

SUPPLY CHAIN SALES PROMOTION:
THE OPERATIONS AND MARKETING INTERFACE

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To the Faculty of Washington State University:

The members of the Committee appointed to examine the dissertation
of SHILEI YANG find it satisfactory and recommend that it be accepted.

Chair

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INCENTIVES OF THE DISSERTATION

With the widespread use of business models in practice, traditional operational decisions have been integrated with other types of decisions, such as pricing, promotions, system design, etc. For any firm, previous myopic cost control operational decision making must be shifted to a multi-dimensional decision making process. It seems natural for us to understand the how operational area interacts with other functional areas.

In academia, focused disciplinary research has been the traditional approach for each individual functional area (e.g., operations, marketing, information systems, and finance). In the past decade, however, interdisciplinary research across functional areas has become a very active research stream. By applying newly acquired knowledge from other functional areas to my specifically trained area, I believe this fusion of ideas can certainly improve our understanding of operations management and hopefully generate more managerial insights for decision making in industry.

SUPPLY CHAIN SALES PROMOTION:
THE OPERATIONS AND MARKETING INTERFACE

Abstract

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Supply chain sales promotion is critical to the organizations in the channel due to complications with hooking up manufacturers, retailers and consumers together. This dissertation analyzes models discussing supply chain sales promotion under collaboration between the operations and marketing disciplines. Borrowing from the marketing empirical research on consumers' slippage behavior, this research focuses on the optimal use of mail-in rebate promotions in conjunction with other promotional tools to maximized supply chain profits.

Related literature is organized in Chapter 2. Following the literature review are three independent modeling chapters. Chapter 3 uses a utility function approach to study the manufacturer's profitability with two promotional strategies: rebates and

manufacturer's suggested retail prices (MSRP). The results show that the manufacturer's optimal strategies are jointly determined by the slippage rate and magnitude of loss aversion. Chapter 4 uses a newsvendor modeling framework to study coordinating issues between the manufacturer and the retailer when the manufacturer provides rebates to consumers and the retailer exerts promotional effort to further spur demand. The results show that a quantity discount contract is enough to coordinate a supply chain under a typical deterministic demand model. For stochastic demand, a quantity discount contract plus buy-back can coordinate the supply chain. Chapter 5 uses an economic order quantity (EOQ) modeling framework to study the retailer's choices of promotional strategies: rebate promotions or everyday low prices. The results show that the retailer's decision making depends upon several important factors including the demand price sensitivity and the regular undiscounted retail price on market.

These research results provide insights for both operations managers and marketers to facilitate proper choosing and designing of sales promotions over a supply chain. Furthermore, scholars interested in cross-disciplinary studies between operations and marketing can utilize the work here as a springboard to explore a wide range of future applications.

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Dedication

This dissertation is dedicated to my grandmother and parents.

CHAPTER 1

INTRODUCTION

Over the past decade, emerging business technologies have provided new opportunities for enhancing the collaboration between marketing and operations. Both practitioners and researchers have increased their focus on the management of the interface between marketing and operations.

Classic operational decisions involve production, procurement and inventory decisions; while classic marketing decisions involve pricing, advertising, promotional decisions. These kinds of decisions making can either be the activities of a single firm or between multiple business entities. The decision making for coordinating different business entities, i.e., manufacturers and retailers, falls within the realm of supply chain management. In the operations literature, supply chain management is called “*the tactical and strategic control of network of firms from raw materials to finished goods*” (Cachon 2006). Below is a figure of the typical supply chain.

[Insert Figure 1.1. here]

However, in the marketing literature, the term “supply chain” has been noticeably replaced by another term, “marketing channel”, which refers to “*the set of interdependent organizations involved in taking a product or service from its point of production to its point of consumption*” (Iyer and Padmanabhan 2003). Although there is no major distinction between the definitions of these two terms, marketers use the word “consumption” to indicate their special focus on consumers, i.e., all marketing events should have an impact on final consumers.

In this dissertation, the consumers' behavior has been embedded into sales promotion.

More specifically, I incorporate sales promotion into the study of a supply chain. As a ubiquitous component of marketing mix, sales promotion can be defined as “*an action-focused marketing event whose purpose is to have a direct impact on the behavior of the firm's customers*” (Blattberg and Neslin 1990). A traditional but more thorough definition of sales promotion is offered by Ulanoff (1985):

Sales promotion consists of all the marketing and promotion activities, other than advertising, personal selling, and publicity, that motivate and encourages the consumer to purchase, by means of such inducements as premiums, advertising specialties, samples, cents-off coupons, sweepstakes, contests, games, trading stamps, refunds, rebates, exhibits, displays, and demonstrations. It is employed, as well, to motivate retailers', wholesalers', and manufacturers' sales forces to sell, through the use of such incentives as awards or prizes (merchandise, cash, and travel), direct payments and allowances, cooperative advertising, and trade shows.

There are three major types of sales promotion: trade deals, retailer promotions, and consumer promotions. Strategically, trade deals and retailer promotions are elements of the push effort, while consumer promotions offered by the manufacturers are part of the pull effort. As Figure 1.2 demonstrates, by including the pull effort, I successfully complete a closed loop in the supply chain.

[Insert Figure 1.2. here]

For each type of promotion, a variety of special promotional tools exists. Table 1.1 lists out the most discussed tools in the marketing literature (Neslin, 2002).

[Insert Table 1.1. here]

In this dissertation, I focus on rebates (i.e., mail-in rebates) as the representative of consumer promotion. (Coupons can be shown to be a special case of rebates in my models.) Retail promotion in my work is characterized into a more general form: retailer promotional effort (more detailed discussion provided in the literature review section). Trade deals between manufacturers and retailers in my work involve wholesale pricing, bill-backs (i.e., channel rebates or retailer rebates in the operations literature), discretionary funds, and possibly some other techniques from the operations literature, for example, buy-back, quantity discount, revenue sharing.

There are three independent modeling sections in this dissertation. In the first section, I use a utility-based model to study consumers' behavior towards the interaction of rebates and reference price. In the second section, I develop coordinating contracts between trading partners under all three types of sales promotions. In the last section, I compare two types of common retailing strategies, everyday low pricing and rebate promotional pricing, in the category of single-firm decision making. The following figure describes my dissertation framework.

[Insert Figure 1.3. here]

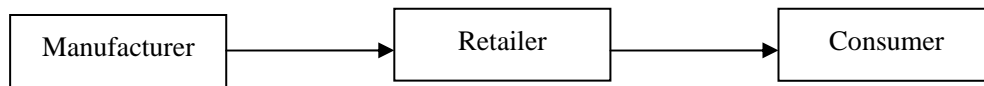


Figure 1.1 A Schematic Framework of the Supply Chain

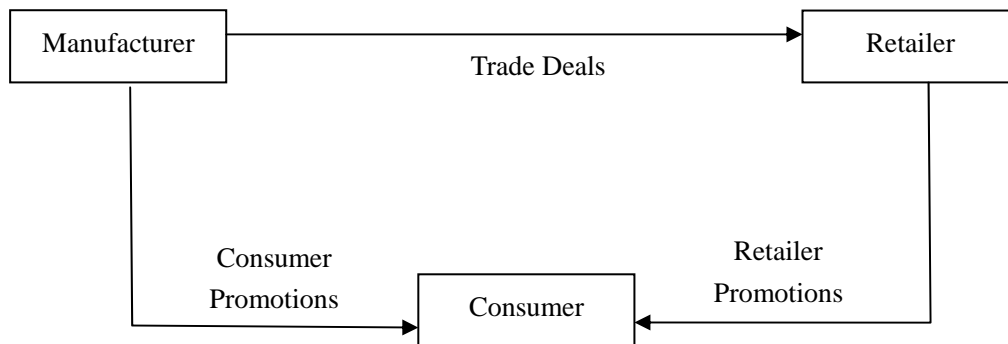


Figure 1.2 A Schematic Framework of the Types of Promotion

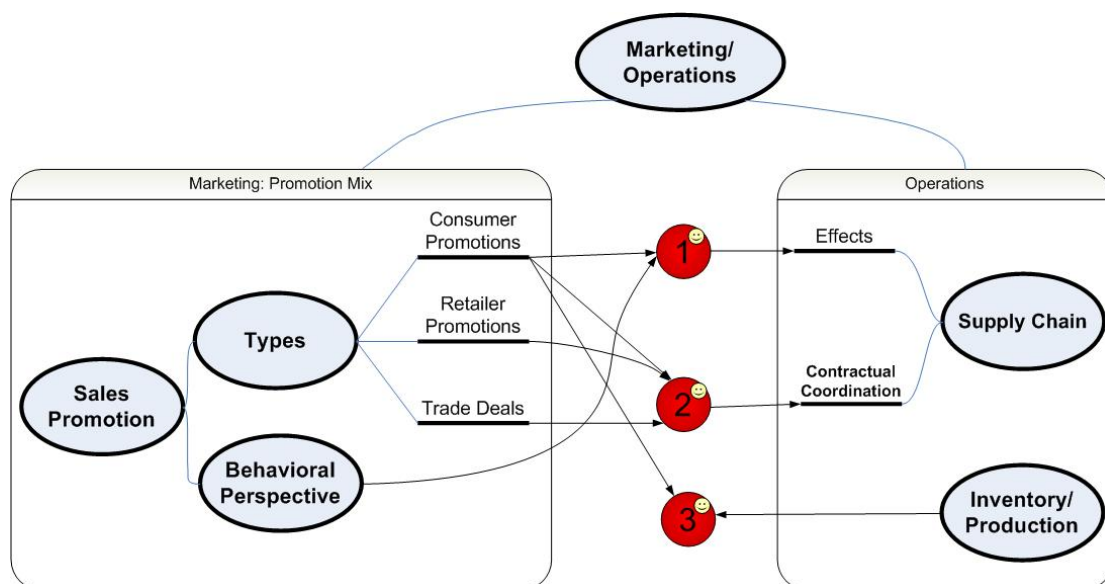


Figure 1.3 A Schematic Framework of the Dissertation Work

Trade Deals	Consumer Promotions	Retailer Promotions
Off-invoice	Coupons	In-store Price Cuts
Discretionary Funds	Rebates	Feature Advertising
Bill-backs	Reward Programs	In-store Displays
	Targeted Promotions	

Table 1.1 Specific Sales Promotion Tools

CHAPTER 2

LITERATURE REVIEW

2.1. Sales Promotion

Sales promotion is certainly the most important element of marketing mix. Statistics for packaged goods companies show that sales promotion comprises nearly 75% of the marketing budget (Neslin 2002). The marketing literature on sales promotion is saturated with both theoretical and empirical works (see Blattberg and Nelsin 1990 for the early work on sales promotion, Nelsin 2002 for an excellent recent review, and Blattberg et al. 1995 for a summary of empirical generalization of promotions).

Consumers represent the ultimate targets of all promotions. Numerous marketing articles focus on how sales promotion impacts the behavior of consumers, particularly their purchasing decisions. For example, Neslin et al. (1985) studies the relationship between consumer promotions and the acceleration of product purchases. Purchase acceleration can behave in two ways: larger purchase quantities and shorter interpurchase times. The authors estimate acceleration effects in two product categories, and they conclude that featured advertising on price cuts is the most effective tool for accelerating purchases. In a recent paper, Zhang et al. (2000) compare two types of promotional incentives: immediate value incentives versus delayed value incentives. They show that delayed incentives are more profitable in markets where consumers exhibit high variety-seeking, while immediate incentives are more profitable in markets where consumers exhibit inertia-proneness.

Among a variety of consumer behavior related topics, the phenomenon of reference

price has been a popular topic in marketing literature. The reference price effect is based on adaptation level, which is “determined by previous and current stimulus to which a person has been exposed” (Blattberg and Neslin 1990). Consumers judge the current available price by comparing it to the adaptation level, which is called reference price. The utility from comparing purchase price relative to the reference price is called transaction utility, or deal value. As a counterpart of transaction utility, acquisition utility is the value derived from the intrinsic utility provided by an item, relative to its purchase price (Neslin, 2002). So the total value of a transaction to a consumer is the sum of acquisition utility and transaction utility. The support for the existence of the reference price effect can be found in a variety of empirical studies (see Kalyanaram and Winer 1995 for a review). Sometimes, however, consistent price promotions may lower the reference prices of consumers, rendering future promotions ineffective. Greenleaf (1995) shows that reference price effects can make the promotion profitable if the profit gains in the current period exceed the losses in the future. The author also proposes a recurring promotion model with dynamic programming to identify the optimal promotional strategy in multiple periods.

There are two broad types of reference prices (Mayhew and Winer 1992): internal and external reference prices. The internal ones are prices stored in the minds of consumers and not presented in the physical environment, such as a historical price, the lowest currently available price, or expected future price. External reference prices are provided by observed stimuli in the purchase environment, such as the regular

price or suggested price displayed on sale tags or featured advertising. Most of the existing literature has focused on internal reference price.

Based on prospect theory, Tversky and Kahneman (1991) extend the reference price effect by adding loss aversion. A typical reference function $R(x)$ satisfying an additive constant loss aversion can be described as

$$R(x) = \begin{cases} u(x) - u(r) & \text{if } x \geq r \\ \lambda [u(x) - u(r)] & \text{if } x < r \end{cases}$$

Where x is a single attribute of a product, such as price

r is the reference point

$U(x)$ is a strictly increasing continuous utility function of x

$\lambda > 1$ is the coefficient of loss aversion

The coefficient λ describes the degree of loss aversion with the restriction $\lambda > 1$ capturing asymmetric response to deviations above and below the reference point. Hardie et al. (1993) implemented this theory to analyze brand choice. In their model, if available price or quality of a certain brand is below the price or quality of reference brand, consumers enjoy additional gains, oppositely they suffer utility losses, which loom larger than gains. In Rosenkranz's (2003) paper, the manufacturer's suggested retail price (MSRP) serves as a reference point, which is a decision variable of manufacturer. The author shows that proper use of MSRP can increase the manufacturer's profits in a distribution channel.

Interestingly, Bell and Lattin (2000) argue that loss aversion may not be a universal

phenomenon due to consumer price responsive heterogeneity. A more price-responsive consumer has a lower price level as a reference point, while a less price-responsive consumer tends to have a higher reference level. The authors show that after controlling for heterogeneity in price responsiveness, the loss aversion effect is no longer statistically significant. A recent empirical paper by Novemsky and Kahneman (2005) also claims that loss aversion is not ubiquitous and that it has certain boundaries. The authors propose that goods that are exchanged as intended do not exhibit a loss aversion effect.

To address complex consumer behaviors, retailers generally employ one of two different types of pricing strategies: everyday low pricing (EDLP) and promotional pricing (HI/LO). EDLP does not necessarily imply no promotions at all, but EDLP stores promote less frequently and less steeply than HI/LO stores. Marketing researchers have postulated a variety of reasons for the coexistence of EDLP and HI/LO. For example, EDLP stores appeal to “expected price shoppers”, while HI/LO stores appeal to “cherry-pickers” (Lattin and Ortmeyer 1991). Moreover, EDLP stores appeal to “large basket” shoppers, while HI/LO stores appeal to “small basket” shoppers (David and Lattin 1998). Ho et al. (1999) find that a rational shopper tends to shop more often but purchase fewer quantities per visit at HI/LO stores. Other researchers (Hoch et al. 1994, Lal and Rao 1997) argue that EDLP and HI/LO are position strategies rather than merely pricing strategies.

The marketing research on retailer promotions or consumer promotions, like that described above, focuses on consumers but ignores intra-firm issues between channel members. Articles on trade promotions need to study the coordination between manufacturers and retailers. As the most important element in promotional mix, trade promotions command half of the marketing budget for many packaged goods firms (Neslin 2002). In spite of the large amount of money spent on trade promotions, the inefficiency of trade deals is a primary concern among manufacturers. The inefficiency of trade promotions are usually attributed to two retailer behaviors: passthrough and forward buying. Manufacturers offer trade promotions to retailers to encourage them to reduce retail prices and, hence, generate incremental sales. However, the retailers may decide not to pass through the full discount to consumers, or they may forward buy the items by carrying inventory to satisfy future demand. Much existing literature in trade promotions focuses on implementing proper strategies or designing efficient tools to help manufacturers to alleviate the passthrough and forward buying problems. For example, Dreze and Bell (2003) suggest that manufacturers can redesign the scan-back deals to leave the retailers weakly better off while leaving themselves strictly better off. Ault et al. (2000) show that the strategic use of instant consumer rebates can increase manufacturers' profits caused by mitigating arbitrage by retailers' forward buying behavior. Kumar et al. (2001) examine how consumer knowledge of trade promotions affect retailers' passthrough behavior, and they suggest that manufacturers can advertise their trade promotions directly to consumers, thus making consumers aware of the ongoing trade

deals. On the other hand, Lal et al. (1996) argue that forward buying has certain benefits – for example, it can decrease the intensity of competition between manufacturers. The authors explain that the forward buying makes the best trade deals unprofitable to manufacturers while making the worst trade deals unacceptable to retailers, consequently decreasing the overall probability of offering trade deals.

2.2. Rebates

This section reviews the literature on rebates, which represent the key element in this dissertation work. In the chapters that follow, rebates exclusively represent consumers' mail-in rebates, and the redemption process typically requires consumers to perform arduous tasks (filling forms, clipping labels and sending them via the mail).

In many papers, rebates have been modeled interchangeably with coupons (i.e., instant rebates). Although in many regards, rebates and coupons are similar (such as sales impact, price discrimination, etc.), one fundamental difference is that coupons are redeemed at the time of purchase and provide an immediate price reduction while rebates can only be redeemed after purchasing the product at the regular price.

Coupons are the most researched form of consumer promotion by far (see Blattberg and Nelsin 1990 p279 for a summary of couponing objectives). As the twin brother of coupons, consumer promotion by rebates does not have much veritable research (Neslin 2002), despite the fact that mail-in rebate business is increasing and the use of traditional cents-off coupons is declining (Bulkeley 1998). In 2005, the total face

value of rebates is estimated to be \$6 billion in the U.S. (Grow 2005).

The most fascinating phenomenon of rebates is consumers' slippage behavior, which occurs when "consumers are enticed to purchase as a result of a rebate offer but subsequently fail to apply for the rebate" (Silk 2004). Business Week (Grow, 2005) reports that "fully 40% of all rebates never get redeemed", which gives rebate issuers a large enough "arbitrage" space. Because this "arbitrage" space is so large, the respective market shares of some companies have even increased by issuing rebates (Bulkeley 1998). Most of the existing marketing literature on rebates can be generally classified into two categories: WHY questions and HOW questions, i.e., explanation for the phenomenon of slippage based on consumers' responses to rebates, and the influences of slippage on promotional strategies. Several early articles (Jolson et al. 1987, Tat et al. 1988) offer some initial explanation for the popularity of rebates. Folkes and Wheat (1995) provide an interesting finding that consumer's future price expectations for products with rebates are higher than those with sales or coupons. Soman (1998) suggests that consumer's purchase decisions of products offering a delayed incentive can be independent of the decisions to redeem the delayed incentive itself. Purchase decisions are influenced by the face value of rebate offer; conversely, redemption decisions are directly dependent on the extent of effort involved. The author further shows that consumers usually underestimate their future effort needed for rebate redemption. Gourville and Soman (2004) offer further insights into the effort-discounting process with an anchoring and adjustment model. Chen et al. (2005)

argue that slippage can be attributed to the different post-purchase states of a consumer. Gilpatric (2005) uses a present-biased preference model to explain the term slippage.

Primarily based on Soman's research, Silk (2004) suggest that there are three characteristics of a rebate offer: value of the reward, length of the redemption period, and redemption effort. Changes in any of these three characteristics have the potential to influence both purchase and redemption. The author finds that the discrepancy between consumers' subjective probabilities of redeeming and their objective probabilities of redeeming causes the slippage. The subjective probability of redeeming represents a consumer's redemption confidence at the time of purchase, which is mainly determined by size of reward and length of redemption period. The objective probability of redeeming represents a consumer's actual redemption behavior after purchase, which is influence by three post-purchase factors (procrastination, prospective forgetting, and redemption effort). Another interesting finding is that increasing the length of the redemption period can have a greater impact on slippage than increasing the redemption effort. Silk and Janiszeweki (2004) provide further support with industry surveys.

Recent analytical papers by quantitative marketing researchers have begun to address how to take advantage of slippage behavior. Moorthy and Soman (2003) provide a way to exacerbate the slippage effects by highlighting the reward and not highlighting

the effort required to redeem. Joseph and Kemieux (2005) explain how the redemption cost influences the designing of rebate promotions. Moorthy and Lu (2004) indicate that rebates are more efficient than coupons in price discriminating between consumer types. Thompson and Noordewier (1992) use a time series approach to study the problems of overusing cash rebates in the automobile industry. Besides the slippage phenomenon, Dogan et al. (2005) show that rebate promotion can serve as an effective market segmentation tool. The authors find that the disadvantaged firm tends to pursue a segmentation strategy by offering rebates more frequently than the advantaged one. Following these works by marketing researchers, operations researchers have begun to apply rebate tools to supply chain management (described in the next two sections).

Here, I list out the generalizations of rebates that can be drawn from literature by both marketing and operations researchers. For each generalization, there are at least three articles sharing the same results. Among them, the slippage phenomenon is uniquely associated with rebates. Coupons may share the same findings (except slippage) with rebates, although my synthesizing work is from the literature on rebates.

- *Price discriminating* --- Gerstner and Hess (1991), Gerstner and Hess (1994), Moorthy and Lu (2004), Chen et al. (2005), Joseph and Lemieux (2005).
- *Slippage/Breakage/Space-out phenomenon* --- Bulkeley (1998), Grow (2005) Lieber (2005), Mitchell (2005), Jolson et al. (1987), Silk and Janiszeweki (2004), Silk (2004), Chen et al. (2005), Moorthy and Soman (2003), Moorthy and Lu (2004), Gilpatric (2005), Khouja (2006): followed by some sub-findings
 - *Redemption cost plays the critical role in designing rebates* --- Soman (1998) Chen et al. (2005), Joseph and Lemieux (2005).
 - *Sales increase with rebate face value* --- Soman (1998), Silk and Janiszeweki (2004), Silk (2004).
 - *Redemption rate decreases with redemption cost* --- Tat et al. (1988), Khouja (2003), Silk and Janiszeweki (2004).
 - *The relationship between rebate face value and redemption rate is mixed* --- Moorthy and Lu (2004), Silk and Janiszeweki (2004) support a positive relationship; in contrast, Soman (1998) and Silk (2004) argue that the effect of face value on redemption is weak.
- *Against forward-buying/inventory stock-up by retailer* --- Bulkeley (1998), Ault et al. (2000), Arcelus and Srinivasan (2003).
- *Improving the manufacturer's profits and the channel profits* --- Gerstner and Hess (1991), Gerstner and Hess (1995), Chen et al., 2005 (2005), Aydin et al. (2005).
- *Increase retail price* --- Gerstner and Hess (1991), Aydin et al. (2005), Arcelus et al. (2006); however, Chen et al. (2005) argue that "the retailer may or may not increase its selling price when the manufacturer offers a rebate".

2.3. Pricing and Production/Inventory Interface

One major component of the marketing/operations interface is the integration of operational decisions with retail pricing. This research area has also been called marketing/manufacturing or pricing/inventory interface. The increased research in this category coincides with the growth of the Internet and E-commerce, which has opened up great opportunities for investigating the pricing mechanism. The latest thorough reviews can be found in Yano and Gilbert (2003) and Chan et al. (2003).

Most of the articles in this category focus on decision making involving only a single firm rather than on coordination issues within and between business entities. The firm under investigation has control over production or inventory decisions, and the price-sensitive demand is usually limited by the quantity produced or procured. The firm's goal is to align the incentives of marketing and production. This section reviews the promotional related articles falling into this category. Although there are many examples of promotional pricing in the marketing literature, operations researchers have produced the majority of the work that aligns promotion decisions with inventory or production decisions.

Sogomonian and Tang (1993) develop a multiple-period deterministic model to maximize a firm's net profit by choosing the timing and level of promotion, as well as the level of production at each period. Their mixed-integer program results in a "nested" longest path problem over a network, which can be solved in polynomial

time. Cheng and Sethi (1999) model a joint inventory-promotion decision problem for a retailer. By using a Markov decision process, they find the optimal promotional timing determined by an inventory threshold. If this threshold is exceeded, then the retailer should promote the product. For the linear ordering cost case, they also find that the retailer should replenish if the inventory falls below a certain base level. Neslin et al. (1995) develop a model to maximize the manufacturer's profits by optimally allocating expenses on advertising directly to the consumers and offering periodic trade deal discounts to the retailer.

Several recent papers on rebates can also be classified into this category. Two papers (Arcelus et al. 2006 and Khouja 2003) use a newsvendor model to study the joint pricing-inventory decision. In Arcelus et al.'s (2006) paper, a profit-maximizing retailer needs to determine the optimal retail pricing and ordering policy when the manufacturer offers the rebates directly to the consumers or a wholesale price discount to the retailer itself. The authors analyze the retailer's behavior through two ratios: passthrough ratio and claw-back ratio (i.e., the proportion of manufacturer's rebates offset by the retail-price increase). In Khouja's (2003) paper, the expected profit for the manufacture is a function of three decision variables (retail price, rebate face value, and the production quantity). The author shows that under certain condition, offering rebates may lead to a large increase in the manufacturer's profit. In another paper, Khouja (2006) implements an EOQ-based model to jointly consider the retailer's optimal pricing, rebate value and lot sizing problems. The

author uses a simple linear deterministic demand function $D = a - bP + cR$, where the ratio $L = c/b$ measures the effectiveness of a one-dollar increase in rebate face value relative to a one-dollar drop in price. The author shows that an increase in the rebate effectiveness leads to a larger optimal face value and greater profit.

This type of decision making is extended into a synchronized decision making of marketing and operations departments within the same firm, which is called “horizontal coordination”. When the two departments are in conflict, there is usually a mismatch in demand and supply, leading to production inefficiencies and unsatisfied consumers. Even when the two independent departments obtain their respective best operating level, it may lead to a suboptimal performance of the firm as a whole. Based on agency theory, Porteus and Whang (1991) suggest optimal compensation plans for one manufacturing and multiple marketing managers. Hess and Lucas (2004) argue that firms without initial knowledge of their potential customers should allocate one-third of their resources to perform marketing research and the rest to manufacture the goods. Pekgun et al. (2005) study a more complex case by adding leadtime. In their paper, the marketing department chooses the price and the manufacturing department chooses the lead time, where both variables influence the demand in a linear way. The authors find that a transfer price contract with bonus payments can achieve coordination. Meanwhile, Balasubramanian and Bhardwaj (2004) argue that conflict between the two departments is not entirely undesirable. They show that the firm’s resulting profits under compromise decisions via bargaining can be higher than

those obtained under perfect interdepartmental coordination.

Obviously, horizontal coordination can be extended into “vertical coordination”, i.e. how to coordinate the manufacturer’s decisions (production, delivery, and inventory) and the retailer’s decisions (pricing and procurement) in a distribution channel. This vertical channel coordination is also called supply chain coordination, which will be discussed next.

2.4. Supply Chain/Channel

The term “supply chain” has been specifically used by operations researchers while the term “marketing channel” is preferred by marketing researchers, though these two terms are used interchangeably without much distinction in this dissertation.

Consistent with the finding by Cachon (2006), I also notice that marketing researchers working on channel coordination almost never cite any literature from operations. The operations researchers on supply chain management do cite a few papers from marketing. The other major distinction is that marketing papers tend to use deterministic demand whereas the operations papers tend to work with stochastic demand. More interestingly, for a demand function $D(P)$, where P is the retail price, the marketing researchers call it a stochastic form because demand is not constant, however, the operations researchers still call it a deterministic form because of lack of random component.

The marketing literature on marketing channels is much more diversified than the operations literature on supply chain management. I will only review the related papers on promotions and some interesting new papers. Gerstner and Hess (1991) provide a foundation for price promotion in a channel. Rebates/coupons offered directly to consumers are called pull price promotions, whereas a temporary wholesale price reduction to the retailer is called push price promotions. Based on the analysis of a segmented consumer market (i.e., high and low segments), the authors find that the manufacturer prefers pull to push; however, the consumers are worse off with push promotions because of the redemption costs. They also find that the channel profit is highest under a combination push-pull, except with small, price-discriminatory rebates. In a later paper, Gerstner et al. (1994) extended the pull price promotion to a version with competitive retailers. Lee and Staelin (1997) define the vertical strategic interaction as “the direction of a channel member’s reaction to the actions of its channel partner within a given demand structures”. There are three types of vertical strategic interactions: substitutability, complementarity, and independence. Two recent papers study the influence of channel structure. Desai and Padmanabhan (2004) discuss the channel of selling extended warranties. The manufacturer has choices on how to sell the extended warranties: indirect selling through retailers, direct selling, or dual distribution. The authors find that the best choice is to use a dual distribution arrangement. Bell et al. (2003) compare two different channel structures: (1) an independent structure without the manufacturer’s

owned flagship retail store and (2) a partially-integrated structure with one flagship store. The authors find that the second structure allows the manufacturer to simultaneously pursue intensive distribution and high levels of retail support for its brand.

Most of the operations literature focuses on trade dealing between the manufacture and the retailer. Only a few papers consider retailer and consumer promotions. One stream studies cooperative advertising (Huang et al. 2002, Li et al. 2002). Yue et al. (2006) extend the Huang et al.' (200) paper by having the manufacturer offer a direct discount to consumers. Only recently have there appeared a couple of papers that explicitly analyze the effects of rebates in a supply chain. Chen et al. (2005) find that as long as some customers attracted by a rebate will forgo the rebate, offering rebates is always beneficial for manufacturers. Unlike the sequential decision making in the Chen et al' (2005) paper, Aydin and Porteus(2005) adopt simultaneous Nash equilibrium decision making. The authors compare consumer rebates to retailer rebates (i.e., channel rebates). Under consumer rebates, the authors find that the optimal profit allocation between the manufacture and the retailer equals the ratio α/β , where α is the effective fraction of rebates and β is the redemption probability. Baysar et al. (2006) compare the effects of cash rebates to consumers and a lump-sum incentive to retailers. They find that with high uncertain market potential, offering rebates may be more profitable for the manufacturer than offering a retailer incentive.

2.5. Contractual Coordination

The above literature on supply chains and marketing channels does not involve manufacturer-retailer contractual relationships. A contract is said to coordinate the supply chain “if the set of supply chain optimal actions is a Nash equilibrium, i.e., no firm has a profitable unilateral deviation from the set of supply chain optimal actions” (Cachon 2003). Furthermore, only verifiable variables can be written into a contract because in the event of a disagreement between the contracting parties, a court must intervene. A channel variable is called observable “if both parties to a bilateral contract can learn the realized value”; it is called verifiable “if outside enforcers (e.g., courts) can also learn the realized value” (Krishnan et al. 2004). Usually both observable and verifiable channel variables are called instruments. In practice, although each firm’s relative power plays an important role in the negotiation process, the majority of the existing work on contractual coordination assumes that the manufacturer has the power to make a “take-it-or-leave-it” offer to the retailer. This assumption appears in this dissertation as well.

Research on contractual coordination to achieve optimal supply chain performance is a very active area. For a review on supply chain/channel coordination with emphasis on contracts, see Cachon (2003) and Iyer and Padmanabhan (2003). The first review is written by an operations researcher, while the second one is written by marketing researchers. Since marketing researchers prefer to use deterministic demand models and the operations researchers prefer to use stochastic newsvendor models, different

forms of popular contracts exist in the respective marketing and operations literatures.

[Insert Table 2.1 here]

Among these favored forms, the two-part tariff is often called a franchising contract in practice. The incremental quantity discount contract in operations is equivalent to the multiple-block wholesale price contract in the marketing literature. Because of deterministic demand assumptions, the marketing literature usually lacks discussions of returns, salvages, or goodwill, which are general components of the newsvendor problem in the operations contracting literature.

Quantity discount contracts have been extensively discussed in both the marketing and operation literatures. Choi et. al. (2003) provide a recent review of coordination with quantity discounts. Quantity discounts incorporated in the operations literature usually arise as part of a minimization of total ordering and inventory-related cost evolving from the classical EOQ model. Alternatively, the marketing literature usually utilizes a price-dependent demand model and employs discount schedules to induce the retailer to lower retail prices. Jeuland and Shugan (1983) is the first paper to specifically discuss the use of quantity discount contracts to coordinate channels.

Recent papers (Weng 1995, Viswanathan and Wang 2003, Choi 2003) have combined the EOQ-based and price-dependent model together. Wang and Wu (2000) and Chen et al. (2001) have extended a one-retailer setting to multiple retailers. As a departure

from above literature on quantity discount coordination, Weng (2004) employs a newsvendor model to study the effect of quantity discounts on channel coordination.

Next, I review some papers directly related to sales promotions. Gerstner and Hess (1995) use the manufacturer's indifference curve to analyze how to mitigate the double marginalization under pull price promotion. They find that pull promotion can improve channel price coordination, even if all consumers use the discount. Jeuland and Shugan (1983) indicate that the quantity discount schedule can involve the sharing of nonprice cost, such as retail displays, consumer advertising, etc. Many other marketing papers on channel coordination fall into the context of franchising agreements (e.g., Lal 1990), where the franchisee needs to pay the franchisor an initial fee plus royalty payments. In Chu and Desai (1995), the retailer can exert long-term customer-satisfying effort and short-term selling effort to increase the demand, while the manufacturer can only exert long-term customer-satisfying effort. Based on a two-period deterministic model, the authors find a two-part tariff with zero wholesale price plus customer satisfying assistance and a lump sum bonus can coordinate the channel.

Operations management has an extensive literature that deals with contract coordination between channel members, but it usually ignores marketing expenses like promotional costs exerted by either manufacturers or retailers. There are only a handful of papers that incorporate sales promotion, which will be discussed below.

Furthermore, few contracting paper in operations consider consumer promotions (i.e., rebates or coupons), which have obvious benefits, such as little verification and negotiation between trading partners.

In recent years, contractual coordination in operations extends the traditional newsvendor setting by allowing the retailer to exert costly effort to increase demand, i.e., retailer promotional effort. The retailer can provide a host of services to spur demand, such as feature advertising, product display, point of sales service, guiding consumer purchase with salespeople, or even providing some value added services (i.e., repackages, repair and maintenance). However, these retailer's efforts are too costly for the manufacturer to observe and usually not verifiable. Hence, in an uncertain demand environment, it is hard for the manufacturer to clearly tell whether a high sales realization is caused by the retailer's effort or simply higher than expected baseline demand. So if the effort cost is written into contracts, the retailer has the incentive to provide less than the contractual level of effort, which is called the moral hazard problem. Of course, some specific effort is verifiable, like shelf-space (Wang and Gerchak 2001), or feature advertising. But, in general, the retailer's promotional effort is not legally contractible. Therefore, the promotional cost cannot be shared between the manufacturer and the retailer. In one revenue-sharing contract paper, Cachon and Lariviere (2005) discuss an extension where the retailer both takes inventory risk and influences demand by exerting costly effort. The authors show that revenue-sharing contract cannot coordinate the supply chain in this situation. Taylor

(2002) is one of the first papers explicitly investigating coordinating contracts under retailer's effort. The author assumes that the retailer's ordering quantity and effort decisions are both made prior to observing the state of market demand, and promotional cost only depends on the level of effort. The author shows that a target channel rebate contract with return credit for each unsold unit (i.e., buy back) can coordinate the supply chain. Krishnan et al. (2004) approach this topic in a more general setting, where the promotional cost depends not only on the level of effort but also on the basic demand. Different from Taylor's assumption, Krishnan et al. assume that retailer can exert promotional effort after observing basic demand. Both papers find similar results: when basic demand is observable and verifiable, a buy-back contract contingent on a sales target achieves coordination; however, if basic demand is observable but not verifiable, a buy-back contract with a markdown allowance to the retailer can coordinate the supply chain. Netessine and Rudi (2000) analyzed the drop-shipping supply chain in a multi-period model with fixed wholesale and retail price. Unlike the traditional shipping scenario in which the retailer takes on the full inventory risk, in drop-shipping, the retailer carries no inventory and focuses on customer acquisition only. As a return, the retailer compensates the wholesaler for inventory carried over, while the wholesaler subsidizes a portion of customer acquisition expenses by the retailer. The authors show that both channel members prefer the drop-shipping agreement over the traditional agreement for most of the conditions, and they also design a new contract scheme to coordinate a drop-shipping supply chain.

The above papers discussing contractual coordination with effort dependent demand all assume that the retail price is exogenously given. When the retailer can choose the retailer price, the problem becomes too complicated by the fact that the incentives provided by the manufacturer to align one action may cause distortions with the other action. The manufacturer could hardly offer any incentives that will not distort all of the retailer's three actions (order quantity, retailer price, and promotional effort). So some other papers only focus on retailer pricing but excluding the promotional effort (see section 3 in Cachon 2003). Two papers also incorporate production/delivery decisions along with marketing retail pricing. Eliashberg and Steinberg (1987) study a two-echelon multiperiod model where the product is delivered continuously to the distributor who can vary its processing rate. The authors find that the coordination can be achieved by the manufacturer's wholesale price contract to the distributor. The optimal wholesale price lies between the manufacturer's per-unit production cost and the average of the maximum possible distributor's price over the season. Unlike their determinist model, Ray et al. (2005) use a stochastic demand model with delivery uncertainty. Via a mean-variance method, Ray et al. propose a new contract that involves revenue sharing between the parties, in lieu of the distributor paying a backordering penalty and charging a low wholesale price.

2.6. Summary

To summarize previous research and position my work more clearly, I provide a summary of various aspects incorporated in the some of the most relevant literature on supply chain/channel.

[Insert Table 2.2 here]

Marketing-Favored Forms	Operations-Favored Forms
One-block wholesale price contract	Wholesale price contract
Multiple-block wholesale price contract	Quantity discount contract
All-units quantity discount contract	Buy-back contract
Two-part tariff contract	Revenue sharing contract
Franchising contract	Channel rebate contract
	Quantity flexibility contract

Table 2.1 Popular Contract Forms

Feature of Model	Jeuilard and Gerstner [1983]	Gerstner and Hess [1991]	Gerstner and Hess [1994]	Chu and Desai [1995]	Lal [1990]	Chen et al. [2005]	Aydin et al. [2005]	Bayraktar et al. [2006]	Yue et al. [2006]	Taylor [2002]	Krishnan et al. [2004]	Neesse and Rudi [2000]	Eliashberg and Steinberg [1987]	Ray et al. [2005]
SETTING														
Deterministic Model	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No	Yes	No
Single Period	Yes	Yes	Yes	Two-P	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Continuous	¹ M-V
Competitive Market	No	No	Yes	No	Yes	No	No	No	No	No	No	No	No	No
TYPES OF PROMOTION														
Trade Deals	Yes	Yes	Yes	² Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Consumer Promotions	No	Yes	Yes	² Yes	No	Yes	Yes	Yes	Yes	No	No	No	No	No
Retailer Promotions	No	No	No	² Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	No	No
CHANNEL MODE OF BEHAVIOR														
Joint Channel Optimization	Yes	No	No	No	No	No	Yes	No	No	No	No	Yes	No	No
Leader/Follower	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
CONTRACTUAL COORDINATION														
Wholesale price	No	N/A	Yes	No	No	N/A	N/A	N/A	N/A	No	No	No	Yes	No
Buy back	No	N/A	No	No	No	N/A	N/A	N/A	N/A	No	No	No	No	No
Revenue/Profit sharing	Yes	N/A	No	No	No	N/A	N/A	N/A	N/A	No	No	No	No	No
Quantity flexibility	No	N/A	No	No	No	N/A	N/A	N/A	N/A	No	No	No	No	No
Channel rebate	No	N/A	No	No	No	N/A	N/A	N/A	N/A	Yes	No	No	No	No
Quantity discount	Yes	N/A	No	No	No	N/A	N/A	N/A	N/A	No	No	No	No	No
Novelty														
Two-part Tariff				Yes	Yes							Yes	Yes	Yes
SPECIFICS														
Retail Pricing	No	Yes	Yes	Yes	Yes	Yes	Yes	³ Yes	Yes	No	No	No	Yes	Yes
Wholesale Pricing	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes	No	Yes	Yes
Inventory Cost	No	No	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes
Demand Uncertainty Model	N/A	N/A	N/A	N/A	N/A	Mul	Mul	Two-State	N/A	Mul	Mul	Add	N/A	Add

Table 2.2 Summary of Most Relevant Literature

- 1 They use a mean-variance approach to maximize profits
- 2 In customer satisfaction respect
- 3 The authors use a linear inverse demand function, so determine optimal Q rather than P

CHAPTER 3

CHANNEL ANALYSIS OF REBATE PROMOTION WITH REFERENCE-DEPENDENT CONSUMERS

3.1. Brief Introduction

This chapter analyzes two popular marketing tools: mail-in rebate promotion (MIR) and the manufacturer's suggested retail price (MSRP). Through a combined strategy of rebates and suggested retail price, the manufacturer can increase the profitability in a reference-dependent consumer market.

Rebates have become the ubiquitous promotional technique for a variety of products, ranging from groceries to electronics. Consumers' *slippage* behavior represents one of the most interesting rebate phenomena. Due to slippage, manufacturers can potentially accrue large profits by expecting that consumers are enticed by the rebate promotions but eventually fail to redeem the rebates. Borrowing from Silk's (2004) empirical analysis, I characterize the slippage phenomenon by two parameters: consumers' subjective redemption confidence r_s at the time of purchase and the objective probability of redeeming r_o after the purchase. The ratio r_s/r_o is defined as *slippage rate* in this chapter¹. The larger the slippage rate, the more significant the slippage effect, which implies that more purchasers fail to redeem. With respect to the situation where $r_s = r_o$, i.e., or all purchasers redeem the rebates, rebates promotion becomes equivalent to coupon promotions.

Previous research (see the literature review in the previous chapter) has demonstrated that rebate promotions cannot increase demand if the retailers counteract direct

¹ Slippage rate can also be defined as $\frac{r_s - r_o}{r_s} = 1 - \frac{r_o}{r_s}$, which is an increasing function of r_s/r_o .

discounts from manufactures to customers by raising the corresponding retail prices.

However, by law, manufacturers cannot dictate prices to retailers; they can only recommend a price at which the product is expected to sell. This recommended retail price is typically called the manufacturer's suggested retail price (MSRP). In this chapter, the MSRP serves as the manufacturer's strategic tool to guide the retailer to price the product.

The MSRP is typically printed on the sales tag, the product tag, or the featured advertising, all of which can easily be observed by the consumers at the time of purchase. For Internet shopping, the MSRP is usually displayed along with the actual retail price. The following example comes from an online camera retailer (mikescamera.com).

[Insert Figure 3.1 here]

At the time of purchase, potential consumers can use the manufacturer's suggested price as a reference point. Based on the reference price literature, I assume that consumers' willingness to buy is increasing when confronted with a lower than suggested retail price, and *vice-versa*. From loss aversion theory, I also assume that consumers react more strongly to a higher than suggested retail price than to a lower one. I use this reference-dependent utility to determine the consumers' market demand.

To the best of my knowledge, this is the first paper to use a utility-based model to

study rebate promotions in a two-echelon supply chain. In my model, the manufacturer can apply two effective marketing tools: rebates and MSRPs. The results show that the optimal strategies for the manufacturer and the retailer are jointly determined by the slippage rate and the magnitude of loss aversion. The slippage rate primarily determines the manufacturer's rebate promotion decisions, while the magnitude of loss aversion primarily determines the retailer's selection of the actual retail price when facing a manufacturer's suggested price.

3.2. Model Environment

This section describes the marketing environment in which I will set up the model.

1. One manufacturer and one retailer comprise an exclusive distribution channel. In the promotional season, the manufacturer sells a product to final consumers through the independent retailer. The manufacturer's unit production cost is not the focus of this chapter and assumed to be zero without loss of generality (see, for example, Lal 1990, and Chu and Desai 1995).
2. The product contains a quality level $s > 0$, which is defined as a summary measure denoting the product's overall attractiveness, exclusive of price. I use s to summarize all of the product's attributes, such as product value, reliability, durability, service, warranty, etc. As such, quality is an overall preference for a particular usage occasion that summarizes multidimensional product attributes.
3. For one unit of product with quality level s , a consumer of type t is willing to

pay up to t_s dollars for the utility derived from consuming the product. A consumer who is more quality sensitive is designated as a higher type. As such, higher type consumers are willing to pay more for the same product than lower type consumers. Alternatively, the consumer's type can be viewed as the importance weight on overall quality relative to an importance weight of 1 on product retail price. I further assume that consumer types are distributed uniformly on $[0, b]$, which captures consumer heterogeneity in the market. Similar assumptions using the uniform distribution can be found in classic marketing literature (Moorthy 1988, Blattberg and Wisniewski 1989, and Rhee 1996). I set the lower limit of the uniform distribution to zero to include the "deal-prone" segment. Some deal-prone consumers have no intention to buy the product at the regular price; however, under heavy promotions, they may obtain the item free after rebates (FAR).

4. Before consumers decide to buy, they can observe the product quality s , the retail price P_r , the rebate face value R , and the MSRP P_s . And, each consumer has a reservation utility zero at the time of purchase, which implies a consumer will purchase the product as long as his overall utility is not negative.
5. Given that the retailer can choose any retail price P_r , consumers can enjoy utility gain $\alpha(P_s - P_r)$ when they observe $P_s > P_r$; however, they suffer utility loss $\beta(P_s - P_r)$ when $P_s < P_r$. Here, α and β are the coefficients for reference price effect, i.e. the importance weight on transaction utility relative to an importance weight of 1 on product retail price. By setting $0 < \alpha, \beta \leq 1$ (see Erdem et al. 2001

for examples of empirical estimation), I assume that the importance weight on transaction utility derived from reference effect cannot be greater than the importance weight on acquisition utility derived from the economic value of purchase. I further assume $\alpha < \beta$ to capture the *loss aversion* effect.

6. In the decision timing, the manufacturer serves as the Stackelberg leader and the retailer serves as the follower (i.e., backward induction is used to obtain the subgame perfect Nash equilibrium (SPNE)). The sequence of decisions begins with the manufacturer determining the wholesale price w and the rebate face value R , and announcing the MSRP P_s . Given the manufacturer's decisions, the retailer then decides the retail price P_r . The manufacturer and the retailer are assumed to be risk neutral, and both seek to maximize their own profits.
7. I assume that consumers have a homogenous subjective redemption confidence r_s at the time of purchase and a homogenous objective probability of redeeming r_o after the purchase. While this assumption may seem strict, part of its validity derives from the realization that every consumer faces the same redemption requirements and the same length of redemption deadline described in the rebate coupon.

[Insert Figure 3.2 here]

Figure 3.2 displays the environment described by the model. Note that the manufacturer strives to dictate behavior to both of the other channel levels: (1) *the*

retailer – directly through w and indirectly through MSRP, and (2) consumers – directly through R and indirectly through MSRP. When both rebate promotions and MSRP are present, the consumer's overall utility² is

$$u = ts - (P_r - r_s R) + \alpha(P_s - P_r)^+ - \beta(P_r - P_s)^+ \quad (3.1)$$

where $(x)^+ = \max\{0, x\}$.

A consumer will purchase the product if $u \geq 0$, where 0 is the reservation utility for each consumer.

u	a consumer's overall utility
s	a summary measure of product quality level
t	consumer types
b	the highest consumer type
α	the coefficient for reference price effect when $P_s > P_r$
β	the coefficient for reference price effect when $P_s < P_r$
r_s	consumers' subjective redemption confidence
r_o	consumers' objective probability of redeeming
P_r	the retail price determined by the retailer
P_s	the MSRP determined by the manufacturer
R	the rebate face value determined by the manufacturer
w	the wholesale price determined by the manufacturer

² This utility function is equivalent to the following one:

$$u = v - \lambda(P_r - r_s R) + \alpha(P_s - P_r)^+ - \beta(P_r - P_s)^+$$

where v reflects that consumers differ in their valuation of product with $v \in [0, 1]$ and $\lambda = 1/bs$ represents the importance weight on acquisition utility.

The following parameter value assumptions will apply in all of the models studied in the ensuing sections:

- (A1) $\beta > \alpha$ to capture the loss aversion effect
- (A2) $r_s \geq r_o$ to capture the slippage phenomenon
- (A3) $w \geq r_o R$ so the manufacturer can obtain positive profit
- (A4) $P_r \geq w$ so the retailer can obtain positive profit
- (A5) $P_r \geq r_s R$ for a logical boundary condition on the rebate value, so consumers cannot potentially make profits from buying the product.
- (A6) $P_s \leq bs$, $P_s \geq w$ and $P_s \geq r_s R$ for other logical boundary conditions

3.3. Model with Rebate Promotion Only

This section formulates the retailer's and the manufacturer's problem under rebate promotions without the MSRP. With $\alpha = \beta = 0$, the consumer's utility function reduces to

$$u = ts - (P_r - r_s R),$$

Resulting in the derived consumer demand function:

$$D(P_r, R) = \int_{\frac{P_r - r_s R}{s}}^b \frac{1}{b} dt = \frac{bs - (P_r - r_s R)}{bs}.$$

Based on the backward induction of SPNE, Proposition 3.1 summarizes the equilibrium results for rebate promotion case without MSRP.

Proposition 3.1. When the manufacturer offers rebates to consumers, the equilibrium is determined by the consumers' slippage behavior.

- (1) If all the purchasers attracted by rebates promotion actually end up redeem the rebates, i.e., $r_o = r_s$, the manufacturer cannot benefit from providing rebates. The equilibrium solution is as shown in Table 3.1.

[Insert Table 3.1 here]

- (2) If the slippage phenomenon exists, i.e. $r_o < r_s$, the manufacturer can benefit from rebates promotion by providing a rebate with $R \in \left[\frac{bs}{r_s - r_o}, \infty \right)$. The equilibrium solution is as shown in Table 3.2.

[Insert Table 3.2 here]

Proof. See Appendix.

As we can see, when $r_o = r_s$, the manufacturer's sales and profit do not improve with rebate promotion. This occurs because when providing rebates the manufacturer increases the wholesale price by $r_o R$ to maintain the same profit margin; in turn, the retailer also increases its retail price by $r_o R$. Hence, the consumer demand does not change. When slippage exists, the manufacturer can achieve arbitrarily large profits if no upper bound exists for R . With a large-ticket rebate, the manufacturer can induce all consumers to buy the product and acquire profits due to the slippage effect.

However, the retail price increases dramatically (i.e., $P_r \geq \frac{r_s}{r_s - r_o} bs$) and is much

higher than the regular price without rebates. Especially when the slippage rate is not

significant, the retail price in equilibrium can reach an extremely high level. I can not explain why consumers should make purchases at such insane prices. This result implies that I need to add the suggested retail price by assuming that consumers are reference-dependent.

3.4. Reference-dependent Model with Rebate Promotion

This section reformulates the retailer's and the manufacturer's problem when consumers use the MSRP P_s as a reference price at the time of purchase. Now I employ the full consumer's utility function (3.1), which can be expressed as:

$$u = \begin{cases} ts - (P_r - r_s R) + \alpha(P_s - P_r) & \text{when } P_s \geq P_r \\ ts - (P_r - r_s R) + \beta(P_s - P_r) & \text{when } P_s < P_r \end{cases} \quad (3.2)$$

The derived demand function based on (3.2) is

$$D(P_r, R, P_s) = \begin{cases} 1 & P_r \leq \frac{r_s R + \alpha P_s}{1 + \alpha} \\ \frac{bs - (1 + \alpha)P_r + r_s R + \alpha P_s}{bs} & \frac{r_s R + \alpha P_s}{1 + \alpha} < P_r \leq P_s \\ \frac{bs - (1 + \beta)P_r + r_s R + \beta P_s}{bs} & P_s < P_r < \frac{bs + r_s R + \beta P_s}{1 + \beta} \\ 0 & P_r \geq \frac{bs + r_s R + \beta P_s}{1 + \beta} \end{cases}.$$

Obviously the lowest $P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}$, since the retailer cannot convince any additional consumers to purchase the product by further reducing its retail price. On the other end, if the retailer chooses $P_r > \frac{bs + r_s R + \beta P_s}{1 + \beta}$, there will be no consumers left to buy the product. The demand function $D(P_r, R, P_s)$ is continuous at $P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}$ and at

$P_r = \frac{bs + r_s R + \beta P_s}{1 + \beta}$. Figure 3.3 shows the kinked demand curve caused by the MSRP P_s .

[Insert Figure 3.3. here]

With loss averse consumers, the demand decreases more rapidly when $P_r > P_s$. In cases where loss aversion does not exit, i.e., $\alpha = \beta$, the demand function will not be kinked at $P_r = P_s$.

The retailer's profit function can now be written as

$$\Pi_r(P_r, w, R, P_s) = (P_r - w) \cdot D(P_r, R, P_s).$$

As the retailer has to take into account the consumer's reference price effect, the retailer's optimal choice depends upon the four subfunctions of $D(P_r, R, P_s)$ and can be characterized by the following lemma:

Lemma 3.1. The retailer chooses a retailer price P_r , depending on w and R , such that:

$$P_r^*(w, R, P_s) = \begin{cases} \frac{r_s R + \alpha P_s}{1 + \alpha} & \leq P_s & \text{for } w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha} \\ \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} & < P_s & \text{for } \frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w < P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \\ P_s & & \text{for } P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta} \\ \frac{w}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} & > P_s & \text{for } P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s. \end{cases}$$

Proof. See Appendix.

We can observe that if the wholesale price w is sufficiently low, the retailer chooses a retail price which is low enough to reach all consumer types such that the customer

demand equals 1. As w increases, the retail price will increase at a rate of $w/2$ until it reaches the MSRP P_s . When $P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta}$, the optimal response of the retailer is to price at P_s (with no loss aversion, the region for the retailer choosing P_s does not exist). Finally, as the wholesale price continues to rise, the retailer chooses to sell only to the higher types of consumers by setting the retail price above P_s .

Anticipating the retailer's reaction to w , R and P_s , the manufacturer's profit can be written as,

$$\Pi_m(w, R, P_s) = (w - r_o R) \cdot D(P_r^*(w, R, P_s), R, P_s).$$

Given the retailer's different choices of $P_r^*(w, R, P_s)$ as characterized above, the manufacturer needs to choose the optimal combination of w , R and P_s to maximize its profits by taking into account the retailer's response.

These optimal strategies of the manufacturer can be summarized in the following proposition.

Proposition 3.2. The manufacturer's optimal strategy is jointly determined by the consumers' slippage behavior and their magnitudes of loss aversion, as shown in Table 3.3.

[Insert Table 3.3. here]

Proof. See Appendix.

Proposition 3.2 produces three major observations:

(1) If all purchasers attracted by rebates actually end up redeeming them, i.e., $r_o = r_s$,

the manufacturer cannot benefit from providing rebates. If consumers are sufficiently

loss averse, i.e., $\beta \geq \frac{2\alpha}{1-\alpha}$, the manufacturer selects a lower MSRP at $P_s = \frac{(3+\beta)bs}{4+2\beta}$

and induces the retailer to adopt this suggested price. If $\beta < \frac{2\alpha}{1-\alpha}$, the manufacturer

sets the MSRP at the ceiling level, i.e., $P_s = bs$, and the retailer chooses a higher retail

price at $P_r = \frac{3}{4}bs$.

(2) If the consumers are sufficiently loss averse, i.e., $\beta \geq \frac{2\alpha}{1-\alpha}$, the manufacturer

should offer rebates as long as some purchasers forgo the redemption; if the

consumers are not sufficiently loss averse, the manufacturer should provide rebates

only after the slippage rate breaks a threshold level $\theta(\alpha, \beta) = \max(1, \frac{(1+\alpha)(1+\beta)}{1+\beta+(\beta-\alpha)})$,

which is strictly less than $1+\alpha$.

(3) When rebates are offered, the manufacturer should always set the MSRP at the

ceiling level, i.e., $P_s = bs$. As the slippage rate gets larger, the manufacturer should

increase the wholesale price and offer a larger rebate, and the retailer should also

increase its retail price accordingly. As a result, both the manufacturer's and the

retailer's profits increase with the slippage rate. Furthermore, when

$\theta(\alpha, \beta) < \frac{r_s}{r_o} \leq 1+\beta$, the manufacturer can induce the retailer to adopt the MSRP at

$P_s = bs$. Finally, as the slippage rate continues to increase, i.e., $\frac{r_s}{r_o} \geq 1+\beta$, the retailer

should select a retail price which is higher the manufacturer's suggested one.

[Insert Figure 3.4. here]

Due to the reference effect, a higher MSRP expands the market demand. However, a higher MSRP also implies a wider range for the retailer to increase the retail price, which can decrease the demand. The manufacturer needs to find a proper balance.

When consumers are sufficiently loss averse, the manufacturer can induce the retailer to adopt the MSRP and hence has more flexibility. Without doubt, in this situation the manufacturer's share of the total profit pie is larger than the share when inducement is not possible.

Without rebate promotions, the retailer will not choose P_r higher than P_s . But with rebate promotions, the retailer may choose $P_r > P_s$ when $\frac{r_s}{r_o} \geq 1 + \beta$. Although a higher than suggested retail price will cause loss aversion among consumers, the medium-ticketed and the large-ticketed rebates can sufficiently offset the loss aversion effect on consumer choices, so the market demand continues to expand. Finally, after the optimal rebate value reaches the ceiling level at $R = \frac{bs}{r_s}$, the manufacturer and the retailer can only attract more consumers by reducing w and R , respectively.

As shown in Appendix, for the situation $\frac{r_s}{r_o} \geq 1 + \frac{1}{\alpha}$, the manufacturer may choose to

issue a large-ticketed rebate ($R = \frac{bs}{r_s}$). At the same time, the manufacturer offers a

sufficiently low wholesale price to the retailer and hence induces the retailer to choose the suggested price as the actual retail price. By doing so, all the consumers will be attracted to buy the product, i.e., $D=1$. Even the deal-prone consumers in the lowest type segment will make a purchase because of free-after-rebate promotion. Although the supply chain is coordinated with a total channel profit $\prod_I = (1 - \frac{r_o}{r_s})bs$, however, in this situation, the retailer gains larger share of the profit pie instead of the manufacturer, which leaves the manufacturer less desirable. Therefore the manufacturer has no incentives to cover all consumer segments (i.e., case a2 is dominated by case d2 as shown in Table A.1)

3.5. Reference-dependent but Loss-neutral Model with Rebate Promotion

Some researchers (Bell and Lattin 2000, Novemsky and Kahneman 2005) have arguments against a loss aversion effect. They show that the loss aversion effect can be overestimated or it is not universal to every product category. To address that case, this section assumes that the consumers are no longer loss averse, i.e., $\alpha = \beta$, such that losses do not loom larger than gains in consumers' minds. The demand function analyzed in section 3.4 now loses its kink at $P_r = P_s$. The function reduces to:

$$D(P_r, R, P_s) = \begin{cases} 1 & P_r \leq \frac{r_s R + \alpha P_s}{1 + \alpha} \\ \frac{bs - (1 + \alpha)P_r + r_s R + \alpha P_s}{bs} & \frac{r_s R + \alpha P_s}{1 + \alpha} < P_r < \frac{bs + r_s R + \alpha P_s}{1 + \alpha}, \\ 0 & P_r \geq \frac{bs + r_s R + \alpha P_s}{1 + \alpha}, \end{cases}$$

and the retailer's optimal retail price is given by:

$$P_r^*(w, R, P_s) = \begin{cases} \frac{r_s R + \alpha P_s}{1 + \alpha} & \text{for } w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha} \\ \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} & \text{for } \frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w \leq P_s. \end{cases}$$

Similar to the proof of Proposition 3.2, the following proposition describes the equilibrium strategies when consumers are loss neutral.

Proposition 3.3. The manufacturer's optimal strategy under the loss-neutral reference-dependent model is jointly determined by the consumers' slippage behavior and the coefficient of transaction utility α , as shown in Table 3.4.. The manufacturer always sets the suggested retail price at the ceiling level $P_s = bs$.

[Insert Table 3.4. here]

Proposition 3.3 produces two major observations:

(1) If the slippage rate is relatively small, $\frac{r_s}{r_o} \leq 1 + \alpha$, the manufacturer will not issue

rebates, while the retailer chooses a lower than suggested retail price $P_r = \frac{3}{4}bs$ to

attract consumers.

(2) If the slippage rate is large enough, i.e., $\frac{r_s}{r_o} > 1 + \alpha$, the manufacturer benefits from

rebate promotions and the retailer always chooses a higher than suggested retail price in equilibrium.

[Insert Figure 3.5. here]

As shown in Figure 3.5, when the consumers are no longer loss averse, the retailer has less pressure to increase P_r . So the manufacturer can no longer induce the retailer to adopt the MSRP. In this situation, a more prominent slippage effect is required to induce the manufacturer to offer a rebate promotion (i.e., $1 + \alpha > \theta(\alpha, \beta)$). This occurs because the manufacturer offers a promotion with the goal to spur more demand and take advantage of the slippage effect; however, the retailer increases its retail price to “hijack” the promotion resulting in a lower demand. For the loss-neutral case, once the manufacturer launches the rebate promotion, the retailer chooses $P_r > P_s$. If the manufacturer still issues a small-ticketed rebate as in the loss-averse case, the market demand decreases for a higher than suggested retail price. So the manufacturer has to issue a medium-ticketed or large-ticketed rebate, which requires larger slippage rate to break even.

3.6. Integrated Channel with Rebate Promotion

This section considers the situation that the manufacturer owns the retailer, i.e., a vertically integrated channel in which the manufacturer can achieve supply chain optimal performance. Because the manufacturer owns the retailer, the manufacturer can dictate the actual retail price. Hence, the manufacturer maximizes its profits by choosing an optimal combination of (P_r, R, P_s) for each segment of the kinked demand function as shown in Figure 3.3. The manufacturer’s optimal strategies can be summarized by the following proposition.

Proposition 3.4. For the integrated channel, the manufacturer always sets the MSRP at the ceiling level $P_s = bs$ to exhaust the benefits by reference price effect. The equilibrium strategies are shown in Table 3.5.

[Insert Table 3.5. here]

Proof. See Appendix.

Proposition 3.4 produces three major observations:

- (1) If the slippage rate is relatively small, $\frac{r_s}{r_o} \leq 1 + \alpha$, the manufacturer will not issue rebates; however, the retailer chooses a lower than suggested retail price $P_r = \frac{3}{4}bs$ to attract consumers.
- (2) If the slippage rate is large enough, i.e., $\frac{r_s}{r_o} > 1 + \alpha$, the manufacturer can benefit from rebate promotions. When the slippage rate continues to increase above $1 + \beta$, the manufacturer offers a large-ticketed rebate.
- (3) If the magnitude of consumers' loss aversion is sufficiently small, such that the slippage rate falls into the interval $\left[1 + \beta, 1 + \frac{1}{\beta}\right)$, the manufacturer should only serve the high consumer segments with $P_r > P_s$. While consumers suffers a traction utility loss which in turn decreases the market demand, the manufacturer can acquire more profits with a large retail price,.

[Insert Figure 3.6. here]

Because of the integrated channel, the manufacturer can acquire more profits even without rebates. Hence, the manufacturer has less incentive to offer rebates and also requires higher a slippage rate (i.e., $1 + \alpha > \theta(\alpha, \beta)$) to make rebate promotion profitable. Once offered, the value of the rebate is larger than the small-ticketed but smaller than the medium-ticketed rebate in the decentralized channel. Furthermore, as opposed to the decentralized channel case, the manufacturer should serve all consumer segments as long as $\frac{r_s}{r_o} \geq 1 + \frac{1}{\beta}$.

3.7. Channel Performance with Rebate Promotion

This section tests the efficiency of rebate promotion in improving the channel performance. The efficiency here is defined as the ratio of decentralized channel profit to the integrated channel profit, i.e., $(\Pi_m + \Pi_r)/\Pi_I$. From the manufacturer's perspective, providing rebates is more attractive if the efficiency ratio in the situation when rebates are provided is higher than the measure in no rebates situation.

When no rebates are offered by the manufacturer in both models, the ratio is

$$\frac{\Pi_m + \Pi_r}{\Pi_I} = \frac{\frac{3(1+\alpha)}{16} b_s}{\frac{(1+\alpha)}{4} b_s} = \frac{3}{4}, \text{ which serves as a benchmark efficiency ratio.}$$

When rebates are offered by the manufacturer in both models, there are three different cases as shown in Figure 3.4 and Figure 3.6.

$$\frac{\Pi_m + \Pi_r}{\Pi_l} = \frac{\frac{(1+\beta)r_s(2r_s + (1+\beta)r_o)}{4(r_s + (1+\beta)r_o)^2} bs}{\frac{r_s}{4r_o} bs} \geq \frac{3}{4} \quad (\text{where } 1 + \alpha < \frac{r_s}{r_o} < 1 + \beta)$$

$$\Leftrightarrow (1+\beta) \frac{r_o}{r_s} (2 + (1+\beta) \frac{r_o}{r_s}) - 3 \geq 0$$

$$\Rightarrow (1+\beta) \frac{r_o}{r_s} (2 + (1+\beta) \frac{r_o}{r_s}) - 3 > 1(2+1) - 3 = 0$$

$$\frac{\Pi_m + \Pi_r}{\Pi_l} = \frac{\frac{3(1+\beta)r_s^2}{4(r_s + (1+\beta)r_o)^2} bs}{\frac{1}{4(1+\beta)} (2 + \beta - (1+\beta) \frac{r_o}{r_s})^2 bs} \leq \frac{3}{4} \quad (\text{where } 1 + \beta \leq \frac{r_s}{r_o} < 1 + \frac{1}{\beta})$$

$$\Leftrightarrow \beta + (1+\beta)^2 ((\frac{r_o}{r_s})^2 - \frac{r_o}{r_s}) \leq 0.$$

It is easy to show that $f(\frac{r_o}{r_s}) = \beta + (1+\beta)^2 ((\frac{r_o}{r_s})^2 - \frac{r_o}{r_s})$ reaches its maximum value

when $\frac{r_o}{r_s} = \frac{1}{1+\beta}$, where $f(\frac{r_o}{r_s} = \frac{1}{1+\beta}) = 0$. Hence $(\Pi_m + \Pi_r)/\Pi_l \leq 3/4$ holds for the

region $1 + \beta \leq \frac{r_s}{r_o} < 1 + \frac{1}{\beta}$.

$$\frac{\Pi_m + \Pi_r}{\Pi_l} = \frac{\frac{3}{16(1+\beta)} (2 + \beta - (1+\beta) \frac{r_o}{r_s})^2 bs}{(1 - \frac{r_o}{r_s}) bs} \geq \frac{3}{4} \quad (\text{where } \frac{r_s}{r_o} \geq 1 + \frac{1}{\beta})$$

$$\Leftrightarrow ((1+\beta) \frac{r_o}{r_s} - \beta)^2 \geq 0$$

Therefore, for the regions $1 + \alpha < \frac{r_s}{r_o} < 1 + \beta$ and $\frac{r_s}{r_o} \geq 1 + \frac{1}{\beta}$, rebate promotion

improves the channel performance; however, in the region $1 + \beta \leq \frac{r_s}{r_o} < 1 + \frac{1}{\beta}$, rebate

promotion does not improve the channel performance in regarding to channel

efficiency.

3.8. Numerical Studies

This section uses numerical studies to further analyze the impact of the slippage phenomenon and loss aversion effect on the manufacturer's profit. Proposition 3.3 provides evidence that the manufacturer's optimal profit increases with the slippage rate under rebate promotion. However, it does not quantify the magnitude of the benefit. Consider an example with the following parameter settings: $bs=\$300$, $\alpha=0.2$, $\beta=0.4$, $r_s=0.9$ and $0.1 \leq r_o \leq r_s$ at an incremental rate of 0.01. The values of Π_m , D , R , P_s , P_r and w are plotted in figure 3.7. From these graphs, we can observe that the manufacturer's profits and the market demand increases smoothly with the slippage rate; however, the curve of the optimal rebate value R increases in a stepwise fashion with the slippage rate. From graph d, we can observe that the retail price almost follows the same pattern as the wholesale price, as expected.

[Insert Figure 3.7. here]

Next, I explore how the slippage and loss aversion jointly affect the manufacturer's profit. By setting β to be flexible from $[\alpha, 0.8]$, we can observe from the three-dimensional graph of Figure 3.8 that the stronger the magnitude of loss aversion, the larger the manufacturer's profits. However, the contribution of loss aversion effects to profits is much smaller than the one brought by slippage effects. Furthermore, the distinct section line on the graph is the section point where the manufacturer changes from offering a small-ticketed rebate to a larger one. With larger magnitude of loss aversion, it is easier for the manufacturer to induce the

retailer to choose the MSRP. In this situation, the manufacturer has higher profit level with a small-ticket rebate, so a larger slippage rate is required for the manufacturer to desire to offer a large-ticketed rebate.

[Insert Figure 3.8. here]

3.9. Conclusions

In this chapter, I analyze the impact of rebates and MSRP on a vertical channel with reference-dependent consumers. Coupled with a rebate promotion, the manufacturer announces a suggested retail price serving as a reference point for consumers. I find that the slippage effect and the loss aversion effect jointly impact the manufacturer's profit. For the decentralized channel, if the consumers are sufficiently loss averse, i.e., $\beta \geq \frac{2\alpha}{1-\alpha}$, the manufacturer should offer rebates as long as some purchasers end up forgoing the rebates. On the other hand, if the consumers are not sufficiently loss averse, the manufacturer chooses to provide rebates only after the slippage rate breaks a threshold level $\theta(\alpha, \beta)$. Under rebate promotions, both the manufacturer's and the retailer's profits increase with the slippage rate and the magnitude of loss aversion. For the loss-neutral case and the integrated channel, the breakeven slippage rate to make rebate promotion profitable increases to $1 + \alpha$. According to industry reports, the slippage rate is ranging from approximately a low rate for 1.7 on electronics (Spencer 2005) to a very high rate for more than 10 in some categories, such as software products (Bulkeley 1998). This reveals why so many companies are issuing

rebates nowadays.

Even for a promoted product facing high redemptions, the companies can increase the slippage rate by adopting appropriate marketing techniques. Rather than increasing the required redemption effort, previous empirical research has provided several effective ways in which the manufacturer can exacerbate the consumers' slippage behavior. Moorthy and Soman (2003) suggests that properly marketing the rebate can exacerbate the slippage by highlighting the reward and not highlighting the effort required to redeem. Silk (2004) suggests that encouraging procrastination and prospective forgetting also have a great impact on slippage by increasing the length of the redemption deadline.

Hopefully, the results in this chapter will provide insights for researchers who would like to further analyze the slippage phenomenon on rebates. One extension would be to associate the objective probability of redeeming r_o to the consumer's type t , i.e. assuming r_o is decreasing with t . With this assumption, rebate promotions can price-discriminate between consumer types after purchase, which implies that high consumer types have low probability to redeem because they usually have high redemption costs and low marginal utility of income. Hence, the manufacturer can possibly achieve higher profits by only serving the high consumer types. Another line of extension would be to apply this model to the research on new product design. In that case, the manufacturer can adjust the product quality level s , which has an

increasing cost $v(s)$ such that the manufacturer needs to determine an optimal quality level. Since advertising is one important element of the promotional mix, researchers can also add the advertising cost to initiate the penetration rate.

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Figure 3.1. An MSRP Example

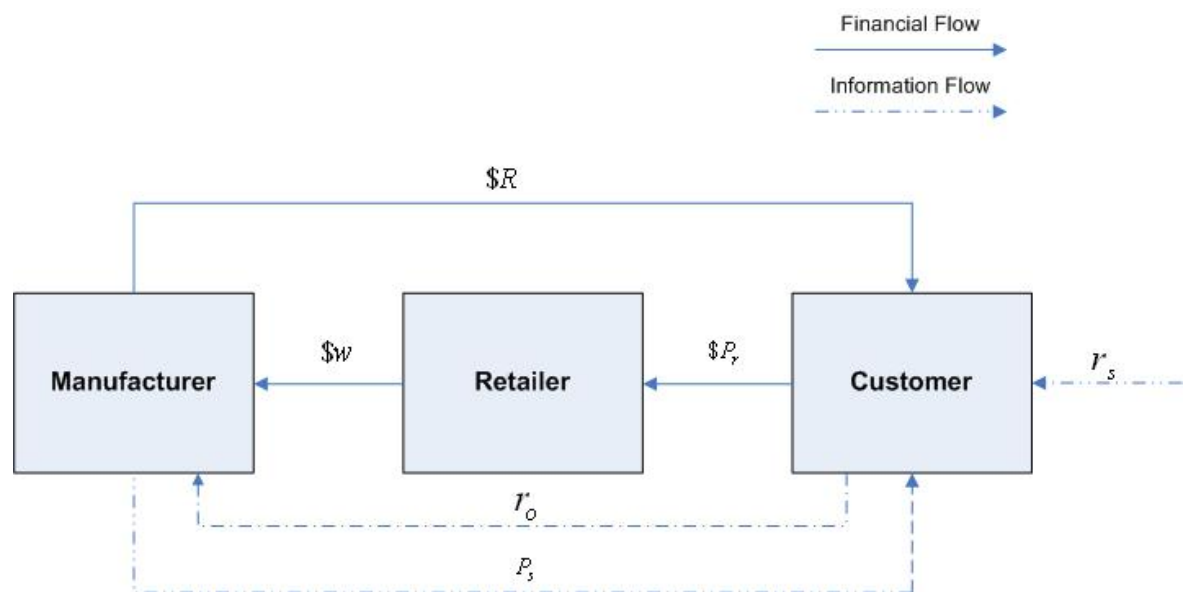


Figure 3.2. A Schematic Framework of the Market Environment

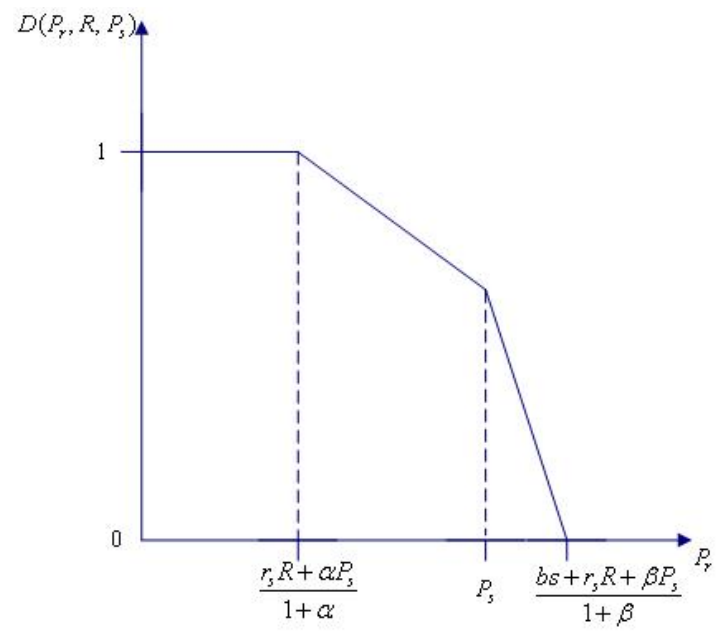


Figure 3.3. The Kinked Demand Curve $D(P_r, R)$

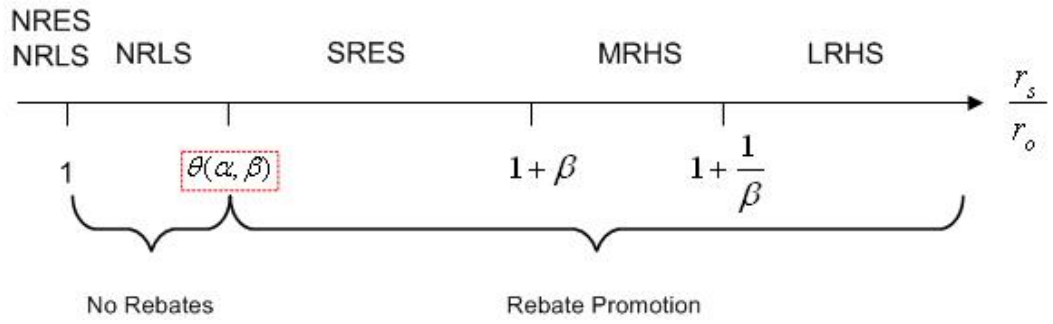


Figure 3.4. A Schematic Framework of Reference-dependent Model

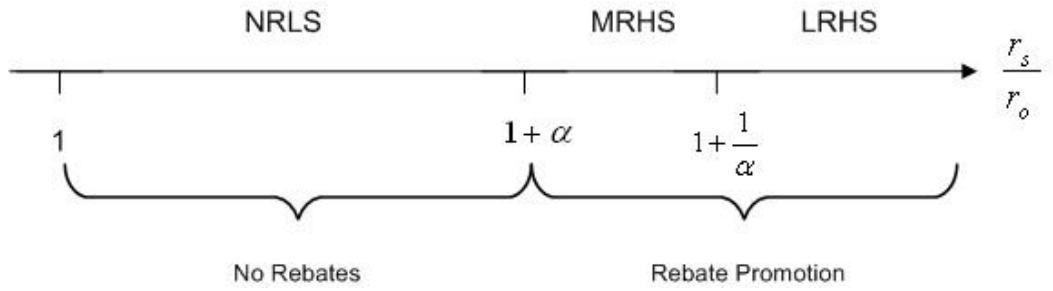


Figure 3.5. A Schematic Framework of Loss-neutral Model

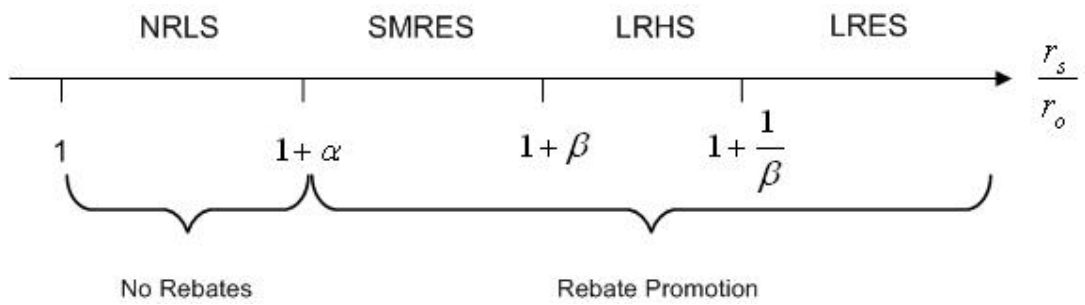
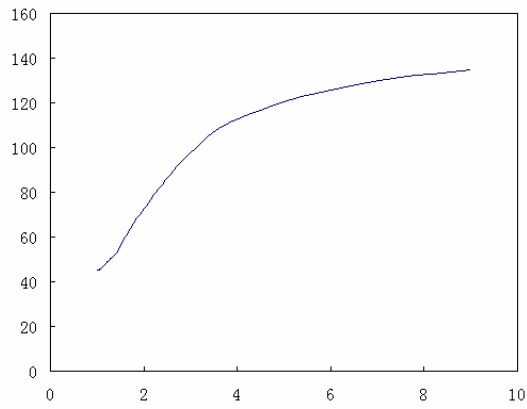
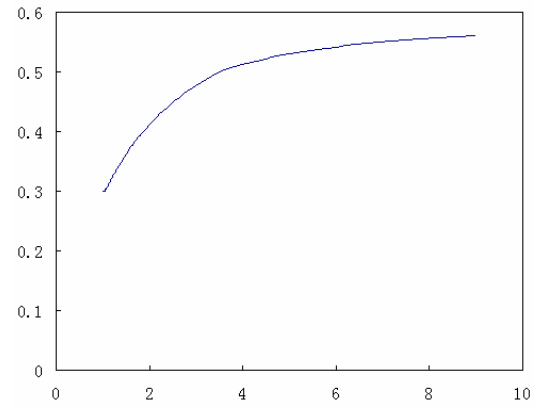


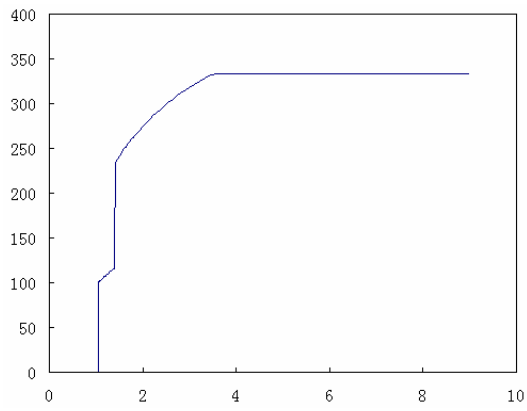
Figure 3.6. A Schematic Framework of Integrated Channel



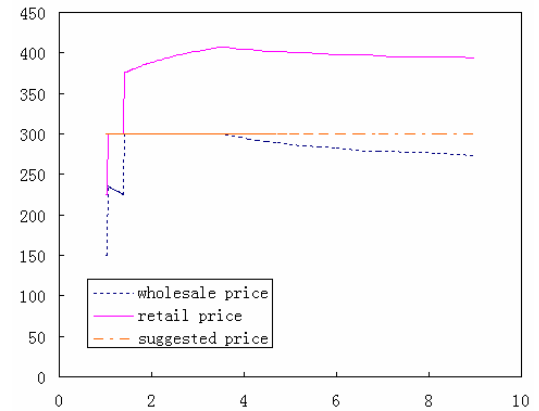
(a) \prod_m versus r_s/r_o



(b) D versus r_s/r_o



(c) R versus r_s/r_o



(d) P_s , P_r and w versus r_s/r_o

Figure 3.7. A Numerical Example

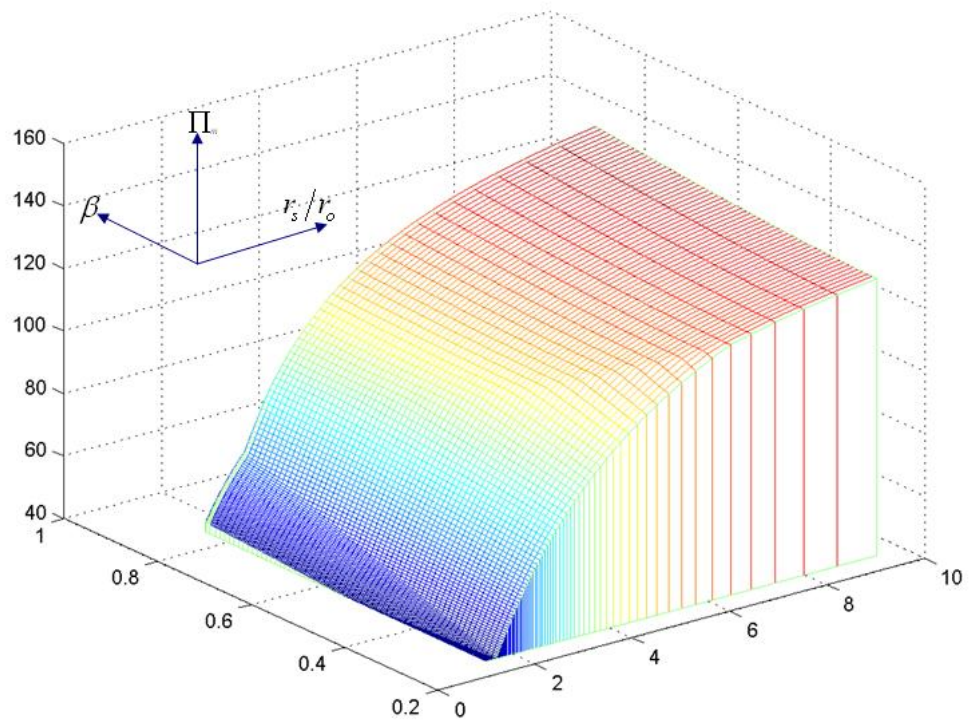


Figure 3.8. The Joint Effects of r_s/r_o and β on The Manufacturer's Profit

$w - r_o R$	$P_r - r_o R$	D	Π_r	Π_m	$\Pi_r + \Pi_m$
$\frac{bs}{2}$	$\frac{3bs}{4}$	$\frac{1}{4}$	$\frac{bs}{16}$	$\frac{bs}{8}$	$\frac{3bs}{16}$

Table 3.1. The Equilibrium Solution of Rebate Promotion Only without Slippage

w	P_r	R	D	Π_r	Π_m	$\Pi_r + \Pi_m$
∞	∞	∞	1	bs	∞	∞

Table 3.2. The Equilibrium Solution of Rebate Promotion Only with Slippage

	NRLS	NRES	SRES	MRHS	LRHS
	b1	c1	c2	d1	d2
Condition	$1 \leq \frac{r_s}{r_o} < \theta(\alpha, \beta)$	$\frac{r_s}{r_o} = 1$	$\theta(\alpha, \beta) \leq \frac{r_s}{r_o} < 1 + \beta$	$1 + \beta \leq \frac{r_s}{r_o} < 1 + \frac{1}{\beta}$	$\frac{r_s}{r_o} \geq 1 + \frac{1}{\beta}$
w	$\frac{bs}{2}$	$\frac{bs}{2}$	$\frac{r_s + 2(1 + \beta)r_o}{2(r_s + (1 + \beta)r_o)} bs$	bs	$(\frac{2 + \beta}{2 + 2\beta} + \frac{r_o}{2r_s}) bs$
R	0	0	$\frac{1 + \beta}{2(r_s + (1 + \beta)r_o)} bs$	$\frac{(1 + \beta)bs}{r_s + (1 + \beta)r_o}$	$\frac{bs}{r_s}$
P_s	bs	$\frac{(3 + \beta)bs}{4 + 2\beta}$	bs	bs	bs
P_r	$\frac{3}{4} bs$	$\frac{(3 + \beta)bs}{4 + 2\beta}$	bs	$\frac{3r_s + 2(1 + \beta)r_o}{2(r_s + (1 + \beta)r_o)} bs$	$(\frac{3(2 + \beta)}{4(1 + \beta)} + \frac{r_o}{4r_s}) bs$
D	$\frac{1 + \alpha}{4}$	$\frac{1 + \beta}{4 + 2\beta}$	$\frac{(1 + \beta)r_s}{2(r_s + (1 + \beta)r_o)}$	$\frac{(1 + \beta)r_s}{2(r_s + (1 + \beta)r_o)}$	$\frac{1}{4}(2 + \beta - (1 + \beta) \cdot \frac{r_o}{r_s})$
Π_r	$\frac{1 + \alpha}{16} bs$	$\frac{1 + \beta}{4(2 + \beta)^2} bs$	$\frac{(1 + \beta)r_s^2}{4(r_s + (1 + \beta)r_o)^2} bs$	$\frac{(1 + \beta)r_s^2}{4(r_s + (1 + \beta)r_o)^2} bs$	$\frac{1}{16(1 + \beta)}(2 + \beta - (1 + \beta) \frac{r_o}{r_s})^2 bs$
Π_m	$\frac{1 + \alpha}{8} bs$	$\frac{1 + \beta}{4(2 + \beta)} bs$	$\frac{(1 + \beta)r_s}{4(r_s + (1 + \beta)r_o)} bs$	$\frac{(1 + \beta)r_s^2}{2(r_s + (1 + \beta)r_o)^2} bs$	$\frac{1}{8(1 + \beta)}(2 + \beta - (1 + \beta) \frac{r_o}{r_s})^2 bs$
$\Pi_r + \Pi_m$	$\frac{3(1 + \alpha)}{16} bs$	$\frac{(1 + \beta)(3 + \beta)}{4(2 + \beta)^2} bs$	$\frac{(1 + \beta)r_s(2r_s + (1 + \beta)r_o)}{4(r_s + (1 + \beta)r_o)^2} bs$	$\frac{3(1 + \beta)r_s^2}{4(r_s + (1 + \beta)r_o)^2} bs$	$\frac{3}{16(1 + \beta)}(2 + \beta - (1 + \beta) \frac{r_o}{r_s})^2 bs$

Table 3.3. The Equilibrium Solution Sets of Reference-dependent Model³

³ At the corner points, i.e., when $r_s/r_o = 1$ with $\beta = 2\alpha/(1 - \alpha)$, $r_s/r_o = \theta(\alpha, \beta)$ and $r_s/r_o = 1 + \beta$, the equilibrium solution can be any combination of the two consecutive solution sets. For example, when

$r_s/r_o = 1 + \beta$, $w^* = \kappa \cdot \frac{r_s + 2(1 + \beta)r_o}{2(r_s + (1 + \beta)r_o)} bs + (1 - \kappa) \cdot bs$, where $\kappa \in [0, 1]$ is a fraction parameter.

	NRLS	MSHS	LRHS
Condition	$\frac{r_s}{r_o} \leq 1 + \alpha$	$1 + \alpha < \frac{r_s}{r_o} < 1 + \frac{1}{\alpha}$	$\frac{r_s}{r_o} \geq 1 + \frac{1}{\alpha}$
w	$\frac{bs}{2}$	bs	$(\frac{2+\alpha}{2+2\alpha} + \frac{r_o}{2r_s})bs$
R	0	$\frac{(1+\alpha)bs}{r_s + (1+\alpha)r_o}$	$\frac{bs}{r_s}$
P_s	bs	bs	bs
P_r	$\frac{3}{4}bs$	$\frac{3r_s + 2(1+\alpha)r_o}{2(r_s + (1+\alpha)r_o)}bs$	$(\frac{3(2+\alpha)}{4(1+\alpha)} + \frac{r_o}{4r_s})bs$
D	$\frac{1+\alpha}{4}$	$\frac{(1+\alpha)r_s}{2(r_s + (1+\alpha)r_o)}$	$\frac{1}{4}(2+\alpha - (1+\alpha) \cdot \frac{r_o}{r_s})$
Π_r	$\frac{1+\alpha}{16}bs$	$\frac{(1+\alpha)r_s^2}{4(r_s + (1+\alpha)r_o)^2}bs$	$\frac{1}{16(1+\alpha)}((2+\alpha - (1+\alpha)\frac{r_o}{r_s})^2bs$
Π_m	$\frac{1+\alpha}{8}bs$	$\frac{(1+\alpha)r_s^2}{2(r_s + (1+\alpha)r_o)^2} \cdot bs$	$\frac{1}{8(1+\alpha)}((2+\alpha - (1+\alpha)\frac{r_o}{r_s})^2bs$
$\Pi_r + \Pi_m$	$\frac{3(1+\alpha)}{16}bs$	$\frac{3(1+\alpha)r_s^2}{4(r_s + (1+\alpha)r_o)^2}bs$	$\frac{3}{16(1+\alpha)}((2+\alpha - (1+\alpha)\frac{r_o}{r_s})^2bs$

Table 3.4. The Equilibrium Solution Sets of Loss-neutral Model.⁴

⁴ At the corner point where $r_s/r_o = 1 + \alpha$, the equilibrium solution can be any combination of the two consecutive solution sets NRLS and MSHS.

	NRLS	SMRES	LRHS	LRES
	b1	c2-1	d	a2
Condi on	$\frac{r_s}{r_o} \leq 1 + \alpha$	$1 + \alpha < \frac{r_s}{r_o} < 1 + \beta$	$1 + \beta \leq \frac{r_s}{r_o} < 1 + \frac{1}{\beta}$	$\frac{r_s}{r_o} \geq 1 + \frac{1}{\beta}$
R	0	$\frac{bs}{2r_o}$	$\frac{bs}{r_s}$	$\frac{bs}{r_s}$
P_s	bs	bs	bs	bs
P_r	$\frac{bs}{2}$	bs	$(\frac{2+\beta}{2+2\beta} + \frac{r_o}{2r_s})bs$	bs
D	$\frac{1+\alpha}{2}$	$\frac{r_s}{2r_o}$	$\frac{1}{2}(2+\beta - (1+\beta)\frac{r_o}{r_s})$	1
Π_I	$\frac{(1+\alpha)}{4}bs$	$\frac{r_s}{4r_o}bs$	$\frac{1}{4(1+\beta)}(2+\beta - (1+\beta)\frac{r_o}{r_s})^2bs$	$(1 - \frac{r_o}{r_s})bs$

Table 3.5. The Equilibrium Solution Sets of Integrated Channel⁵

⁵ At the corner points, i.e., when $r_s/r_o = 1 + \alpha$ and $r_s/r_o = 1 + \beta$, the equilibrium solution can be any combination of the two consecutive solution sets

CHAPTER 4

COORDINATING CONTRACTS UNDER SALES PROMOTION

4.1. Brief Introduction

In a decentralized supply chain, channel members acting independently usually cannot achieve optimal performance of the supply chain due to the double marginalization problem (Spengler 1950). To improve supply chain performance, the coordination mechanism between upstream manufacturers and downstream retailers has been studied extensively in recent years. A contract is widely used between independent channel members to prevent a unilateral deviation from the set of globally optimal actions. This chapter examines contracting coordination issues under sales promotion in a supply chain. I build a three-way promotion loop in a supply chain by including all three types of sales promotions (consumer promotions, retailer promotions, and the trade dealings). Because sales promotion is indispensable in business, such three-way promotions frequently occur in practice. When the manufacturer launches a consumer promotion (such as rebates or coupons), the retailer usually performs multiple follow-up promotional tasks (such as in-store displays, feature advertising, etc) to leverage the manufacturer's consumer promotion and spur even more market demand.

Among various techniques of consumer promotions, mail-in rebates offered by the manufacturer can bypass the retailer and reach consumers directly. Usually the consumers are eligible to redeem the rebates as long as they purchase the required products. However, there has been a tendency in recent years to apply rebate promotions only to a limited set of retailers or even a single cooperative retailer. The

following rebate promotion is provided by Logitech and requires purchasing from Amazon.com only.

[Insert Figure 4.1. here]

Apparently, there are some cooperative promotions uniquely existing between these two supply chain partners. So a properly designed contract can certainly improve the performance of the sales promotion.

We consider the following two-echelon system in a single selling season (newsvendor-like) environment. The manufacturer chooses the rebate face value and the wholesale price, where both are observable and verifiable (i.e., contract instruments). Facing the manufacturer's rebate promotion, the retailer acting as a newsvendor chooses order quantity and promotional effort level before the selling season starts. However, due to the moral hazard problem (see p.27 on literature review for reference), the retailer's promotional effort cannot be written into contract, hence, cost sharing is not possible in contracting. As shown in previous literature, traditional contracts (i.e., wholesale, buy-back, revenue sharing, channel rebates) offered by the manufacturer are not sufficient to coordinate the supply chain, in part because these contracts fail to align the retailer's incentives (i.e., the order quantity and the promotional effort level). I show that a quantity discount contract with buy-back is sufficient to coordinate the supply chain with stochastic market demand.

To the best of my knowledge, this is one of the first papers in the coordination literature that specifically studies the manufacturer's rebate promotion and the retailer's promotional effort simultaneously in a general setting. The rest of the chapter is organized as follows. Model development (descriptions, assumptions and notations) are presented in section 4.2. Section 4.3 analyzes the deterministic demand model which is usually favored by the marketing literature. Section 4.4 analyzes the stochastic demand model which usually exists in operations literature. Section 4.5 contains the numerical examples. Finally, section 4.6 concludes this chapter. The flowchart below reveals a layout of the discussed contracts in the rest of the chapter.

[Insert Figure 4.2. here]

4.2. Model Development

This section describes the basic model setting. Given the short life cycle of many products (such as software and electronics) and the short-term nature of promotions, a one-period model is employed. This approach is consistent with the contracting literature where one-period models are widely used. This model may also serve as an approximation for time-restricted promotions for longer life-cycle products. In this model, the manufacturer can only sell products to final consumers through the retailer, i.e., no direct sales can occur.

The retail price is exogenously given by the market, i.e. the retailer cannot dictate the pricing. The exogenous retail price has been used previously in contracting literature (Taylor 2001, Krishnan et al. 2004, Netessine and Rudi 2000). This assumption can be justified under a sufficiently competitive market where retailers are price takers.

Alternatively, in the durable goods market, manufacturers may have control over the retail price by employing manufacturer suggested retail price (MSRP) or resale price maintenance (see Gurnani and Xu 2006 for explicit resale price maintenance discussion).

As two different types of sales promotion, rebate promotion and retailer promotional effort (see p.27 on literature review for reference) should have dissimilar effects on consumer demand. I assume that the rebate influences consumer demand in an additive fashion; however, the retailer's effort could influence demand in a multiplicative way, i.e.,

$$D(R, e) = (ar_s R + \xi)e$$

where

a is a scaling coefficient for the impact of the rebate promotion

r_s is the consumers' subjective redemption confidence at the time of purchase

R is the rebate face value, a decision variable of the manufacturer

e is the level of promotional effort, a decision variable of the retailer.

ξ is the demand given by a random variable with density $f(\xi)$ and distribution

$F(\xi)$.

This functional form of demand can be justified from the existing marketing literature (Neslin 2002), as retailer efforts (features and displays) have been shown to add significantly to the effectiveness of temporary price reduction. Even if there is no accompanying price discount, features and displays can increase sales dramatically (Inman et al. 1990). I believe effects of rebates on sales are similar to the effects of price discount, but in a delayed manner to the consumer. So the retailer's promotional effort is assume to be stochastically related to the demand, however effect of rebate promotion is deterministically related to the demand.

The manufacturer serves as the Stackelberg leader and the retailer serves as the follower. The manufacturer first sets a linear wholesale price w , announces the rebate face value R , and may offer the retailer a conditional ex post transfer payment T (such as channel rebate, buy-back credit, markdown allowance). Given the manufacturer's decisions, the retailer then places an order with the manufacturer and chooses the effort level before observing the state of underlying demand ξ . With symmetric information, the manufacturer and the retailer are risk neutral, and both seek to maximize their own profits. Neither the manufacturer nor the retailer incurs any goodwill penalty cost if inventories are insufficient to meet market demand, and I also assume the product has no salvage value.

Given the value of w and R from manufacturer, the retailer's profit function is

given as

$$\Pi_r(Q, e) = -w \cdot Q + p \cdot E[\min(Q, (ar_s R + \xi)e)] - V(e) + T,$$

where

Q is the order quantity, a decision variable of the retailer

w is the wholesale price, a decision variable of the manufacturer

p is the exogenous retail price

T is the conditional ex post transfer from the manufacturer to the retailer

$V(e)$ is the retailer's cost of exerting e level of effort, which is convex, increasing,

and continuously differentiable in e for any $e \geq 0$, with $V(0) = 0$.

Anticipating the retailer's proper profit maximizing reaction (Q^*, e^*) , the

manufacturer's profit function can be written as

$$\Pi_m(w, R) = (w - c) \cdot Q^*(w, R) - r_o R \cdot E[\min(Q^*(w, R), (ar_s R + \xi)e^*(w, R))] - T,$$

where

c is the manufacturer's unit production cost

r_o is consumers' objective probability of redeeming the rebate after the purchase.

The logical boundary conditions are listed below:

$$(A1) \quad 0 < c < w < p,$$

$$(A2) \quad R \geq 0, \quad w > c + r_o R, \quad e \geq 0,$$

$$(A3) \quad 0 < r_o \leq r_s \leq 1,$$

$$(A4) \quad f(\xi) > 0 \quad \text{for all } \xi > 0.$$

4.3. The Deterministic Demand Model

When the market demand is certain, the retailer's order quantity Q is equivalent to the market demand D . So the original problem reduces to a pricing and promotion problem to find the optimal demand. I use $u = E(\xi)$ represent a constant basic demand. The retailer's and the manufacturer's profit functions become

$$\Pi_r = (p - w)(a_r R + u)e - V(e),$$

$$\Pi_m = (w - c - r_o R)(a_r R + u)e,$$

respectively. For an integrated channel, the profit function follows as

$$\Pi_I(R, e) = (p - c - r_o R)D(R, e) - V(e) = (p - c - r_o R)(a_r R + u)e - V(e).$$

Since $V(e)$ is convex in e and $D(R, e)$ is linear in R and e , $\Pi_I(R, e)$ is strictly concave in both R and e . The above profit function is assumed to be well behaved such that a unique maximizing solution (R^*, e^*) exists with finite arguments, i.e., the Hessian matrix of $\Pi_I(R, e)$ is negative definite. For all $w > c + r_o R$, $\frac{\partial \Pi_r}{\partial e} < \frac{\partial \Pi_I}{\partial e}$.

So the retailer always exerts a lower than optimal promotional effort; hence, a simple wholesale price contract cannot coordinate the supply chain unless the retailer keeps all realized profit. It is easy to show that a contract of sharing rebate cost or sharing revenue does not coordinate either.

4.3.1. Quantity Discount Contract

Consider a quantity discount contract where the manufacturer offers the retailer a varying wholesale price according to the quantity ordered by the retailer. The larger the quantity ordered, the lower the wholesale price. From the demand function,

$D = (ar_s R + u)e$, there is a one-to-one relationship between e and D for any given value of R by the manufacturer. So the retailer's promotional effort level can be represented by a function of market demand and rebate face value, i.e.,

$e(D, R) = \frac{D}{ar_s R + u}$. For integrated channel, the profit function can be written as

$$\Pi_I(D, R) = (p - c - r_o R)D - V[e(D, R)].$$

Theorem 4.1. There exists an all-units quantity discount contract $w(D, R)$ that coordinates the supply chain.

(a) The quantity discount schedule is given by

$$w(D, R) = k_1 p + (1 - k_1)(c + r_o R) + \frac{k_2}{D} - k_1 \frac{V[e(D, R)]}{D},$$

where $k_1 \in (0, 1)$ and k_2 are profit-splitting parameters between the manufacturer and the retailer.

(b) The resulting profits to the manufacturer and the retailer are

$$\Pi_m = k_1 \Pi_I(D^*, R^*) + k_2 \quad \text{and} \quad \Pi_r = (1 - k_1) \Pi_I(D^*, R^*) - k_2, \text{ respectively.}$$

Under this specification, the wholesale price is jointly determined by the market

demand and the rebate value. Furthermore, as long as the demand elasticity of

$V[e(D, R)]$, i.e., $\frac{\partial V / \partial D}{V / D}$, is greater than one, $w(D)$ is indeed a quantity discount

schedule for any $k_2 \geq 0$. This property is intuitive: as the order quantity increase, the

promotional cost increases by a larger percentage. The property $\frac{\partial V / \partial D}{V / D} > 1$ holds for

most realistic promotional effort cost function. For example, assume $V(e) = b e^2 / 2$

(see Taylor, 2002), where $b > 0$ can be interpreted as the costliness of effort, we have

$$\frac{\partial V / \partial D}{V / D} = 2 > 1.$$

With this quantity discount contract, the retailer's profit function becomes

$$\Pi_r(D) = (p - w(D, R))D - V[e(D, R)] = (1 - k_1)\{(p - c - r_o R)D - V[e(D, R)]\} - k_2.$$

The retailer now faces the same decision problem as the one in integrated channel.

Thus, the profit maximizing behavior of the retailer is consistent with the channel

profit maximizing behavior, implying that the retailer will choose the

channel-optimizing order quantity as well as the channel-optimizing level of

promotional effort. The manufacturer's profit is also linearly related to the channel

profit, implying that the manufacturer will choose the channel-optimizing rebate face

value contingent that D^* is chosen by the retailer. Therefore, this quantity discount

scheme $w(D, R)$ can coordinate the supply chain by inducing the retailer to order

more and resulting in exerting the optimal promotional effort. The intuition behind

this is that the discount scheme has been designed so that the retailer's marginal cost

is equal to its marginal revenue p at the point D^* , i.e.,

$$\frac{\partial}{\partial D}(w(D, R)D + V(D, R)) = k_1 p + (1 - k_1)(c + r_o R + \frac{\partial V[e(D, R)]}{\partial D}), \text{ where}$$

$$\frac{\partial V[e(D, R)]}{\partial D} \Big|_{D=D^*} = p - c - r_o R. \text{ The discount scheme indicates that supply chain}$$

coordination involves a sharing of rebate cost, i.e., the retailer needs to share

$100(1 - k_1)\%$ of each redeemed rebate.

The quantity discount schedule in Theorem 4.1 is a continuous one. A coordinating

discrete discount schedule can also be developed. Previous theoretic results (Weng

1995) already predict that one price break at D^* is sufficiently enough to coordinate the supply chain under a deterministic model. With the assumption that a discrete discount policy would appeal to the retailer only if its profit will increase by no smaller than $(1 + \lambda) \times 100\%$, the following corollary explains the coordinating mechanism with a discrete schedule.

Theorem 4.2. There exists a discrete quantity discount contract that coordinates the supply chain.

(a) The quantity discount schedule is given by $\{(w_1, R_1), (w_2, R_2)\}$ with price break at D^* such that

$w_2 = w_2(w_1, R_1, R_2) = p - \frac{1}{D^*} \left((1 + \lambda) \left((p - w_1) D_1 - V[e(D_1, R_1)] \right) + V[e(D^*, R_2)] \right)$, where D_1 is the solution of the equation $\left. \frac{\partial V[e(D, R_1)]}{\partial D} \right|_{D=D_1} = p - w_1$.

(b) The resulting profits to the manufacturer and retailer are

$$\Pi_m^d = \Pi_I(D^*, R^*) - (1 + \lambda) \left((p - w_1) D_1 - V[e(D_1, R_1)] \right) \text{ and}$$

$$\Pi_r^d = (1 + \lambda) \left((p - w_1) D_1 - V[e(D_1, R_1)] \right), \text{ respectively.}$$

The legality issue of proposed quantity discount contracts can be justified by arguing a cost savings by producing for a large order size (Jeuland and Shugan 1983). Hence, as long as the promotional cost structures of different retailers are similar, then retailers will not pay different prices for the same order quantities. Thus, my proposed contracts are legal under Robinson Patman Act, which prohibits offering different terms to different retailers in the same retailer class. However, if the retailers have

significantly different promotional cost structures, the proposed discount schemes may not be directly applicable because different retailer will end up paying a different unit wholesale price.

4.3.2. Two-part Tariff Contract

In practice, quantity discount are often implemented as a set of two-part tariff contract, especially in the extent of franchised chains. A typical set of two-part tariff contract involves a fixed payment and per-unit charges, i.e. the retailer pays an initial fee F for buying any amount of the product plus a constant wholesale price w . The following two-part tariff contract achieves channel coordination,

$$F(R) = k_1 \left[(p - c - r_o R) D(e^*, R) - V(e^*) \right] + k_2,$$

$$w(R) = c + r_o R$$

where $k_1 \in (0,1)$ and k_2 are profit-splitting parameters,

e^* is the optimal promotional effort in the integrated channel.

The cost of the rebate has been shared in the fixed initial fee by the retailer. The main idea behind this contract is that the retailer keeps all realized revenues such that it will exert the correct amount of promotional effort. The retailer's profit function is

$$\begin{aligned} \Pi_r &= (p - w(R)) D(e, R) - V(e) - F(R) \\ &= (p - c - r_o R) D(e, R) - V(e) - k_1 \left[(p - c - r_o R) D(e^*, R) - V(e^*) \right] - k_2. \end{aligned}$$

Since the above function is linearly related to the integrated channel profits, the retailer's profit maximizing is equivalent to the channel's maximizing problem. Hence, the retailer will choose the channel optimal promotional effort level. For the

manufacturer,

$$\Pi_m = (w - c - r_o R)D(e, R) + F = k_1 \left[(p - c - r_o R)D(e^*, R) - V(e^*) \right] + k_2.$$

So as long as the retailer chooses the optimal decisions (i.e., e^*), the manufacturer's profit is also maximized. So the supply chain achieves coordination, and the split of profits between the manufacturer and the retailer is exactly the same as the quantity discount contract.

The proposed continuous quantity discount contract and the two-part tariff contract also function properly in situations where the retailer (like Wal-Mart) has more bargaining power, and acts as a leader by offering a contract to the manufacturer. The same quantity discount scheme still coordinate the supply chain, and the two-part tariff contract can also work after adjusting the fixed fee to

$$F(e) = k_1 \left[(p - c - r_o R)D(e, R^*) - V(e) \right] + k_2.$$

4.4. The Stochastic Demand Model

When the market demand is stochastic, we have the following demand function

$$D(R, e) = (a_r R + \xi)e.$$

Let the density function and distribution function of $D(R, e)$ be $\phi(y | R, e)$ and

$\Phi(y | R, e)$, respectively. From the distribution of ξ , it is straightforward to show that

$$\begin{aligned} \phi(y | R, e) &= \frac{1}{e} f\left(\frac{y}{e} - a_r R\right), \text{ and} \\ \Phi(y | R, e) &= \int_{a_r R}^{\frac{y}{e}} \frac{1}{e} f\left(\frac{y}{e} - a_r R\right) dy = F\left(\frac{y}{e} - a_r R\right). \end{aligned}$$

As a benchmark, suppose that the manufacturer owns the retailer, i.e., the case of integrated channel. For the integrated channel, the manufacturer faces a newsvendor problem with three decision actions: the production quantity Q , the level of promotional effort e , and the rebate face value R . Let $S(Q, R, e)$ be expected sales, $E[\min(Q, D)]$,

$$\begin{aligned}
S(Q, R, e) &= E[\min(Q, (ar_s R + \xi)e)] \\
&= Q(1 - \Phi(Q | R, e)) + \int_{ar_s R}^Q y \phi(y | R, e) dy \\
&= Q - \int_{ar_s R}^Q \Phi(y | R, e) dy \\
&= Q - \int_{ar_s R}^Q F\left(\frac{y}{e} - ar_s R\right) dy \\
&= Q - e \int_0^{\frac{Q}{e} - ar_s R} F(y) dy.
\end{aligned}$$

4.4.1. Centralized Supply Chain

As a benchmark, suppose the manufacturer owns the retailer. The profit function of the integrated channel is

$$\begin{aligned}
\Pi_I(Q, R, e) &= -cQ + (p - r_o R) \cdot E[\min(Q, (ar_s R + \xi)e)] - V(e) \\
&= (p - r_o R - c)Q - (p - r_o R)e \int_0^{\frac{Q}{e} - ar_s R} F(y) dy - V(e).
\end{aligned}$$

Lemma 4.1. $\Pi_I(Q, R, e)$ is strictly concave in Q , R and e .

Proof: See Appendix

We assume that the function $V(e)$ and the demand distribution are chosen such that the channel profit function Π_I is well-behaved, i.e., the existence of an optimal solution (Q^I, R^I, e^I) is assured in the feasible area (i.e., satisfying all assumptions A1-A4). The optimal solution should satisfy the following first-order conditions:

$$Q^I = (ar_s R^I + \bar{Q}^I) e^I \quad (4.1)$$

$$R^I = \frac{p-c}{r_o} - \frac{Q^I - e^I \int_0^{\bar{Q}} F(y) dy}{ar_s e^I} \quad (4.2)$$

$$\frac{\partial}{\partial e} V(e) \big|_{e=e^I} = (p - r_o R^I) \left(\frac{Q^I}{e^I} \cdot F(\bar{Q}^I) - \int_0^{\bar{Q}^I} F(y) dy \right) \quad (4.3)$$

$$\text{where } \bar{Q}^I = F^{-1} \left(\frac{p - r_o R^I - c}{p - r_o R^I} \right).$$

By embedding (4.1) into (4.2) and (4.3), we can get

$$R^I = \frac{p-c}{2r_o} - \frac{\bar{Q}^I - \int_0^{\bar{Q}^I} F(y) dy}{2ar_s} \quad (4.4)$$

$$\begin{aligned} \frac{\partial}{\partial e} V(e) \big|_{e=e^I} &= (p - r_o R^I) \left((ar_s R^I + \bar{Q}^I) F(\bar{Q}^I) - \int_0^{\bar{Q}^I} F(y) dy \right) \\ &= ar_s (p - r_o R^I - c) R^I + (p - r_o R^I) \int_0^{\bar{Q}^I} y dF(y) \end{aligned} \quad (4.5)$$

So R^I can be obtained by solving (4.4)¹. Note that the optimal rebate value is not related to the cost structure of $V(e)$. With R^I , we can get e^I and Q^I sequentially from (4.5) and (4.1). Let Π_I denote the corresponding maximum profits for the integrated channel.

$$\begin{aligned} \Pi_I(Q^I, R^I, e^I) &= (p - r_o R^I - c) Q^I - (p - r_o R^I) e^I \int_0^{\frac{Q^I - ar_s R^I}{e^I}} F(y) dy - V(e^I) \\ &= (p - r_o R^I - c) Q^I + e^I \frac{\partial}{\partial e} V(e) \big|_{e=e^I} - (p - r_o R^I) Q^I F(\bar{Q}^I) - V(e^I) \\ &= e^I \frac{\partial}{\partial e} V(e) \big|_{e=e^I} - V(e^I). \end{aligned}$$

The above profit function is in the same form as the one in Taylor (2002), which does not include rebate promotions.

¹ Multiple complex solutions of equation 4.4 exist, depending on the demand distribution. For example, if the basic demand is uniformly distributed, equation 4.4 is a cubic function, which has at least one real number root. For most of the realistic parameter settings, equation 4.4 has only one solution falling in the feasible area. In particular, if a feasible solution does not exist, the optimal value of the rebate is zero.

Theorem 4.3. For different rebate value R , the maximum profit of the centralized supply chain strictly increases with the optimal promotional effort level; however, may not necessarily increase with the optimal production quantity.

By $\Pi_I(Q^I(R), R, e^I(R)) = e^I(R) \frac{\partial}{\partial e} V(e)|_{e=e^I(R)} - V(e^I(R))$, it is easy to show that $\Pi_I(Q^I(R), R, e^I(R))$ is strictly increase with $e^I(R)$ because of the strict convexity of $V(e)$. So the maximum supply chain profit strictly increases with the optimal $e^I(R)$ without regarding to the value of R . However, in an example with $p=10$, $c=2$, $r_s=0.9$, $r_o=0.6$, $a=0.1$, $b=1$ by assuming $V(e) = be^2/2$ and $\xi \square Uniform(0,1)$, it can be easily verify that when $R=4$, $Q^I(R)=4.47$ and $\Pi_I(R)=8.32$; when $R=5$, $Q^I(R)=4.70$ and $\Pi_I(R)=8.14$. Therefore, there exist examples where maximum supply chain profit decreases with the optimal $Q^I(R)$. It also implies the optimal production quantity may not necessarily increase with the optimal promotional effort level for different rebate values, although for any fixed R it is true.

4.4.2. Buy-back Only Contract

In a decentralized supply chain, the upstream manufacturer uses the downstream retailer to reach consumers. Since the decision makings of both channel members are independent, the classical contract offered by the manufacturer certainly causes incentive distortions to the retailer. A coordinating contract must align both members incentives and the terms offered by the manufacturer can induce the retailer to choose the optimal promotional effort e^I and the order quantity Q^I . Given the assumption

that the retailer's promotional level is not contractible, a possible solution can only contract on order quantity or market sales.

Under a wholesale price contract, the retailer's profit function is

$$\Pi_r^w(Q, e) = -wQ + p \cdot S(Q, R, e) - V(e) = (p - w)Q - pe \int_0^{\frac{Q}{e} - ar_s R} F(y) dy - V(e).$$

For any given order quantity Q and rebate value R , the following first-order condition of promotional effort is necessary for coordination (but not sufficient),

$$\frac{\partial \Pi_r^w(Q, e)}{\partial e} = p \left(\frac{Q}{e} \cdot F\left(\frac{Q}{e} - ar_s R\right) - \int_0^{\frac{Q}{e} - ar_s R} F(y) dy \right) - \frac{\partial}{\partial e} V(e) = 0.$$

However it is greater than $\partial \Pi_f(Q, e, R) / \partial e$ for any positive rebate value. As a result, the retailer exerts a higher than optimal effort. Therefore a wholesale price contract does not coordinate the supply chain.

Next, consider a buy-back contract where the manufacturer charges the retailer a wholesale price w but pays the retailer credit b per unit remaining at the end of the season. The retailer's profit function is

$$\begin{aligned} \Pi_r^b(Q, e) &= -wQ + p \cdot S(Q, R, e) + b(Q - S(Q, R, e)) - V(e) \\ &= (p - w)Q - (p - b)e \int_0^{\frac{Q}{e} - ar_s R} F(y) dy - V(e). \end{aligned}$$

For any given order quantity Q and rebate value R , the retailer chooses the following promotional effort to maximize its profit,

$$\frac{\partial \Pi_r^b(Q, e)}{\partial e} = (p - b) \left(\frac{Q}{e} \cdot F\left(\frac{Q}{e} - ar_s R\right) - \int_0^{\frac{Q}{e} - ar_s R} F(y) dy \right) - \frac{\partial}{\partial e} V(e) = 0.$$

Compared to the channel profit function, i.e.,

$$\frac{\partial \Pi_r(Q, R, e)}{\partial e} = (p - r_o R) \left(\frac{Q}{e} \cdot F\left(\frac{Q}{e} - ar_s R\right) - \int_0^{\frac{Q}{e} - ar_s R} F(y) dy \right) - \frac{\partial}{\partial e} V(e) = 0,$$

the retailer's promotion effort function is not distorted with $b = r_o R$. Via buy-back, the retailer's self-interest promotional decision is successfully aligned together with the channel incentives. Note that although the effort decision is no longer distorted with buy-back, the order quantity is still distorted unless the manufacturer is willing to earn a non-positive profit by only charging the marginal cost. On condition that the retailer chooses a lower than optimal order quantity, the retailer's actual promotional effort cannot reaches the optimal level. For any wholesale price w and rebate value R given by the manufacturer, let $e^b(w, R)$ and $Q^b(w, R)$ denote the retailer's optimal effort level and order quantity. From the first-order conditions, we can obtain

$e^b(w, R)$ and $Q^b(w, R)$ from equation (4.6) and (4.7), respectively,

$$\begin{aligned} \frac{\partial}{\partial e} V(e) \big|_{e=e^b(w, R)} &= (p - r_o R) \left((ar_s R + \bar{Q}^b(w, R)) F(\bar{Q}^b(w, R)) - \int_0^{\bar{Q}^b(w, R)} F(y) dy \right) \\ &= a(p - w)r_s R + (p - r_o R) \int_0^{\bar{Q}^b(w, R)} y dF(y), \end{aligned} \quad (4.6)$$

$$Q^b(w, R) = (ar_s R + \bar{Q}^b(w, R)) e^b(w, R), \text{ where } \bar{Q}^b(w, R) = F^{-1}\left(\frac{p - w}{p - r_o R}\right). \quad (4.7)$$

And, the resulting retailer's profit is

$$\Pi_r^b(Q^b(w, R), e^b(w, R)) = e^b(w, R) \frac{\partial}{\partial e} V(e) \big|_{e=e^b(w, R)} - V(e^b(w, R)). \quad (4.8)$$

With the retailer's effort level $e^b(w, R)$ and order quantity $Q^b(w, R)$, the manufacturer's profit function can be written as

$$\begin{aligned} \Pi_m^b(w, R) &= (w - c)Q^b(w, R) - r_o RS(Q^b(w, R), R, e^b(w, R)) - b(Q^b(w, R) - S(Q^b(w, R), R, e^b(w, R))) \\ &= (w - c - r_o R)Q^b(w, R) \end{aligned}$$

We let (w^b, R^b) denote the manufacturer's optimal pair that maximizes the above

profit function, and Π_m^b is the corresponding manufacturer's maximum profit. With (w^b, R^b) chosen by the manufacturer, the retailer's maximum profit Π_r^b can be obtained.

Theorem 4.4. Suppose $V(e) = be^k$ ($k \geq 2$), where $b > 0$ can be interpreted as the costliness of effort. The efficiency of the buy-back contract $(\Pi_m^b + \Pi_r^b)/\Pi_l$ and the manufacturer's optimal decisions on (w, R) is not influenced by the value of b .

Proof: See Appendix

It also can be shown that identical results for the parameter b also hold under a wholesale price contract.

The following lemma also holds for any given rebate face value.

Lemma 4.2. For any given rebate value R , $Q^b(w^b(R), R)$ is strictly less than $Q^l(R)$.

Proof: See Appendix

Lemma 4.2 characterize the optimal quantity decision for any given rebate face value, which serves as a base for the discrete quantity discount in the following section.

In this section, I show that a buy-back contract by itself is not enough to coordinate a supply chain. However, a buy-back contract does not distort the retailer's promotional decision. Based on this, two coordinating contracts are proposed in the following.

4.4.3. Continuous Quantity Discount Contract with Buy-back

Inspired by Cachon and Lariviere (2005), where the authors find that a continuous quantity discount contract can coordinate the supply chain with the retailer's promotional effort, I propose a continuous quantity discount schedule with buy-back contract that can coordinate a supply chain.

Theorem 4.5. There exists a continuous all-unit quantity discount contract with buy-back $(w(Q,R),b)$ that coordinates the supply chain.

(a) The quantity discount schedule is given by

$$w(Q,R) = k_1(p - r_o R) \frac{S(Q,R,e^I)}{Q} + r_o R + (1 - k_1)c + \frac{k_2}{Q}$$

and the buy-back credit is given by $b(R) = r_o R$.

(b) The resulting profits to the manufacturer and the retailer are

$\Pi_m = k_1 \Pi_I(Q^I, R^I, e^I) + k_1 V(e^I) + k_2$ and $\Pi_r = (1 - k_1) \Pi_I(Q^I, R^I, e^I) - k_1 V(e^I) - k_2$, respectively.

where e^I is the optimal effort level in the integrated channel

$k_1 \in (0,1)$ and k_2 are profit-splitting parameters

Proof: See Appendix.

With this quantity discount contract with buy-back, the retailer keeps all the revenues such that it will choose the optimal promotional effort as in the integrated channel.

Coordination occurs because the retailer's effort decision is not distorted, and its order quantity decision is adjusted contingent that e^I is chosen; subsequently, the manufacturer's rebate value decision is adjusted contingent that e^I and Q^I are

chosen. Moreover, for a special case, if $k_2 = -k_1 V(e')$, the profit can be shared exactly between the manufacturer and the retailer with a percentage rate k_1 .

4.4.4. Discrete Quantity Discount Contract with Buy-back

The continuous quantity discount contract with buy-back achieves coordination because the retailer's expected profit is proportional to the supply chain's expected profit under the proposed contract. As long as the promotional cost structures of different retailers are similar, my proposed contract is legal under Robinson Patman Act.

However, although continuous discount schedule is popular in academia (See Jeuland and Shugan 1983, Cachon and Lariviere 2005 for examples), the infinite number of price breaks associated with continuous discount is definitely not welcomed by the managers in practice. In a field study by Munson and Rosenblatt (1998), the authors say "none of the participants have seen continuous schedules in practice", and they suggest researchers should especially "shy away from continuous discount schedules".

So I create a discrete discount schedule with one price break, and then test whether the manufacturer can design a quantity discount contract with buy-back which can sufficiently coordinate the supply chain.

In my proposed contract, the manufacturer offers a quantity discount schedule with only one price breaks at Q_d , i.e., the manufacturer offers two pairs of wholesale price

and rebate value as follows: if the retailer's order quantity Q is less than Q_d , the manufacturer charges a basic wholesale price w_1 and announce a rebate promotion with R_1 ; if Q is greater than or equal to Q_d , the manufacturer charges a discounted wholesale price w_2 (where $w_2 < w_1$) and announce a rebate promotion with R_2 . After the selling season ends, the retailer can return the leftovers to the manufacturer with $b = r_o R$. The objective of the manufacturer is to offer a quantity discount schedule such that the retailer will always order at the level $w = w_2$, which is equivalent to maximizing the manufacturer's profit at $w = w_2$ subject to the constraint that the retailer's maximum profit earned at the level $w = w_2$ is no smaller than its profit earned by ordering at the $w = w_1$ level.

The retailer would be willing to order at a discounted wholesale price w_2 only if its profit would not decrease by ordering $Q \geq Q^d$. The retailer's profit function can be given by,

$$\begin{aligned}\Pi_r(Q, e | Q \geq Q^d) &= -w_2 Q + pS(Q, R, e) + b(Q - S(Q, R, e)) - V(e) \\ &= (p - w_2)Q - (p - r_o R)e \int_0^{\frac{Q}{e} - ar_s R} F(y) dy - V(e).\end{aligned}$$

Let Q' be the unconstrained optimal order quantity, i.e.,

$$Q'(w_2, e(w_2, R_2), R_2) = \left(F^{-1}\left(\frac{p - w_2}{p - r_o R_2}\right) + ar_s R_2 \right) e(w_2, R_2).$$

Because the retailer's profit function is piecewise concave in order quantity, the retailer will choose either Q' or Q_d . Directly from Lemma 4.2, we have $Q'(w_2, R_2) < Q'(R_2)$. Hence, to induce the retailer to choose the same optimal order quantity Q' in the integrated channel, we must have Q' as the price breakpoint, i.e. $Q_d = Q'$, such that the retailer's profit is

maximized at $Q = Q'$.

I assume that a quantity discount policy would appeal to the retailer only if its profit will increase by no smaller than $(1 + \lambda) \times 100\%$. I propose the following contract.

Theorem 4.6. There exists a discrete quantity discount contract with buy-back that coordinates the supply chain.

(a) The quantity discount schedule is given by $\{(w_1, R_1), (w_2, R_2)\}$ with price break at Q' such that

$$w_2 = w_2(w_1, R_1, R_2) = p - \frac{1}{Q'} \left((1 + \lambda) \Pi_r^b(w_1, R_1) + (p - r_o R_2) e^d(R_2) \int_0^{\frac{Q'}{e^d(R_2)} - ar_s R_2} F(y) dy + V(e^d(R_2)) \right),$$

where $\Pi_r^b(w_1, R_1)$ and $e^d(R_2)$ are obtained from (4.8) and (4.9) in appendix, respectively.

(b) The buy-back credit is given by $b(R) = r_o R$ accordingly.

(c) The resulting profits to the manufacturer and retailer are

$$\Pi_m^d = \Pi_I - (1 + \lambda) \Pi_r^b(w_1, R_1) \text{ and } \Pi_r^d = (1 + \lambda) \Pi_r^b(w_1, R_1), \text{ respectively.}$$

Proof: See Appendix.

Note that under coordination, the arbitrary profit splitting can be achieved by choosing a sufficiently large w_1 (which results in $\Pi_r^d = (1 + \lambda) \Pi_r^b(w_1, R_1) = 0$) or by choosing a sufficiently large λ (which results in $\Pi_m^d = \Pi_I - (1 + \lambda) \Pi_r^b(w_1, R_1) = 0$). It should be pointed out that in Theorem 4.6, the manufacturer does not need to maximize its own profit at the level $w = w_1$ as long as it can induce the retailer to order at a discounted wholesale price level $w = w_2$ by offering a properly designed

contract. In an extreme case, the manufacture can keep almost all gains by passing onto the retailer only a “just enough” portion to induce ordering at a discounted price level. However, in Lim and Ho (2006), the authors show experimentally that the retailer will not always order at the cheapest wholesale price level designed by the manufacturer. In a quantity discount schedule with one price break, the retailer has the possibility of ordering at the level $w = w_1$ because of some non-pecuniary reasons. Hence, the manufacturer has the incentive to maximize its own profits by decentralized decision makings at the level $w = w_1$, the following theorem illustrates the corresponding results.

Theorem 4.7. For any sufficiently small λ , there exists a discrete quantity discount policy that maximizes the manufacturer’s profit.

(a) The quantity discount schedule is given by $\{(w^b, R^b), (w_2, R^I)\}$ with price break at

Q^I , where $w_2 = r_o R^I + c + \frac{\Pi_I - (1 + \lambda)\Pi_r^b}{Q^I}$, and the buy-back credit is given by

$b(R) = r_o R$ accordingly.

(b) The necessary condition is $\lambda \leq \frac{\Pi_I - \Pi_m^b - \Pi_r^b}{\Pi_r^b}$.

(c) The manufacturer’s profit increased by $\left(\frac{\Pi_I - (1 + \lambda)\Pi_r^b}{\Pi_m^b} - 1 \right) \cdot 100\%$, and the

retailer’s profit increased by $\lambda \cdot 100\%$.

(d) The manufacturer’s profit share will increase if $\lambda < \frac{\Pi_I}{\Pi_m^b + \Pi_r^b} - 1$

At the basic price level $w = w_1$, the manufacturer will announce $w_1 = w^b$ and $R_1 = R^b$ to maximize its profit. The corresponding profits for the retailer and the manufacturer are Π_r^b and Π_m^b , respectively, as denoted in the buy-back contract. At the discounted price level w_2 , the manufacturer maximizes its profit by choosing $R_2 = R^I$. Then, from Theorem 4.6, we have

$$w_2 = w_2(w^b, R^b, R^I) = p - \frac{1}{Q^I} \left((1 + \lambda) \Pi_r^b(w^b, R^b) + (p - r_o R^I) e^d(R^I) \int_0^{\frac{Q^I}{e^d(R^I)} - a_r R^I} F(y) dy + V(e^d(R^I)) \right)$$

$$\Rightarrow w_2 = p - \frac{1}{Q^I} \left((1 + \lambda) \Pi_r^b + (p - r_o R^I) e^I \int_0^{\bar{Q}} F(y) dy + V(e^I) \right) = r_o R^I + c + \frac{\Pi_I - (1 + \lambda) \Pi_r^b}{Q^I}.$$

So the manufacturer's maximum profit is given by

$$\Pi_m^d = (w_2 - r_o R_2 - c) Q^I = \Pi_I - (1 + \lambda) \Pi_r^b.$$

However, this achieved manufacturer's profit should not be less than Π_m^b ; otherwise, the manufacturer as a contract provider would not be willing to offer such a contract.

The manufacturer's profit should satisfy the following condition

$$\Pi_m^d = \Pi_I - (1 + \lambda) \Pi_r^b \geq \Pi_m^b \Rightarrow \lambda \leq \frac{\Pi_I - \Pi_m^b - \Pi_r^b}{\Pi_r^b}$$

So as long as the retailer is not too aggressive, i.e. its profit increasing rate is not greater than $(\Pi_I - \Pi_m^b - \Pi_r^b) / \Pi_r^b$, there always exists a cooperative way to coordinate the supply chain.

Although it is a special case of Theorem 4.6, Theorem 4.7 is more realistic for the situation when the retailer is sensitive to non-pecuniary reasons. It can be easily seen that if the manufacturer charges a wholesale price w_1 equal to the retail price p , the retailer has to place an order at the level w_2 . However, the retailer might reject the

contract because of the unreasonable wholesale price setting. So the contract by Theorem 4.7 is less likely to be rejected by the retailer. On the other hand, as the numerical example in next section shows, the manufacturer does not need to discount $w_1 = w_b$ significantly to achieve supply chain coordination,

Please note that the existence of Theorem 4.7. needs to satisfies a requirement that $Q^b < Q^I$, which cannot be obtained directly from Lemma 4.2. (To be proved).

4.5. Numerical Studies

In this section, I use numerical examples to gain more insights of coordinating contracts. This base parameter set is tested: $p=10$, $c=2$, $r_s=0.9$, $r_o=0.6$, $a=0.1$, $b=1$ with the assumption $V(e)=be^2/2$ and $\xi \square Uniform(0,1)$. All the following results are obtained by modifying the base set one parameter at a time.

In Theorem 4.5 and 4.6, I propose two quantity discount (continuous/discrete) contracts with buy-back that can coordinate the supply chain. Two measurements are used to test the performance of a contract: the efficiency of the contract,

$(\Pi_m^b + \Pi_r^b)/\Pi_I$, and the manufacturer's profit share, $\Pi_m^b/(\Pi_m^b + \Pi_r^b)$.

[Insert Figure 4.3. here]

The above numerical results demonstrate that coordination achieved by the proposed contracts can improve the supply chain performance significantly. The maximum efficiency of the buy-back contract and wholesale price contract is around 74% . Furthermore, the efficiency of the buy-back contract is very robust to parameter changes because the retailer's optimal decision on promotional effort is not distorted through a buy-back credit $b = r_o R$. However, the efficiency of the wholesale price contract varies as parameter changes due to the fact that neither of retailer's decisions have been corrected. As Theorem 4.4. shows, the costliness of effort (b) does not influence the performance of both contracts.

[Insert Figure 4.4. here]

As the figures in 4.4 demonstrate, when the impact of rebate promotion on market demand is very small, i.e., when parameter a is sufficiently small, the manufacturer will not issue rebates (it also holds under integrated channel). In this situation, the buy-back contract becomes wholesale price contract because $b = r_o R = 0$. Furthermore, The retail price parameter p influences the supply chain in a similar way as the parameter a does because retail price restricts the upper bound of the rebate value. Hence, when p is sufficiently small, the impact of a tiny rebate on market demand is very small and the manufacturer chooses not to issue rebates. Interestingly, there exists a special relatively small segment for parameters a and p under which the manufacturer chooses to issue rebates under coordinated channel or uncoordinated

channel with buyback only contract, but not with wholesale price contract. This implies that the manufacturer with wholesale price only is less likely to offer rebate promotions. The above figures also show that when rebate benefits are significant, i.e., a larger rebate impact (a), a higher retail price (p), and a higher slippage rate (r_s/r_o), the manufacturer's profit share will increase. For the buy-back contract, the ratio of optimal rebate value R^b/R^l sticks around 1.04 with very small varying; however, for the wholesale price contract, R^w/R^l increases with potential rebate benefits. Moreover, because the rebates help the manufacturer by increasing the order quantity from the retailer, contrary to general belief, the numerical example suggests that even if all rebates are redeemed (i.e., by letting $r_o = 1$), the manufacturer would still prefer providing rebates to consumers as long as neither a nor p is sufficiently small.

[Insert Figure 4.5. here]

The above two figures report the sensitivity of the discrete quantity discount contract with buy-back based on Theorem 4.7. Without doubt, as the retailer's profit reservation parameter λ increases, the manufacturer's profit share decreases accordingly. The top figure implies that the proposed contract can achieve supply chain coordination with arbitrary profit splitting, which is determined by channel members' relative bargaining power. Furthermore, the manufacturer does not need to discount the wholesale price significantly to achieve arbitrary profit splitting. The bottom figure shows the optimal unit back-back credit ($b = r_o R$) decreases with r_o for

both buy-back only contract and quantity discount contract with buy-back. It is intuitive that the smaller the probability of redeeming r_o , the larger the optimal rebate value R , so the numerical example implies that the optimal R increases at a higher rate compared to the decreasing rate of r_o .

4.6. Conclusions

In this chapter, I study a three-way sales promotion that is very popular in practice. Under the situation where the manufacturer can influence the consumer demand directly through mail-in rebates while the retailer simultaneously exerts promotional effort to further spur demand, I find that trade dealing via quantity discounts plus buy-back is sufficient to coordinate the supply chain. For the deterministic demand model, even a quantity discount contract itself achieves coordination. The results show that the performance of a simple wholesale price contract under sales promotion is not robust and also far from a perfect situation. A successful coordination can result in significant supply chain improvement, which leads the retailer to order more and exert higher promotional effort, however, a coordination does not necessarily lead the manufacturer to issue larger-ticketed rebate.

Hopefully, some of the results in this chapter can provide insights for researchers who would like to further analyze the coordination issue involving consumer mail-in rebates. One direct extension is to change the rebate and effort-dependent demand model to a rebate and price-dependent one. In this case, the retailer can choose the

retail price instead of assuming an exogenously given one. I believe analogous contracts can coordinate the supply chain if $D = ar_s R + bp + \xi$. An interesting different game would be to adopt the following timeline of decisions: first the manufacturer chooses a wholesale price and the retailer determines the order quantity, then the manufacturer announces the rebate promotion value and the retailer determines the promotional effort level. Based on this, I can investigate the potential that the manufacturer uses rebate promotion to coordinate a multiple retailers via structure coordination. In this scenario, the manufacturer issues rebates which are only valid at a certain flagship retailer store. Clearly, this rebate promotion will influence the pricing of other retailers during a relatively long promotion period (consider Google Checkout discount as an example). Hence, the manufacturer can use this partial forward integration instead of contracting schemes to improve the performance of a supply chain. Another line of extension would be to change rebate promotions to the idea of price match where the retailer price match the price difference to the customers if the price drops in a short period or the other authorized retailer has a lower price.

The analytical results are based on a specification of market demand. Other types of demand functions may generate different managerial insights. Moreover, the coordination scheme is certainly not unique. Exploration of other possible coordinating contracts deserves future analysis, especially under a competitive market environment.

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Figure 4.1. An Example of Restricted Rebates Promotion

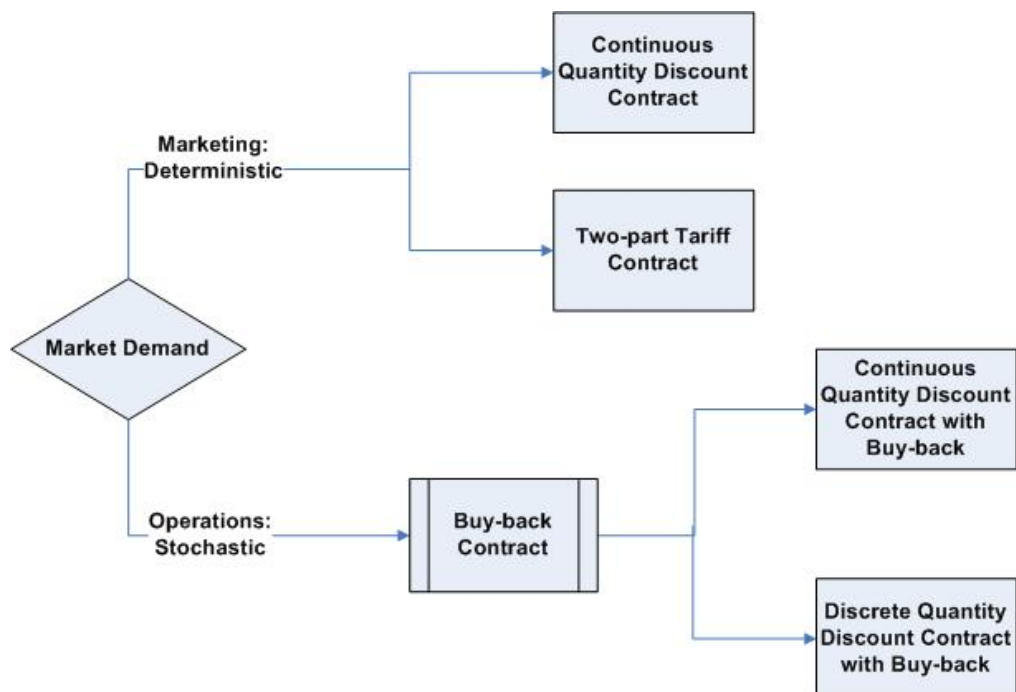


Figure 4.2. The Layout of Proposed Contracts

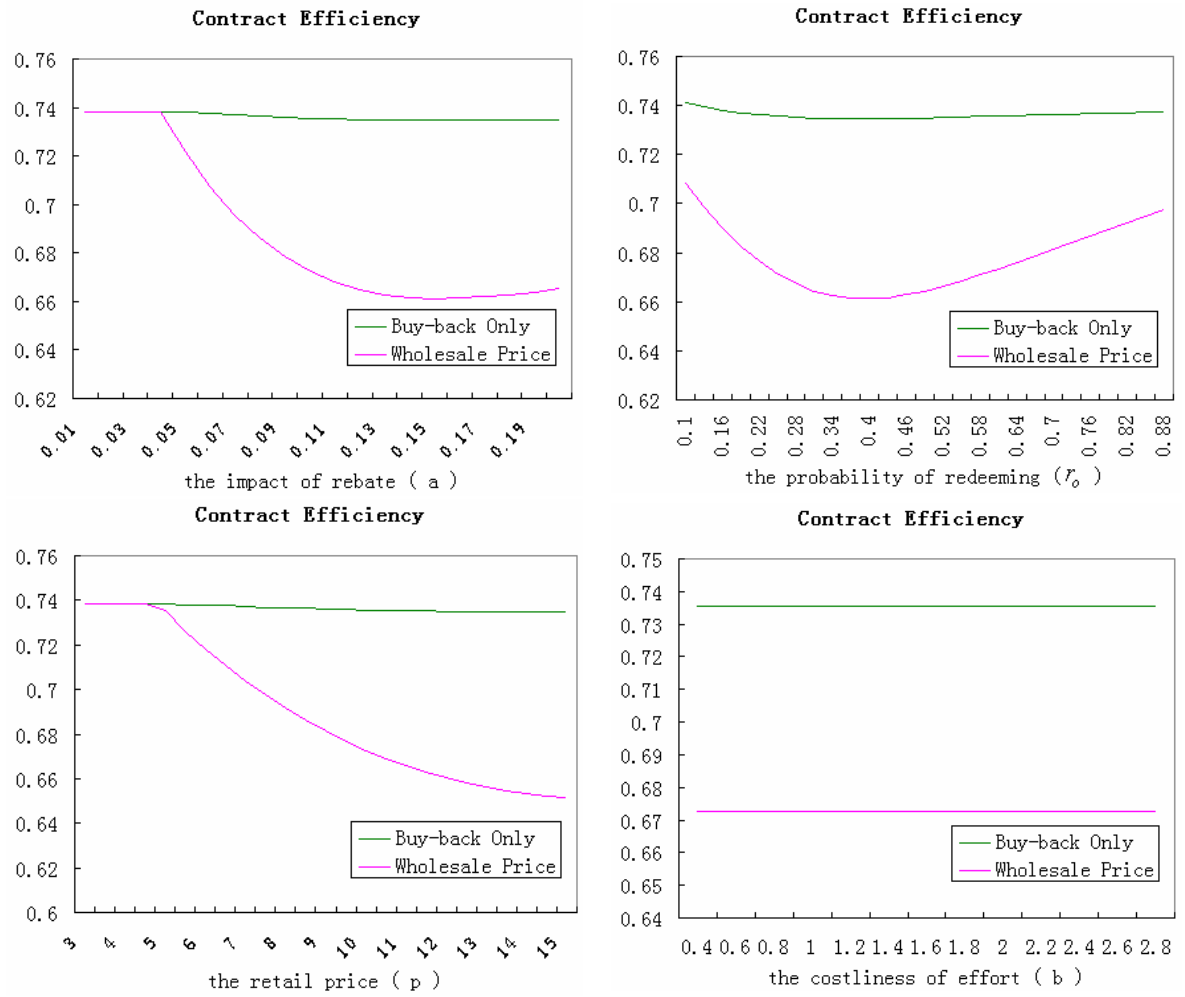


Figure 4.3. Numerical Examples of Contract Efficiency



Figure 4.4. Sensitivity Analysis One

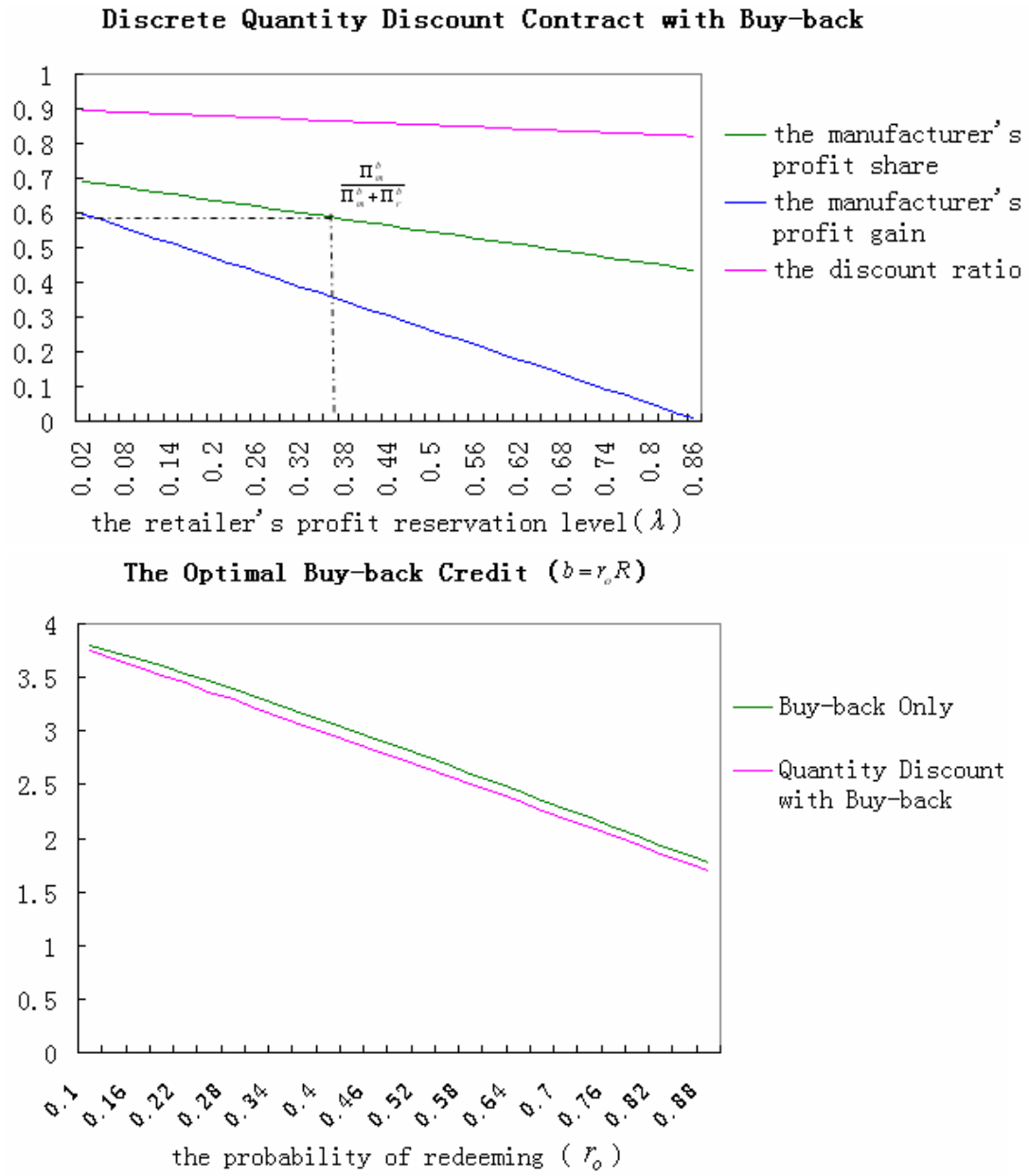


Figure 4.5. Sensitivity Analysis Two

CHAPTER 5

RETAILER'S PROMOTIONAL CAMPAIGN: WHY WAL-MART NEVER ISSUES REBATES

5.1. Brief Introduction

Mail-in rebates have become common promotional techniques in the modern industry.

Given the high slippage rate of rebates, many manufacturers not only have spurred demand but have also generated free money via rebate promotions. With the recent rebate boom, many manufacturers have subcontracted the administrative rebate process to some third-party rebate fulfillment businesses. The popularity of rebates is not only limited to manufacturers; many retailers also provide their own rebates to attract consumers. Some retailers, like Staples, have launched paperless rebate systems to decrease the rebates processing cost and also build customer loyalty. However, the world's largest retailer, Wal-Mart, never issues rebates.

The question arising here actually addresses the core of a retailer's decision making on promotional strategy. Typically, the retailer has two choices: one is to be an everyday low price provider, like Wal-Mart; the other is to adopt higher base retail prices but offer higher promotional discounts. Apparently, everyday low price (EDLP) has many potential benefits, such as relatively consistent demand, low advertising cost, and low managerial and inventory cost. Marketing researchers have provided a variety of reasons to explain the coexistence of EDLP and other promotional strategies. As a departure from traditional literature on the marketing and operations interface, which typically involves retail pricing with inventory decisions, in this chapter I focus on the comparison of two promotion vehicles: rebate promotions and an EDLP policy under the environment incorporating typical economic order quantity

(EOQ) assumptions.

This chapter show that the retailer's decision making on promotional strategies depends upon several factors. Among the most important of these are the demand price sensitivity and the regular undiscounted retail price on market. I argue that choices between rebates promotion and EDLP are positioning strategies rather than purely pricing strategies.

5.2. Model Development

This section describes and formulates the model. For a typical rebate offer, there are three characteristics: value of the reward, length of the redemption period, and redemption effort. In my model of characterizing a rebate, I focus on the role of the rebate face value R and the required redemption effort e . According to the empirical research of Soman (1998) and Silk (2004), consumers' purchase decisions of products offering a rebate can be independent of the decisions to redeem the rebate. In particular, at the time of purchase, consumers tend to underweight the latent future redemption effort and be highly confident of redeeming a rebate. Such misperception of consumers can even be exacerbated by highlighting the reward benefits and not highlighting the effort required to redeem (Soman 1998, Moorthy and Soman 2003). So I assume that consumers' subjective probabilities of redeeming, which determine their purchase decisions, are only related to the reward size but not related to the

actual redemption effort required. The subjective probability of redeeming r_s is strictly increasing with the rebate face value R , implying that a larger reward increases the effectiveness of a rebate offer and generates more market demand. Similar to Soman (1998), I assume a linear deterministic demand, i.e.,

$$D_R(R) = a - b(p_o - r_s(R) \cdot R),$$

where D_R is the consumer demand in the market during the promotional period

p_o is the regular undiscounted retail price on market

a is the market potential parameter

b is the price sensitivity parameter

$r_s(R)$ is the consumer's subjective probability of redeeming.

In the demand model, $p_o - r_s(R) \cdot R$ can be interpreted as the net effective retail price including the rebate incentive. Different from Khouja (2006), the retail price p_o is not a decision variable in my model but exogenously given, which can be justified under a sufficiently competitive market where retailers are price takers. Such a phenomenon is also common in practice where retailers provide rebates but do not necessarily increase their retail price during the promotional period.

As mentioned previously, a high redemption confidence does not necessarily translate into actual redemption behavior (Silk 2004). At the time of redemption, consumers become more accurately aware of the required redemption effort. Thus, the size of the reward has a weaker effect on the redemption decisions because consumers reevaluate the rebate value relative to the extent of required redemption effort. So I assume that

consumers' objective probabilities of redeeming increases in rebate value R but strictly declines in required effort level e . In my model, the effort level e is a real number greater than or equal to 1, which reflects the inherent difficulty level that the retailer imposes on the redemption of rebate. For example, $e=1$ might represent a requirement of only submitting the purchasing information online; $e=2$ might represent a requirement of filling out forms and cutting and mailing the original UPC; while $e=3$ might require purchase of extra products to qualify for a rebate besides the regular redemption effort, etc. Rebate slippage is caused by the difference between consumers' subjective probabilities of redeeming and their objective probabilities of redeeming. Apparently, for any given rebate size, a high required redemption effort can result in a high slippage rate. However, a slippage rate caused by redemption effort can have an upper limit. So I assume that the consumer's objective probability of redeeming, denoted by $r_o(R, e)$ is convex, decreasing in e , and $r_o(R, e=1) = r_s(R)$.

Furthermore, simple redemption requirement usually has a lower unit rebate processing cost for the retailer. For example, the processing of a Staples' easy rebate does not require any manual work by Staples but processing of a regular mail-in paper rebate requires a certain level of manual processing or even involves a payment to some special rebate fulfillment businesses. On the other hand, "experiencing a high effort redemption process dramatically decreases the proportion of rebate buyers that purchase the offer again" (Silk, 2004), i.e., a higher effort level hurts the customers' loyalty. So I build an effort-induced unit cost $c(e)$ for the retailer. This cost $c(e)$ is

an overall cost measure, which may include the rebate processing cost, the loss of future sales, and the damage to the customers' loyalty in the long run. So I assume $c(e)$ is convex and increasing in e .

Assuming no supplier capacity constraints, all replenishment orders incur a fixed setup cost s . In addition, the retailer incurs inventory holding cost which at any point in time is proportional to its inventory level and the retail price. However, because of a single-period modeling, the retailer is assumed not to carry inventory from one promotional season to the next one. Therefore, for a retailer providing rebate promotion in a certain promotional period, its profit function can be written as follows:

$$\Pi_R(R, Q, e) = p_o D_R(R) - r_o(R, e)(R + c(e))D_R(R) - \frac{Q}{2} h p_o - s \frac{D_R(R)}{Q}, \quad (5.1)$$

where Q is the order quantity

s is the setup cost per order placed by the retailer

h is the inventory holding cost per unit per dollar during the promotional period.

Instead, if the retailer chooses to adopt a direct price cut, i.e., offering an everyday low price rather than a rebate promotion, the market demand function and the retailer's profit function are

$$D_p(\lambda) = a - b\lambda p_o,$$

$$\Pi_p(\lambda, Q) = \lambda p_o D_p(\lambda) - \frac{Q}{2} h \lambda p_o - s \frac{D_p(\lambda)}{Q}, \quad (5.2)$$

respectively, where λ is the price reduction percentage.

The logical boundary conditions are listed below:

(A1) $a - b p_o > 0$, which guarantees no negative demand under the undiscounted retail price even if there is no promotions.

(A2) $R \leq p_o$, $e \geq 1$, $\lambda \leq 1$

(A3) $D_R \geq Q$ or $D_p \geq Q$, which guarantees the order quantity per time will not be greater than the total market demand during the season.

5.3. Analysis of Rebate Promotions Using Specific Functional Forms of r_s , r_o , and $c(e)$

To obtain managerial insights, I begin by assuming that $r_s(R) = R/p_o$, which implies that consumers' redemption confidence and the attractiveness of a rebate offer increases linearly in the ratio of the rebate value R to the regular retail price p_o of the product. At the extreme, a free-after-rebate product ($R = p_o$) has a 100% redemption confidence. However, even these 100% rebates do not elicit 100% redemption because of the redemption effort involved. I further assume that consumers' objective probability of redeeming is $r_o(R, e) = \frac{R}{p_o} \cdot \frac{1}{e}$. Obviously, there is no slippage behavior when $e = 1$ in my setting. The unit rebate induced cost $c(e)$ is

assumed to be $c(e) = cp_o e^2$, where c is a sufficiently small number and can be interpreted as the retailer's costliness parameter of a rebate offer. Under these assumptions, the retailer's profit function (5.1) becomes,

$$\begin{aligned}\Pi_R(R, Q, e) &= p_o D_R(R) - r_o(R, e)(R + c(e))D_R(R) - \frac{Q}{2}hp_o - s\frac{D_R(R)}{Q} \\ &= p_o D_R(R) - \frac{R}{p_o} \cdot \frac{1}{e}(R + cp_o e^2)D_R(R) - \frac{Q}{2}hp_o - s\frac{D_R(R)}{Q}\end{aligned}$$

where $D_R(R) = a - b(p_o - \frac{R}{p_o} \cdot R)$.

By embedding the demand function inside the profit function, the sufficient conditions for optimality are obtained by taking the first-order derivatives with respect to Q and e , respectively,

$$Q_R^* = \sqrt{\frac{2sD_R(R)}{hp_o}} \quad (5.3)$$

$$e^* = \sqrt{\frac{R}{cp_o}} \quad (5.4)$$

For any given rebate value R , we can obtain the following Hessian matrix,

$$H = \begin{bmatrix} -2sD \cdot \frac{1}{Q^3} & 0 \\ 0 & -\frac{2RD}{p_o} \cdot \frac{1}{e^3} \end{bmatrix}.$$

So the Hessian matrix is negative definite for any given R .

Furthermore, from (5.4), we can obtain

$$r_o(R, e^*) = \frac{R}{p_o} \cdot \frac{1}{e^*} = \sqrt{\frac{cR}{p_o}} = ce^*,$$

which implies that for any given R , the consumer's objective probability of

redeeming is increasing with the optimal redemption effort level. Although this seems counter-intuitive, the explanation is that the higher the optimal redemption effort level implies a larger rebate face value, which resulting a higher objective probability of redeeming.

By embedding (5.3) and (5.4) into the retailer's profit function, we have

$$\begin{aligned}\Pi_R(R) &= p_o D_R(R) - 2\sqrt{\frac{c}{p_o}} R^{\frac{3}{2}} D_R(R) - \sqrt{2shp_o D_R(R)} \\ &= p_o \left(a - bp_o + \frac{b}{p_o} R^2\right) - 2\sqrt{\frac{c}{p_o}} R^{\frac{3}{2}} \left(a - bp_o + \frac{b}{p_o} R^2\right) - \sqrt{2shp_o \left(a - bp_o + \frac{b}{p_o} R^2\right)}.\end{aligned}$$

For $\Pi_R(R)$ to be concave requires that

$$\frac{\partial^2 \Pi_R(R)}{\partial R^2} = 2b - \sqrt{\frac{c}{p_o}} R \left(\frac{3}{2}(a - bp_o) + \frac{35}{2} \frac{b}{p_o} R^2 \right) - b \sqrt{\frac{2sh}{p_o D_R(R)}} \left(1 - \frac{bR^2}{p_o D_R(R)} \right) \leq 0.$$

Although the above condition does not hold for all parameter settings, it can be easily tested that the above condition holds for a wide range of realistic parameter values. So from the first order condition of R , the optimal rebate value R^* should satisfy the following equation,

$$R^{\frac{1}{2}} \left\{ 2bR^{\frac{1}{2}} - \sqrt{\frac{c}{p_o}} \left(3(a - bp_o) + 7 \frac{b}{p_o} R^2 \right) - b \sqrt{\frac{2shR}{ap_o - bp_o^2 + bR^2}} \right\} = 0 \quad (5.5)$$

Hence, $R=0$ is a possible candidate for the optimal solution. If the retailer decides

not to offer rebates, the optimal order quantity Q^* is given by $Q^* = \sqrt{\frac{2s(a - bp_o)}{hp_o}}$,

which leads to a profit of $\Pi_R = p_o(a - bp_o) - \sqrt{2shp_o(a - bp_o)}$. If the retailer can achieve

higher profits by offering rebates, the optimal rebate face value R^* can be solved

from the equation below,

$$2bR^{\frac{1}{2}} - \sqrt{\frac{c}{p_o}} \left(2(a - bp_o) + 7\frac{b}{p_o}R^2 \right) - b\sqrt{\frac{2shR}{ap_o - bp_o^2 + bR^2}} = 0$$

Due the complicity of the polynomial function, a closed form of optimal R cannot be obtained.

5.4. Analysis of EDLP Policy

Similar to the analysis of rebate promotion, if the retailer chooses to adopt a direct price cut (EDLP policy), from the profit function (5.2), we can obtain that

$$Q_p^* = \sqrt{\frac{2sD_p(\lambda)}{\lambda hp_o}}. \text{ By embedding it inside the profit function, we have}$$

$$\Pi_p(\lambda) = \lambda p_o D_p(\lambda) - \sqrt{2shp_o \lambda D_p(\lambda)},$$

which reaches a minimum value when $\lambda p_o D_p(\lambda) = \frac{sh}{2}$. Given that sh is a very small number compared to the revenue $\lambda p_o D_p(\lambda)$, the profit function $\Pi_p(\lambda)$ is strictly increasing with $\lambda p_o D_p(\lambda)$. So we only need to maximize $\lambda p_o D_p(\lambda)$ for the purpose

of maximizing the retailer's profit. Hence, $\lambda^* = \frac{a}{2bp_o}$ is the optimal price-cut

percentage for the retailer when $a \leq 2bp_o$, which leads to the optimal order quantity

$$Q_p^* = \sqrt{\frac{2bs}{h}}. \text{ Therefore, the maximum profit for the retailer is}$$

$$\Pi_p = \frac{a}{2\sqrt{b}} \left(\frac{a}{2\sqrt{b}} - \sqrt{2sh} \right).$$

On the other hand, when $a > 2bp_o$, $\lambda^* = \frac{a}{2bp_o}$ becomes greater than 1. However, by

the restriction of $\lambda \leq 1$ in (A2), the retailer cannot freely increase the retail price due

to the pressure from his competitors or manufacturers. Due to the concavity of λ , the retailer chooses not to offer any price-cut promotion, i.e., $\lambda^* = 1$, which results in a maximum profit of $\Pi_p = p_o(a - bp_o) - \sqrt{2shp_o(a - bp_o)}$.

5.5. Sensitivity Analysis and Discussions

In this section, I use numerical examples to gain further insights. Consider a product with the market potential $a = 20,000$ and price sensitivity $b = 0.02 \times a$ (i.e., a dollar change in effective retail price will cause the demand to change by 2%). The other parameters in the base set include $p_o = 30$, $c = 0.1$, $h = 0.01$, and $s = 2000$. All the following analytical results are obtained by modifying the base set by one or two parameters at a time.

For a linear demand function, the most important parameters are the market potential a and the price sensitivity b . Previous studies (Gerstner et al. 1994, Moorthy and Lu 2004, Chen et al. 2005) have confirmed that rebate/coupon promotion is an effective technique for price-discriminating by making products appealing to price-sensitive consumers. Because of this price sensitivity, the retailer can charge customers different prices through slippage. My results also imply that rebate promotion is more effective than direct price-cut promotion (EDLP) when consumers are highly price-sensitive.

[Insert Table 5.1. here]

[Insert Figure 5.1. here]

Table 5.1 and Figure 5.1 (where b varies from $0.015 \times a$ to $0.025 \times a$) show that the benefits of rebates promotion increases with consumers' price sensitivity. Some product categories have low price sensitivity. In this situation, the EDLP promotion is at least as effective as rebates promotion and retailer chooses not to provide rebates. Because consumers are not price-sensitive enough, a rebate promotion cannot attract more customers and generate significant revenue increases to cover the high rebate promotion costs. In Blattberg and Neslin (1990), the authors argue that frequent promotion can increase price sensitivity, which is a limitation of promotions. However, I argue that a product category with high price sensitivity can also be beneficial to the retailer to implement rebate promotions where the retailer can vary the level of redemption effort to cause slippage.

The numerical results further suggest that the market potential parameter a plays a less important role on the choices of promotions. As the Figure 5.2 shows, the benefits brought by rebate promotions under a high market potential is not as significant as the benefits under a high price sensitivity. Figure 5.3 shows that the optimal rebate face value increases with the market potential parameter but at an extremely small rate, i.e., the optimal rebate value is insensitive to parameter a .

[Insert Figure 5.2. here]

[Insert Figure 5.3. here]

Usually, products carrying direct price reduction or coupon promotions which offer discounts up front are normally small-ticketed. In contrast, rebate promotions are more prominent on medium-ticketed to large-ticketed products. Figure 5.4 confirms this phenomenon. The regular undiscounted retail price p_o restricts the upper bound of rebate value R . Hence, with a small p_o , the impact of a tiny rebate on market demand is not significant enough to offset the rebate-related cost. So the retailer chooses not to issue rebates but adopts an EDLP policy. As the regular retail price p_o increases, the use of rebates can result in a significant increase in profits.

[Insert Figure 5.4. here]

If the regular retail price is not sufficiently small, the retailer chooses to provide rebates promotion. Figure 5.5 shows that the optimal rebate face value increases linearly with p_o .

[Insert Figure 5.5. here]

Figure 5.6 reports the joint effects of the regular retail price p_o and the price sensitivity parameter b on the optimal rebate value. Give a sufficiently large p_o , the

retailer will choose to offer rebates even at a low value of price sensitivity, i.e., the profitable range of parameter b for the retailer to offer rebates expands as p_o increases.

[Insert Figure 5.6. here]

It should be noted that above the results are based on a reasonable range of values. For example, if p_o is extremely large, the retailer tends to issue an extremely big rebate but at the same time has to require an extraordinarily high redemption effort level for the purpose of slippage. However, an extremely complicated rebate redemption process is definitely not welcomed by customers, which will significantly hurt the customer loyalty and make them avoid products carrying such offers. Moreover, extremely complicated rebate redemption may also increase the rebate processing cost and the cost of handling customer's complaints. All of these consequences can cause a variation on the retailer's costliness parameter c of rebate offer. As c increases, the profitability of a rebate promotion decreases and the retailer chooses the EDLP instead of rebate promotion. Figure 5.7 and Figure 5.8 show that both the optimal rebate value and optimal redemption effort level decrease rapidly in the rebate costliness parameter c . At an extreme, if parameter c is sufficiently small, the retailer may provide free-after-rebate offer, i.e., the optimal rebate value is equal to the regular price. Such rebate offers are not rare in practice (see <http://www.free-after-rebate.net> for examples). For free-after-rebate products,

consumers subjective probabilities r_s are equal to 1 which implies high rebate effectiveness; however, their actual objective probabilities of redeeming is low due to the relatively higher redemption effort level.

[Insert Figure 5.7. here]

[Insert Figure 5.8. here]

From the above analysis, the retailer can increase its profitability dramatically by providing a properly designed rebate offer, and the magnitude of profit increase depends on several important factors. These important parameters are usually inherent within the retailer itself and also product categories, so the choices of rebates promotion or EDLP policy are usually implemented as positioning strategies rather than purely pricing strategies.

5.6. Comparative Example

To illustrate the retailer's decision making on retailing strategies, consider two different fictitious retailers: retailer A (Wal-Mart type) and retailer B (Staples type). Both retailers are planning on a seasonal sale for the SanDisk Extreme III SD card in July, 2007. The manufacturer's suggested retail price for this SD card is \$99, which serves as the regular undiscounted retail price p_o . Without the loss of generality, I assume that the market potential for both retailers are the same, i.e., $a = 20,000$, while

the price sensitivity parameter b is $0.006 \times a$ and $0.008 \times a$ for retailer A and B, respectively. Other parameter values are $c=0.15$, $h=0.01$, and $s=5000$. Thus, the only difference between the two retailers is the price sensitivity of their respective customers.

By solving equation (5.5), the optimal rebate values for retailer A and retailer B are \$43.68 and \$70.50, respectively. Embedding the rebate values into (5.3) and (5.4), the optimal solutions using both policies can be obtained as in Table 5.2.

[Insert Table 5.2 here]

From Table 5.2, obviously retailer A should adopt the EDLP policy, while retailer B should adopt a rebate promotion. Hence, depending on the different values of inherent marketing parameters, the choices of rebate promotion or EDLP policy are positioning strategies rather than purely pricing strategies.

5.7. Conclusions

In this chapter, I use an EOQ based model to compare two different promotional policies: rebate promotion and EDLP via direct price-cut. For rebate promotion, the retailer needs to jointly determine the optimal order quantity, the rebate face value and the level of redemption effort. For EDLP, the retailer needs to determine the optimal

order quantity and the price reduction percentage. I show that rebate promotions can result in a significant increase in profits depending on several important factors, such as the price sensitivity parameter, the regular undiscounted retail price, and the rebate costliness parameter. The different values of these factors induce the retailer to make a choice between rebate promotions and EDLP. Customers visiting Wal-Mart are typically “expected price shoppers” and are less likely to chase deals all over town once they are in store. Hence, such customers typically have lower price sensitivity, so as a positioning strategy Wal-Mart chooses to adopt an EDLP policy. Most of the products offered at Wal-Mart stores are small-ticketed non-durable goods, which are not suitable for rebate promotion by my analysis.

Although the rebate face value and required redemption effort play an important role on consumers’ purchase and redemption behaviors, there are some other factors contributing to creating slippage behavior which have not been studied in this chapter. For example, Gourville and Soman (2004) suggests an anchoring and self adjustment, while Silk (2004) provides procrastination and forgetting as additional explanation for slippage. Furthermore, the benefits of rebates are not restricted to the increasing profits brought by slippage. Rebate promotions also provide the retailer interest free loans during the long redemption and processing period even if customers successfully receive the rebate checks.

Another limitation in this chapter is the use of a linear demand model, which is not

suitable for extreme values, so future researchers using richer models should be able to develop more analytical results. Another interesting approach would be to follow the idea in chapter three and use consumer utility function to generate market demand and actual redemption rate.

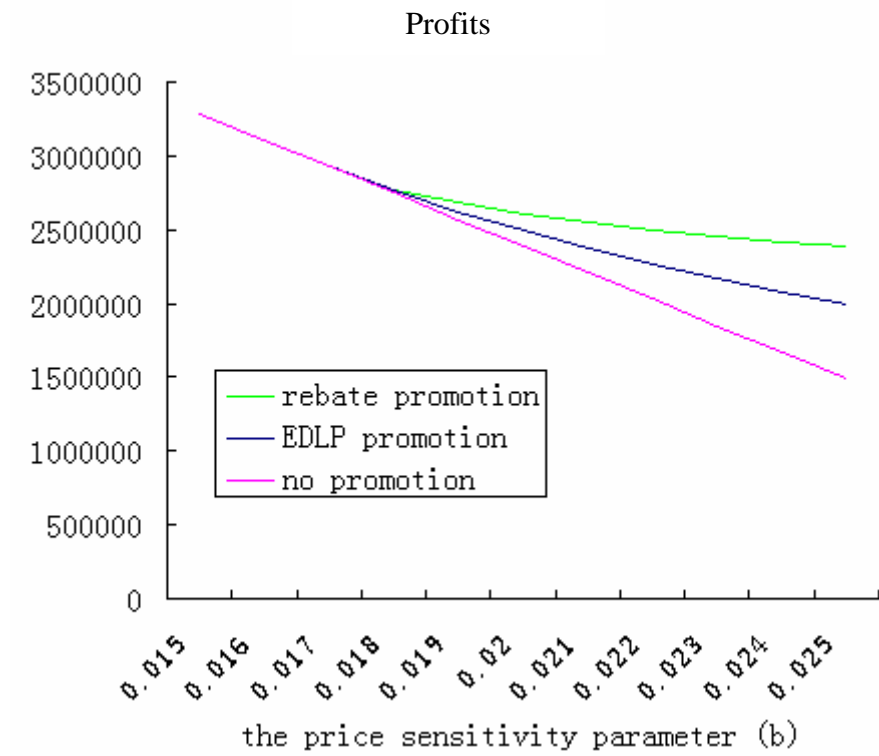


Figure 5.1. Price Sensitivity Parameter b vs Profits

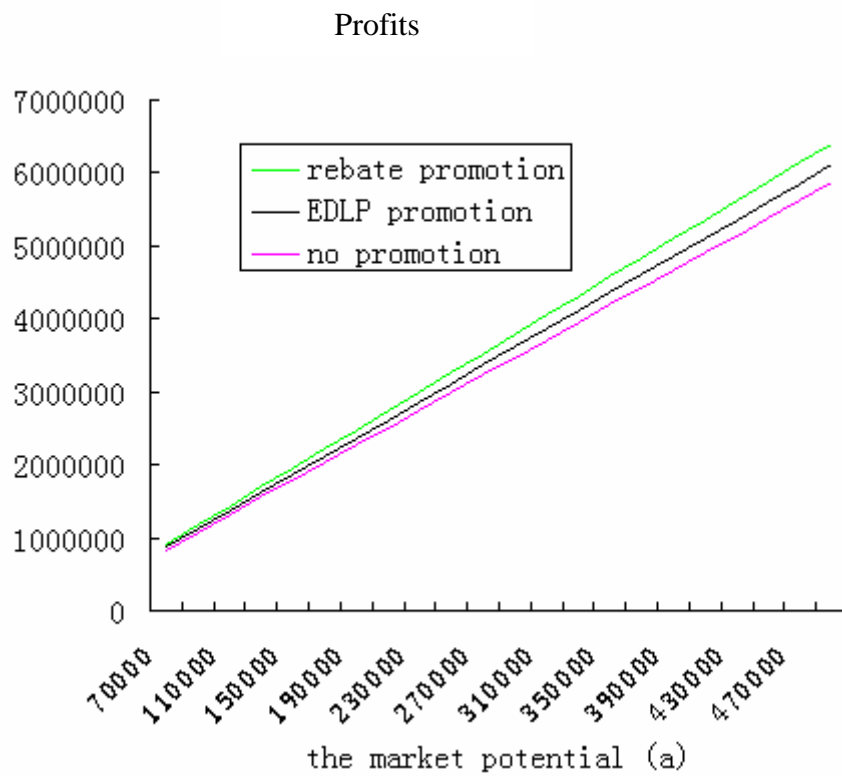


Figure 5.2. Market Potential Parameter a vs Profits

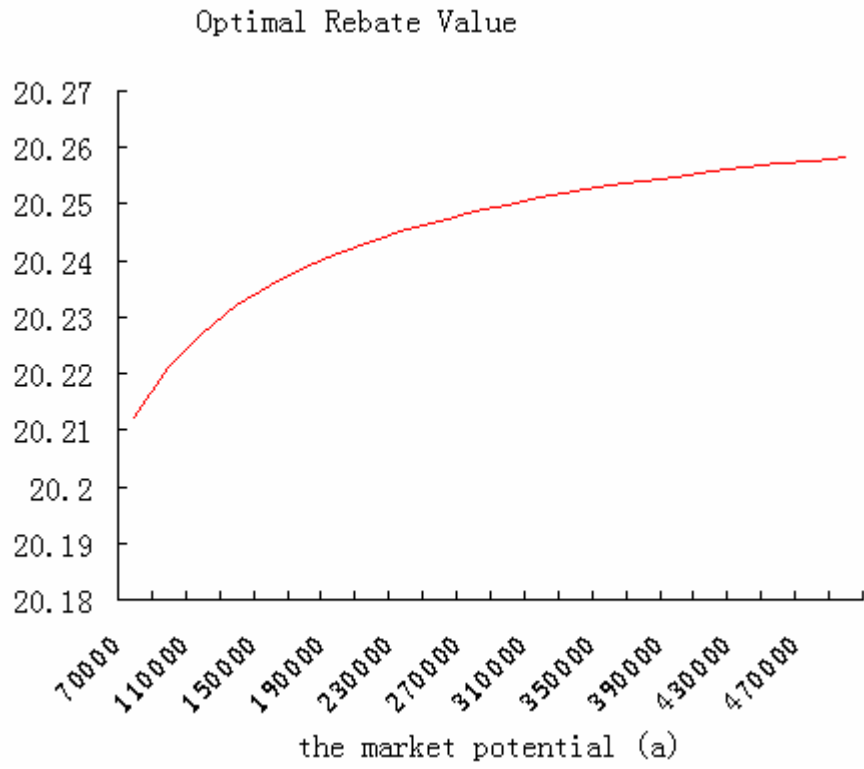


Figure 5.3. Market Potential Parameter a vs Optimal Rebate Value

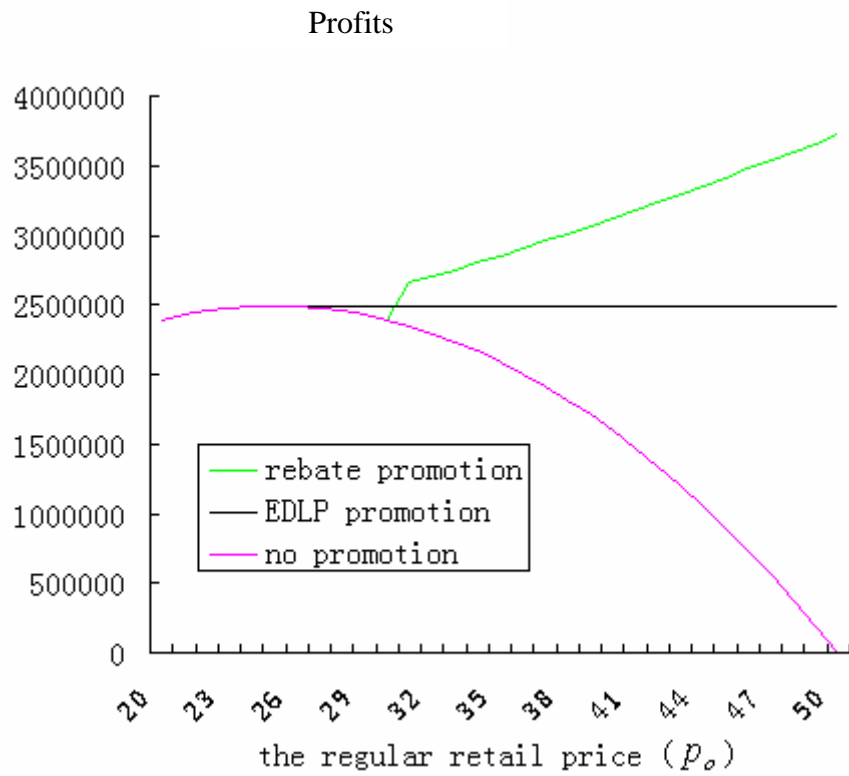


Figure 5.4. Regular Retail Price p_o vs Profits

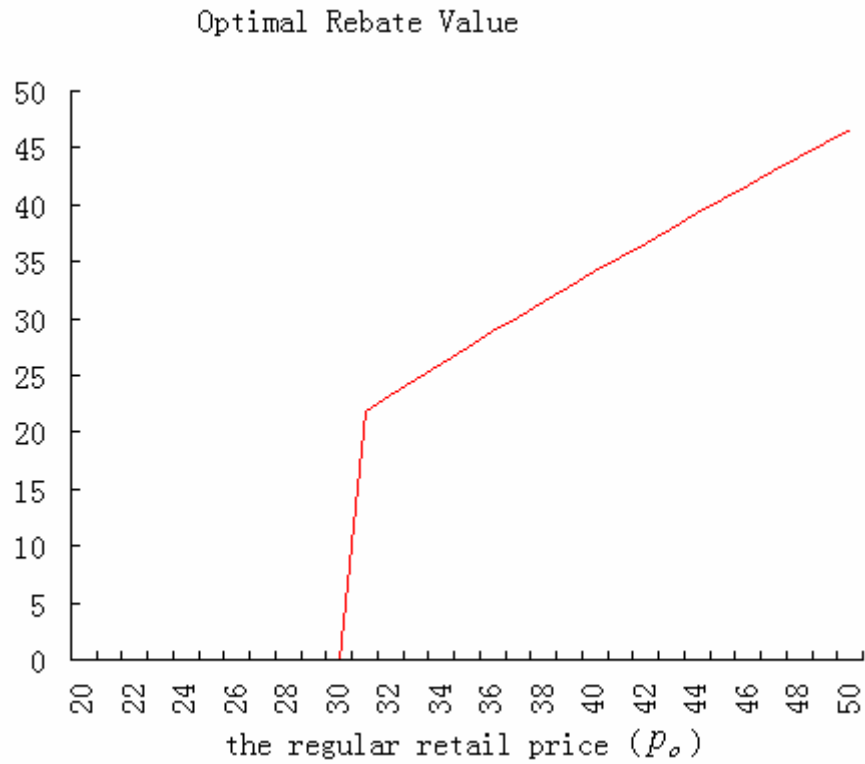


Figure 5.5. Regular Retail Price p_o vs Optimal Rebate Value

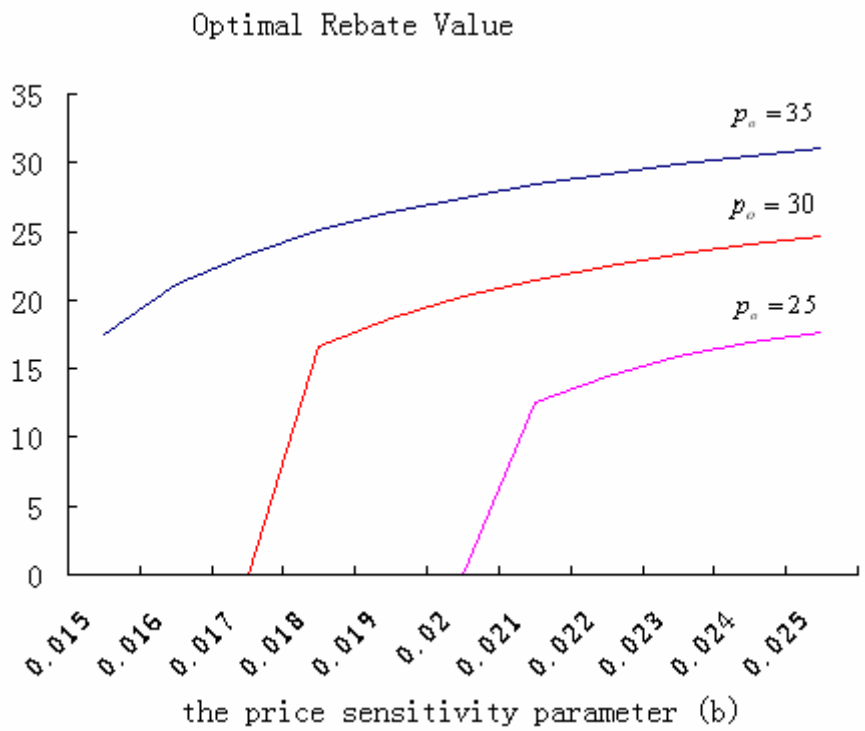


Figure 5.6. The Joint Effects of Regular Retail Price p_o and Price Sensitivity

Parameter b

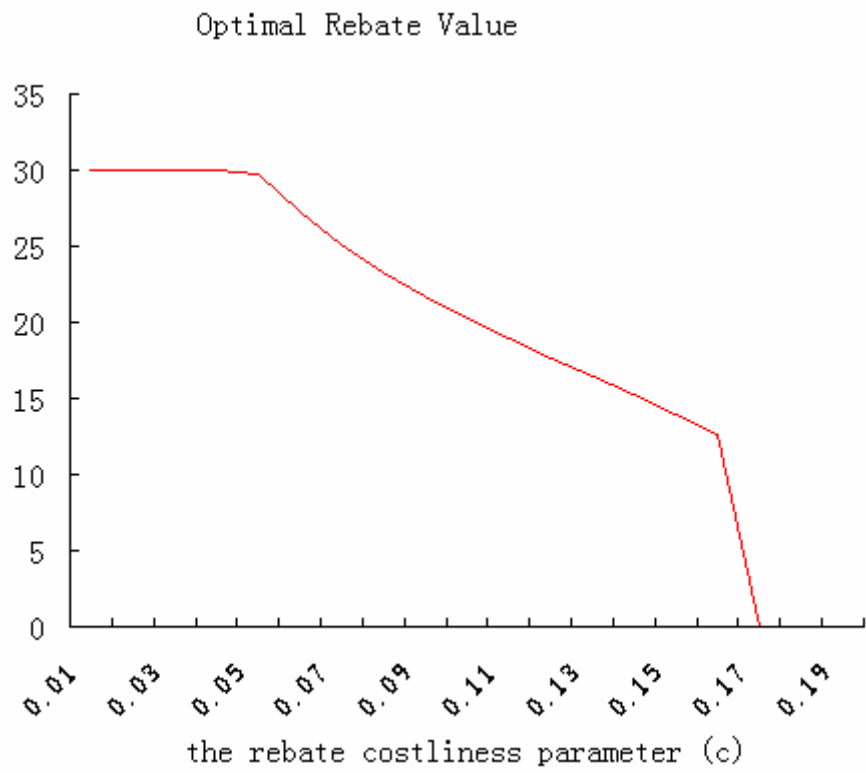


Figure 5.7. Rebate Costliness Parameter c vs Optimal Rebate Value

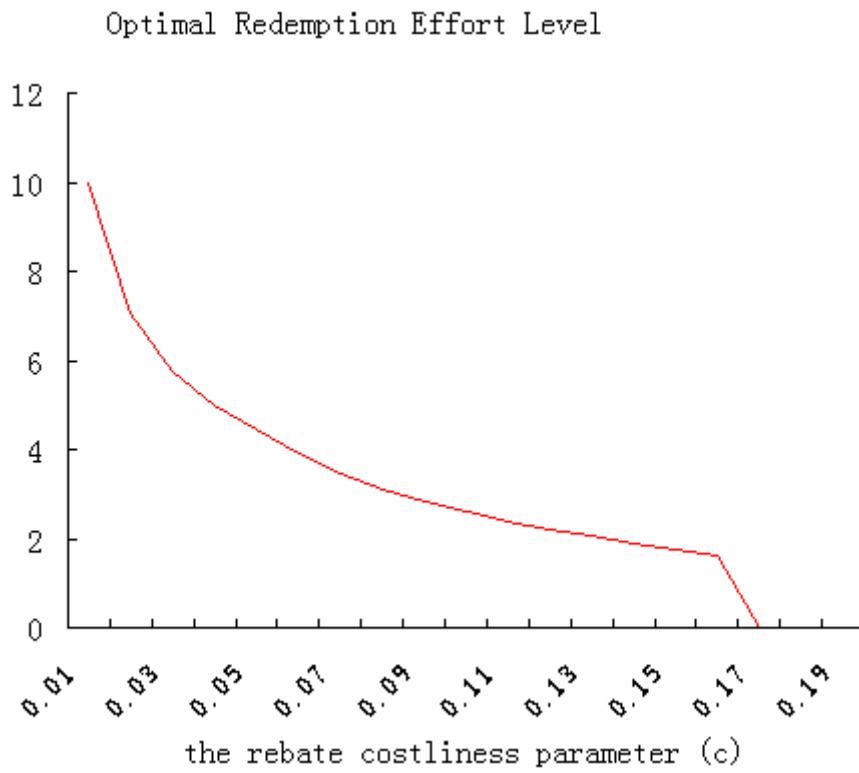


Figure 5.8. Rebate Costliness Parameter c vs Optimal Redemption Effort Level

b	R	Π_R	Π_p	Π_{none}
0.015	0	3,288,511	3,288,511	3,288,511
0.016	0	3,108,829	3,108,828	3,108,828
0.017	0	2,929,156	2,930,330	2,929,156
0.018	16.58	2,763,035	2,767,237	2,749,493
0.019	18.72	2,679,694	2,621,319	2,569,841
0.02	20.24	2,610,459	2,490,000	2,390,202
0.021	21.45	2,551,759	2,371,193	2,210,576
0.022	22.44	2,501,396	2,263,192	2,030,966
0.023	23.28	2,457,846	2,164,588	1,851,374
0.024	24.01	2,419,980	2,074,204	1,671,802
0.025	24.64	2,386,930	1,991,056	1,492,254

Table 5.1. Effects of Price Sensitivity Parameter b

	Π_R	Π_p	Π_{none}	R	e	r_s	r_o	λ
Retailer A	\$788,206	\$824,167	\$794,914	\$43.68	1.715	0.441	0.257	0.842
Retailer B	\$634,215	\$616,875	\$405,423	\$70.50	2.179	0.712	0.327	0.631

Table 5.2. Optimal Solutions of the Comparative Example

APPENDIX

Proof of Proposition 3.1:

With the demand function, we can proceed in two different cases.

(a) If $P_r = r_s R$, the market demand is equal to 1. The retailer's profit function is given

by $\Pi_r(w, R) = (P_r - w) \cdot 1 = r_s R - w$. Hence, for $w \leq r_s R$, $P_r^a = r_s R$; otherwise, if $w > r_s R$, which leads to a negative profit, the retailer will not choose $P_r = r_s R$.

(b) If $r_s R < P_r < bs + r_s R$, The retailer's profit function is given by

$$\Pi_r(w, R, P_r) = (P_r - w) \cdot D(P_r, R) = (P_r - w) \cdot \frac{bs - (P_r - r_s R)}{bs}$$

Since $\Pi_r(w, R, P_r)$ is concave in P_r , from FOC, we get $P_r^b(w, R) = \frac{bs + w + r_s R}{2}$.

This solution is in the relevant interval if $r_s R < P_r < bs + r_s R$ holds, which leads to

$$r_s R < \frac{bs + w + r_s R}{2} < bs + r_s R \Leftrightarrow r_s R - bs < w < bs + r_s R$$

Note that the upper bound for w in case a is larger than the lower bound for w in case

b. Obviously, there is an interval for w in which P_r^a and P_r^b are both interior solutions. The retailer's best interior solution is the one which leads to higher profits.

A comparison of the retailer's profits in that region shows that

$$\Pi_r(w, R) - \Pi_r(w, R, P_r^*) = r_s R - w - \frac{(bs + r_s R - w)^2}{4bs} = -\frac{(bs + w - r_s R)^2}{4bs} \leq 0.$$

Hence, the interior solution is P_r^a for $w \leq r_s R - bs$, and it is P_r^b for

$$r_s R - bs < w < bs + r_s R.$$

c) If $P_r \geq bs + r_s R$, apparently the retail will not choose this region because of the zero consumer demand.

From the retailer's response, the manufacturer chooses his optimal combination of w and R for each case.

a) with $w \leq r_s R - bs$, the retailer chooses $P_r^a = r_s R$ and the manufacturer's profit can be written as,

$$\Pi_m(w, R) = (w - r_o R) \cdot 1 \leq r_s R - bs - r_o R = (r_s - r_o)R - bs$$

So obviously, there are two cases: $r_s = r_o$ and $r_s > r_o$

Case a1: If $r_s = r_o$, which leads to a negative manufacturer's profit, so the manufacturer does not have a feasible solution in this interval.

Case a2: If $r_s > r_o$, the manufacturer's profit is strictly increasing in R without bound.

The manufacturer's optimal solution is $w^* = r_s R - bs$.

By (A4), $w^* = r_s R - bs \geq r_o R \Leftrightarrow R \geq \frac{bs}{r_s - r_o}$. So if $r_s > r_o$, $R \in \left[\frac{bs}{r_s - r_o}, \infty \right)$, which leads to

$P_r = r_s R$ and $w = r_s R - bs$. Hence, the manufacturer chooses the highest feasible

$R^* = \infty$, $P_r^* = \infty$ and $w^* = \infty$, which results in a profit of $\Pi_m = \infty$.

b) For $r_s R - bs < w < bs + r_s R$, given the information that the retailer will choose

$P_r^b(w, R) = \frac{bs + w + r_s R}{2}$, the manufacturer's profit function is given by

$$\Pi_m(w, R) = (w - r_o R) \cdot \frac{bs - (P_r^b - r_s R)}{bs} = (w - r_o R) \cdot \frac{bs - w + r_s R}{2bs}$$

In order to solve the manufacturer's problem we proceed in two steps, first, we

characterize the optimal wholesale price, $w^*(R)$, for a given rebate face value R, and

next, we find the optimal R, by embedding $w^*(R)$ in the manufacturer's objective function and maximizing it over R.

The manufacturer's objective is concave in w, so from FOC, we get

$w^*(R) = \frac{bs + (r_s + r_o)R}{2}$, which is greater than $r_o R$. By embedding $w^*(R)$ in the

manufacturer's objective function, the manufacturer's profit follows as

$$\Pi_m(R) = \frac{[bs + (r_s - r_o)R]^2}{8bs}$$

So obviously, there are two cases: $r_s = r_o$ and $r_s > r_o$

Case 1: $r_s = r_o$

It is straightforward to verify that I can get $w - r_o R = \frac{bs}{2}$ in equilibrium, which is equivalent to the optimal wholesale price decision.

Case 2: $r_s > r_o$

Since its profit is strictly increasing in R , the manufacturer will choose the highest feasible R . From the restriction of relevant region, we have

$$r_s R - bs < w < bs + r_s R \Leftrightarrow r_s R - bs < \frac{bs + (r_s + r_o)R}{2} < bs + r_s R \Leftrightarrow R < \frac{3bs}{r_s - r_o}$$

Hence, the manufacturer chooses the corner solution $\frac{3bs}{r_s - r_o}$. However, if the

manufacturer chooses $R = \frac{3bs}{r_s - r_o}$, the retailer will choose $P_r = r_s R$, which is the

situation under case a. Hence, if $r_s > r_o$, the manufacturer does not have a feasible solution for the interval $r_s R - bs < w < bs + r_s R$.

Proof of Lemma 3.1.: Being confronted with the four intervals of the demand function

$D(P_r, R, P_s)$, the retailer chooses the optimal $P_r(w, R, P_s)$ for any given w , R and P_s of the manufacturer.

a) For $P_r \leq \frac{r_s R + \alpha P_s}{1 + \alpha}$, the retailer's optimal retail price is straightforward,

$$P_r^a = \frac{r_s R + \alpha P_s}{1 + \alpha} \quad \text{for } w \leq \frac{r_s R + \alpha P_s}{1 + \alpha}$$

Otherwise, if $w > \frac{r_s R + \alpha P_s}{1 + \alpha}$, which leads to negative profits, the retailer will not choose P_r in this interval.

b) For $\frac{r_s R + \alpha P_s}{1 + \alpha} < P_r \leq P_s$, the retailer's profit function is given by

$$\Pi_r(P_r) = (P_r - w) \left(\frac{bs - (1 + \alpha)P_r + r_s R + \alpha P_s}{bs} \right)$$

The above objective is concave in P_r , so from FOC, we get

$$P_r^b = \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)}$$

This solution is in the relevant interval if $\frac{r_s R + \alpha P_s}{1 + \alpha} < P_r \leq P_s$ holds, which leads to

$$\frac{r_s R + \alpha P_s}{1 + \alpha} < \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} \leq P_s \Leftrightarrow \frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w \leq P_s - \frac{r_s R + bs - P_s}{1 + \alpha}$$

Note that the upper bound for w in case a is greater than the lower bound for w in case

b. Obviously, there is an interval for w in which $P_r^a(R)$ and $P_r^b(w, R)$ are both interior solutions. The retailer's best interior solution is the one which leads to higher profits. A comparison of the retailer's profits in that region shows that

$$\Pi_r(w, P_r^a) - \Pi_r(w, P_r^b) = -\frac{1}{4(1 + \alpha)bs} ((1 + \alpha)w + bs - r_s R - \alpha P_s)^2 \leq 0$$

Hence, the interior solution is P_r^a for $w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha}$, and it is P_r^b for

$$\frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w \leq P_s - \frac{r_s R + bs - P_s}{1 + \alpha}.$$

c) For $P_s < P_r < \frac{bs + r_s R + \beta P_s}{1 + \beta}$, the retailer's maximization function is

$$\Pi_r(P_r) = (P_r - w) \left(\frac{bs - (1 + \beta)P_r + r_s R + \beta P_s}{bs} \right)$$

The above objective is concave in P_r , so from FOC, we have

$$P_r^c = \frac{w}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)}$$

This solution is in the relevant interval if $P_s < P_r \leq \frac{bs + r_s R + \beta P_s}{1 + \beta}$ holds, which leads to

$$P_s < \frac{w}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} < \frac{bs + r_s R + \beta P_s}{1 + \beta} \Leftrightarrow P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w < \frac{\beta P_s + r_s R + bs}{1 + \beta}$$

From (A6), we have $w \leq P_s$. Since the RHS $\frac{\beta P_s + r_s R + bs}{1 + \beta} \geq \frac{\beta P_s + P_s}{1 + \beta} = P_s$, so the appropriate interval is $P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s$.

Note that the upper bound for w in case a is always less than the upper bound for w in case c: by (A6), it is easy to show $\frac{r_s R + \alpha P_s}{1 + \alpha} \leq P_s$; however, for the upper bound for w

in case a and the lower bound for w in case c the following relation holds:

$$P_s - \frac{r_s R + bs - P_s}{1 + \beta} \leq \frac{r_s R + \alpha P_s}{1 + \alpha} \Leftrightarrow P_s \leq \frac{1 + \alpha}{2 + \alpha + \beta} bs + r_s R$$

So when $P_s \leq \frac{1 + \alpha}{2 + \alpha + \beta} bs + r_s R$, there is an interval for w in which P_r^a and P_r^c are

both interior solutions. The retailer's best interior solution is the one which leads to

higher profits. A comparison of the retailer's profits in that region shows that

$$\begin{aligned} & \Pi_r(w, P_r^a) - \Pi_r(w, P_r^c) \\ &= \left(\frac{r_s R + \alpha P_s}{1 + \alpha} - w \right) - \left(\frac{r_s R + \beta P_s}{1 + \beta} - w \right) - \frac{1}{4(1 + \beta)bs} ((1 + \beta)w + bs - r_s R - \beta P_s)^2 \\ &= -\frac{(\beta - \alpha)(P_s - r_s R)}{(1 + \alpha)(1 + \beta)} - \frac{1}{4(1 + \beta)bs} ((1 + \beta)w + bs - r_s R - \beta P_s)^2 \leq 0 \end{aligned}$$

Hence, the interior solution is P_r^c for $P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s$.

d) Since we assume $\beta > \alpha$, the upper bound for w in case b is less than the lower

bound in case c. So there exists an interval $P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta}$ for which $P_r^d = P_s$ is a corner solution to the retailer's optimization problem.

Note that for the upper bound for w in case a and the lower bound for w in case d the following relation holds:

$$P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq \frac{r_s R + \alpha P_s}{1 + \alpha} \Leftrightarrow P_s \leq \frac{bs}{2} + r_s R$$

So when $P_s \leq \frac{bs}{2} + r_s R$, there is an interval for w in which P_r^a and P_r^d are both interior solutions. The retailer's best interior solution is the one which leads to higher profits. A comparison of the retailer's profits in that region shows that

$$\begin{aligned} \Pi_r(w, P_r^a) - \Pi_r(w, P_r^d) &= \left(\frac{r_s R + \alpha P_s}{1 + \alpha} - w \right) - (P_s - w) \left(\frac{bs - P_s + r_s R}{bs} \right) \\ &= (P_s - r_s R) \left(\frac{P_s - w}{bs} - \frac{1}{1 + \alpha} \right) \leq (P_s - r_s R) \left(\frac{r_s R + bs - P_s}{1 + \alpha} / bs - \frac{1}{1 + \alpha} \right) = -\frac{(P_s - r_s R)^2}{(1 + \alpha)bs} \leq 0 \end{aligned}$$

Hence, the interior solution is P_r^d for $P_s - \frac{r_s R + bs - P_s}{1 + \alpha} < w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta}$

e) For $P_r \geq \frac{bs + r_s R + \beta P_s}{1 + \beta}$, obviously the retailer does not have a feasible P_r from this interval with zero consumer demand.

f) Now that all interior solutions are calculated, we have to compare the retailer's profits associated with those solutions to the profits at the corner of the intervals. It is straightforward to exclude $P_r = 0$ and $P_r = \frac{bs + r_s R + \beta P_s}{1 + \beta}$ as optimal retail prices for any combination (w, R, P_s) of the manufacturer because both cannot lead to positive profits for the retailer.

First consider P_r^a for $w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha}$. Only $P_r = P_s$ is a candidate for corner

solution. A comparison of profits shows that

$$\begin{aligned} \Pi_r(w, P_r^a) - \Pi_r(w, P_r = P_s) &= (P_s - r_s R) \left(\frac{P_s - w}{bs} - \frac{1}{1 + \alpha} \right) \\ &\geq (P_s - r_s R) \left(\left(P_s - \frac{\alpha P_s + r_s R - bs}{1 + \alpha} \right) / bs - \frac{1}{1 + \alpha} \right) = \frac{(P_s - r_s R)^2}{(1 + \alpha)bs} \geq 0 \end{aligned}$$

Hence, if $w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha}$, the retailer still chooses P_r^a .

Next consider P_r^b for $\frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w < P_s - \frac{r_s R + bs - P_s}{1 + \alpha}$. The only possible

candidate $P_r = \frac{bs + r_s R + \beta P_s}{1 + \beta}$ as a corner solution is already excluded.

Next consider P_r^d for $P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta}$. As we already show in

case d), for the candidate $P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}$ as a corner solution,

$$\Pi_r(w, P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}) - \Pi_r(w, P_r^d) \leq 0. \text{ Hence, if } P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1 + \beta},$$

the retailer still chooses P_r^d .

Next consider P_r^c for $P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s$. As we already show in case c), for

the candidate $P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}$ as a corner solution,

$$\Pi_r(w, P_r = \frac{r_s R + \alpha P_s}{1 + \alpha}) - \Pi_r(w, P_r^c) \leq 0. \text{ Hence, if } P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s, \text{ the retailer}$$

still chooses P_r^c .

Also, it is straightforward to show that $P_r^a(R) = \frac{r_s R + \alpha P_s}{1 + \alpha}$ is less than or equal to P_s

from (A6). The retailer chooses P_r^b when $\frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w < P_s - \frac{r_s R + bs - P_s}{1 + \alpha}$, so we

$$\text{have } P_r^b = \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} < \frac{P_s}{2} - \frac{r_s R + bs - P_s}{2(1 + \alpha)} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} = P_s.$$

And, the retailer chooses For P_r^c when $P_s - \frac{r_s R + bs - P_s}{1 + \beta} < w \leq P_s$, so we have

$$P_r^c = \frac{w}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} > \frac{P_s}{2} - \frac{r_s R + bs - P_s}{2(1 + \beta)} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} = P_s$$

Proof of Proposition 3.2: From the retailer's response in Lemma1, the manufacturer chooses his optimal combination of w , R and P_s for each segment.

a) with $w \leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha}$, the retailer's strategy is given by $P_r^a(R) = \frac{r_s R + \alpha P_s}{1 + \alpha}$ and

the manufacturer's profit function is:

$$\begin{aligned} \Pi_m(w, R, P_s) &= (w - r_o R) \cdot D(P_r^a, R, P_s) = (w - r_o R) \cdot 1 \\ &\leq \frac{\alpha P_s + r_s R - bs}{1 + \alpha} - r_o R = \frac{r_s - (1 + \alpha)r_o}{1 + \alpha} R + \frac{\alpha P_s - bs}{1 + \alpha} \end{aligned}$$

Case a1: if $\frac{r_o}{r_s} \geq \frac{1}{1 + \alpha}$, the optimal $R = 0$ and $w = \frac{\alpha P_s - bs}{1 + \alpha} \leq 0$

So if $r_s - (1 + \alpha)r_o \leq 0$, the manufacturer does not have a feasible solution in this interval.

Case a2: if $\frac{r_o}{r_s} < \frac{1}{1 + \alpha}$, the profit is strictly increasing in P_s and R , so the

manufacturer chooses $P_s^* = bs$. The highest feasible R is determined by (A6):

$$r_s R \leq P_s \Leftrightarrow R \leq \frac{bs}{r_s}$$

This leads to $R^* = \frac{bs}{r_s}$ and $w^* = \frac{\alpha}{1 + \alpha} bs$.

By (A3), $w^* = \frac{\alpha}{1 + \alpha} bs \geq r_o R = \frac{r_o}{r_s} bs \Leftrightarrow \frac{r_o}{r_s} \leq \frac{\alpha}{1 + \alpha}$. So if $\frac{r_o}{r_s} \leq \frac{\alpha}{1 + \alpha}$, the manufacturer

will choose $P_s^* = bs$, $R^* = \frac{bs}{r_s}$ and $w^* = \frac{\alpha}{1 + \alpha} bs$, which results in a profit of

$$\Pi_m = \left(\frac{\alpha}{1 + \alpha} - \frac{r_o}{r_s} \right) bs.$$

b) For $\frac{\alpha P_s + r_s R - bs}{1 + \alpha} < w < P_s - \frac{r_s R + bs - P_s}{1 + \alpha}$, given the information that the retailer will

choose $P_r^b = \frac{w}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)}$, the manufacturer's profit function is

$$\Pi_m(w, R, P_s) = (w - r_o R) \cdot D(P_s^b, R, P_s) = (w - r_o R) \cdot \frac{bs + r_s R + \alpha P_s - (1 + \alpha)w}{2bs}$$

We proceed in two steps, first, we characterize the optimal wholesale price, $w^*(R, P_s)$,

for given values R and P_s , and next, we find the optimal R and P_s , by embedding

$w^*(R, P_s)$ in the manufacturer's objective function and maximizing it over R and P_s .

The manufacturer's objective is concave in w , so from FOC, we obtain

$$w^*(R, P_s) = \frac{r_o R}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)}$$

By embedding $w^*(R, P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(R, P_s) = \frac{1}{8(1 + \alpha)bs} (bs + \alpha P_s + (r_s - (1 + \alpha)r_o)R)^2$$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s^* = bs$ and

next we determine the feasible R . By (A6), we have $R \leq \frac{P_s}{r_s}$

From the restriction of relevant region, we have

$$\begin{aligned} \frac{\alpha P_s + r_s R - bs}{1 + \alpha} &< w < P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \\ \Leftrightarrow \frac{\alpha P_s + r_s R - bs}{1 + \alpha} &< \frac{r_o R}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} < P_s - \frac{r_s R + bs - P_s}{1 + \alpha} \\ \Leftrightarrow R &< \frac{3bs - \alpha P_s}{r_s - (1 + \alpha)r_o} \text{ and } R < \frac{(4 + \alpha)P_s - 3bs}{3r_s + (1 + \alpha)r_o} \\ \Rightarrow \begin{cases} R < \frac{3bs - \alpha P_s}{r_s - (1 + \alpha)r_o} \text{ and } R < \frac{(4 + \alpha)P_s - 3bs}{3r_s + (1 + \alpha)r_o} & \text{if } \frac{r_o}{r_s} < \frac{1}{1 + \alpha} \\ R < \frac{(4 + \alpha)P_s - 3bs}{3r_s + (1 + \alpha)r_o} & \text{if } \frac{r_o}{r_s} \geq \frac{1}{1 + \alpha} \end{cases} \end{aligned}$$

Let $R_1 = \frac{3bs - \alpha P_s}{r_s - (1 + \alpha)r_o} = \frac{(3 - \alpha)}{r_s - (1 + \alpha)r_o} bs$, $R_2 = \frac{P_s}{r_s} = \frac{bs}{r_s}$, and

$$R_3 = \frac{(4 + \alpha)P_s - 3bs}{3r_s + (1 + \alpha)r_o} = \frac{(1 + \alpha)}{3r_s + (1 + \alpha)r_o} bs$$

It is straightforward to show that $R_1 \geq R_2 \geq R_3$ when $\frac{r_o}{r_s} \geq 1 - \frac{3}{1+\alpha}$. Since $1 - \frac{3}{1+\alpha} < 0$,

so it is always true for $\frac{r_o}{r_s} \geq 1 - \frac{3}{1+\alpha}$.

Case b1: $\frac{r_o}{r_s} \geq \frac{1}{1+\alpha}$

We have $R_2 \geq R_3$, so the condition $R < \frac{(1+\alpha)bs}{3r_s + (1+\alpha)r_o}$ needs to be satisfied. In this

situation, Π_m is nonincreasing in R . Hence, the manufacturer chooses $R^* = 0$,

$P_s^* = bs$, $w^* = \frac{bs}{2}$, and $P_r^* = \frac{3bs}{4}$, which results in a profit of $\Pi_m = \frac{(1+\alpha)}{8}bs$.

Case b2: $\frac{r_o}{r_s} < \frac{1}{1+\alpha}$

The manufacturer chooses the corner solution $R_3 = \frac{(1+\alpha)}{3r_s + (1+\alpha)r_o}bs$. However, if the

manufacturer chooses $R_3 = \frac{(1+\alpha)}{3r_s + (1+\alpha)r_o}bs$, the retailer will choose $P_r = P_s$, which is

the situation under case c. Hence if $\frac{r_o}{r_s} < \frac{1}{1+\alpha}$, the manufacturer will not choose a

solution in b.

c) For $P_s - \frac{r_s R + bs - P_s}{1+\alpha} \leq w \leq P_s - \frac{r_s R + bs - P_s}{1+\beta}$, given the information that the retailer

will choose $P_r = P_s$, the manufacturer's profit function is

$$\begin{aligned} \Pi_m(w, R, P_s) &= (w - r_o R) \cdot D(P_s, R) = (w - r_o R) \cdot \frac{bs - P_s + r_s R}{bs} \\ &\leq \left(P_s - \frac{r_s R + bs - P_s}{1+\beta} - r_o R \right) \cdot \frac{bs - P_s + r_s R}{bs} \end{aligned}$$

In order to solve the optimization problem we proceed in two steps, first, we

characterize the optimal rebate face value $R^*(P_s)$ for a given P_s . The manufacturer's

objective function is concave in R , so from FOC, we obtain

$$R^*(P_s) = \frac{((3+\beta)r_s + (1+\beta)r_o)P_s - (2r_s + (1+\beta)r_o)bs}{2r_s(r_s + (1+\beta)r_o)}$$

By embedding $R^*(P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(P_s) = \frac{(1+\beta)((r_s - r_o)P_s + r_o bs)^2}{4r_s(r_s + (1+\beta)r_o)bs}$$

Case c1: $r_s = r_o$

$$\Pi_m(R, P_s) = \frac{(2+\beta)(P_s - r_s R) - bs}{1+\beta} \cdot \frac{bs - (P_s - r_s R)}{bs}. \text{ Obviously this profit function is}$$

equivalent to $\Pi_m(P'_s = P_s - r_s R) = \frac{(2+\beta)P'_s - bs}{1+\beta} \cdot \frac{bs - P'_s}{bs}$, so issuing rebates will not be

beneficial. Hence, the manufacturer chooses $R^* = 0$, $P_r^* = P_s^* = \frac{(3+\beta)bs}{4+2\beta}$, and

$$w^* = \frac{bs}{2}, \text{ which results in a profit of } \Pi_m = \frac{1+\beta}{8+4\beta} bs.$$

Case c2: $r_s > r_o$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s^* = bs$,

which leads to $R^* = \frac{1+\beta}{2(r_s + (1+\beta)r_o)} bs$, which satisfies $R \leq \frac{bs}{r_s}$ from (A6). So we

obtain $P_r^* = P_s^* = bs$ and $w^* = \frac{r_s + 2(1+\beta)r_o}{2(r_s + (1+\beta)r_o)} bs$, which results in a profit of

$$\Pi_m = \frac{(1+\beta)r_s}{4(r_s + (1+\beta)r_o)} bs$$

d) For $P_s - \frac{r_s R + bs - P_s}{1+\beta} < w \leq P_s$, given the information that the retailer will choose

$P_r = \frac{w}{2} + \frac{bs + r_s R + \beta P_s}{2(1+\beta)}$, the manufacturer's profit function is

$$\Pi_m(w, R, P_s) = (w - r_o R) \cdot \frac{bs - (1+\beta)P_r + r_s R + \beta P_s}{bs} = (w - r_o R) \cdot \frac{bs + r_s R + \beta P_s - (1+\beta)w}{2bs}$$

We proceed in two steps, first, we characterize the optimal wholesale price, $w^*(R, P_s)$,

for a given rebate face value R, and next, we find the optimal R, by embedding

$w^*(R, P_s)$ in the manufacturer's objective function and maximizing it over R and P_s .

The manufacturer's objective is concave in w , so from FOC, we obtain

$$w^*(R, P_s) = \frac{r_o R}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)}$$

By embedding $w^*(R, P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(R, P_s) = \frac{1}{8(1 + \beta)bs} (bs + \beta P_s + (r_s - (1 + \beta)r_o)R)^2$$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s^* = bs$ and

next we determine the feasible R . By (A6), we have $R \leq \frac{bs}{r_s}$.

From the restriction of relevant region, we have

$$\begin{aligned} P_s - \frac{r_s R + bs - P_s}{1 + \beta} &< w \leq P_s \\ \Leftrightarrow P_s - \frac{r_s R + bs - P_s}{1 + \beta} &< \frac{r_o R}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} \leq P_s \\ \Leftrightarrow \frac{(4 + \beta)P_s - 3bs}{3r_s + (1 + \beta)r_o} &< R \leq \frac{(2 + \beta)P_s - bs}{r_s + (1 + \beta)r_o} \end{aligned}$$

We have $\frac{(2 + \beta)bs - bs}{r_s + (1 + \beta)r_o} < \frac{bs}{r_s} \Leftrightarrow \frac{r_o}{r_s} > \frac{\beta}{1 + \beta}$.

Case d1: $\frac{r_o}{r_s} > \frac{\beta}{1 + \beta}$, the manufacturer chooses $R = \frac{(1 + \beta)bs}{r_s + (1 + \beta)r_o}$, which leads to

$$w^* = bs, \quad P_r^* = bs + \frac{r_s}{2(r_s + (1 + \beta)r_o)} bs > P_s = bs, \quad \text{and} \quad \Pi_m = \frac{(1 + \beta)r_s^2}{2(r_s + (1 + \beta)r_o)^2} bs.$$

Case d2: $\frac{r_o}{r_s} \leq \frac{\beta}{1 + \beta}$, the manufacturer chooses $R = \frac{bs}{r_s}$, which leads to

$$w^* = \left(\frac{2 + \beta}{2 + 2\beta} + \frac{r_o}{2r_s} \right) bs, \quad P_r^* = \left(\frac{3(2 + \beta)}{4(1 + \beta)} + \frac{r_o}{4r_s} \right) bs > P_s = bs \quad \text{and}$$

$$\Pi_m = \frac{1}{8(1 + \beta)} \left((2 + \beta - (1 + \beta)\frac{r_o}{r_s})^2 bs \right).$$

By far the optimal strategies of the manufacturer and the retailer have been computed

for every interval.

[Insert Table A.1. here]

A comparison of the resulting profits helps to decide which strategy the manufacturer will eventually to be chosen. Based on the conditions for each candidate strategy set, we draw the following figure to help visualize the potential candidate sets.

[Insert Figure A.1. here]

First, consider the situation where $r_o = r_s$, i.e. no slippage phenomenon, issuing rebates will not help the manufacturer to improve sales or profits. Both cases b1, c1 and d1 satisfies the condition, so we need to compare the manufacturer's profits.

$$\Pi_m^{d1} - \Pi_m^{c1} = \frac{(1+\beta)}{2(2+\beta)^2} \cdot bs - \frac{1+\beta}{4(2+\beta)} bs = \frac{1+\beta}{2(2+\beta)} \left(\frac{1}{2+\beta} - \frac{1}{2} \right) bs < 0$$

$$\Pi_m^{d1} - \Pi_m^{b1} = \frac{(1+\beta)}{2(2+\beta)^2} \cdot bs - \frac{1+\alpha}{8} bs = -\frac{\beta^2 + \alpha(2+\beta)^2}{8(2+\beta)^2} bs < 0$$

$$\Pi_m^{b1} - \Pi_m^{c1} = \frac{1+\alpha}{8} bs - \frac{1+\beta}{4(2+\beta)} bs \leq 0 \Leftrightarrow \beta \geq \frac{2\alpha}{1-\alpha}$$

So if $\beta \geq \frac{2\alpha}{1-\alpha}$, the manufacturer will choose strategy set c1; otherwise he will choose b1.

Next, consider the situation where $\frac{1}{1+\alpha} \leq \frac{r_o}{r_s} < 1$, we need to compare cases b1, c2,

and d1.

$$\Pi_m^{d1} - \Pi_m^{b1} = \frac{(1+\beta)r_s^2}{2(r_s + (1+\beta)r_o)^2} \cdot bs - \frac{1+\alpha}{8} bs \leq 0 \Leftrightarrow \frac{r_o}{r_s} \geq \frac{2}{\sqrt{(1+\alpha)(1+\beta)}} - \frac{1}{1+\beta}$$

Since $\frac{2}{\sqrt{(1+\alpha)(1+\beta)}} - \frac{1}{1+\beta} \leq \frac{1}{1+\alpha}$, so case b1 dominates case d1 when

$$\frac{1}{1+\alpha} \leq \frac{r_o}{r_s} < 1.$$

$$\Pi_m^{d1} - \Pi_m^{c2} = \frac{(1+\beta)r_s^2}{2(r_s + (1+\beta)r_o)^2} \cdot bs - \frac{(1+\beta)r_s}{4(r_s + (1+\beta)r_o)} bs \leq 0 \Leftrightarrow \frac{r_o}{r_s} \geq \frac{1}{1+\beta}$$

So case b1 also dominate case d1 when $\frac{1}{1+\alpha} \leq \frac{r_o}{r_s} < 1$.

$$\Pi_m^{b1} - \Pi_m^{c2} = \frac{1+\alpha}{8} bs - \frac{(1+\beta)r_s}{4(r_s + (1+\beta)r_o)} bs \leq 0 \Leftrightarrow \frac{r_o}{r_s} \leq \frac{1}{1+\alpha} + \frac{\beta-\alpha}{(1+\beta)(1+\alpha)}$$

And we have $\frac{1}{1+\alpha} + \frac{\beta-\alpha}{(1+\beta)(1+\alpha)} < 1 \Leftrightarrow \beta < \frac{2\alpha}{1-\alpha}$

So if $\beta \geq \frac{2\alpha}{1-\alpha}$, the manufacturer will always choose c2; otherwise, if

$\frac{1}{1+\alpha} \leq \frac{r_o}{r_s} \leq \frac{1}{1+\alpha} + \frac{\beta-\alpha}{(1+\beta)(1+\alpha)}$, the manufacturer will choose c2, and if

$\frac{1}{1+\alpha} + \frac{\beta-\alpha}{(1+\beta)(1+\alpha)} < \frac{r_o}{r_s} < 1$, the manufacturer will choose b1.

For the situation $\frac{1}{1+\beta} < \frac{r_o}{r_s} < \frac{1}{1+\alpha}$, we already prove case c2 dominates d1 if

$$\frac{r_o}{r_s} \geq \frac{1}{1+\beta}.$$

Next, consider the situation $\frac{\beta}{1+\beta} < \frac{r_o}{r_s} \leq \frac{1}{1+\beta}$, we already prove case d1 dominates c2

if $\frac{r_o}{r_s} \leq \frac{1}{1+\beta}$.

Next, consider the situation $\frac{r_o}{r_s} \leq \frac{\beta}{1+\beta}$, we first compare case d2 with case c2.

$$\begin{aligned} \Pi_m^{c2} - \Pi_m^{d2} &= \frac{(1+\beta)r_s}{4(r_s + (1+\beta)r_o)} bs - \frac{1}{8(1+\beta)} (2 + \beta - (1+\beta)\frac{r_o}{r_s})^2 bs \leq 0 \\ &\Leftrightarrow (1 - (1+\beta)\frac{r_o}{r_s}) \left[(\beta - (1+\beta)\frac{r_o}{r_s})^2 - 2 - 2(1+\beta)\frac{r_o}{r_s} \right] \leq 0 \end{aligned}$$

So case d2 dominates c2 if $\frac{r_o}{r_s} \leq \frac{\beta}{1+\beta}$

Last we compare case d2 with case a2 at $\frac{r_o}{r_s} \leq \frac{\alpha}{1+\alpha}$,

$$\begin{aligned}\Pi_m^{a2} - \Pi_m^{d2} &= \left(\frac{\alpha}{1+\alpha} - \frac{r_o}{r_s}\right)bs - \frac{1}{8(1+\beta)} \left((2+\beta - (1+\beta)\frac{r_o}{r_s})^2\right)bs < 0 \\ \Leftrightarrow -(1+\beta)^2 \left(\frac{r_o}{r_s}\right)^2 - 2(1+\beta)(2-\beta)\frac{r_o}{r_s} + 8(1+\beta)\frac{\alpha}{1+\alpha} - (2+\beta)^2 &< 0\end{aligned}$$

By embedding $\frac{\alpha}{1+\alpha} \leq \frac{1}{2}$, we can get $8(1+\beta)\frac{\alpha}{1+\alpha} - (2+\beta)^2 \leq 4(1+\beta) - (2+\beta)^2 < 0$.

So strategy in case d2 dominates the one in case a2. Therefore, for the segment

$0 \leq \frac{r_o}{r_s} \leq \frac{\beta}{1+\beta}$, the manufacturer chooses the optimal strategy set in case d2.

Proof of Proposition 3.4:

For the centralized channel, where the manufacturer owns the retailer, the manufacturer chooses his optimal combination of (P_r, R, P_s) for each segment of the kinked demand function.

(a) For $P_r \leq \frac{r_s R + \alpha P_s}{1 + \alpha}$, the manufacturer's profit function is:

$$\Pi_m(P_r, R, P_s) = (P_r - r_o R) \cdot 1 \leq \frac{r_s R + \alpha P_s}{1 + \alpha} - r_o R = \frac{r_s - (1 + \alpha)r_o}{1 + \alpha} R + \frac{\alpha P_s}{1 + \alpha}$$

Case a1: if $\frac{r_o}{r_s} \geq \frac{1}{1 + \alpha}$, the optimal $R^* = 0$, $P_r^* = \frac{\alpha}{1 + \alpha} b_s$ and $P_s^* = b_s$, which results

in a profit $\Pi_m = \frac{\alpha}{1 + \alpha} b_s$.

Case a2: if $\frac{r_o}{r_s} < \frac{1}{1 + \alpha}$, the profit is strictly increasing in P_s and R , so the

manufacturer chooses $P_s^* = b_s$. The highest feasible R is determined by (A6):

$$r_s R \leq P_s \Leftrightarrow R \leq \frac{b_s}{r_s}$$

This leads to $R^* = \frac{b_s}{r_s}$ and $P_r^* = b_s$, which results in a profit

$$\Pi_m = \frac{r_s - (1 + \alpha)r_o}{1 + \alpha} \cdot \frac{b_s}{r_s} + \frac{\alpha}{1 + \alpha} b_s = (1 - \frac{r_o}{r_s}) b_s$$

(b) For $\frac{r_s R + \alpha P_s}{1 + \alpha} < P_r < P_s$, the manufacturer's profit function is

$$\Pi_m(P_r, R, P_s) = (P_r - r_o R) \cdot D(P_r^b, R, P_s) = (P_r - r_o R) \cdot \frac{b_s - (1 + \alpha)P_r + r_s R + \alpha P_s}{b_s}$$

We proceed in two steps, first, we characterize the optimal retail price, $P_r^*(R, P_s)$, for

given values R and P_s , and next, we find the optimal R and P_s , by embedding

$P_r^*(R, P_s)$ in the manufacturer's objective function and maximizing it over R and P_s .

The manufacturer's objective is concave in P_r , so from FOC, we obtain

$$P_r^*(R, P_s) = \frac{r_o R}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)}$$

By embedding $P_r^*(R, P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(R, P_s) = \frac{1}{4(1 + \alpha)bs} (bs + \alpha P_s + (r_s - (1 + \alpha)r_o)R)^2$$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s = bs$ and

next we determine the feasible R . By (A6), we have $R \leq \frac{P_s}{r_s}$

From the restriction of relevant region, we have

$$\begin{aligned} & \frac{r_s R + \alpha P_s}{1 + \alpha} < P_r < P_s \\ \Leftrightarrow & \frac{r_s R + \alpha P_s}{1 + \alpha} < \frac{r_o R}{2} + \frac{bs + r_s R + \alpha P_s}{2(1 + \alpha)} < P_s \\ \Rightarrow & \begin{cases} R < \frac{bs - \alpha P_s}{r_s - (1 + \alpha)r_o} \text{ and } R < \frac{(2 + \alpha)P_s - bs}{r_s + (1 + \alpha)r_o} & \text{if } \frac{r_o}{r_s} < \frac{1}{1 + \alpha} \\ R < \frac{(2 + \alpha)P_s - bs}{r_s + (1 + \alpha)r_o} & \text{if } \frac{r_o}{r_s} \geq \frac{1}{1 + \alpha} \end{cases} \end{aligned}$$

Let $R_1 = \frac{bs - \alpha P_s}{r_s - (1 + \alpha)r_o} = \frac{(1 - \alpha)bs}{r_s - (1 + \alpha)r_o}$, $R_2 = \frac{P_s}{r_s} = \frac{bs}{r_s}$, and

$$R_3 = \frac{(2 + \alpha)P_s - bs}{r_s + (1 + \alpha)r_o} = \frac{(1 + \alpha)bs}{r_s + (1 + \alpha)r_o}$$

It is straight forward to show that $R_1 \geq R_2 \geq R_3$ when $\frac{r_o}{r_s} \geq \frac{\alpha}{1 + \alpha}$.

Case b1: $\frac{r_o}{r_s} \geq \frac{1}{1 + \alpha}$

We have $R_2 \geq R_3$, so the condition $R < \frac{(1 + \alpha)bs}{r_s + (1 + \alpha)r_o}$ needs to be satisfied. In this

situation, Π_m is nonincreasing in R . Hence, the manufacturer chooses $R^* = 0$,

$P_s^* = bs$ and $P_r^* = \frac{bs}{2}$, which results in a profit of $\Pi_m = \frac{(1 + \alpha)}{4}bs$.

Case b2: $\frac{\alpha}{1+\alpha} \leq \frac{r_o}{r_s} < \frac{1}{1+\alpha}$

In this situation, Π_m strictly increasing in R and $R_1 \geq R_2 \geq R_3$. So the

manufacturer chooses the corner solution $R_3 = \frac{(1+\alpha)bs}{r_s + (1+\alpha)r_o}$. However, if the

manufacturer chooses $R_3 = \frac{(1+\alpha)bs}{r_s + (1+\alpha)r_o}$, we have $P_r = P_s$, which is the case under c.

Hence, if $\frac{\alpha}{1+\alpha} \leq \frac{r_o}{r_s} < \frac{1}{1+\alpha}$, the manufacturer does not have a feasible solution in case

b.

Case b3: $\frac{r_o}{r_s} < \frac{\alpha}{1+\alpha}$

In this situation, Π_m strictly increasing in R and $R_1 < R_2 < R_3$. So the

manufacturer chooses the corner solution $R_1 = \frac{(1-\alpha)bs}{r_s - (1+\alpha)r_o}$. However, if the

manufacturer chooses $R_1 = \frac{(1-\alpha)bs}{r_s - (1+\alpha)r_o}$, we have $D=1$, which is the case under a.

Hence, if $\frac{r_o}{r_s} < \frac{\alpha}{1+\alpha}$, the manufacturer does not have a feasible solution in case b.

(c) For $P_r = P_s$, the manufacturer's profit function is

$$\Pi_m(R, P_s) = (P_s - r_o R) \cdot \frac{bs - P_s + r_s R}{bs}$$

In order to solve the optimization problem we proceed in two steps, first, we

characterize the optimal rebate face value $R^*(P_s)$ for a given P_s . The manufacturer's

objective function is concave in R , so from FOC, we obtain

$$R^*(P_s) = \frac{(r_s + r_o)P_s - r_o bs}{2r_o r_s}$$

By embedding $R^*(P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(P_s) = \frac{((r_s - r_o)P_s + r_o bs)^2}{4r_o r_s bs}$$

Case c1: $r_s = r_o$

$\Pi_m(R, P_s) = (P_s - r_s R) \cdot \frac{bs - (P_s - r_s R)}{bs}$. Obviously this profit function is equivalent to

$\Pi_m(P'_s = P_s - r_s R) = P'_s \cdot \frac{bs - P'_s}{bs}$, so issuing rebates will not be beneficial. Hence, the

manufacturer chooses $R^* = 0$ and $P_r^* = P_s^* = \frac{bs}{2}$, which results in a profit of

$$\Pi_m = \frac{bs}{4}.$$

Case c2: $r_s > r_o$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s^* = bs$,

which leads to $R^* = \frac{bs}{2r_o}$. By (A6), we have $R \leq \frac{bs}{r_s}$. So we have $\frac{bs}{2r_o} \leq \frac{bs}{r_s} \Rightarrow \frac{r_o}{r_s} \geq \frac{1}{2}$.

Case c2-1: $\frac{1}{2} \leq \frac{r_o}{r_s} < 1$

The manufacturer chooses $R^* = \frac{bs}{2r_o}$ and $P_r^* = P_s^* = bs$, which results in a profit of

$$\Pi_m = \frac{r_s}{4r_o} bs.$$

Case c2-2: $\frac{r_o}{r_s} < \frac{1}{2}$

Similarly, it is easy to show that Π_m is strictly increasing in R as long as $\frac{r_o}{r_s} < 1$.

So the manufacturer chooses $R^* = \frac{bs}{r_s}$ and $P_r^* = P_s^* = bs$, which results in a profit of

$\Pi_m = (1 - \frac{r_o}{r_s})bs$. Obviously, c1 is exactly the same with a2 but with a shorter covering

region. So we can omit case c2-2.

(d) For $P_s < P_r < \frac{bs + r_s R + \beta P_s}{1 + \beta}$, the manufacturer's profit function is

$$\Pi_m(P_r, R, P_s) = (P_r - r_o R) \cdot \frac{bs - (1 + \beta)P_r + r_s R + \beta P_s}{bs}$$

We proceed in two steps, first, we characterize the optimal retail price, $P_r^*(R, P_s)$, for given values R and P_s , and next, we find the optimal R and P_s , by embedding

$P_r^*(R, P_s)$ in the manufacturer's objective function and maximizing it over R and P_s .

The manufacturer's objective is concave in P_r , so from FOC, we obtain

$$P_r^*(R, P_s) = \frac{r_o R}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)}$$

By embedding $w^*(R, P_s)$ in the manufacturer's objective function, we have

$$\Pi_m(R, P_s) = \frac{1}{4(1 + \beta)bs} (bs + \beta P_s + (r_s - (1 + \beta)r_o)R)^2$$

Π_m is strictly increasing in P_s . Hence, the manufacturer will choose $P_s = bs$ and

next we determine the feasible R . By (A6), we have $R \leq \frac{bs}{r_s}$.

From the restriction of relevant region, we have

$$\begin{aligned} P_s < P_r &< \frac{bs + r_s R + \beta P_s}{1 + \beta} \\ \Leftrightarrow P_s &< \frac{r_o R}{2} + \frac{bs + r_s R + \beta P_s}{2(1 + \beta)} < \frac{bs + r_s R + \beta P_s}{1 + \beta} \\ \Leftrightarrow R &> \frac{(2 + \beta)P_s - bs}{r_s + (1 + \beta)r_o} \end{aligned}$$

Hence, $\frac{(2 + \beta)bs - bs}{r_s + (1 + \beta)r_o} < R \leq \frac{bs}{r_s}$, which implies the condition $\frac{(1 + \beta)bs}{r_s + (1 + \beta)r_o} < \frac{bs}{r_s}$ needs

to be satisfied; otherwise there is no feasible solution. So we have

$$\frac{(1 + \beta)bs}{r_s + (1 + \beta)r_o} < \frac{bs}{r_s} \Rightarrow \frac{r_o}{r_s} > \frac{\beta}{1 + \beta}$$

So if $\frac{r_o}{r_s} > \frac{\beta}{1 + \beta}$, manufacturer chooses $P_s = bs$, $R = \frac{P_s}{r_s}$, and $P_r^* = (\frac{2 + \beta}{2 + 2\beta} + \frac{r_o}{2r_s})bs$,

which result in a profit of $\Pi_m = \frac{1}{4(1+\beta)} ((2+\beta - (1+\beta)\frac{r_o}{r_s})^2 bs$.

(e) For $P_r \geq \frac{bs + r_s R + \beta P_s}{1+\beta}$, the manufacturer cannot achieve positive profits.

By far the optimal strategies of the manufacturer and the retailer have been computed for every interval.

[Insert Table A.2 here]

A comparison of the resulting profits helps to decide which strategy the manufacturer will eventually to be chosen.

[Insert Figure A.2 here]

First, when $r_o = r_s$, it is obvious case c1 is dominated by case b1. So we can combine

segments $r_o = r_s$ and $\frac{1}{2} \leq \frac{r_o}{r_s} < 1$ together.

Next, for region $\frac{r_o}{r_s} \geq \frac{1}{1+\alpha}$, we need to compare cases a1, b1, c2-1 and d.

$\Pi_l^{a1} - \Pi_l^{b1} = \frac{\alpha}{1+\alpha} bs - \frac{(1+\alpha)}{4} bs = -\frac{(1-\alpha)^2}{4(1+\alpha)} bs \leq 0$. This implies that without rebate

promotion, the profit with a lower retail price to cover all consumer segments is less

profitable than a higher retail price to cover only a portion of the whole market.

$\Pi_l^{c2-1} - \Pi_l^{b1} = \frac{r_s}{4r_o} bs - \frac{(1+\alpha)}{4} bs = (\frac{r_s}{r_o} - (1+\alpha)) \frac{bs}{4} \leq 0$

$\Pi_l^d - \Pi_l^{b1} = \frac{1}{4(1+\beta)} (2+\beta - (1+\beta)\frac{r_o}{r_s})^2 bs - \frac{(1+\alpha)}{4} bs \leq 0 \Leftrightarrow \frac{r_o}{r_s} \geq 1 + \frac{1}{1+\beta} - \sqrt{\frac{1+\alpha}{1+\beta}}$

So we need to prove $\frac{1}{1+\alpha} \geq 1 + \frac{1}{1+\beta} - \sqrt{\frac{1+\alpha}{1+\beta}}$,

Let $f(x) = \sqrt{\frac{1+\alpha}{x}} - \frac{1}{x} - \frac{\alpha}{1+\alpha}$ with $x \in (1+\alpha, 2]$.

$$\Rightarrow f'(x) = -\frac{1}{2} \sqrt{\frac{1+\alpha}{x^3}} + \frac{1}{x^2} = \frac{1}{x^2} \left(1 - \frac{1}{2} \sqrt{(1+\alpha)x}\right) \geq 0 \Rightarrow f(x=1+\beta) > f(x=1+\alpha) = 0$$

Hence, we have proved strategy in case b1 dominates the rest when $\frac{1}{1+\alpha} \leq \frac{r_o}{r_s} < 1$.

Next, consider the situation where $\frac{1}{2} \leq \frac{r_o}{r_s} < \frac{1}{1+\alpha}$, we need to compare case a2, c2-1

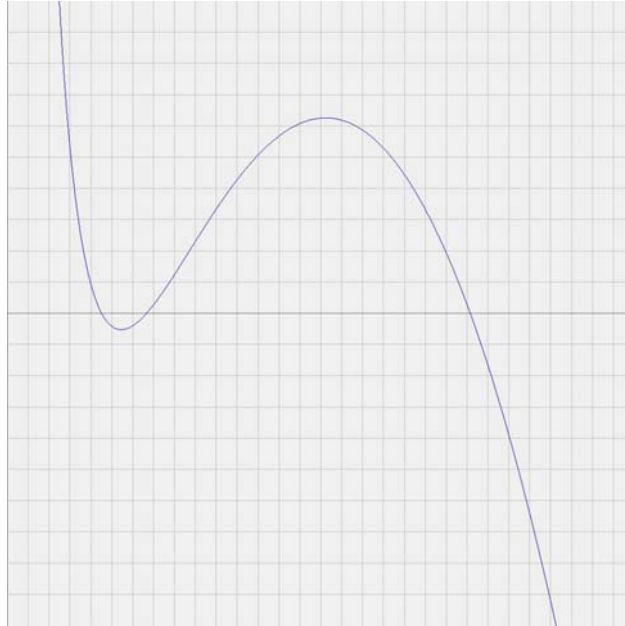
and d.

$$\prod_I^{a2} - \prod_I^{c2-1} = \left(1 - \frac{r_o}{r_s}\right)bs - \frac{r_s}{4r_o}bs = -\frac{(2r_o - r_s)^2}{4r_sr_o} \leq 0$$

$$\prod_I^{a2} - \prod_I^d = \left(1 - \frac{r_o}{r_s}\right)bs - \frac{1}{4(1+\beta)} \left((2+\beta - (1+\beta)\frac{r_o}{r_s})^2 bs\right) \leq 0$$

$$\Leftrightarrow 4\left(1 - \frac{r_o}{r_s}\right)(1+\beta) - (2+\beta - (1+\beta)\frac{r_o}{r_s})^2 \leq 0 \Leftrightarrow -(\beta - (1+\beta)\frac{r_o}{r_s})^2 \leq 0$$

$$\prod_I^{c2-1} - \prod_I^d = \frac{r_s}{4r_o}bs - \frac{1}{4(1+\beta)} \left((2+\beta - (1+\beta)\frac{r_o}{r_s})^2 bs\right)$$



It is easy to verify that $\frac{r_o}{r_s} = \frac{1}{1+\beta} \Rightarrow \Pi_I^{c1} - \Pi_I^d = 0$, $\frac{r_o}{r_s} = \frac{1}{2} \Rightarrow \Pi_I^{c21} - \Pi_I^d \leq 0$, and

$\frac{r_o}{r_s} = 1 \Rightarrow \Pi_I^{c2-1} - \Pi_I^d \geq 0$. So if $\frac{1}{2} \leq \frac{r_o}{r_s} \leq \frac{1}{1+\beta}$, the manufacturer chooses case d;

otherwise if $\frac{1}{1+\beta} < \frac{r_o}{r_s} < \frac{1}{1+\alpha}$, he chooses case c2-1.

Last, for the situation where $\frac{\beta}{1+\beta} < \frac{r_o}{r_s} < \frac{1}{2}$, case d dominates case a2.

Proof of Lemma 4.1:

Taking the derivatives with respect to Q , R , and e , respectively, we get

$$\frac{\partial \Pi_I}{\partial Q} = (p - r_o R - c) - (p - r_o R)F\left(\frac{Q}{e} - ar_s R\right)$$

$$\frac{\partial \Pi_I}{\partial R} = -r_o Q + r_o e \int_0^{\frac{Q}{e} - ar_s R} F(y) dy + ar_s e (p - r_o R) F\left(\frac{Q}{e} - ar_s R\right)$$

Because $\frac{\partial \Pi_I}{\partial Q}$ strictly decreases with Q and $\frac{\partial \Pi_I}{\partial R}$ strictly decreases with R ,

Π_I is strictly concave in both Q and R .

$$\begin{aligned} \frac{\partial \Pi_I}{\partial e} &= -(p - r_o R) \left(\int_0^{\frac{Q}{e} - ar_s R} F(y) dy + e \cdot F\left(\frac{Q}{e} - ar_s R\right) \cdot \left(-\frac{Q}{e^2}\right) \right) - \frac{\partial}{\partial e} V(e) \\ &= (p - r_o R) \left(\frac{Q}{e} F\left(\frac{Q}{e} - ar_s R\right) - \int_0^{\frac{Q}{e} - ar_s R} F(y) dy \right) - \frac{\partial}{\partial e} V(e) \\ \Rightarrow \frac{\partial^2 \Pi_I}{\partial e^2} &= -\frac{Q^2}{e^3} (p - r_o R) f\left(\frac{Q}{e} - ar_s R\right) - \frac{\partial^2}{\partial e^2} V(e) \end{aligned}$$

Because $V(e)$ is convex in e , so Π_I is also strictly concave in e .

Proof of Theorem 4.4:

With the optimal choices of the retailer, the manufacturer's profit function follows as

$$\Pi_m^b(w, R) = (w - c - r_o R)Q^b(w, R) = (w - c - r_o R)(ar_s R + \bar{Q}^b(w, R)) \cdot e^b(w, R),$$

where

$$\begin{aligned} \frac{\partial}{\partial e} V(e) \big|_{e=e^b(w, R)} &= a(p - w)r_s R + (p - r_o R) \int_0^{\bar{Q}^b(w, R)} y dF(y), \\ \Rightarrow e^b(w, R) &= b^{\frac{1}{1-k}} \left\{ \frac{1}{k} a(p - w)r_s R + (p - r_o R) \int_0^{\bar{Q}^b(w, R)} y dF(y) \right\}^{\frac{1}{k-1}} \end{aligned}$$

which imply the manufacturer's functions can always be written as a form of

$\Pi_m^b(w, R) = Z_1(w, R) \cdot b^{\frac{1}{1-k}}$. So the manufacturer's optimal choices (w^b, R^b) are not affected by the value of b . The retailer's profit function is uniquely determined by the promotional effort level, hence,

$$\begin{aligned} \Pi_r^b(Q^b(w^b, R^b), e^b(w^b, R^b)) &= e^b(w^b, R^b) \frac{\partial}{\partial e} V(e) \big|_{e=e^b(w^b, R^b)} - V(e^b(w^b, R^b)) \\ &= b(k-1)b^{\frac{k}{1-k}} \left\{ \frac{1}{k} a(p - w^b)r_s R^b + (p - r_o R^b) \int_0^{\bar{Q}^b(w^b, R^b)} y dF(y) \right\}^{\frac{k}{k-1}} \\ &= b^{\frac{1}{1-k}} \cdot (k-1) \left\{ \frac{1}{k} a(p - w^b)r_s R^b + (p - r_o R^b) \int_0^{\bar{Q}^b(w^b, R^b)} y dF(y) \right\}^{\frac{k}{k-1}} \end{aligned}$$

So the retailer's profit can be written as a form of $\Pi_r^b(w^b, R^b) = Z_2(w^b, R^b) \cdot b^{\frac{1}{1-k}}$.

Similarly, the integrated channel profit can also be represented by

$$\Pi_I(R^I) = Z_3(R^I) \cdot b^{\frac{1}{1-k}}. \text{ Therefore, } \frac{\Pi_m^b + \Pi_r^b}{\Pi_I} = \frac{Z_1(w^b, R^b) + Z_2(w^b, R^b)}{Z_3(R^I)}.$$

Proof of Lemma 4.2:

First we prove it by contradiction. For any given R , we assume $Q^b(w^b(R), R) \geq Q^I(R)$.

Because $\partial \Pi_r^b(Q, e)/\partial e$ has the exact form of $\partial \Pi_I(Q, e, R)/\partial e$ as follows

$$\begin{aligned} Z(Q, e) &= (p - r_o R) \left(\frac{Q}{e} \cdot F\left(\frac{Q}{e} - ar_s R\right) - \int_0^{\frac{Q}{e} - ar_s R} F(y) dy \right) - \frac{\partial}{\partial e} V(e) \\ \Rightarrow \frac{\partial e}{\partial Q} &= - \frac{\partial Z / \partial Q}{\partial Z / \partial e} = - \frac{(p - r_o R) \frac{Q}{e^2} f\left(\frac{Q}{e} - ar_s R\right)}{\left\{ (p - r_o R) \frac{Q^2}{e^3} \cdot f\left(\frac{Q}{e} - ar_s R\right) + \frac{\partial^2}{\partial e^2} V(e) \right\}} > 0. \end{aligned}$$

Hence, we can get $e^b(w, R) \geq e^I(R)$. Since $V(e)$ is convex, so

$$\left. \frac{\partial}{\partial e} V(e) \right|_{e=e^b(w, R)} \geq \left. \frac{\partial}{\partial e} V(e) \right|_{e=e^I(R)}. \text{ And, the first order condition of optimal promotional}$$

effort can be denoted by $\frac{\partial}{\partial e} V(e) = (p - r_o R) \left((ar_s R + \bar{Q}) \cdot F(\bar{Q}) - \int_0^{\bar{Q}} F(y) dy \right) = Z(\bar{Q})$. It is

easy to show that $Z(\bar{Q})$ is strictly increasing with the variable \bar{Q} . Hence, we should

have $\bar{Q}^b(w^b(R), R) \geq \bar{Q}^I(R)$. However,

$$\bar{Q}^b(w^b(R), R) = F^{-1}\left(\frac{p - w^b(R)}{p - r_o R}\right) < F^{-1}\left(\frac{p - r_o R - c}{p - r_o R}\right) = \bar{Q}^I(R). \text{ Thus, we prove}$$

$$Q^b(w^b(R), R) < Q^I(R).$$

Alternatively, $Q^b(w^b(R), R) < Q^I(R)$ may be proved as follows by taking

$\Pi_r^b(w^b(R), R) < \Pi_I(R)$ for granted. For any given R , we have

$$\Pi_r^b(Q^b(w^b(R), R), e^b(w^b(R), R)) = e^b(w^b(R), R) \frac{\partial}{\partial e} V(e) \Big|_{e=e^b(w^b(R), R)} - V(e^b(w^b(R), R))$$

$$\Pi_I(Q^I, R, e^I(R)) = e^I(R) \frac{\partial}{\partial e} V(e) \Big|_{e=e^I(R)} - V(e^I(R))$$

Because of $\Pi_r^b(w^b(R), R) < \Pi_I(R)$, from the proof in theorem 4.2., we can get

$e^b(w^b(R), R) < e^I(R)$. Hence, for any $w^b(R) > r_o R + c$, the following condition holds

$$Q^b(w^b(R), R) = (ar_s R + \bar{Q}^b(w^b(R), R)) e^b(w^b(R), R) < (ar_s R + \bar{Q}^I(R)) e^I(R) = Q^I(R).$$

Proof of Theorem 4.5:

First we prove that $\frac{S(Q, R, e^I)}{Q}$ is strictly decreasing in Q .

$$\begin{aligned}\frac{\partial}{\partial Q} \left(\frac{S(Q, R, e^I)}{Q} \right) &= \frac{e}{Q^2} \int_0^{\frac{Q - ar_s R}{e}} F(y) dy - \frac{1}{Q} F\left(\frac{Q - ar_s R}{e}\right) < 0 \\ \Leftrightarrow \int_0^{\frac{Q - ar_s R}{e}} F(y) dy &< \frac{Q}{e} F\left(\frac{Q - ar_s R}{e}\right) \\ \Leftrightarrow \frac{1}{e} F\left(\frac{Q - ar_s R}{e}\right) &< \frac{1}{e} F\left(\frac{Q - ar_s R}{e}\right) + \frac{Q}{e^2} f\left(\frac{Q - ar_s R}{e}\right)\end{aligned}$$

So $w(Q, R)$ is indeed a quantity discount schedule for any $k_2 \geq 0$.

With quantity discount and buy-back contract, the retailer's profit function is

$$\begin{aligned}\Pi_r(Q, e) &= -w(Q, R)Q + pS(Q, R, e) + b(R)(Q - S(Q, R, e)) - V(e) \\ &= -cQ + (p - r_o R)S(Q, R, e) - V(e) - k_1(-cQ + (p - r_o R)S(Q, R, e^I)) - k_2\end{aligned}$$

Take the first derivative with respect to e , we have

$$\frac{\partial \Pi_r(Q, e)}{\partial e} = (p - r_o R) \frac{\partial S(Q, R, e)}{\partial e} - \frac{\partial V(e)}{\partial e} = 0$$

Hence, the retailer chooses the optimal effort level e^I . With the chosen optimal effort level,

$$\frac{\partial \Pi_r(Q, e^I)}{\partial Q} = (1 - k_1) \left(-c + (p - r_o R) \frac{\partial S(Q, R, e^I)}{\partial Q} \right).$$

Hence, the retailer also chooses the optimal order quantity Q^I .

Apparently, with the anticipation of the retailers choices, the manufacturer's profit function is

$$\begin{aligned}\Pi_m(R) &= (w(Q^I, R) - c)Q^I - r_o RS(Q^I, R, e^I) - b(R)(Q^I - S(Q^I, R, e^I)) \\ &= -k_1 c Q^I + k_1 (p - r_o R) S(Q^I, R, e^I) + k_2 \\ &= k_1 \Pi_r(Q^I, R, e^I) + k_2 + k_1 V(e^I)\end{aligned}$$

Hence, the manufacturer's decision on rebate value is R^I .

Proof of Theorem 4.6:

At the undiscounted price level w_1 , similar to lemma 4.2, we can obtain the optimal order quantity for the retailer satisfies the condition $Q^b(w_1, R_1) < Q^I(R_1)$. Obviously, the manufacturer can always find a R_1 such that $Q^I(R_1) \leq Q^I$, for example, simply by choosing $R_1 = R^I$.

At the discounted price level w_2 , the retailer chooses Q^I as his optimal order quantity. Because

$$\frac{\partial \Pi_r(Q^I, e)}{\partial e} = (p - r_o R_2) \left(\frac{Q^I}{e} \cdot F\left(\frac{Q^I}{e} - ar_s R_2\right) - \int_0^{\frac{Q^I}{e} - ar_s R_2} F(y) dy \right) - \frac{\partial}{\partial e} V(e) = 0 \quad (4.9)$$

So for any given rebate value R_2 , the retailer's promotional decision is not distorted and not related to w_2 , denote by $e^d(R_2)$, which can be solved from (4.9).

Hence, the manufacturer's problem is to maximize the following profit function,

$$\Pi_m(w_2, R_2) = (w_2 - r_o R_2 - c)Q^I,$$

with the constraint that

$$\Pi_r = (p - w_2)Q^I - (p - r_o R_2)e^d(R_2) \int_0^{\bar{Q}^d(R_2)} F(y) dy - V(e^d(R_2)) \geq (1 + \lambda)\Pi_r^b(w_1, R_1)$$

where $\bar{Q}^d(R_2) = \frac{Q^I}{e^d(R_2)} - ar_s R_2$

$$\text{Hence, } w_2 \leq p - \frac{1}{Q^I} \left((1 + \lambda)\Pi_r^b(w_1, R_1) + (p - r_o R_2)e^d(R_2) \int_0^{\bar{Q}^d(R_2)} F(y) dy + V(e^d(R_2)) \right),$$

$$\text{or } w_2 \leq p - \frac{1}{Q^I} \left((1 + \lambda)\Pi_r^b(w_1, R_1) + (p - r_o R_2)Q^I F(\bar{Q}^d(R_2)) - e^d(R_2) \frac{\partial}{\partial e} V(e) \Big|_{e=e^d(R_2)} + V(e^d(R_2)) \right).$$

So the manufacturer's problem is equivalent to maximize

$$\Pi_m(R) = \left\{ (p - r_o R_2 - c)Q^I - (p - r_o R_2)e^d(R_2) \int_0^{\bar{Q}^d(R_2)} F(y) dy - V(e^d(R_2)) \right\} - (1 + \lambda)\Pi_r^b(w_1, R_1).$$

The first term of the above function is in exactly the same form as the integrated

channel. So the manufacturer will announce $R_2 = R^I$. As long as the manufacturer

choose the optimal R^I , the retailer's promotional effort will be adjusted accordingly to the level e^I since the retailer's promotional decision is not distorted. With these optimal choices, the manufacturer's wholesale price is

$$\begin{aligned} w_2 &= \frac{1}{Q^I} \left((r_o R^I + c) Q^I + e^I \frac{\partial}{\partial e} V(e) \Big|_{e=e^I} - V(e^I) - (1 + \lambda) \Pi_r^b(w_1, R_1) \right) \\ &= r_o R^I + c + \frac{\Pi_I - (1 + \lambda) \Pi_r^b(w_1, R_1)}{Q^I} \end{aligned}$$

and his maximum profit is denoted by $\Pi_m^d = \Pi_I - (1 + \lambda) \Pi_r^b(w_1, R_1)$.

However, the discounted wholesale price should be less than the undiscounted one,

i.e., $w_2 < w_1$. Hence,

$$\begin{aligned} w_2 &= p - \frac{1}{Q^I} \left((1 + \lambda) \Pi_r^b(w_1, R_1) + (p - r_o R_2) e^d(R_2) \int_0^{\bar{Q}^d(R_2)} F(y) dy + V(e^d(R_2)) \right) < w_1 \\ \Leftrightarrow \lambda &> \frac{1}{\Pi_r^b(w_1, R_1)} \left\{ (p - w_1) Q^I - (p - r_o R_2) e^d(R_2) \int_0^{\bar{Q}^d(R_2)} F(y) dy - V(e^d(R_2)) \right\} - 1 \end{aligned}$$

Given if the manufacturer chooses a w_1 sufficiently close to the retail price p , the above condition can always be satisfied.

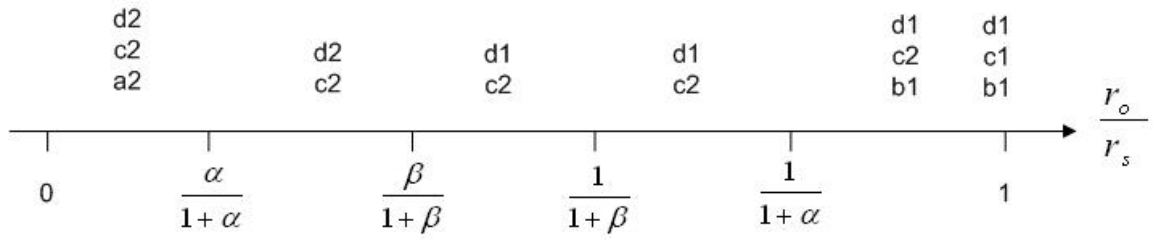


Figure A.1. The Manufacturer's Candidate Strategy Sets in Decentralized Channel

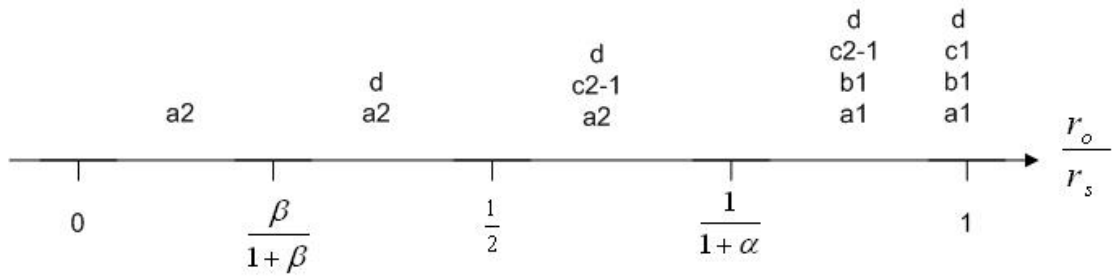


Figure A.2. The Manufacturer's Candidate Strategy Sets in Integrated Channel

	a2	b1	c1	c2	d1	d2
Condition	$\frac{r_o}{r_s} \leq \frac{\alpha}{1+\alpha}$	$\frac{r_o}{r_s} \geq \frac{1}{1+\alpha}$	$\frac{r_o}{r_s} = 1$	$\frac{r_o}{r_s} < 1$	$\frac{r_o}{r_s} > \frac{\beta}{1+\beta}$	$\frac{r_o}{r_s} \leq \frac{\beta}{1+\beta}$
w	$\frac{\alpha}{1+\alpha}bs$	$\frac{bs}{2}$	$\frac{bs}{2}$	$\frac{r_s + 2(1+\beta)r_o}{2(r_s + (1+\beta)r_o)}bs$	bs	$(\frac{2+\beta}{2+2\beta} + \frac{r_o}{2r_s})bs$
R	$\frac{bs}{r_s}$	0	0	$\frac{1+\beta}{2(r_s + (1+\beta)r_o)}bs$	$\frac{(1+\beta)bs}{r_s + (1+\beta)r_o}$	$\frac{bs}{r_s}$
P_s	bs	bs	$\frac{(3+\beta)bs}{4+2\beta}$	bs	bs	bs
P_r	bs	$\frac{3}{4}bs$	$\frac{(3+\beta)bs}{4+2\beta}$	bs	$\frac{3r_s + 2(1+\beta)r_o}{2(r_s + (1+\beta)r_o)}bs$	$(\frac{3(2+\beta)}{4(1+\beta)} + \frac{r_o}{4r_s})bs$
D	1	$\frac{1+\alpha}{4}$	$\frac{1+\beta}{4+2\beta}$	$\frac{(1+\beta)r_s}{2(r_s + (1+\beta)r_o)}$	$\frac{(1+\beta)r_s}{2(r_s + (1+\beta)r_o)}$	$\frac{1}{4}(2+\beta - (1+\beta) \cdot \frac{r_o}{r_s})$
Π_r	$\frac{1}{1+\alpha}bs$	$\frac{1+\alpha}{16}bs$	$\frac{1+\beta}{4(2+\beta)^2}bs$	$\frac{(1+\beta)r_s^2}{4(r_s + (1+\beta)r_o)^2}bs$	$\frac{(1+\beta)r_s^2}{4(r_s + (1+\beta)r_o)^2}bs$	$\frac{1}{16(1+\beta)}(2+\beta - (1+\beta)\frac{r_o}{r_s})^2bs$
Π_m	$(\frac{\alpha}{1+\alpha} - \frac{r_o}{r_s})bs$	$\frac{1+\alpha}{8}bs$	$\frac{1+\beta}{4(2+\beta)}bs$	$\frac{(1+\beta)r_s}{4(r_s + (1+\beta)r_o)}bs$	$\frac{(1+\beta)r_s^2}{2(r_s + (1+\beta)r_o)^2}bs$	$\frac{1}{8(1+\beta)}(2+\beta - (1+\beta)\frac{r_o}{r_s})^2bs$
$\Pi_r + \Pi_m$	$(1 - \frac{r_o}{r_s})bs$	$\frac{3(1+\alpha)}{16}bs$	$\frac{(1+\beta)(3+\beta)}{4(2+\beta)^2}bs$	$\frac{(1+\beta)r_s(2r_s + (1+\beta)r_o)}{4(r_s + (1+\beta)r_o)^2}bs$	$\frac{3(1+\beta)r_s^2}{4(r_s + (1+\beta)r_o)^2}bs$	$\frac{3}{16(1+\beta)}(2+\beta - (1+\beta)\frac{r_o}{r_s})^2bs$

Table A.1. The Candidate Solution Sets in Decentralized Channel

	a1	a2	b1	c1	c2-1	d
Conditio	$\frac{r_o}{r_s} \geq \frac{1}{1+\alpha}$	$\frac{r_o}{r_s} < \frac{1}{1+\alpha}$	$\frac{r_o}{r_s} \geq \frac{1}{1+\alpha}$	$\frac{r_o}{r_s} = 1$	$\frac{1}{2} \leq \frac{r_o}{r_s} < 1$	$\frac{r_o}{r_s} > \frac{\beta}{1+\beta}$
R	0	$\frac{bs}{r_s}$	0	0	$\frac{bs}{2r_o}$	$\frac{bs}{r_s}$
P_s	bs	bs	bs	$\frac{bs}{2}$	bs	bs
P_r	$\frac{\alpha}{1+\alpha}bs$	bs	$\frac{bs}{2}$	$\frac{bs}{2}$	bs	$(\frac{2+\beta}{2+2\beta} + \frac{r_o}{2r_s})bs$
D	1	1	$\frac{1+\alpha}{2}$	$\frac{1}{2}$	$\frac{r_s}{2r_o}$	$\frac{1}{2}(2+\beta-(1+\beta)\frac{r_o}{r_s})$
Π_I	$\frac{\alpha}{1+\alpha}bs$	$(1-\frac{r_o}{r_s})bs$	$\frac{(1+\alpha)}{4}bs$	$\frac{1}{4}bs$	$\frac{r_s}{4r_o}bs$	$\frac{1}{4(1+\beta)}(2+\beta-(1+\beta)\frac{r_o}{r_s})^2bs$

Table A.2. The Candidate Solution Sets in Integrated Channel

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