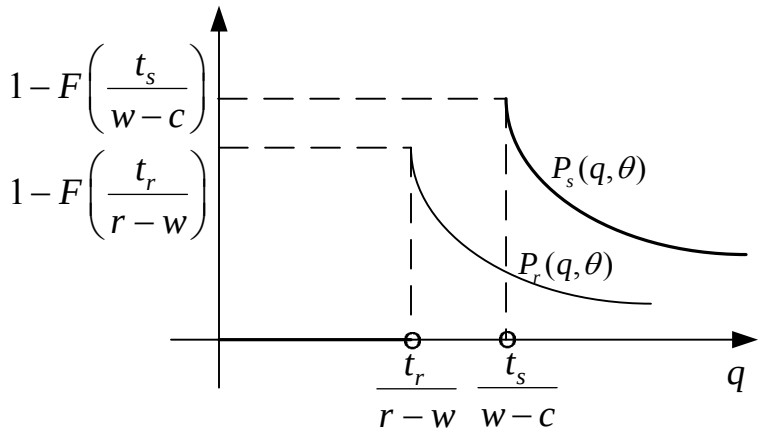

Figure 3.2: The probabilities for the supplier (bold line) and the retailer to achieve their target profits under a buy back contract with $w < \hat{w}$.

Theorem 3.2:

The three sets of buy back contracts summarized in Table 3.3 are Pareto optimal. The second column of the table identifies the conditions for the contract parameter set $\theta = [w, b]$ to be Pareto optimal. The associated Pareto-optimal order quantity and maximal probabilities of achieving the target profits for the supplier and retailer are given by $q^*(\theta)$, $P_s(\theta)$ and $P_r(\theta)$, respectively.

Table 3.3: Pareto-optimal buy back contracts for the supply chain with the profit satisficing objective.

Sets	$\theta = [w, b]$	$q^*(\theta)$	$P_s(\theta)$	$P_r(\theta)$
BB1	$w = \hat{w}$	$\frac{t_r + t_s}{r - c}$	$1\text{-}F\left(\frac{t_r + t_s}{r - c}\right)$	$1\text{-}F\left(\frac{t_r + t_s}{r - c}\right)$
BB2	$c < w < \hat{w}$, $b = w$	$\frac{t_s}{w - c}$	$1\text{-}F\left(\frac{t_s}{w - c}\right)$	$1\text{-}F\left(\frac{t_r}{r - w}\right)$
BB3	$\hat{w} < w < r$, $b = w - (c - v)$	$\frac{t_r}{r - w}$		

Proof.

We first examine the buy back contracts with wholesale price $w = \hat{w}$. In this case, the participation constraint's for both the supplier and the retailer reduce to $q \geq \frac{t_r + t_s}{r - c}$. Since both $P_s(q, \hat{w}, b)$ and $P_r(q, \hat{w}, b)$ decrease in q, the Pareto-optimal order quantity is $q^*(\hat{w}, b) = \frac{t_r + t_s}{r - c}$. As a result, the associated maximal probabilities for both the supplier and the retailer are given by $P_s^*(\hat{w}, b) = P_r^*(\hat{w}, b) = 1\text{-}F\left(\frac{t_r + t_s}{r - c}\right)$. This case corresponds to Pareto-optimal set *BB1* in Table 3.3.

Next we consider the buy back contracts with wholesale price $w < \hat{w}$. Based on Figure 3.2, it is clear that $q^*(\theta) = \frac{t_s}{w-c}$ is the unique feasible Pareto-optimal order quantity. The associated probability functions of achieving the supplier's and the retailer's profit targets are, respectively,

$$P_s\left(q^*(\theta)\right) = 1 - F\left(\frac{t_s}{w-c}\right) \tag{3.14}$$

$$P_r\left(q^*(\theta)\right) = 1 - F\left(\frac{(w-b)t_s + (w-c)t_r}{(w-c)(r-b)}\right) \tag{3.15}$$

Simple calculation shows that

$$\frac{\partial P_r\left(q^*(\theta)\right)}{\partial b} = -f\left(\frac{(w-b)t_s + (w-c)t_r}{(w-c)(r-b)}\right)\frac{(w-c)t_r - (r-w)t_s}{(w-c)(r-b)^2} > 0 \tag{3.16}$$

Hence, the Pareto-optimal buy back price satisfies $b = w$. The retailer's probability then reduces to

$$P_r\left(q^*(\theta)\right) = 1 - F\left(\frac{t_r}{r-w}\right) \tag{3.17}$$

Since $P_s\left(q^*(\theta)\right)$ increases in w but $P_r\left(q^*(\theta)\right)$ decreases in w, any $w < \hat{w}$ is Pareto optimal. This case corresponds to the Pareto-optimal set *BB2* in Table 3.3.

The results for the case of $w > \hat{w}$, corresponding to set *BB3* in Table 3.3, can be proven similarly. Finally, one can verify that the three parameter sets identified in Table 3.3 are all Pareto optimal since the resulting maximal probabilities $P_s(\theta)$ and $P_r(\theta)$ do not dominate each other. □

Notice that wholesale price w in a buy back contract plays an important role for the contract to be Pareto optimal. When $c < w < \hat{w}$, the buy back price b needs to be at its upper bound w in order for the buy back contract to be Pareto optimal. On the other hand, when $\hat{w} < w < r$, the buy back price b needs to be at its lower bound $w - (c - v)$ in order for the buy back contract to be Pareto optimal. When $w = \hat{w}$, the buy back contract is Pareto optimal regardless of the buy back price *b*, as long as the general requirement of $w - (c - v) \le b \le w$ holds.

3.4 Pareto-optimal Quantity Flexibility Contracts under Profit Satisficing Objective

With a quantity flexibility contract, the supplier charges the retailer a constant unit wholesale price w but offers the retailer the flexibility of adjusting the initial order quantity. Suppose the retailer place an initial order quantity of q. Depending on demand from customers, the retailer can adjust the initial quantity q to be anywhere within

$[dq, uq]$ without extra financial charge, where $0 \le d \le 1$ and $u \ge 1$ represent the downward and upward adjustment parameters, respectively. Therefore, the retailer's actual order quantity will be dq , D, and uq if the realized demand is $D \le dq$, $dq \le D \le uq$, and $D \ge uq$, respectively. The supplier is responsible to supply up to quantity uq for the supply chain.

Each quantity flexibility contract could be characterized by its parameter set $\theta = [w, u, d]$. For convenience, let $\alpha = \frac{d}{u}$ denote the adjustment ratio. Since the stocking level in the channel will be uq, $0 \le \alpha \le 1$ represents the fraction for which the retailer is responsible. In addition, a higher α indicates less flexibility for the retailer. In another word, a higher α indicates more flexibility for the supplier. To avoid confusion, we use the term flexibility from the perspective of the retailer. A quantity flexibility contract with the least flexibility, i.e., α =1, degenerates into a wholesale price contract. In this case, the retailer has to stick to the initial order quantity. It is worthwhile noticing that α =1 requires that $u = d$ =1.

For notational simplicity, we define two "wholesale prices" and four order quantities:

$$w_1 = \frac{c + \alpha\beta r}{1 + \alpha\beta} \tag{3.18}$$

$$w_2 = \frac{\alpha v + \alpha\beta r + c - v}{\alpha(1 + \beta)} \tag{3.19}$$

$$q_1(\theta) = \frac{t_s}{u(w-c)} \tag{3.20}$$

$$q_2(\theta) = \frac{t_r}{u(r-w)} \tag{3.21}$$

$$q_3(\theta) = \frac{t_s}{d(w-v) - u(c-v)} \tag{3.22}$$

$$q_4(\theta) = \frac{t_r}{d(r-w)} \tag{3.23}$$

It can be easily verified that $c < w_1 \le \hat{w} \le w_2$, and $w_1 = w_2 = \hat{w}$ when $\alpha = 1$. Note that w_1 and w_2 defined in (3.18) and (3.19) are not actual wholesale prices; they are actually equation constraints for the parameter set θ. Finally, the relative magnitudes of the four order quantities depend on the parameter set θ except that inequalities $q_1(\theta) \le q_3(\theta)$ and $q_2(\theta) \le q_4(\theta)$ hold under all situations.

Under a quantity flexibility contract with parameter set $\theta = [w, u, d]$, if the retailer places an initial order quantity of q, her random profit is given by:

$$\Pi_r(q,\theta) = (r-w)\ uq - (r-w)\ (uq-D)^+ - (w-v)\ (dq-D)^+ \tag{3.24}$$

It can be verified that the probability function for the retailer to achieve her target profit t_r is given by:

$$P_r(q,\theta) = \begin{cases} 0 & if\ q < q_2(\theta) \\ 1 - F\left(\dfrac{t_r}{r-w}\right) & if\ q_4(\theta) > q \geq q_2(\theta) \\ 1 - F\left(\dfrac{t_r + (w-v)dq}{r-v}\right) & if\ q \geq q_4(\theta) \end{cases} \tag{3.25}$$

Clearly, the maximal probability for the retailer to achieve her target is $1 - F\left(\dfrac{t_r}{r-w}\right)$, which is always less than 1.

Now we consider the supplier. The profit function of the supplier, given the retailer's initial order quantity q, is given as:

$$\Pi_s(q,\theta) = (w-c)\ uq - (w-v)\ (uq-D)^+ + (w-v)\ (dq-D)^+ \tag{3.26}$$

Hence, the supplier's minimum profit is $(w-v)dq - (c-v)uq$, which occurs when $D \leq dq$. It turns out that the supplier's probability of achieving his target depends on if his minimum profit is positive or not. If the contract parameter set θ is chosen such that $w \leq v + \dfrac{c-v}{\alpha}$, the minimum profit is non-positive. The supplier's probability of achieving his target profit t_s is given as:

$$P_s(q,\theta) = \begin{cases} 0 & if\ q < q_1(\theta) \\ 1 - F\left(\dfrac{t_s + (c-v)uq}{w-v}\right) & if\ q \geq q_1(\theta) \end{cases} \tag{3.27}$$

which is always less than 1. On the other hand, if the contract parameter set θ is chosen such that $w > v + \frac{c-v}{\alpha}$, the minimum profit is positive. The supplier's probability function of achieving his target profit t_s is then given as:

$$P_s(q,\theta) = \begin{cases} 0 & if\ q < q_1(\theta) \\ 1 - F\left(\frac{t_s + (c-v)uq}{w-v}\right) & if\ q_3(\theta) > q \geq q_1(\theta) \\ 1 & if\ q \geq q_3(\theta) \end{cases} \tag{3.28}$$

Therefore, if $w > v + \frac{c-v}{\alpha}$, the supplier's probability of achieving his target can be as high as 1.

Now we proceed to design Pareto-optimal quantity flexibility contracts for the supply chain under the satisficing objectives. We have the following Theorem 3.3.

Theorem 3.3:

The two sets of quantity flexibility contracts summarized in Table 3.4 are Pareto optimal. The second column of the table identifies the conditions for the contract parameter set $\theta = [w, u, d]$ to be Pareto optimal. The associated Pareto-optimal order quantity and maximal probabilities of achieving the target profits for the supplier and retailer are given by $q^*(\theta)$, $P_s(\theta)$ and $P_r(\theta)$, respectively.

Table 3.4: Pareto-optimal quantity flexibility contracts for the supply chain under the profit satisficing objective.

Sets	$\theta = [w, u, d]$ *or* $[w, \alpha]$	$q^*(\theta)$	$P_s(\theta)$	$P_r(\theta)$
QF1	$\frac{c+\alpha\beta r}{1+\alpha\beta} \le w < \frac{c+\beta r}{1+\beta}$	$\frac{t_s}{u(w-c)}$	$1\text{-}F\left(\frac{t_s}{w-c}\right)$	$1\text{-}F\left(\frac{t_r}{r-w}\right)$
QF2	$w = \frac{c+\beta r}{1+\beta},\ \alpha = 1$	$\frac{t_r+t_s}{r-c}$	1	$1\text{-}F\left(\frac{t_r+t_s}{r-c}\right)$

Proof.

To design Pareto-optimal quantity flexibility contracts, it is worthwhile noticing that the supplier's probability function takes different forms depending on if the parameter set θ satisfies inequality $w > v + \frac{c-v}{\alpha}$ or not. Hence, we design Pareto-optimal quantity flexibility contracts for Case 1 of $w > v + \frac{c-v}{\alpha}$ and Case 2 of $v + \frac{c-v}{\alpha}$, respectively.

Case 1: $w > v + \frac{c-v}{\alpha}$

Under Case 1, the probability function of the supplier to achieve his target profit is given by (3.28). Furthermore, it can be seen below that Pareto optimality depends on the relative magnitudes of $q_1(\theta)$, $q_2(\theta)$, $q_3(\theta)$ and $q_4(\theta)$. Therefore, we consider the following three sub-cases.

Case 1.1: $w \geq w_2$

Under Case 1.1, we have $q_1(\theta) \leq q_3(\theta) \leq q_4(\theta)$. The probability functions $P_s(q,\theta)$ and $P_r(q,\theta)$ for the supplier and the retailer are plotted in Figure 3.3. Note that it is also possible that $q_3(\theta) \leq q_2(\theta)$ under Case 1.1.

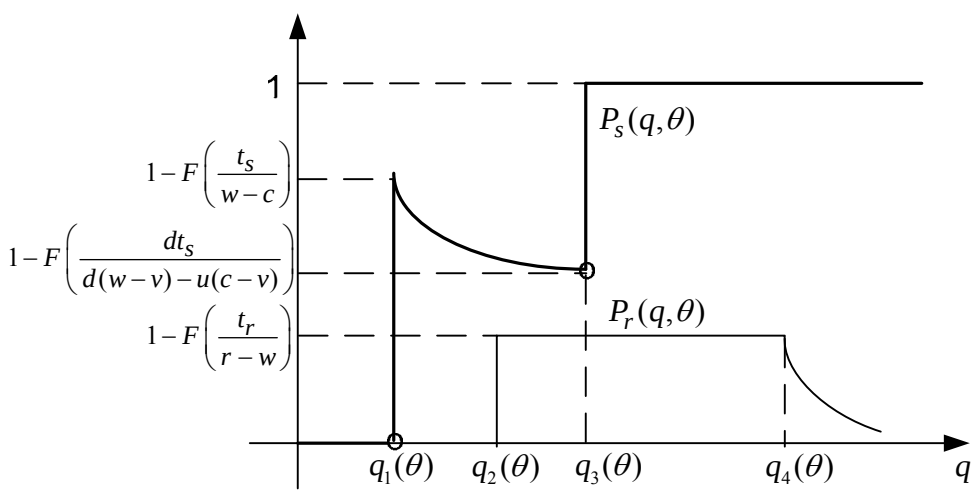

Figure 3.3: The probability functions for the supplier (bold line) and the retailer to achieve their target profits under Case 1.1.

It can be seen from Figure 3.3 that the Pareto-optimal order quantity set is given by:

$$q^*(\theta) \in \left[\max\left(q_2(\theta), q_3(\theta)\right), q_4(\theta)\right] \tag{3.29}$$

The associated probabilities for the supplier and the retailer are:

$$P_s\left(q^*(\theta),\theta\right)=1 \text{ and } P_r\left(q^*(\theta),\theta\right)=1-F\left(\frac{t_r}{r-w}\right) \tag{3.30}$$

Obviously that $P_r\left(q^*(\theta),\theta\right)$ decreases with respect to w. Hence we choose $w^*(u,d)=w_2$. The associated retailer's probability is:

$$P_r\left(q^*(w_2,u,d),\theta\right)=1-F\left(\frac{\alpha(t_r+t_s)}{\alpha(r-v)-(c-v)}\right), \tag{3.31}$$

which is maximized at $\alpha=1$. Note that $w_2=\hat{w}$ when $\alpha=1$. Moreover, substituting $\alpha=1$ into (3.31), we have the Pareto-optimal set *QF2* in Table 3.4.

Case 1.2: $w \le w_1$

Under Case 1.2, we have $q_2(\theta) \le q_4(\theta) \le q_1(\theta) \le q_3(\theta)$. The probability functions of the supplier and the retailer are shown in Figure 3.4.

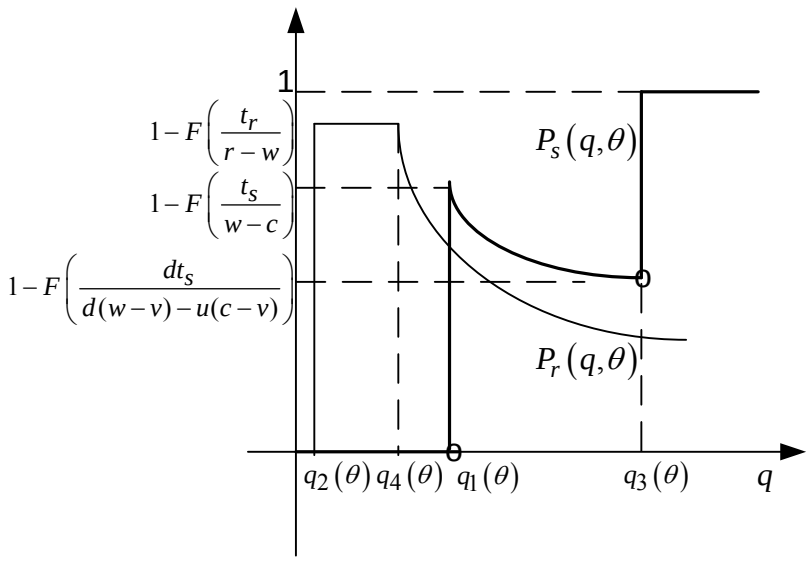

Figure 3.4: The probability functions for the supplier (bold line) and the retailer to achieve their target profits under Case 1.2.

Therefore, both $q_3(\theta)$ and $q_1(\theta)$ are Pareto-optimal order quantities. It can be verified that the order quantity $q_3(\theta)$ will eventually lead to the Pareto-optimal set *QF2* in Table 4. On the other hand, if the retailer orders $q_1(\theta)$, the profit probabilities for the supplier and the retailer are given by:

$$P_s\left(q_1(\theta),\theta\right)=1-F\left(\frac{t_s}{w-c}\right) \tag{3.32}$$

$$P_r\left(q_1(\theta),\theta\right)=1-F\left(\frac{1}{r-v}\left(t_r+\frac{w-v}{w-c}\alpha t_s\right)\right) \tag{3.33}$$

Since both $P_s\left(q_1(\theta),\theta\right)$ and $P_r\left(q_1(\theta),\theta\right)$ increase with respect to w, we choose $w^*(u,d)=w_1$. The associated Pareto-optimal order quantity $q_1\left(\theta\right)$ then becomes:

$$q^*(w_1,u,d)=\frac{dt_s+ut_r}{ud(r-c)} \tag{3.34}$$

The associated probabilities for the supplier and the retailer are:

$$P_s\left(q^*(w_1,u,d),\theta\right)=1\text{-}F\left(\frac{t_r+\alpha t_s}{\alpha(r-c)}\right) \tag{3.35}$$

$$P_r\left(q^*(w_1,u,d),\theta\right)=1\text{-}F\left(\frac{t_r+\alpha t_s}{r-c}\right) \tag{3.36}$$

Because the two probabilities vary in different directions as α changes, any α is Pareto optimal. In summary, the resulted Pareto-optimal set is a special case of the Pareto-optimal set *QF1* in Table 3.4 when $w=w_1$.

Case 1.3: $w_1 \le w < w_2$

Under Case 1.3, we have $q_1\left(\theta\right) \le q_4\left(\theta\right) \le q_3\left(\theta\right)$. Once again, it can be verified that both $q_3\left(\theta\right)$ and $q_1\left(\theta\right)$ are Pareto optimal, and $q_3\left(\theta\right)$ will eventually lead to the Pareto-

optimal set *QF2* in Table 3.4. If the retailer orders $q_1(\theta)$, the supplier's probability of achieving his target profit $P_s(q_1(\theta),\theta)$ is less than 1, and the retailer's probability is $P_r(q_1(\theta),\theta)=1-F\left(\frac{t_r}{r-w}\right)$. To make sure that the resulted Pareto-optimal set will not be dominated by set *QF2*, we need to have:

$$P_r(q_1(\theta),\theta) > 1-F\left(\frac{t_s+t_r}{r-c}\right) \tag{3.37}$$

which is equivalent to $w<\hat{w}$. Therefore, we only need to consider the case when $w_1 \le w < \hat{w}$, for which $q_2(\theta) \le q_1(\theta)$. The probability functions of the supplier and the retailer are plotted in Figure 3.5.

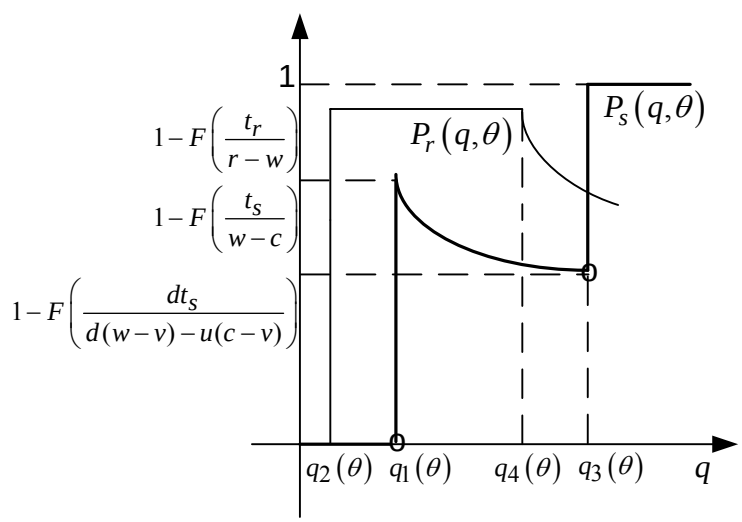

Figure 3.5: The probability functions for the supplier (bold line) and the retailer to achieve their target profits under Case 1.3.

It can be seen from Figure 3.5 that the probabilities of the supplier and the retailer associated with order quantity $q_1(\theta)$ are:

$$P_s\left(q_1(\theta),\theta\right)=1-F\left(\frac{t_s}{w-c}\right) \tag{3.38}$$

$$P_r\left(q_1(\theta),\theta\right)=1-F\left(\frac{t_r}{r-w}\right) \tag{3.39}$$

Because the two probabilities vary in different directions as w changes, any w is Pareto optimal. In summary, this Pareto-optimal set is given as *QF1* in Table 3.4.

Case 2: $w \le v+\frac{c-v}{\alpha}$

Under Case 2, the probability of the supplier to achieve his target profit is given by (3.27), which is always less than 1. Furthermore, the retailer's probability will be no larger than $1-F\left(\frac{t_r}{r-w}\right)$. Therefore, to guarantee the resulted Pareto-optimal set is not dominated by set *QF2* in Table 3.4, we need to have:

$$1-F\left(\frac{t_r}{r-w}\right) > 1-F\left(\frac{t_s+t_r}{r-c}\right) \tag{3.40}$$

which is equivalent to $w < \hat{w}$ or $q_2(\theta) \le q_1(\theta)$. Now we consider the two subcases as follows.

Case 2.1: $w_1 \le w < \hat{w}$

Under Case 2.1, we have $q_2(\theta) \le q_1(\theta) \le q_4(\theta)$. It can be verified that the order quantity $q_1(\theta)$ is Pareto optimal, with the supplier's and the retailer's probabilities given by:

$$P_s(q_1(\theta),\theta) = 1 - F\left(\frac{t_s}{w-c}\right) \text{ and } P_r(q_1(\theta),\theta) = 1 - F\left(\frac{t_r}{r-w}\right) \tag{3.41}$$

Therefore, any w in Case 2.1 is Pareto optimal, which gives the Pareto-optimal set *QF1* in Table 3.4.

Case 2.2: $w \le w_1$

Under Case 2.2, we have $q_2(\theta) \le q_4(\theta) \le q_1(\theta)$. It can be verified that the order quantity $q_1(\theta)$ is Pareto optimal, with the supplier's and the retailer's probabilities given by:

$$P_s(q_1(\theta),\theta) = 1 - F\left(\frac{t_s}{w-c}\right) \tag{3.42}$$

$$P_r\left(q_1(\theta),\theta\right)=1-F\left(\frac{1}{r-v}\left(t_r+\frac{w-v}{w-c}\alpha t_s\right)\right) \tag{3.43}$$

Because both probabilities increase with respect to w, we have $w^*(u,d) = w_1$. Substituting $w^*(u,d)=w_1$ into $q_1(\theta)$ and (A12), we end up with a special case of the Pareto-optimal set *QF1* in Table 3.4 when $w=w_1$. □

So far we have derived the Pareto-optimal quantity flexibility contracts, for which we have the following observations. First, for a quantity flexibility contract to be Pareto optimal, its contract parameters must satisfy $w_1 \le w \le \hat{w}$. When $w=\hat{w}$, the adjustment ratio α has to be 1. This means a quantity flexibility contract will be Pareto optimal if it degenerates into a wholesale price contract with wholesale price $w=\hat{w}$. Second, the Pareto-optimal quantity flexibility contracts depend on the downward and upward adjustment parameters only through the adjustment ratio α. However, the associated Pareto order quantity may depend on the upward adjustment parameter only, which is the case for the Pareto-optimal set *QF1*. Finally, the Pareto-optimal quantity flexibility contracts depend on the supplier's and the retailer's target profits only through the target ratio β.

Example: Consider a supply chain with a supplier and a retailer who faces a Gamma demand with mean 400 and coefficient of variation 0.5. The supplier procures the good at a unit cost c = 15 and the retailer sells at a unit price r = 20. Any unsold unit has a

salvage cost $v = 8$. The terms of trade between the supplier and the retailer are specified by a quantity flexibility contract. Both the supplier and the retailer adopt the satisficing objective. The target profits for the supplier and the retailer are set at $t_s = 1200$ and $t_r = 800$, respectively. What are the Pareto-optimal quantity flexibility contracts?

Figure 3.6 illustrates the Pareto optimality. The region bounded by the solid line and the dotted line represents the Pareto-optimal quantity flexibility contracts under the satisficing objectives. We use the dotted line to emphasize the fact that the dotted line is excluded from the region.

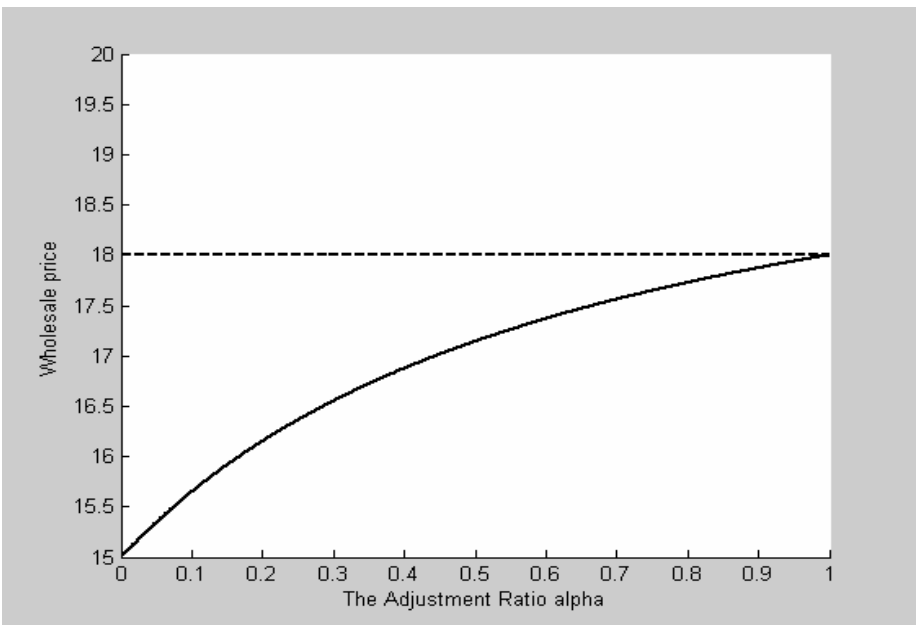

Figure 3.6: The Pareto-optimal quantity flexibility contracts for the supply chain under profit satisficing objective (the region bounded by the dotted line and the solid line).

3.5 Supply Chain Coordination under Profit Satisficing Objective

So far we have designed the Pareto-optimal contracts for the three contractual forms under the profit satisficing objective. In this section, our goal is to identify the contractual forms which are capable of coordination of the supply chain under the profit satisficing objective.

To this ends, we adopt the general definition of supply chain coordination with contracts proposed by Gan *et al* (2004). A supply chain is said to be coordinated with a contract if the optimizing actions of the agents under the contract lead to Pareto optimality, i.e., no agent in the chain can be better off without making any other agent worse off. We have the following Theorem 3.4 on supply chain coordination with contracts under the profit satisficing objective.

Theorem 3.4:

Let t_s and t_r be the profit targets for the supplier and the retailer, respectively. If both of them adopt the profit satisficing objective, the supply chain is coordinated if and only if:

(1) The retailer chooses the optimal order quantity $q^* = \frac{t_r + t_s}{r - c}$, which is the solution to the optimization problem for the supply chain::

$$\max_q P\left\{\Pi(q) \geq t_s + t_r\right\} \tag{3.44}$$

where $\Pi(q) = \Pi_s(q,\theta) + \Pi_r(q,\theta)$ is the total profit of the supply chain.

(2) The supplier and the retailer follow the optimal profit allocation rule $\Psi^* = [\Pi_s^*, \Pi_r^*]$, where Π_s^* and Π_r^* are profits allocated to the supplier and the retailer, respectively: For any demand realization $\Pi(q^*)$, set $\Pi_s^* \geq t_s$ and $\Pi_r^* \geq t_r$ if $\Pi(q^*) \geq t_s + t_r$; Otherwise, set either $\Pi_s^* = t_s$ and $\Pi_r^* = \Pi(q^*) - t_s$, or $\Pi_r^* = t_r$ and $\Pi_s^* = \Pi(q^*) - t_r$.

Proof.

ONLY IF: We show that the optimal profit allocation rule Ψ^* is necessary by contradiction. First note with the optimal allocation rule Ψ^*, we have $P\{\Pi_s^* \geq t_s\}$=1 and $P\{\Pi_r^* \geq t_r\}$=1 if $\Pi(q^*) \geq t_s + t_r$. Otherwise, we have $P\{\Pi_s^* \geq t_s\}$=1 or $P\{\Pi_r^* \geq t_r\}$=1. Now consider ANY other allocation rule $\Psi = [\Pi_s', \Pi_r']$. If the chain profit $\Pi(q^*) \geq t_s + t_r$, rule Ψ implies either $\Pi_s' < t_s$ or $\Pi_r' < t_r$, and hence, $P\{\Pi_s' \geq t_s\}$=0 or $P\{\Pi_r' \geq t_r\}$=0. If $\Pi(q^*) < t_s + t_r$, rule Ψ implies $\Pi_s' < t_s$ and $\Pi_r' < t_r$, and hence, $P\{\Pi_s' \geq t_s\}$ =0 and $P\{\Pi_r' \geq t_r\}$=0. In either case, the probability pair $[P\{\Pi_s' \geq t_s\}, P\{\Pi_r' \geq t_r\}]$ is dominated by $[P\{\Pi_s^* \geq t_s\}, P\{\Pi_r^* \geq t_r\}]$. Therefore the profit allocation rule Ψ^* is Pareto optimal.

Next we show that order quantity q^* is necessary. Consider ANY other order quantity $q' \neq q^*$. If $q' < q^*$, the maximum chain profit is $\Pi(q') = (r-c)q' < t_s + t_r$. This implies that it is impossible for both the supplier and the retailer to achieve their profit targets simultaneously, no matter what is the demand realization. Therefore, $q' < q^*$ cannot be

Pareto optimal. Now we consider the case of $q' > q^*$. We have the following based on Figure 2.1:

$$\begin{cases} \Pi(q') > \Pi(q^*) = t_r + t_s & if\ D > \overline{q} \\ \Pi(q') < \Pi(q^*) = t_r + t_s & if\ \overline{q} > D > q^* \\ \Pi(q') < \Pi(q^*) < t_r + t_s & if\ D < q^* \end{cases} \tag{3.45}$$

where:

$$q^* < \overline{q} = \frac{t_r + t_s + (c - v)q'}{r - v} < q' \tag{3.46}$$

If $D > \overline{q}$ or $D < q^*$, the order quantities q^* and q' lead to the same probability pair with the allocation rule Ψ^*. If $\overline{q} > D > q^*$, the order quantity q^* leads to the probability pair [1,1] for the supplier and retailer. Hence, q^* Pareto dominates q' under the case of $\overline{q} > D > q^*$. In summary, the order quantity q^* is Pareto optimal.

IF: Let $\lambda = P\{\Pi(q^*) < t_r + t_s, \Pi_s^* = t_s\}$, based on the profit allocation rule Ψ^*, we have:

$$P\{\Pi_s^* \geq t_s\} = P\{\Pi(q^*) \geq t_r + t_s\} + (1 - P\{\Pi(q^*) \geq t_r + t_s\})\lambda \tag{3.47}$$

Similarly, we also have:

$$P\{\Pi_r^* \geq t_r\} = P\{\Pi(q^*) \geq t_r + t_s\} + (1 - P\{\Pi(q^*) \geq t_r + t_s\})(1-\lambda) \tag{3.48}$$

Since both $P\{\Pi_s^* \geq t_s\}$ and $P\{\Pi_r^* \geq t_r\}$ increase in $P\{\Pi(q^*) \geq t_r + t_s\}$ and $P\{\Pi(q^*) \geq t_r + t_s\}$ is maximized at q^*, [$P\{\Pi_s^* \geq t_s\}$, $P\{\Pi_r^* \geq t_r\}$] is Pareto optimal. □

Based on Theorem 3.4, the coordinating contract for the supply chain with the profit satisficing objective has a very simple structure. First, the retailer's order quantity needs to be chosen so that the probability for the whole chain to achieve the chain profit target $t_s + t_r$ is maximized. Second, the supply chain's profit should allocated such that at least one of the agents (either the supplier or the retailer), and both agents whenever possible, to achieve their profit targets.

We can reinstate Theorem 3.4 in another way as in the following theorem.

Theorem 3.5:

The supply chain is coordinated if and only if the supplier's and the retailer's profit probabilities is given by the optimal probability pair:

$$[1\text{-}(1-\lambda)\ F\left(\frac{t_r + t_s}{r-c}\right),\ 1\text{-}\lambda\ F\left(\frac{t_r + t_s}{r-c}\right)] \tag{3.49}$$

where λ is the probability that the supplier will be assigned his target profit when the realized total profit is not enough to satisfy both agents' target profits.

Proof.

The probability that the realized total profit is at least sum of the target profits is 1-$F\left(\frac{t_r+t_s}{r-c}\right)$. The probability that the realized total profit is less than the sum is $F\left(\frac{t_r+t_s}{r-c}\right)$. Therefore we have:

$$P_s^* = 1 - F\left(\frac{t_r+t_s}{r-c}\right) + \lambda\ F\left(\frac{t_r+t_s}{r-c}\right) = 1 - (1-\lambda)\ F\left(\frac{t_r+t_s}{r-c}\right) \tag{3.50}$$

$$P_s^* = 1 - F\left(\frac{t_r+t_s}{r-c}\right) + (1-\lambda)\ F\left(\frac{t_r+t_s}{r-c}\right) = 1 - (1-\lambda)\ F\left(\frac{t_r+t_s}{r-c}\right) \tag{3.51}$$

Obviously the Pareto-optimal wholesale price contract can coordinate the supply chain under the profit satisficing objective which results in the probability pair [1, 1-$F\left(\frac{t_r+t_s}{r-c}\right)$]. This is a special case (λ=1) when the supplier always gets his target profit when the realized total profit is not enough for both. However, the Pareto-optimal buy back contracts (see Table 3.3) cannot coordinate the supply chain. This is because all three resulted probability pairs are dominated by the optimal probability pair in (3.49).

As for Pareto-optimal quantity flexibility contracts (See Table 3.4), the probability pair from set *QF1* is dominated by [1-$F\left(\frac{t_r+t_s}{r-c}\right)$, 1]. Therefore, the Pareto-optimal quantity

flexibility contracts in set *QF1* cannot coordinate such a supply chain. However, the Pareto-optimal quantity flexibility contract in set *QF2*, which is a wholesale price contract with wholesale price $\hat{w}$, coordinates the supply chain. Hence, for a quantity flexibility contract to coordinate the supply chain under the profit satisficing objective, it has to degenerate into a wholesale price contract.

The results above provide an important additional justification for the popularity of wholesale price contracts besides their simplicities and lower administration costs. Recall that wholesale price, buy back and quantity flexibility contracts have one, two, and three contractual parameters, respectively. Intuitively, more contractual parameters mean more design freedom, which is indeed the case for the supply chain with the objective of expected profit maximization (Cachon 2003). However, we show that wholesale price contracts are better than buy back contracts when it comes to coordination of the supply chain under the profit satisficing objective. Furthermore, for a quantity flexibility contract to coordinate such a supply chain, it has to degenerate into a wholesale price contract.

It is also worthwhile noticing the interesting wholesale price $\hat{w}$, for which we have the following observations. First, it is a linear combination of procurement cost c and selling price r, with 1 and the target ratio β as the weights, respectively. Such a wholesale price is both surprisingly simple and economically makes sense: a wholesale price need be more than the production cost but less than the retailing price. Second, for a wholesale price contract to coordinate the supply chain, its wholesale price has to be $\hat{w}$. Third, by setting the wholesale price to $\hat{w}$, a buy back contract will be Pareto optimal regardless of

the buy back price b. Finally, for a quantity flexibility to coordinate the supply chain, it has to degenerate into a wholesale price contract with wholesale price $\hat{w}$.

However, the resulted probability pair from the coordinating wholesale price and quantity flexibility contracts is [1, 1-$F\left(\frac{t_r + t_s}{r - c}\right)$], a special case of the optimal probability pair when λ=1. This is equivalent to say that the supplier always gets his target profit and the retailer gets the rest, which happens when the supplier has a relatively high bargaining power.

There is another type of Pareto-optimal profit allocation where the retailer always gets her target profit of t_r and the supplier gets the rest. This is likely to happen if the retailer is in a relatively dominant position in a supply chain, such as Wal-Mart. The resulted optimal probability pair will be [1-$F\left(\frac{t_r + t_s}{r - c}\right)$, 1]. This can be implemented through the simple slotting fee contracts (Lariviere and Padmanabhan, 1997), in which the retailer receives a fixed slotting fee and the supplier gets the rest profit (or loss). By setting the slotting fee equal to the retailer's profit target t_r, the slotting fee contract is Pareto-optimal and coordinates the supply chain under the profit satisficing objective.

3.6 Pareto-optimal Wholesale Price Contract under Profit and Revenue Satisficing Objective

How to design a contract and how to coordinate a supply chain when each agent adopts both profit and revenue satisficing objectives? In this section, we consider the same supply chain considered in the previous sections. The difference is that both supplier and retailer now have two equally important targets to achieve simultaneously, i.e., the profit and revenue targets. The profit targets for the supplier and the retailer are set externally at t_s^p and t_r^p, respectively. The revenue targets for the supplier and the retailer are set externally at t_s^e and t_r^e, respectively. For simplicity, we impose the following assumption:

$$t_r^e - t_r^p < t_s^e \tag{3.52}$$

which represents the situation where the retailer's revenue target is not significantly larger than the supplier's revenue target.

Both the supplier and the retailer in the supply chain adopt the profit and revenue satisficing objective. The supplier wants to maximize the probability of achieving the predetermined target profit t_s^p and target revenue t_s^e simultaneously, i.e., to maximize $P_s^{pe}(q,\theta) = P\{\Pi_s^p(q,\theta) \geq t_s^p \ and \ \Pi_s^e(q,\theta) \geq t_s^e\}$. For simplicity, we call it the supplier's profit and revenue probability. Similarly, the objective of the retailer is to maximize

$P_r^{pe}(q,\theta) = P\{\Pi_r^p(q,\theta) \geq t_r^p \ and \ \Pi_r^e(q,\theta) \geq t_r^e\}$ and we call it the retailer's profit and revenue probability.

Under a wholesale price contract with parameter set θ, the supplier's profit and revenue are $\Pi_s^p(q,w) = (w-c)q$ and $\Pi_s^e(q,w) = wq$, respectively, both of which are deterministic. Hence, the supplier's participation constraint is $q \geq \max\left\{\frac{t_s^p}{w-c}, \frac{t_s^e}{w}\right\}$. Once the participation constraint is satisfied, the supplier's profit and revenue probability is always 1.

The retailer is a newsvendor with procurement cost w. Recall the results from Section 2.4. If the wholesale price $w \geq \frac{t_r^e - t_r^p}{t_r^r} r$, or equivalently $\frac{t_r^e}{t_r^p} \leq \frac{\text{r}}{\text{r-w}}$, we have the followings:

$$q^*(w) = \frac{t_r^p}{r-w} \tag{3.53}$$

$$P_r^{pe*} = 1 - F\left(\frac{t_r^p}{r-w}\right) \tag{3.54}$$

If the wholesale price $w \leq \frac{t_r^e - t_r^p}{t_r^e} r$, or equivalently $\frac{t_r^e}{t_r^p} \geq \frac{\text{r}}{\text{r-w}}$, we have the followings:

$$q^*(w) = \frac{t_r^e - t_r^p}{w} \tag{3.55}$$

$$P_r^{pe*} = 1 - F\left(\frac{(w-v)t_r^e + vt_r^p}{w(r-v)}\right) \tag{3.56}$$

Therefore, we need to consider cases depending on the wholesale price. In the first case of $w \le \frac{t_r^e - t_r^p}{t_r^e} r$, to satisfy the supplier's participation constraint, we need to have:

$$q^*(w) = \frac{t_r^e - t_r^p}{w} \ge \max\left\{\frac{t_s^p}{w-c}, \frac{t_s^e}{w}\right\} \tag{3.57}$$

which is impossible because of the assumption (3.52). So we only need consider the second case of $w \ge \frac{t_r^e - t_r^p}{t_r^e} r$. The implication is that the wholesale price should be designed such that the retailer does not need to worry about her revenue target. The Pareto-optimal *w* is the minimum which is constrained by:

$$w \ge \frac{t_r^e - t_r^p}{t_r^r} r \text{ and } \frac{t_r^p}{r-w} \ge \max\left\{\frac{t_s^p}{w-c}, \frac{t_s^e}{w}\right\} \tag{3.58}$$

which gives:

$$w^* = \max \{ \frac{t_r^e - t_r^p}{t_r^e} r , \frac{t_s^e}{t_s^e + t_r^p} r , \frac{t_r^p c + t_s^p r}{t_r^p + t_s^p} \} \tag{3.59}$$

which is reduced to:

$$w^* = \max \{ \frac{t_s^e}{t_s^e + t_r^p} r , \frac{t_r^p c + t_s^p r}{t_r^p + t_s^p} \} \tag{3.60}$$

This is because we have $\frac{t_s^e}{t_s^e + t_r^p} r \geq \frac{t_r^e - t_r^p}{t_r^e} r$ when the assumption t_r^e - t_r^p < t_s^e is satisfied. To further simplify (3.60), we have two separate cases. In the first case of $\frac{r-c}{r} \geq \frac{t_r^p + t_s^p}{t_r^p + t_s^e}$, we have:

$$w^* = \frac{t_s^e}{t_s^r + t_r^p} r \tag{3.61}$$

$$q^*(w) = \frac{t_s^e + t_r^p}{r} \tag{3.62}$$

$$P_r^{pr*} = 1 - F\left(\frac{t_s^e + t_r^p}{r} \right) \tag{3.63}$$

In the second case of $\frac{r-c}{r} \leq \frac{t_r^p + t_s^p}{t_r^p + t_s^e}$, we have:

$$w^* = \frac{t_r^p c + t_s^p r}{t_r^p + t_s^p} \tag{3.64}$$

$$q^*(w) = \frac{t_s^p + t_r^p}{r - c} \tag{3.65}$$

$$P_r^{pr*} = 1 - F\left(\frac{t_s^p + t_r^p}{r - c}\right) \tag{3.66}$$

The results are very interesting and we have the following observations:

1. The optimal wholesale price should be designed such that the retailer's revenue target does not matter: when the retailer achieves her profit target, she achieves her revenue target automatically.
2. The optimal wholesale price depends on the ratio of profit margin (*r*-*c*) and revenue margin (*r*).
3. Given a fixed ratio between the profit margin and revenue margin, if the supplier's revenue target t_s^e is set aggressively such that $\frac{r-c}{r} \geq \frac{t_r^p + t_s^p}{t_r^p + t_s^e}$, the optimal wholesale price will increase in t_s^e. In addition, his maximal profit and revenue probability will decrease in t_s^e.
4. Given a fixed ratio between the profit margin and revenue margin, if the supplier's revenue target t_s^e is set modestly such that $\frac{r-c}{r} \leq \frac{t_r^p + t_s^p}{t_r^p + t_s^e}$, the optimal

wholesale price and his maximal profit and revenue probability will be independent of the revenue target t_s^e.

5. The optimal wholesale price and the optimal order quantity take surprisingly simple form: they are independent of the demand distribution.
6. It is also worthwhile noticing that the salvage value does not matter.

3.7 Conclusions

In this chapter, we focus on a basic supply chain with a single supplier and a single retailer. Both the supplier and the retailer have exogenous profit targets and their objective to maximizing their probabilities to achieve their respective targets. There are two levels of decisions to make. In the first level, how to design the contract for the supply chain? In the second level, how much should retailer order?

We study three representative contractual forms: linear tariff contracts (including wholesale price contracts as special cases), buy back contracts and quantity flexibility contracts. Our results are very interesting and provide much managerial insights. It turns out that wholesale price contracts can coordinate such a basic supply chain under the profit satisficing objective, while buy back contracts cannot. Furthermore, for quantity flexibility contracts to achieve coordination, they have to degenerate into wholesale price contracts. In the literature, questions have been raised on why wholesale price contracts are popular although they are incapable of supply chain coordination. The only argument for wholesale price contracts is that they are simpler and cheaper to administer. However,

all those results and arguments are based on the assumption that each agent will adopt the objective of expected profit maximization. Our results, on the other hand, provide an important justification for the wide use of wholesale price contracts because they can coordinate the supply chain under the profit satisficing objective. As a matter of fact, many firms are motivated to achieve some performance target, especially the profit target. Such a fact may well contribute to the wide use of wholesale price contracts.

The second contribution of this chapter is to provide a necessary and sufficient condition for supply chain coordination under the profit satisficing objective. First, the supplier and the retailer need work together such that the probability to achieve the sum of target profits is maximized for the whole supply chain. Second, the supplier and retailer need allocate the profit properly. If the total profit is more than enough, each one gets at least his/her profit target. Otherwise, either the supplier or the retailer will get his/her target profit while the other gets the rest.

The third contribution of this chapter is to study contract design under both profit and revenue satisficing objectives. As a first attempt, we study wholesale price contracts only. The results again are interesting. One of them is that the wholesale price should be designed such that the retailer does not need worry about the revenue target.

In summary, in this chapter, we present the first study on contract design and supply chain coordination under the profit satisficing objective. There are a number of future research projects which could be done. For example, how to design Pareto-optimal buy

back contracts, quantity flexibility contracts, among others, for the supply chain under the profit and revenue satisficing objective? Can the resulted Pareto-optimal contracts coordinate the supply chain? Furthermore, since we only consider a basic supply chain with one supplier and one retailer, it would be a natural step to extend the analysis to a more complex supply chain. For example, how to design contract and how to achieve supply chain coordination when multiple suppliers sell to a single retailer, when a supplier sells to multiple retailers, and finally, multiple suppliers sell to multiple retailers? The expected results will help more realistic supply chain management.

Chapter 4

Contract Design and Supply Chain Coordination under Multiple Objectives

4.1 Introduction and Literature Review

The traditional supply chain management, especially supply chain coordination, focuses on the objective of expected profit maximization, which is risk-neutral and long-term oriented. In the previous chapter, we study contract design and supply chain coordination under the profit satisficing objective, which may be risk-averse and more short-term oriented. It is shown that very different behavior will be induced under the profit satisficing objective comparing with those results obtained under the objective of expected profit maximization.

It is natural to argue that in practice, firms are concerned with both short-term and long-term results. Actually, objectives of firms include expected profit maximization, expected utility maximization and expected profit maximization subject to downside-risk constraint (Gan *et al*, 2005). If a firm, or some division of a firm, is concerned with profit, it is very likely for the firm to adopt both the profit satisficing objective and the objective of expected profit maximization. If a supply chain consists of such firms, how do we design contract and achieve supply chain coordination, if possible? This is the core question we want to answer in this chapter.

To our best knowledge, there is little study on contract design and supply chain coordination under multiple objectives. There have been some initial attempts the framework of newsvendor, though. Parlar and Weng (2003) develop a model that unifies and integrates two objectives in the framework of a newsvendor. This is done by considering the probability of exceeding expected profit level, which is a function of the order quantity decision variable. Therefore, Parlar and Weng (2003) actually combine two objective into one.

In this chapter, the supply chain consists of agents each adopting multiple objectives simultaneously, namely, the profit satisficing objective and the objective of expected profit maximization. The terms of trade between the supplier and the retailer are specified by quantity flexibility contracts and we are interested in the design of Pareto-optimal quantity flexibility contracts. The reason we choose to study quantity flexibility contracts is its three degrees of freedom, which implies more flexibility in contract design. It is also hoped that more degrees of freedom will benefit the ultimate goal: supply chain coordination.

This chapter begins with review of Pareto-optimal quantity flexibility contracts under the traditional objective of expected profit maximization, which is in Section 4.2. In Section 4.3, borrowing the results in the previous chapter, we obtain Pareto-optimal quantity flexibility contracts under both the profit satisficing objective and the objective of expected profit maximization. We prove the existence of such Pareto-optimal quantity flexibility contracts. In Section 4.4, we study a supply chain where different agent adopts

different objective, which is likely in practice. We then conclude this chapter in Section 4.5.

4.2 Pareto-optimal Quantity Flexibility Contracts under the Objective of Expected Profit Maximization

In this section, we review the Pareto-optimal quantity flexibility contracts in a supply chain consisting of a supplier and a retailer who sells a short life-cycle goods/service to the customers. Both the supplier and the retailer adopt the objective of expected profit maximization only. The supplier procures or produces the good at a constant unit cost c. The retailer procures the good from the supplier and sells to her customers at a constant unit retail price r. The customer demand D follows a probability density function (PDF) $f(\cdot)$ and a non-decreasing cumulative density function (CDF) $F(\cdot)$. If the supply chain fails to provide sufficient stock, unmet demand is lost. For simplicity, we assume zero shortage cost throughout the paper. Any unsold inventory can be salvaged at a constant unit salvage price v. To avoid trivial situations, we assume $v < c < r$.

Since each agent adopts the objective of expected profit maximization only, this will also be the objective of the centralized channel. Given a stocking level of q, the random profit of the centralized channel is given by:

$$\Pi_c(q) = (r-c)\ q - (r-v)\ (q-D)^+ \tag{4.1}$$

To maximize the expected profit, the optimal stocking level is given by:

$$q_{PM} = F^{-1}\left(\frac{r-c}{r-v}\right) \tag{4.2}$$

Now we consider the decentralized system where the terms of trade between the supplier and the retailer are specified by a quantity flexibility contract. With a quantity flexibility contract, the supplier charges the retailer a constant unit wholesale price w but offers the retailer limited flexibility of adjusting the initial order quantity when selling season starts. Suppose the retailer initially orders quantity q. Depending on demand from customers, the retailer can adjust the initial quantity q to be anywhere within $[dq, uq]$ without extra financial charge, where $0 \le d \le 1$ and $u \ge 1$ represent the downward and upward adjustment parameters, respectively. Therefore, the retailer's actual order quantity will be dq, D, and uq if the realized demand is $D \le dq$, $dq \le D \le uq$, and $D \ge uq$, respectively. The supplier is responsible to supply up to quantity uq for the supply chain. Finally note that if the maximum allowed downward and upward adjustments are desired to be symmetrical around the initial order quantity, it is required that $u + d$ =2.

Each quantity flexibility contract can be characterized by its parameter set $\theta = [w, u, d]$. For convenience, let $\alpha = \frac{d}{u}$ denote the adjustment ratio. Since the stocking level in the channel will be uq, $0 \le \alpha \le 1$ represents the fraction for which the retailer is responsible.

In addition, a higher α indicates less flexibility for the retailer. A quantity flexibility contract degenerates into a wholesale price contract if α =1, i.e., $u = d$ =1.

Quantity flexibility contracts are capable of coordination of the supply chain under the objective of expected profit maximization (Lariviere 1999). Therefore, a quantity flexibility contract is Pareto optimal if and only if it coordinates the supply chain, i.e., it can lead to the of expected profit maximization for the centralized channel. This can be done by inducing the retailer to place an initial order quantity of $\frac{q_{PM}^{*}}{u}$. In this way, the channel's stocking level will be q_c, which maximizes the expected profit for the centralized channel.

If the retailer places an initial order quantity of q, her random profit will be given by:

$$\Pi_r(q,\theta) = (r-w)\ uq - (r-w)\ (uq-D)^{+} - (w-v)\ (dq-D)^{+} \tag{4.3}$$

It can be verified that the retailer's optimal order quantity q_{PM}^{*} under the objective of expected profit maximization is unique and is implicitly defined by (Lariviere 1999):

$$(r-w)\ u\bar{F}(uq_{PM}^{*}) - (w-v)\ dF(dq_{PM}^{*}) = 0 \tag{4.4}$$

Substituting $q^* = \frac{q_{PM}^*}{u}$ into (4.4), we can have the following condition for a quantity flexibility contract to be Pareto optimal:

$$w(\alpha) = v + \frac{c-v}{\frac{c-v}{r-v} + \alpha F\left(\alpha q_{PM}^*\right)} \tag{4.5}$$

Since $0 \le \alpha \le 1$, we have $0 \le \alpha F\left(\alpha q_{PM}^*\right) \le \frac{r-c}{r-v}$ and $c \le w(u,d) \le r$. Hence the Pareto-optimal quantity flexibility contracts are guaranteed to be economical sensible. Finally note that (4.5) is also the condition for a quantity flexibility contract to coordinate the supply chain under the objective of expected profit maximization.

4.3 Pareto-optimal Quantity Flexibility Contracts under Multiple Objectives

In this section, we consider the supply chain where each agent adopts multiple objectives, namely, he/she wants to maximize the expected profit and to maximize the probability of achieving his/her target profit level at the same time. Obviously, these two objectives are both desirable but may conflict with each other in business practice.

If each agent adopts the multiple objectives, an ideal contract should be able to coordinate the supply chain under both objectives simultaneously. However, quantity flexibility contracts are incapable of coordination for both objectives simultaneously. Recall that to coordinate a supply chain under the satisficing objective, quantity

flexibility contracts have to degenerate into wholesale price contracts. However, it is well known that wholesale price contracts are incapable of coordination of the supply chain under the objective of expected profit maximization.

Therefore, we focus on the design of quantity flexibility contracts which are Pareto optimal for the two objectives simultaneously. In the previous chapter, we obtain the sets of Pareto-optimal quantity flexibility contracts under the profit satisficing objective. In the previous section, we have the sets of Pareto-optimal quantity flexibility contracts under the objective of expected profit maximization. Therefore, the intersection of the two sets, if not empty, will be Pareto optimal for the two objectives simultaneously. We have the following theorem.

Theorem 4.1:

There always exist a set of quantity flexibility contracts which are Pareto optimal for the supplier chain under the multiple objectives simultaneously, namely the satisficing objective and the objective of expected profit maximization. Furthermore, such Pareto-optimal quantity flexibility contracts tend to decrease the adjustment ratio α as the target ratio β increases.

Proof.

First note that if $\alpha = 1$, $w = \hat{w}$ is required for a quantity flexibility contract to be Pareto optimal under the satisficing objective. However, $w = c$ is required for a quantity

flexibility contract to be Pareto optimal under the objective of expected profit maximization. Therefore, $\alpha \neq 1$ is required for a quantity flexibility contract to be Pareto optimal for the multiple objectives. Similarly, we can verify that $\alpha \neq 0$ is also required.

Therefore, we only need to consider the case $0<\alpha<1$. To be Pareto optimal for the objective of expected profit maximization only, the quantity flexibility contract parameters must satisfy (4.5). To be Pareto-optimal for the satisficing objective, the quantity flexibility contract parameters must satisfy the conditions specified in set *QF1* in Table 3.4. Therefore, to be Pareto optimal for both objectives simultaneously, the quantity flexibility contract parameters must satisfy:

$$G(\alpha,\beta) \leq w(\alpha) = v + \frac{c-v}{\frac{c-v}{r-v} + \alpha F(\alpha q_{PM}^{*})} < H(\beta) \tag{4.6}$$

where $G(\alpha,\beta) = \frac{c+\alpha\beta r}{1+\alpha\beta}$ and $H(\beta) = \frac{c+\beta r}{1+\beta}$. It can be easily verified that $c = w(1) < G(\alpha,\beta) < H(\beta) < w(0) = r$ for any $0<\alpha<1$. Hence both $H(\beta)$ and $G(\alpha,\beta)$ must intersect with $w(\alpha)$, say at α_1 and α_2, respectively. Since $G(\alpha,\beta) < H(\beta)$ and $w(\alpha)$ decreases with respect to α, we have $0<\alpha_1<\alpha_2<1$. Therefore, if the ratio α is chosen to be within $(\alpha_1,\alpha_2]$, inequality (4.6) is satisfied, i.e., the quantity flexibility contracts will be Pareto optimal for both objectives simultaneously.

Finally note that as β increases, both $G(\alpha,\beta)$ and $H(\beta)$ increase. Since $w(\alpha)$ decreases with respect to α, the α's in the Pareto-optimal set, i.e., the α's satisfying (4.6) , tend to decrease. □

We can also understand the comparative statistics between β and α intuitively. Recall that the target ratio β is the ratio between the supplier's and the retailer's target profits. A larger β indicates that the supplier has an aspiration for a relatively larger target profit. If it is up to the supplier to design the quantity flexibility contract, he will try to induce the retailer to order more. To this end, the supplier tends to offer the retailer a greater flexibility, i.e., a smaller α in the quantity flexibility contract.

Furthermore, note that under the objective of expected profit maximization, Pareto-optimal quantity flexibility contracts are equivalent to coordinating quantity flexibility contracts. Therefore, Theorem 4.1 shows that we can always design quantity flexibility contracts which can coordinate the supply chain under the objective of expected profit maximization, and are Pareto optimal under the satisficing objective at the same time. Since both objectives are desirable but may conflict with each other in business practice, this is an advantage of quantity flexibility contracts over other simpler contractual forms, such as the wholesale price contracts. In another word, more degrees of freedom in quantity flexibility contracts benefit a supply chain under multiple objectives.

Example: Consider a supply chain with a supplier and a retailer who faces a Gamma demand with mean 400 and coefficient of variation 0.5. The supplier procures the good at

a unit cost $c = 15$ and the retailer sells at a unit price $r = 20$. Any unsold unit has a salvage cost $v = 8$. The terms of trade between the supplier and the retailer are specified by a quantity flexibility contract. The target profits for the supplier and the retailer are set at $t_s = 1200$ and $t_r = 800$, respectively. What are the quantity flexibility contracts which are Pareto optimal for the objective of expected profit maximization, the satisficing objective, and both objectives simultaneously?

Figure 4.1 illustrates the associated Pareto optimality. The bold solid line represents the Pareto-optimal quantity flexibility contracts under the objective of expected profit maximization. The region bounded by the solid line and the dotted line represents the Pareto-optimal quantity flexibility contracts under the profit satisficing objective. Therefore, the segment of the bold solid line inside the region represents the Pareto-optimal quantity flexibility contracts under both objectives simultaneously. For the Pareto-optimal quantity flexibility contracts under both objectives in our example, the wholesale price should be in the range $17.6 \leq w \leq 18.0$ and the corresponding adjustment ratio should be in the range $0.66 \leq \alpha \leq 0.71$. For instance, if we choose the adjustment ratio to be $\alpha = 0.7$, the wholesale price should be $w = 17.7$. If the maximum allowed downward and upward adjustments are symmetrical around the initial order quantity ($u + d = 2$), we then have $u = 1.18$ and $d = 0.82$. Hence, the quantity flexibility contract is designed such that the retailer can adjust her initial order quantity upward or downward by up to 18%.

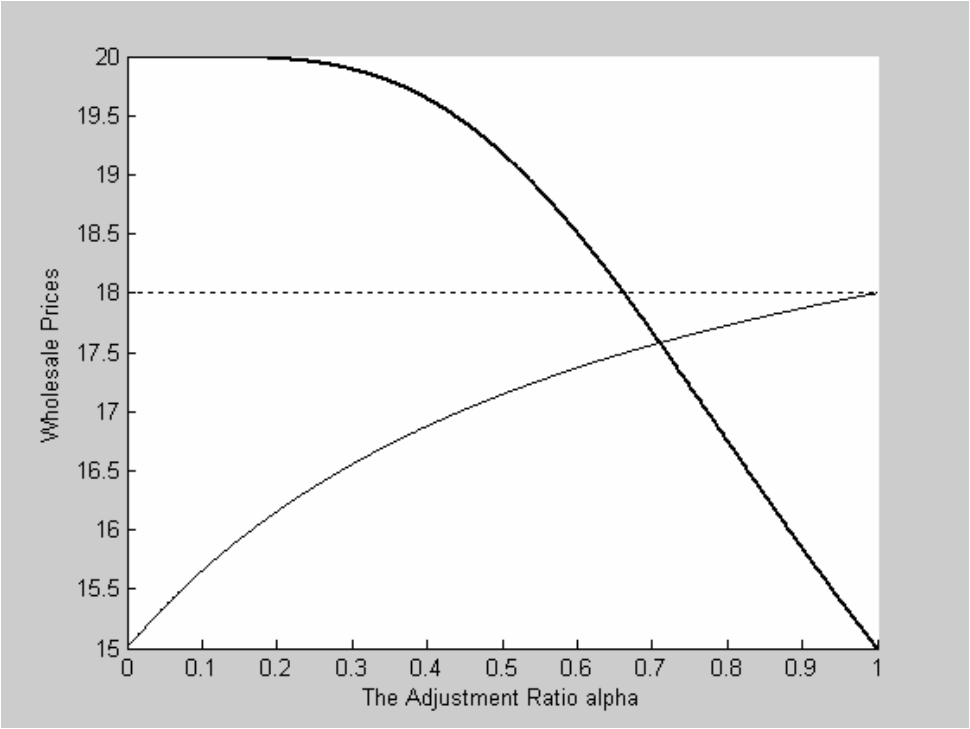

Figure 4.1: The Pareto-optimal quantity flexibility contracts for the supply chain under the objective of expected profit maximization (bold solid line), under the profit satisficing objective (the region bounded by the dotted line and the solid line), and under multiple objectives simultaneously (the segment of the solid bold line inside the region).

4.4 Pareto-optimal Wholesale Price Contract under Different Objectives

So far, we have considered contract design and supply chain coordination when each involved agent adopts identical objective(s). In Chapter 3, we study the situation where each agent adopts the profit satisficing objective. In the previous Section 4.3, we study the situation where each agent adopts multiple objectives simultaneously, i.e., the profit satisficing objective and the objective of expected profit maximization.

However, it is very likely that different agent in the same supply chain will adopt different types of objectives. This certainly is the case when one agent is risk-neutral and the other agent is risks-averse, and/or when one agent is long-term oriented and the other agent is short-term oriented. Depending on the combinations, there are a number of possible scenarios for supply chains.

For the purpose of illustration, we consider the same decentralized supply chain as the one in the previous sections. Suppose the supplier adopts the profit satisficing objective and the retailer adopts the objective of expected profit maximization. The terms of trade between the two agents are specified by a simple wholesale price contract. For simplicity, we assume the demand follows a uniform distribution on $[0, z]$.

If the supply chain is centralized and both agents adopt the objective of expected profit maximization, it can be verified that the Pareto-optimal order quantity is given by:

$$q_{PM}^{*} = F^{-1}\left(\frac{r-c}{r-v}\right) = \frac{(r-c)z}{r-v} \tag{4.7}$$

and the associated maximal expected profit is given by:

$$\Pi^{o} = \frac{z(r-c)^{2}}{2(r-v)} \tag{4.8}$$

Now we consider the supply chain where the retailer adopts the objective of expected profit maximization while the supplier adopts profit satisficing objective. How to design the Pareto-optimal wholesale price contract for such a supply chain?

Given a wholesale price contract with wholesale price w, to maximize her expected profit, the retailer will order:

$$q_r = F^{-1}\left(\frac{r-w}{r-v}\right) \tag{4.9}$$

For the supplier, his participation constraint is $q \geq \frac{t_s}{w-c} = q_s$ given his target profit level t_s. Once the participation constraint is satisfied, the probability to achieve his target profit will always be 1, and thus is maximized. Here we also assume $\frac{\Pi^o}{2} \leq t_s \leq 2\Pi^o$. Note that setting his target profit at least half of Π^o indicates the supplier has a relatively larger bargaining power. In addition, it is reasonable to set his target profit no more than twice of Π^o.

Therefore, the Pareto-optimal order quantity for the supply chain is $q^* = \max(q_r, q_s)$. The associated probability for the supplier to achieve his target is 1, and the associated expected profit of the retailer is given as:

$$E\Pi_r(q^*) = (r-w)q^* - (r-v)\int_0^{q^*} F(x)dx \tag{4.10}$$

Under the assumption of $\frac{\Pi^o}{2} \leq t_s \leq 2\Pi^o$, it can be verified that $q^* = \frac{t_s}{w-c}$ and

$$\frac{\partial E\Pi_r}{\partial w} = \frac{t_s}{z}\frac{(r-v)t_s - z(r-c)(w-c)}{(w-c)^3} \tag{4.11}$$

which increases first and then decreases with regard to w. By setting (4.11) equal to 0, we obtain the Pareto-optimal wholesale price:

$$w^* = c + \frac{t_s(r-v)}{z(r-c)} \tag{4.12}$$

which maximizes $E\Pi_r(q^*)$. It can be verified that w^* lies in the interval of $[c + \frac{r-c}{4}, r]$, and hence is economically sensible. Furthermore, the Pareto-optimal order quantity becomes:

$$q^* = \frac{(r-c)z}{r-v} \tag{4.13}$$

which is independent of t_s. It is also interesting to note that $q^* = q^*_{PM}$, which is Pareto-optimal for the supply chain where both agents adopt the objective of expected profit maximization.

Of course, the results we obtained above are unlikely to hold for general cases. It would be interesting to design Pareto-optimal and/or coordinating contracts for the supply chain where different agents adopt other different objectives, and/or some agent adopts multiple objectives.

4.5 Conclusions

In this chapter, we continue our study on contract design for a basic supply chain involving not only the profit satisficing objective but also other types of objectives. Our focus is the study the optimality under multiple objectives. Generally speaking, multiple objectives, on one hand, include satisficing objectives and maximizing objectives. On the other hand, multiple objectives can be defined in terms of profit, revenue, market share and so on.

We first design Pareto-optimal quantity flexibility contract under two objectives simultaneously, i.e., the profit satisficing objective and objective of expected profit maximization. We prove the existence of quantity flexibility contracts which are Pareto under the two objectives. However, no quantity flexibility contracts can coordinate such a supply chain under the two objectives. Next, we illustrate the design of wholesale price

contract when different agents adopt different objectives. Specifically, the supplier adopts the profit satisficing objective and the retailer adopts the objective of expected profit maximization.

Our study in this chapter is relatively limited in that we only consider some special cases of contract design and supply chain coordination under multiple objectives. In particular, it remains unclear how to achieve supply chain coordination under multiple objectives. On the other hand, this implies a lot of more work need to be done. One direction is to consider a variety of different objectives for different agents in this basic supply chain. Another direction is to consider more realistic supply chains, which usually include multiple supplier and retailers. The expected results will provide important managerial insights.

Chapter 5

Summary and Future Research Directions

We believe that satisficing objectives are at least as important as maximizing objectives, which are widely studied in Operations Management and Supply Chain Management literature. In this dissertation, our focus has been on satisficing objectives. To our best knowledge, this dissertation is the first systematic study on newsvendors and supply chains under satisficing objectives.

In Chapter 1, we provide background on supply chain management and satisficing objectives. After literature review, it becomes clear that there are few studies on supply chain management involving satisficing objectives, which provides the motivation for this research. We also outline the structure of this dissertation at the end of Chapter 1.

Since the newsvendor model is a fundamental building block in Operations Management and Supply Chain Management, in Chapter 2, we first study a single newsvendor under satisficing objectives. Review on the previous results on a single newsvendor under the profit satisficing objective, we first extend the study to a single newsvendor under the revenue satisficing objective. In the framework of newsvendor, the revenue satisficing objective alone is a bit trivial because a larger order quantity always benefit. We then study a single newsvendor under the profit and revenue satisfying objective. Under such an objective, the newsvendor endeavors to maximize the joint

probability to achieve both a profit target and a revenue target. It turns out that there are important ratios. The first ratio is between the revenue margin and the profit margin of the product, for which we call it the margin ratio. The second ratio is between the revenue target and the profit target for the newsvendor, for which we call it target ratio. If the target ratio less than the margin ratio, the joint probability is same as the profit probability. In another word, the newsvendor needs only concern about the profit target: once the profit target is obtained, the revenue target will be achieved automatically. The revenue target will become a concern for the newsvendor only when the target ratio is larger than the margin ratio. As a critical extension to the classical newsvendor model, we then consider a single price-setting newsvendor first under the profit satisficing objective alone and then the profit and revenue satisficing objective. We consider both the additive and multiplicative demand models. Our results are very different from those under the objective of expected profit maximization. In most cases, we obtain analytic results on the optimal order quantity, optimal retail price and the associated maximal probability to achieve one or more target performance levels.

The second contribution of Chapter 2 is the study of inventory competition for newsvendors under the profit satisficing objective. The previous research on this topic has been extensive but there are two drawbacks. First, there is no categorization for the proposed models. Second, there is no research on inventory competition for newsvendors who are not risk neutral. We categorize the models into two classes. In the Model I, the stocking level of a newsvendor will affect all newsvendors' demands, including her own. In the Model II, the stocking level of a newsvendor will affect all other newsvendors'

demands, but not her own. We study both models under the profit satisficing objective. Our results again are very interesting. For example, when identical newsvendors engage in "inventory competition" in Model I, there is a threshold for the number of newsvendors, below which the newsvendors will "ignore" the competition.

At the end of Chapter 2, we also study the target setting problem in the framework of newsvendors. To our best knowledge, ours is the first attempt on target setting. It turns out that analytic results are generally not available. However, by consider a couple of special cases, we provide some managerial insights. For example, when a manager assign target profit to two newsvendors, the newsvendor with higher profit margin will be assigned a higher target profit.

Using the results from Chapter 2 as a foundation, we focus our attention to supply chains. In Chapter 3, we consider a basic supply chain with one supplier and one retailer, each with satisficing objective(s). We first study the supply chain under the profit satisficing objective only. We derive Pareto-optimal wholesale price contracts, buy back contracts and quantity flexibility contracts, which have increasing degrees of freedom from one to three. After that, we prove a necessary and sufficient condition for supply chain coordination. The condition mandates that first, the supplier and the retailer work as a centralized system and the optimal order quantity should be to maximize the probability to achieve the sum of their target profits. Second, the profit allocation scheme should be such that if the total profit is not enough, either one gets his/her target profit exactly while the other gets the rest. Based on this necessary and sufficient condition, it

turns out that wholesale price contracts can coordinate such a supply chain. Due to the nature of wholesale price contracts, the supplier always gets his target profit and his profit probability is 100%. However, buy back contracts, a more advanced contractual form, cannot coordinate. Furthermore, quantity flexibility contracts have to degenerate into wholesale price contracts to coordinate such a supply chain. We then propose the use slotting fee contracts, which is capable of supply chain coordination under the profit satisficing objective. This is done by setting the slotting fee equal to the target profit of the retailer. In this case, the retailer always gets her target profit while the supplier gets the rest.

Our results are in contrast to the findings under the objective of expected profit maximization, where more advanced contracts, such as buy back contracts and quantity flexibility contracts, are strongly advocated to overcome the inefficiency due to double marginalization (Spengler, 1950). Our results, therefore, state that wholesale price contracts are one of the best under the profit satisficing objective, which is an important justification for the wide use of wholesale price contracts.

In the last section of Chapter 3, we also briefly study a basic supply chain where both agents adopt the profit and revenue satisficing objective. As the initial step, we only consider the wholesale price contract. Among the findings, one is that the wholesale price should be designed such that the retailer needs not to worry about her revenue target. Therefore, her joint probability to achieve both her targets will be her profit probability to achieve her target profit.

In Chapter 4, we study contract design and supply chain coordination when the involved agents adopt multiple objectives and/or different objectives, which is more realistic. We first study the design of quantity flexibility contracts under both the profit satisficing objective and the objective of expected profit maximization simultaneously. It is shown that while quantity flexibility contracts cannot coordinate such a supply chain, there always exist quantity flexibility contracts which are Pareto-optimal under theose two objectives at the same time. This result implies that although more degrees of freedom do not benefit supply chain coordination under the profit satisficing objective alone, they do indeed benefit supply chains under multiple objectives. In the next part of Chapter 4, we study a supply chain where different agents adopt different types of objectives. As an example, we derive Pareto-optimal wholesale price contracts for a supply chain where the supplier adopts the profit satisficing objective and the retailer adopts the traditional objective of expected profit maximization.

In general, there are two types of objectives, one is the type of maximizing objectives and the other is the type of satisficing objectives. On the other hand, the performance measures firms are concerned with include profit, revenue, market share, and so on. Intuitively, as far as what kind of objective a firm will adopt under a specific situation, there are a variety of possibilities. In this dissertation, we focus on the profit satisficing objective, the revenue satisficing objective, the profit and revenue satisficing objective, the objective of expected profit maximization, and different combinations among the three. Hence, there are a lot of possible future research directions when we consider other

possible combinations. At the end of each chapter, we provide some concrete examples of future research directions. However, some common core questions remain similar. The first core question is: how different objective will affect the behavior of the involved agents? The second core question is: how to design contract to align the incentives of the agents such that the supply chain will achieve the best possible performance?

In a broad perspective, there are two important research directions. One is to study the objectives of firms. There have been some empirical studies in this direction, which generally support the importance of satisficing objectives. However, we believe more empirical studies need to be done, especially in the framework of Operations Management and Supply Chain Management. The second research direction is to study target setting in firms. Little research has been done in this topic. As argued before, one common practice is: first, performance target levels are set for firms quarterly or annually, either internally or externally by Wall Street analysts; second, firms make operational decisions to achieve the target levels. This dissertation is more concerned with the second step, although the first step is briefly touched upon in Chapter 2.

Although various research directions could be done and should be done, we have obtained some insights in this dissertation. First, when we study "best" operational decisions of firms, it is a critical as the very first step to know what kinds of objectives firms adopt in practice. Second, the use of advanced contractual forms should be cautious. More advanced contracts are certainly more costly to administer. However, whether they will achieve the desired benefit depends on what the objective of the involved firms in

the supply chain. Our research results in this dissertation provide a perfect example. More advanced contracts than the simple wholesale price contracts, such as buy back contracts and quantity flexibility contracts, can induce retailers to order more, as argued in most of previous literature. However, this is based on the assumption that firms are risk-neutral and they adopt the objective of expected profit maximization. When the firms become sufficiently risk-averse, such as when they adopt satisficing objectives, those advanced contracts will not benefit the supply chain. The reason is intuitive: ordering more may means higher expected profit, but it will also result in higher risk. Therefore, to design a "better" contract to achieve "better" supply chain performance, it is critical to know the behavior, especially the attitude towards risk.

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