FINITE ELEMENT MODELING OF ELECTROMAGNETIC AND THERMO-MECHANICAL PHENOMENA IN MICROWAVE AND LASER SYSTEMS

by

RAVINDRA AKARAPU.

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

WASHINGTON STATE UNIVERSITY
Department of Mechanical Engineering

DECEMBER 2003
To the Faculty of Washington State University:

The members of the Committee appointed to examine the thesis of RAVINDRA AKARAPU find it satisfactory and recommend that it be accepted.

______________________________
Chair

______________________________

______________________________
ACKNOWLEDGEMENT

I would like to thank my committee chair and advisor Dr. Ben. Q. Li under whose guidance this research was conducted. He has continuously supported me in times of need providing fresh ideas and useful insights. I would also like to thank my other committee members Dr. Albert Segall and Dr. Juming Tang for being in my committee and providing helpful suggestions.

I would like to thank Dr. Frank Lui, Dr. Slava, Dr. Cheng and Ram Bhuvan Pandit from the Department of Biological Systems Engineering, WSU and Guodong Cai from Pennsylvania State University who provided invaluable support for my research work. I would also like to thank my friends Rajesh Prasannavenkatesan, Jagan Padbidri, Imtiyaz Shaikh, Romit Dhar and my office mates Yuolong Huo, Xin Ai, Yan Shu, Bing Xu, Xiaoming Cui and Kei Okamoto at ETRL 207 for all the helpful and interesting discussions we have had regarding this research. I would also like to thank all other friends in Pullman for their moral support, love and affection.

I would also like to acknowledge the financial support received from the Department of Energy (Grant # DE-FC07-01ID14189), NASA (Grant # NAG8-1693), NSF (Grant # DMI-0093903), WSU IMPACT and School of Mechanical and Materials Engineering, WSU for the completion of my degree.
In this work a finite element model, which can simulate electromagnetic and thermo-mechanical phenomena in real time systems, has been developed. This system has been used to simulate, two varied applications, (1) Microwave heating of food, (2) Laser cutting of ceramics.

An integrated model is developed by coupling the electromagnetic model with thermal module of the thermo-mechanical model to simulate the three-dimensional electromagnetic wave propagation and thermal phenomena during microwave heating of biological materials. The model development is based on the edge finite element formulation for electromagnetic fields and the node finite element formulation for thermal phenomena. The electromagnetic module is validated by comparing well with analytical solutions of established test cases. The sensitivity of the shape of the tetrahedral elements is presented to illustrate the limitation of the present model to accurately predict the temperatures for complex microwave systems. With the model, the heating of whey protein gel in microwave horn applicator and single mode applicator, for
different situations, is extensively studied and results are compared with experiments to some extent. The applicability of the model in designing and optimizing the microwave heating systems is made clear. The basic principles of Laser, their types, propagation and interaction with materials are presented.

Using thermo-mechanical model a full 3-D transient model is developed for the ablation phenomena and thermal stress evolution during laser cutting of ceramic plates. The computational methodology is based on the Galerkin finite element method along with the use of a fixed grid algorithm to treat the thermal ablation resulting from an applied laser source. The present model is able to model any complex ablation operations involving discontinuity in geometries, as encountered in laser cutting and laser drilling operations. This is an advantage over the front tracking method by which the ablation moving interface is precisely tracked in time and which is useful for simple geometries. The laser ablation model is coupled with a thermal stress model to predict the evolution of thermal stresses, which arise due to a rapid change in thermal gradient near the laser beams. The thermal and bending stresses obtained from the stress analyses are post-processed to predict volume and surface fracture probabilities. Model predictions compare well with the available data in literature for a simple configuration. Results obtained from model for both single and dual pulsed laser cutting are discussed.
# TABLE OF CONTENTS

Acknowledgements ......................................................................................... iii

Abstract ........................................................................................................ iv

1. INTRODUCTION........................................................................................................ 1

2. FINITE ELEMENT FORMULATION FOR 3D ELECTROMAGNETIC WAVE SIMULATIONS ........................................................................................................ 6

  2.1. Maxwell’s equation ......................................................................................... 6

  2.2. Finite element formulation ........................................................................... 9

  2.3. Summary ...................................................................................................... 13

3. VALIDATION OF FINITE ELEMENT MODEL ....................................................... 14

  3.1. Semi-infinite Dielectric heating ..................................................................... 14

  3.2. Semi-infinite Magnetic heating ..................................................................... 14

  3.3. Shorted rectangular Waveguide ................................................................... 16

    3.3.1. Loss less Medium .................................................................................. 16

    3.3.2. Lossy Medium ....................................................................................... 17

    3.3.3. Filled with Both Lossy and Lossless Medium ....................................... 19

  3.4. Mesh Shape Sensitivity .............................................................................. 21

  3.5. Summary ...................................................................................................... 22

4. COUPLED THERMAL AND ELECTRO-MAGNETIC ANALYSIS OF MICROWAVE HEATING ........................................................................................................ 24

  4.1. Microwaves ................................................................................................. 24
4.2 Principle behind Microwave Heating .........................................................25
4.3. Different Microwave Applicators .............................................................26
4.4. Simulation of Microwave Heating of Whey Protein Gel .........................27
  4.4.1. Heating with Packet in Air .................................................................28
4.5. Microwave heating of food in single mode cavity applicator .................38
4.6. Advantages of Coupled Modeling for Microwave Heating Systems ......43
4.7. Summary .....................................................................................................43

5. LASERS AND ELECTRO-MAGNETIC ANALYSIS OF LASER BEAMS ......44
  5.1 Lasers and their Types .............................................................................44
  5.2 Electromagnetic Field Modes of Laser ..................................................49
  5.3 Laser Beam Propagation and Interaction with Materials ....................50
  5.4 Summary .....................................................................................................52

6. FINITE ELEMENT FORMULATION OF THERMO-MECHANICAL MODEL
   FOR LASER CUTTING OF CERAMICS ......................................................53
  6.1 Governing Equations .............................................................................53
  6.2 Finite Element Formulation .....................................................................54
  6.3 Treatment of Ablation ............................................................................57
  6.4. Stress calculation and smoothing .........................................................59
  6.5 Fast fracture reliability analysis .............................................................59
    6.5.1 Volume And Surface Flaw Reliability Analysis ...............................60
  6.6 Summary .....................................................................................................64

7. RESULTS OF LASER CUTTING SIMULATIONS ......................................65
  7.1 Comparison with Existing Data ..............................................................66
7.2 Single and Dual Laser Cutting Results ........................................69

7.3. Single Laser drilling .................................................................77

7.4 Fracture Probability Predictions ..............................................80

  7.4.1 Transversely Loaded Circular Disk ......................................80

  7.4.2 Thermal Fracture Prediction for Laser Cutting of Alumina ........81

  7.4.3 Thermal Fracture Prediction during Laser Scanning of Alumina

   with CW Laser ........................................................................82

7.5 Summary .................................................................................84

8. CONCLUSIONS AND FUTURE WORK .......................................85

BIBLIOGRAPHY ...........................................................................88

A. EDGE SHAPE FUNCTIONS FOR TETRAHEDRAL ELEMENT ..........92
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The line plot of modulus of the electric field along the direction of propagation in comparison with the analytical solution ........................................15</td>
</tr>
<tr>
<td>2.</td>
<td>The line plot of modulus of the magnetic field along the direction of penetration in comparison with the analytical solution ...........................................15</td>
</tr>
<tr>
<td>3.</td>
<td>The finite element mesh used for the rectangular wave guide simulations ........16</td>
</tr>
<tr>
<td>4.</td>
<td>The schematic of shorted rectangular wave guide ....................................17</td>
</tr>
<tr>
<td>5.</td>
<td>The lines plots of modulus of the electric field along the axis of the shorted wave-guide, with loss less media ($\varepsilon=2.0$), for 4-12 elements per wavelength in comparison with analytical solution .........................................................18</td>
</tr>
<tr>
<td>6.</td>
<td>The line plot of modulus of the electric field along the axis of the shorted wave-guide, with lossy media ($\varepsilon=2.0+0.2j$), in comparison with analytical solution …18</td>
</tr>
<tr>
<td>7.</td>
<td>The schematic of shorted rectangular waveguide with a discontinuity due to dielectric piece .................................................................19</td>
</tr>
<tr>
<td>8.</td>
<td>The line plot of modulus of the electric field along the axis of the shorted wave-guide, with dielectric piece in later half ($\varepsilon=2.0+0.2j$), in comparison with analytical solution .........................................................20</td>
</tr>
<tr>
<td>9.(a)</td>
<td>The contour plots of modulus of the electric field for the shorted wave-guide for all cases .................................................................21</td>
</tr>
<tr>
<td>9.(b)</td>
<td>The effect of shape of tetrahedral elements on the accuracy of field calculation .................................................................23</td>
</tr>
<tr>
<td>10.</td>
<td>Frequency Spectrum ..............................................................................25</td>
</tr>
</tbody>
</table>
11. Microwave heating ..........................................................26
12. The schematic of microwave horn applicator .......................27
13. The mesh of microwave horn applicator ...............................29
14. The electromagnetic field distribution in a microwave horn with a whey protein gel packet ......................................................30
15. The steady state temperature profiles, full block and block cut at center ........ 31
16. Comparison of experimental measured heating pattern and the calculated temperature distribution, on the center plane, in a Whey Protein Gel package heated in the microwave horn .................................................32
17. The temperature profile after heating whey-protein gel placed in air with 5KW power for 1min using transient analysis .................................33
18. The temperature profile after heating whey-protein gel placed in air with 5KW power for 1min using coupled transient analysis .........................35
19. The mesh used for simulation of food heating in single mode applicator ...........36
20.(a). The front and top views of the single mode applicator used for heating food .................................37
20.(b). The schematic of the single mode applicator ..............................37
21. Temperature band plot of food packet placed in air ..................39
22. Temperature Band plot of food packet placed in water .................41
23. Electron relaxing to lower state .............................................45
24. The working of simple ruby laser ............................................47
25. The electric field variation of different TEM modes .........................50
26. Illustration of procedures used to deactivate the elements and re-assigning the boundary conditions during deactivation of ablated elements .................58
27. Geometric arrangement of lasers and work piece during laser ablation or cutting operations. .................................................................66
28. Schematic of dual laser beam cutting, along with coordinate system used for numerical analyses .........................................................66
29. Normalized temperature Vs Normalized length in comparison with Standard values ...........................................................................67
30. Basic mesh used for laser cutting simulation. ........................................69
31. Computed results for temperature, morphology of ablation fronts during a single laser beam cutting operation ......................................71
32. Evolution of cutting groove shapes during a single laser beam cutting ......71
33. Computed thermal stresses and temperature distribution along the x-direction (Y= 0.1mm ,Z=0.625mm) ahead of the cutting laser during ablation ..........72
34. Effects of laser beam diameters on the laser ablation processes ..................73
35. Computed results for temperature, morphology of ablation fronts during a dual laser beam cutting operation .............................................75
36. Time development of groove shapes ablated ......................................76
37. Computed thermal stresses and temperature distribution along the x-direction (Y= 0.1mm ,Z=0.625mm) ahead of the cutting laser during ablation ..........77
38. Basic mesh used for drill simulations .................................................78
39. The hole morphology .....................................................................79
40. The cross section of groove shapes ...............................................79
41. Temperature and stress distributions ..................................................80
42. Volume Flaw Probability of Transversely Loaded Circular Disk ...............81
43. The mesh used for laser cutting calculations ........................................82
44. The thermal fracture probabilities of elements during the laser cutting ..........83
45. The mesh used for CW laser scanning of ceramic ...................................83
46. The fracture probabilities of elements which fall between (0.008,0.0,Z_{layer})
   and (0.01067,0.00267, Z_{layer}) for different layer of elements .....................84
47. Tetrahedral edge element with edge and node numbering.........................93
LIST OF TABLES

TABLE

PAGE

1. Dielectric properties of Whey-protein gel with temperature…………………………34

2. Properties of parameters for laser cutting simulation …………………………………….76
Dedication

This thesis is dedicated to my parents for their love and affection.
1. INTRODUCTION

Integrated modeling has become valuable during recent years to simulate multi-physics phenomena in real time systems. In this work an integrated model, which can simulate electromagnetic and thermo-mechanical phenomena is developed. This model has been used to model, two very different systems, (1) Microwave heating of food, (2) Laser cutting of ceramics. These two systems are introduced here with the relevant modeling philosophy.

1.1 Microwave Heating

Microwave, as an electro-heating source, has been widely used in the processing industries for packaged foods and other materials. The commonly used household microwave oven is a typical example of microwave heating processes. Other industrial microwave processing systems include large-scale microwave guides and cavities, which are used for the sintering operations as required for making ceramic components or for sterilizing the packaged foods.

Unlike the electromagnetic heating resulting from magnetic induction, which occurs in electrically conducting materials through free electrons, microwave heating is derived from the excitation of dipoles in dielectric or insulating materials. As the microwave penetrates through the dielectric materials, the energy carried by the electromagnetic waves is absorbed by the electrical dipoles of the materials. The heating in the dielectric materials occurs when resonance absorption takes place over certain tuned frequency ranges.

Understanding of the electromagnetic and thermal phenomena in microwave processing systems is of crucial importance for both process design and optimization. Because of the interactive nature of the problems, a predictive model for microwave thermal processing requires the
consideration of both the microwave energy absorption and the thermal responses of the dielectric materials.

In this work, a computational model is presented, with a full integration of 3-D electromagnetic heating and thermal phenomena, for microwave thermal processing applications. The model development is based on the frequency domain formulation of the full 3-D electromagnetic fields described by the Maxwell equations. To ensure the divergence-free requirement of the magnetic field, the vector finite element formulation, which uses the element-edge based vector shape functions, is employed. The electric heating source is then calculated from the field distributions and fed as an input to a thermal model, which is developed based on the traditional finite element formulation that uses the node-based formulation. The model development, self-consistent testing, and comparison of the model predictions with both analytical solutions and experimental measurements taken on microwave heating of packaged foods are presented. Chapters 2-5 deal with the electromagnetic modeling. These chapters deal with Finite element formulation, its validation and implementation in microwave heating and laser beam propagation.

1.2 Laser Cutting of Alumina

The field of ceramics is extremely vast and varied in both composition and material properties. In words of Kingery, ceramics can be defined as materials “which have as their essential component, and are composed in large part of, inorganic nonmetallic materials” [35]. This definition characterizes ceramics as a class of materials entirely different from metals. In general, ceramics are characterized by low strength in tensile loading, brittleness, and susceptibility to thermal and mechanical shock. The brittleness and resulting low tensile strength have limited ceramics use in industry. However, ceramics also have the following favorable properties, even
at high temperature: high hardness and wear resistance, good creep strength, good corrosion resistance, and high electrical resistance [35-39]. Interest in ceramics has increased dramatically during past two decades due to three developments: a high demand for unique materials and properties, improved fabrication processes, and an increased understanding of ceramics behavior and properties. Alumina is a ceramic of special interest.

Alumina, a well known structural ceramic, is often used to sustain mechanical loading and to electrically insulate. Alumina was first commercially used as a spark plug insulator [35]. Improvement of fabrication facilities and techniques, which allow for precise dimensional and grain size control improve quality of alumina. Alumina is currently used in a wide variety of applications due to improvement of fabrication process and an increased understanding of its properties. Some of the uses are high pressure nozzles, high strength insulators, prosthetic hip replacements, spark plug insulators, and thread guides [35]. Alumina is also being extensively used in the field of microelectronics as an electric insulator. The small electronic circuits require alumina’s innate material properties of low conductivity, very smooth surfaces and edges, and chemical inertness.

The need to improve the machining processes of alumina is increasing as technologies advance and the world is becoming “miniaturized”. The demand for small detailed substrates is high, especially in the microelectronics industry. So the substrates must be cut accurately and efficiently without premature fracturing. Finding a process to precisely machine these brittle components is a significant challenge because mechanical forces associated with traditional processing such as milling, turning, grinding, and punching all cause fracture while some cause contamination of alumina due to cutting debris and fluids. Laser machining is a viable alternative in cutting alumina, with advantages over traditional processes.
Lasers have now been widely used as a manufacturing tool for high precision cutting and drilling operations [40-43]. The very basic idea of laser machining comes from the fact that a focused light beam generates a highly localized heating power, by which materials are melted and subsequently blown away by a gas jet. One of the potential advantages with laser machining is that the cutting is a near-zero force operation, thereby eliminating the relatively high force loads and contamination common with traditional machining methods. Such an advantage is of critical importance when it comes to cut the materials inherently brittle such as alumina-based thin ceramic plates. To take a full advantage of these unique near-zero force features, laser machining operations must be designed such that localized laser heating does not generate an infernal spot, around which an extremely high temperature gradient produces intolerable thermal stresses, thereby causing the materials to fracture prematurely before a laser completes its path.

Although a complex phenomenon, premature fractures are related to the tensile stresses caused by a deleterious combination of scrap weight, cooling gases, dross build-up surrounding the erosion-front, and spurious thermal stresses in the vicinity of the cut. Separation burrs and chips result from the mixed-mode crack growth that occurs through the compressively stressed underside and/or from the twisting of the scrap as the uncut and supporting section diminishes. The size and shape of the separation burr and chips will vary depending on a number of factors that include the size and shape of the artifact, laser path, thickness, materials properties, the proximity of laser cuts, and various laser process parameters. Since each manufactured substrate may have multiple cutouts, the entire component must be rejected even if the damage is restricted to one section. Any modifications to the laser machining procedure that controls or
reduces the extent and severity of damage from the laser machining process will be beneficial to both manufacturers and their customers.

A numerical model can be useful in providing a fundamental understanding of thermal and mechanical development ultimately to quantify the fracture mechanism for designing better process to delay or control fracture. When verified against experimental measurements, the model should also be able to be used as valuable tool for optimizing the existing procedures or designing new processes so as to alleviate or eliminate separation burrs/chips during laser machining. The work in this Part B of the thesis presents a finite element model for the ablation phenomena and thermal stress evolution and prediction of volume and surface fracture probabilities during laser cutting or drilling of ceramics plates. Case studies of dual pulse laser cutting and single pulse laser drilling are discussed.

Chapter 6 deals with the detailed formulation of the finite element model rite from the temperature calculation to fracture prediction. Chapter 7 presents all the results and relevant discussions.
2. FINITE ELEMENT FORMULATION FOR 3D ELECTROMAGNETIC WAVE SIMULATIONS

In this chapter a brief introduction is given for the Maxwell’s equations and the boundary conditions existing at the interface of two materials. The detailed finite element discretization is done using “Galerkin Finite Element Method”.

2.1 Maxwell’s Equations

The distribution of static charges produces electric field and steady currents produce magnetic field. The time varying currents produce both time varying electric and magnetic fields. The electric and magnetic fields produced by any of the above cases obey the following differential equations collectively called as Maxwell’s equations. The detailed description of the Maxwell’s equations can be found in standard textbooks [1, 2, and 3].

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday’s Law of Induction)} \quad (2.1)
\]

\[
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_f \quad \text{(Ampere’s Law with Maxwell’s term)} \quad (2.2)
\]

\[
\nabla \cdot \mathbf{B} = 0 \quad \text{(No name)} \quad (2.3)
\]

\[
\nabla \cdot \mathbf{D} = \rho_f \quad \text{(Gauss Law)} \quad (2.4)
\]

where,

\( \mathbf{E} \) is the electric field intensity (volts/meter)

\( \mathbf{D} \) is the electric flux density (coulombs/meter\(^2\))

\( \mathbf{H} \) is the magnetic field intensity (amperes/meter)

\( \mathbf{B} \) is the magnetic flux density (webers/ meter\(^2\))
$J_f$ is the free electric current density (amperes/meter$^2$)

$\rho_f$ is the charge density (coulombs/meter$^3$)

Along with the above Maxwell’s equations it is worthy to recollect the Law of Conservation of Energy.

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

(Law of Conservation of Energy) \hspace{1cm} (2.5)

The Eq (2.1) is Faraday’s law of induction, which says “Changing magnetic field induces electric field”. The Eq (2.2) is Ampere’s law, suitably modified for time varying electromagnetic fields, says “The circulation of Magnetic Field along any closed path represents the electrical current which threads through it”. The Eq (2.4) is Gauss law stating that the surface integral of Electric field around any closed surface gives the charge enclosed in the volume.

For Electro- and Magnetostatic Fields the Maxwell’s equations reduce to

$$\nabla \times E = 0$$ \hspace{1cm} (2.6)

$$\nabla \times H = J_f$$ \hspace{1cm} (2.7)

$$\nabla \cdot B = 0$$ \hspace{1cm} (2.8)

$$\nabla \cdot D = \rho_f$$ \hspace{1cm} (2.9)

It is evident that in this case there is no interaction between the electric and magnetic fields, and therefore, electrostatic case can be separately described by Eq (2.6) & Eq (2.9) or a magnetostatic case described by Eq (2.7) & Eq (2.8).

When the field quantities are harmonically oscillating functions with single frequency, a time-harmonic field is obtained. Using the complex phasor notation [4], Maxwell’s equations can be written as follows,
\( \nabla \times \mathbf{E} + j \omega \mathbf{B} = 0 \) \hspace{1cm} (2.10)

\( \nabla \times \mathbf{H} - j \omega \mathbf{D} = \mathbf{J}_f \) \hspace{1cm} (2.11)

\( \nabla \cdot \mathbf{J} = -j \omega \rho \) \hspace{1cm} (2.12)

\( \nabla \cdot \mathbf{B} = 0 \) \hspace{1cm} (2.13)

\( \nabla \cdot \mathbf{D} = \rho_f \) \hspace{1cm} (2.14)

where \( \mathbf{E}, \mathbf{H}, \mathbf{B}, \mathbf{D}, \mathbf{J}, \rho \) are complex quantities as opposed to the previous equations and these symbols refer to complex fields hereafter.

The three independent equations among the five Maxwell’s equations described above are in indefinite form since the number of unknowns is more than number of equations. Maxwell’s equations become definite when constitutive relations between field quantities are specified. The constitutive relations describe the macroscopic properties of the medium being considered. For a simple medium, they are

\[ \mathbf{D} = \varepsilon \mathbf{E} \] \hspace{1cm} (2.15)

\[ \mathbf{B} = \mu \mathbf{H} \] \hspace{1cm} (2.16)

\[ \mathbf{J} = \sigma \mathbf{E} \] \hspace{1cm} (2.17)

where the constitutive parameters \( \varepsilon, \mu, \) and \( \sigma \) denote, respectively, the permittivity (farads/meter), permeability (henrys/meter), and conductivity (siemens/meter) of the medium. These parameters are tensors for anisotropic media and scalars for isotropic media. For the inhomogeneous media, they are functions of positions, while for homogeneous media they are not. At high frequencies, the inertia of the atomic system causes the polarization vectors \( \mathbf{P} \) and \( \mathbf{M} \) to lag behind the fields. As a result, \( \varepsilon \) and \( \mu \) must be represented by complex quantities in order
to account for this difference in time phase. The real parts will be designated by $\varepsilon', \mu'$ and the imaginary parts by $\varepsilon'', \mu''$; thus

$$\varepsilon = \varepsilon' - j\varepsilon'' \quad (2.18)$$
$$\mu = \mu' - j\mu'' \quad (2.19)$$

The imaginary part of the complex permittivity contributes for dielectric relaxation losses.

The electric and magnetic field quantities will be discontinuous at the boundary between two different media or at the surface, which carries charge density $\sigma$ or current density $K$. These discontinuities, deduced from the integral form of Maxwell’s equations, can be summarized for linear media as

\[
\begin{align*}
(i) \quad & (\varepsilon_1 \mathbf{E}_1 - \varepsilon_2 \mathbf{E}_2) \cdot \mathbf{n} = \sigma_f \\
(ii) \quad & \mathbf{E}_1 \times \mathbf{n} - \mathbf{E}_2 \times \mathbf{n} = 0 \\
(iii) \quad & (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n} = 0 \\
(iv) \quad & \frac{1}{\mu_1} \mathbf{B}_1 \times \mathbf{n} - \frac{1}{\mu_2} \mathbf{B}_2 \times \mathbf{n} = \mathbf{K}_f \times \mathbf{n}
\end{align*}
\]

\[
(2.20)
\]

2.2 Finite Element Formulation

To solve for the electric field or magnetic field for time harmonic electromagnetic waves, described by Eqs (2.10-2.14), the equations are decoupled and inhomogeneous second order differential vector wave equation is formed in electric or magnetic field, Eq (2.21), by taking curl of Eq (2.10) and substituting in Eq (2.11) or vice versa.

\[
\begin{align*}
\nabla \times \left( \frac{1}{\mu_r} \nabla \times \mathbf{E} - k_0^2 \varepsilon_r \mathbf{E} \right) &= -j\omega \mu_0 \mathbf{J} \\
\nabla \times \left( \frac{1}{\mu_r} \nabla \times \mathbf{H} - k_0^2 \varepsilon_r \mathbf{H} \right) &= \frac{1}{\mu_r} \nabla \times \mathbf{J}
\end{align*}
\]

\[
(2.21)
\]

where $\mu_r (=\mu/\mu_0)$ is the relative magnetic permeability, $\varepsilon_r (\varepsilon_r = \varepsilon' + j\varepsilon'' = \varepsilon' - j \sigma/\varepsilon_0 \omega)$ the complex permittivity, and $k_0$ the system parameter, $k_0^2 = \omega^2 \mu_0 \varepsilon_0$. The symbols used are the same as in the
standard electromagnetic textbooks [1-4]. Specifically, \( \mu_0 \) stands for the magnetic permeability of free space, \( \varepsilon_0 \) the permittivity of free space, \( \sigma_e \) the effective conductivity, \( \omega \) the frequency of the harmonic field and \( j=(-1)^{(1/2)} \).

In the present work vector wave equation in electric field has been solved. So the Finite Element Formulation starts with three dimensional vector wave equation in electric field, Eq (2.21). After multiplying the equation by a vector weighting function \( W \) and integrating over the microwave cavity (or computational domain), one obtains the following integral, which is often referred as weak form [5, 6],

\[
\iiint_W W \left( \nabla \times \frac{1}{\mu_r} \nabla \times E - k^2 \varepsilon_r E \right) dV = -\iiint_W j \omega \mu_0 W \cdot J dV
\]  

(2.22)

Making use of the vector Green’s theorem identity,

\[
\iiint_W \left[ \frac{1}{\mu_r} \left( \nabla \times W \right) \cdot \left( \nabla \times E \right) - W \cdot \left( \nabla \times \frac{1}{\mu_r} \nabla \times E \right) \right] \frac{1}{\mu_r} dV = \iiint_S \frac{1}{\mu_r} \left( W \times \nabla \times E \right) \cdot n dS
\]  

(2.23)

the second order differential terms can be integrated by parts,

\[
\iiint_W \left( \nabla \times E \cdot \nabla \times W - k^2 \varepsilon_r W \cdot E \right) dV = \iiint_S \left[ \frac{1}{\mu_r} \left( W \times \nabla \times E \right) \cdot n \right] dS - \iiint_W j \omega \mu_0 W \cdot J dV
\]  

(2.24)

\[
\mathbf{n} \times (\nabla \times E) + \gamma \mathbf{n} \times (\mathbf{n} \times E) = \mathbf{U}^{inc}
\]  

(2.25)
The surface integral in the above Eq (2.24), using the natural boundary condition of the form given in Eq (2.25), can be transformed as

\[
\iiint_V \left( \frac{1}{\mu_r} \nabla \times \mathbf{E} \cdot \nabla \times \mathbf{W} - k^2 \mathbf{E} \cdot \nabla \times \mathbf{W} \right) dV = -\iiint_S \gamma (\mathbf{n} \times \mathbf{E}) \cdot (\mathbf{n} \times \mathbf{W}) + \mathbf{W} \cdot \mathbf{U}^{inc} dS - \iiint_V j_\omega \mathbf{N} \mathbf{J} dV \tag{2.26}
\]

By discretizing the above equation using traditional finite elements obtained by interpolating nodal values, the following three major difficulties are encountered [7].

- The working variable is discontinuous across the interface between different media.
- Occurrence of nonphysical spurious solutions as the divergence condition of electric or magnetic field is not completely satisfied.
- The inability of modeling field singularities at conducting and/or material corners, edges, and tips.

A better choice, for all the above problems, is to use the vector edge-based element shape functions. This approach is taken in the present study. For this purpose, tetrahedral elements are used to discretize the above the computational domain \( V \). Over each tetrahedron, vector electric field is defined along the edges and the vector edge shape functions \( \mathbf{N}_k(\mathbf{r}) \) are constructed as follows [A],

\[
\mathbf{N}_k(\mathbf{r}) = \begin{cases} 
\mathbf{f}_k + \mathbf{g}_k \times \mathbf{r} & \text{r in the tetrahedron} \\
0 & \text{otherwise}
\end{cases}
\tag{2.27}
\]

with
\[ f_k = \frac{b_k}{6V_e} (r_{(7-k)1} \times r_{(7-k)2}) \quad (2.28) \]

\[ g_k = \frac{b_kb_{(7-k)}}{6V_e} e_{(7-k)} \quad (2.29) \]

where \( k = 1, 2, \ldots, 6 \), \( V_e \) = volume of tetrahedron, \( e_k \) = unit vector of the \( k \)th edge and \( b_k \) = length of the \( k \)th edge of the tetrahedron.

With the vector edge shape function so chosen, the electric field \( \mathbf{E} \) within a tetrahedron can be expanded as,

\[ \mathbf{E} = \sum_{i=1}^{6} E_i \mathbf{N}_i \quad (2.30) \]

Substituting this into Eq. (15) and carrying out the necessary numerical integration over an element, one has the following matrix equation for the discretized electric fields defined along the element edges,

\[ [K^e]\{\mathbf{E}^e\} + [B^e]\{\mathbf{U}^{inc^e}\} = \{\mathbf{F}^e\} \quad (2.31) \]

where the matrices are calculated using the following expressions,
Assembling the elemental equations we have the final global matrix equation as follows,

\[
\begin{align*}
K_{ij}^1 &= \iiint_{V_e} \left( \frac{1}{\mu_r} \nabla \times \mathbf{N}_i \cdot \nabla \times \mathbf{N}_j - k^2 \varepsilon_r \mathbf{N}_i \cdot \mathbf{N}_j \right) dV \\
B_{ij}^e &= \iint_{S_e} \mathbf{S}_i \cdot \mathbf{S}_j dS \\
C_{ij} &= \iint_{S_e} \gamma_e (\mathbf{n} \times \mathbf{N}_i) \cdot (\mathbf{n} \times \mathbf{N}_j) dS = \iint_S \gamma_e (\mathbf{S}_i) \cdot (\mathbf{S}_j) dS \\
F_i^e &= -\iiint_{V_e} j\omega\mu_0 \mathbf{N}_i \cdot \mathbf{J} dV \\
K_{ij}^e &= K_{ij}^1 + C_{ij}
\end{align*}
\]

(2.32)

which can be solved, after applying essential boundary condition, for unknown vector fields defined by \( \mathbf{E} \).

### 2.3 Summary

In this chapter a brief discussion on the Maxwell’s equations has been presented and detailed description of finite element formulation for three-dimensional vector wave equation using vector elements is given. In chapter 2 numerical comparisons for some examples with well known analytical solutions have been used to validate the model and study mesh sensitivity. Real time simulations for microwave heating of food, spread of laser beam and prediction of absorption coefficient for laser heating are presented in chapters 3 and 4.
3. VALIDATION OF FINITE ELEMENT MODEL

In previous chapter a detailed formulation of the finite element model for solving 3D vector wave equation has been formulated. In this chapter the finite element model is validated by comparing with some simple examples having well established analytical solutions. The study of mesh sensitivity is also done using these examples.

3.1 Semi-Infinite Dielectric Heating.

The validation of the finite element model starts with a simple model, where a thin dielectric slab of infinite length is irradiated by a microwave signal emanating from microwave horn [8]. The electric field in Z direction, of wave propagating in Y direction, is assumed to be constant in X direction by analyzing thin dielectric. The analytical solution of the electric field, $E_z$, is given by

$$E_z(y) = E_0 e^{-\gamma y} \quad \text{with} \quad \gamma = \sqrt{-\omega^2 \varepsilon_0 \mu (\varepsilon' - j \sigma') / (\omega \varepsilon_0)} \quad (3.1)$$

The comparison of results of finite element code with the analytical solution is given in Fig 1.

3.2 Semi-Infinite Magnetic Heating.

In the case of magnetic induction a semi-infinite case, where it is assumed that the magnetic field decays very sharply as it penetrates in the material so as it is though the material were infinite. The analytical solution of the magnetic field, $H_x$, is given by

$$H_x(y) = H_x e^{-\gamma_m y} \quad \text{with} \quad \gamma_m = \sqrt{2 / \sigma \mu} \quad (3.2)$$

The comparison of results of finite element code with the analytical solution is given in Fig 2.
Figure 1: The line plot of modulus of the electric field along the direction of propagation in comparison with the analytical solution.

Figure 2: The line plot of modulus of the magnetic field along the direction of penetration in comparison with the analytical solution.
3.3 Shorted Rectangular Waveguide [6].

Additional comparison is made against the analytical solution of electromagnetic fields in a shorted wave-guide of rectangular cross section filled with lossless media or lossy media or both. The finite element mesh, of tetrahedral elements, used for these simulations is shown in Fig 3.

![Figure 3: The finite element mesh used for the rectangular wave guide simulations.](image)

3.3.1 Loss less Medium

A shorted rectangular waveguide, shown in Fig 4, filled with lossless material, is excited at one end by a TE$_{10}$ wave and shorted at the other. The analytical solution, for the only existing electric field component $E_y$, is

$$E_y = E_0 \left( \frac{-e^{-2j\beta z}e^{ij\beta y}}{1-e^{-2j\beta z}e^{ij\beta y}} + \frac{1}{1-e^{-2j\beta z}e^{-ij\beta y}} \right)$$  

(3.3)

where,
\[ \beta \] is the wave number of the wave guide.

L is the length of the waveguide.

\[ E_0 \] is the magnitude of the incident field.

This example has been used for studying the mesh sensitivity. The numerical results obtained by using 4-12 elements per wavelength are plotted in comparison with the analytical solution in Fig 5. From the plot it can be concluded that at least of 10 elements per wavelength should be used.

![Figure 4: The schematic of shorted rectangular wave guide.](image)

### 3.3.2 Lossy Medium.

The same shorted rectangular waveguide, previously considered and shown in Fig 4, is now filled with lossy material (dielectric constant \( \varepsilon_r = 3.0 \) and conductivity \( \sigma = 2.0E-02 \)). The comparison of the numerical solution with analytical solution, given in equation 3.3, is shown in Fig 6.
Figure 5: The lines plots of modulus of the electric field along the axis of the shorted waveguide, with loss less media ($\varepsilon=2.0$), for 4-12 elements per wavelength in comparison with analytical solution.

Figure 6: The line plot of modulus of the electric field along the axis of the shorted waveguide, with lossy media ($\varepsilon=2.0+0.2j$), in comparison with analytical solution.
3.3.3 Filled with Both Lossy and Lossless Medium.

The shorted rectangular waveguide, now half filled with the dielectric material (complex permittivity \( \varepsilon = 1.0 + 0.2j \)), is shown in Fig 7.

The general equation for standing electromagnetic wave in discontinuous shorted waveguide is

\[
E_0^y = Ae^{j\beta_{0z}} + Be^{-j\beta_{0z}} \quad \text{in air} \\
E_d^y = Ce^{j\beta_{dz}} + De^{-j\beta_{dz}} \quad \text{in water}
\]  

The boundary conditions are

1. At \( Z = 0 \) \( E_y^0 = E_{\text{inc}}^0 \) implies \( A + B = E_{\text{inc}}^0 \) (incident electric field).
2. At \( Z = L \) \( E_y^d = 0 \) implies \( C = -De^{-j\beta_d L} \) (shorted).
3. At \( Z = L_i \) (interface) \( E_y^0 = E_y^d \) implies \( Ae^{j\beta_{0z}} + Be^{-j\beta_{0z}} = Ce^{j\beta_{dz}} + De^{-j\beta_{dz}} \)
4. At \( Z = L_i \) (interface) \( H_x^0 = H_x^d \) implies \( \beta_0 Ae^{j\beta_{0z}} - \beta_0 Be^{-j\beta_{0z}} = \beta_d Ce^{j\beta_{dz}} - \beta_d De^{-j\beta_{dz}} \)

The Eqs (3.4) & (3.5) when solved with boundary conditions Eq (3.6) analytical solution is obtained. Fig 8 shows the comparison of the numerical and analytical solution. This example is
also used to validate the power absorption calculation. Using $A=1570.0$ (value of the incident field for 1W power) and using 2, 3, 4 of Eqs (3.6) to solve for the total field yields a power absorption of 0.813W in the dielectric piece. The numerical solution gave a result of 0.846W, which is reasonable comparison. The contour plot of the modulus of electric field for all the three cases is shown in Fig 9(a).

![Contour plot of modulus of electric field](image)

**Figure 8:** The line plot of modulus of the electric field along the axis of the shorted wave-guide, with dielectric piece in later half ($\varepsilon=2.0+0.2j$), in comparison with analytical solution.
Figure 9(a): The contour plots of modulus of the electric field for the shorted wave-guide for cases (1) [Max: 1.507, Min: 0], (2) [Max: 1.0415, Min: 0], (3) [Max: 1.147, Min: 0].

3.4 Mesh Shape Sensitivity

The tetrahedral edge elements, used in the above finite element model, are very sensitive to their shape. The condition of the field representation deteriorates when tetrahedral edge elements do not have approximately equal sided lengths [45]. The shape quality of the tetrahedral elements used can be quantitatively measured by [46]

\[
\eta(T) = \frac{12(3v)^{2/3}}{\sum_{0\leq i<j\leq 3} l_{ij}^2}
\]  

(3.7)

where \( \eta \) is the shape quality number, \( v \) is the volume, \( l_{ij} \) is the length of the edge. The quality is 1.00 for tetrahedral with equal edges and the values drops down for distorted one. This shape quality indicator indicates how distorted the element is.
This mesh shape sensitivity is illustrated by calculating the electric field in shorted rectangular wave with lossless medium. The waveguide, 48cm long (Z direction) with cross-section of 8cm (X direction) by 4cm (Y direction) as shown in Fig 4, is excited by TE_{10} at one end. This problem is solved using three meshes

- Mesh1 with 48 division in Z direction, 8 divisions in X direction and 4 divisions in Y direction.
- Mesh2 with 96 division in Z direction, 8 divisions in X direction and 4 divisions in Y direction.
- Mesh3 with 144 division in Z direction, 8 divisions in X direction and 4 divisions in Y direction.

The shape quality number for the typical element (almost all the elements are of same shape in all meshes) in meshes Mesh1, Mesh2 and Mesh3 are 0.84, 0.706, and 0.485 respectively. The electric field plotted along the center line of the waveguide in comparison with the analytical solution given in Eq (3.3) is shown in Figure 9(b). This plot shows how the field is deteriorated with elements of bad shape even if the number of elements is increased.

This simulation illustrates the use of the edge elements with approximately equal sided edges. This can be achieved by unstructured mesh and local refinement as opposed to structured meshing; domain is first divided in to hexahedral elements that are further divided into five tetrahedrons [6], used in this work.
Figure 9(b): The effect of shape of tetrahedral elements on the accuracy of field calculation.

3.5 Summary
In this chapter the finite element code, formulated in the previous chapter, has been validated by comparing with well defined analytical solutions of test problems. Now that some confidence has been established with the model it is used to solve real time problems like microwave heating, modeling of laser beam spreading and absorption coefficient of dielectric for a given laser.
4. COUPLED THERMAL AND ELECTRO-MAGNETIC ANALYSIS
OF MICROWAVE HEATING

In this chapter a brief introduction is given about microwaves, principle behind microwave heating followed by description of different applicators. Finally coupled analysis of microwave heating of whey protein gel in microwave horn applicator is presented.

4.1 Microwaves

Electromagnetic radiation begins with a phenomenon that occurs when electric current flows through a conductor, such as a copper wire. The motion of the electrons through the wire produces a field of energy that surrounds the wire and floats just off its surface. This floating zone or cloud of energy is actually made up of two different fields of energy, one electric and one magnetic. The electric and magnetic waves that combine to form an electromagnetic wave travel at right angles to each other and to the direction of motion. If the current flowing through the wire is made to oscillate at a very rapid rate, the floating electromagnetic field will break free and be launched into space. Then, at the speed of light, the energy will radiate outward in a pulsating pattern, much like the waves in the pond. It is theorized that these waves are made up of tiny packets of radiant energy called photons. Streams of photons, each carrying energy and momentum, travel in waves like an undulating string of cars on a speeding roller coaster. These electromagnetic waves are classified based upon the frequency range. Such a classification is depicted in Fig 10. Looking at the figure microwaves can be defined as waves of electromagnetic radiation that oscillate from approximately 3MHZ to 3GHZ. Since that the microwaves are defined lets move on to the next section describing their generation.
4.2 Principle behind Microwave Heating

The main mechanism due to which microwaves, high frequency electromagnetic waves, heat or dissipate energy in a material is dipolar relaxation [8]. When an electric field is applied to a dielectric material the material gets polarized either by orienting the existing dipoles or newly formed dipoles with the electric field. The polarization is lost by removal of the electric field. As the frequency of the applied electric field is increased the dipoles are unable to restore the original positions during the field reversals and as a consequence the dipolar polarization lags the
applied external electric field. As the frequency increases further a point is reached where the re-orientation polarization fails to follow the applied electric field and contributes less to total polarization. The fall of the effective polarization manifests itself as a fall in the dielectric constant and a rise in the loss factor, which results in dissipation of energy in the dielectric. This is how the microwaves heat the dielectric materials. The amount of heating depends on the loss factor. This loss mechanism is what that helps in heating food by microwaves. The microwave heating mechanism is depicted in Fig 11.

![Microwave heating](image)

**Figure 11: Microwave heating**

### 4.3 Different Microwave Applicators

The microwaves generally are transmitted through special channels called waveguides, the dimensions of which depend upon the operating frequency. The microwave applicators are varied arrangement of the waveguides and other passive microwaves elements like microwave horn radiators and cavities to impart energy to the load. The microwave applicators are generally classified into following categories [8, 10, 11, 12]

**Traveling Wave Applicator:** As the name suggests in these applicators the microwave energy, passed through a well defined geometry like waveguide, is absorbed by well-matched work load in it during one pass of the wave, with residue being dissipated in an absorbing terminal load.
**Single Mode Applicator:** These applicators are designed to operate with single or a few resonant modes, fundamental mode or higher order mode, in a small well defined volume.

**Multimode Cavity Applicator:** The multimode applicators, designed for large volumes (relative to wave length), typically consist of a metallic box and a coupling mechanism, such as a simple iris or a number of apertures forming an array, to couple the microwave energy from the source to metallic enclosure.

**Radiation Applicators:** These applicators operate on basic principle of microwave antenna radiation in which the outgoing/propagating microwave energy propagates freely in the load like free space wave, with out being substantially bounded to certain geometry.

**Microwave Horn Applicator:** These applicators use horn antennas to beam energy into conveyer tunnel, which carries foodstuffs or other packages to be processed.

### 4.4 Simulation of Microwave Heating of Whey Protein Gel

The simulation microwave heating of whey protein gel in a microwave horn applicator, shown in Fig 12, is studied.

![Figure 12: The schematic of microwave horn applicator.](image)

The simulation of heating of whey protein gel placed in the rectangular cavity with air around is done both by transient and coupled analysis and compared with experiments.
4.4.1 Heating with Packet in Air

The electromagnetic analysis is done and the power dissipation is calculated and input as heat generation for the thermal calculations. First a steady state thermal calculation has been done to compare the temperature profiles with experiments. Finally a transient coupled simulation has been done to predict the temperatures after heating for one minute with 5KW power.

A typical design of microwave cavity, which is discretized using the tetrahedral elements, for industrial microwave processing of food packages is shown in Figure 13. The computations used 309,120 edge elements and the direct solvers are used for computations. The simulation of heating of whey protein gel packet (140mm x 90mm x 25mm) is carried using the model.

The vector plot, contour of dominant $E_y$ field, contour of the modulus of the electric field, on the central X-Z plane, for the case where the food packet is placed in the center of the rectangular cavity, is shown in the Figure 14. Looking at the contour plot of modulus of the electric field it can be concluded that the electric field strength first increases and then decreases as we move away from center. The corresponding temperature distribution viewed as a whole field or a cut-through field is shown in Figure 15(a) and (b). Inspection of the temperature distribution illustrates that for this configuration, the temperature is higher at the two ends of the package and lower at the middle edges. This is consistent with the electric energy distribution profile Figure 14 (c) in that the higher energy absorption occurs at the narrow sides of the package and the lower at the elongated sides. Comparison of Figures 15(a) and (b) shows that the highest temperature occurs near the edges on the middle plane (Figure 15(b)). This type of the heating behavior is quite different from induction heating of metals where the highest temperature occurs at the surface facing the exciting coils. Also, the thermal field is symmetric, as expected from the applied conditions and geometric arrangements. The steady state temperature profiles for
different positions of food in cavity are presented in Fig 15 (a)-(j). The contours of temperature for different positions after heating for 1 minute with power of 5KW are shown in Fig 15 (k)-(n). It can be seen that the food material is heated more after moving it 10cm to right.

To validate the computational model predictions, experiments were also conducted using the Whey Protein gel package. The steady state analysis has been done and the obtained temperature profile is compared with the infrared image in Fig 16. The transient temperature analysis, for heating of whey protein gel food packet in air with power of 5KW for about 1 minute, was done and the temperature contour profile is shown in Fig 17(a). The line plot of temperatures of hot and cold spots with time is shown in Fig 17(b). The comparison of the temperatures, on top and middle face of food packet, with experiments is presented in Fig 17 (c)-(f).

Figure 13: The mesh of microwave horn applicator.
Figure 14. The electromagnetic field distribution in a microwave horn with a whey protein gel packet: (a) vector plot of real electric field distribution, (b) contour plot of dominant component, real $E_y$ field (Max : 4000V/m, Min : 2538.2 V/m) (c) contour plot of modulus $\sqrt{E \cdot E^*}$ of the electric field (Max : 4002.1V/m, Min : 0 V/m). Conditions used for simulations: frequency= 915MHz, $E_y$=4000V/m imposed at the opening ports, in-phase at the two ports

To incorporate the change of dielectric properties with temperature coupled analysis is done. The way coupled analysis works can be depicted in the following flowchart. The coupled analysis has been done for heating of whey protein gel food packet in air with power of 5KW for about 1 minute.
Figure 15: The steady state temperature profiles for input power of 243W, full block and block cut at center, at:
(a,b) center (lowest, highest temperature=418.7K, 487.8K);
(c,d) 5cm moved right (lowest, highest temperature=474.9K, 632.7K);
(e,f) 7cm moved right (lowest, highest temperature=485.3K, 757.5K);
(g,h) 10cm moved right (lowest, highest temperature=518.6K, 934.6K);
(i,j) 14 cm moved right (lowest, highest temperature=431K, 715.7K).
Figure 15: The contour temperature profiles after heating with a power of 5KW for 1 min (k) packet moved 5cm right (Max: 560.6K; 307.6K), (l) packet moved 7cm to right (Max: 764.2K; Min: 308.9K), (m) packet moved 10cm to right (Max: 1184.0K; Min: 309K), (n) packet moved 14cm to right (Max: 711.8K, 304.2K).

Figure 16: Comparison of experimental measured heating pattern and the calculated temperature distribution, on the center plane, in a Whey Protein Gel package heated in the microwave horn: (a) Infrared image of the packed heated by microwave and (b) predicted temperature distribution in the package by the integrated electromagnetic and thermal model.
Figure 17(a): The temperature contour profile and band plot of food packet after heating with power of 5KW for 1 minute (Max Temp = 381.78K, Min Temp = 314.42K)

Figure 17(b): Variation of temperatures with time at hot and cold spots in comparison with lumped average temperature with the inset of lumped average temperature with time till the steady state is reached.

The variation of dielectric properties for whey-protein gel with temperature is given in Table 1.

The temperature profiles predicted by transient and coupled analysis are almost same. The contour profile for coupled analysis is shown in Fig 18(a) and the line plot of temperatures of hot and cold spots with time is shown in Fig 18(b). Fig 18(c) and (d) compare temperature band plots of the top surface of the food packet for transient and coupled analysis.
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Dielectric constant</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>59.03</td>
<td>34.80</td>
</tr>
<tr>
<td>30</td>
<td>57.77</td>
<td>39.17</td>
</tr>
<tr>
<td>40</td>
<td>57.00</td>
<td>44.10</td>
</tr>
<tr>
<td>50</td>
<td>56.03</td>
<td>49.50</td>
</tr>
<tr>
<td>60</td>
<td>54.53</td>
<td>54.63</td>
</tr>
<tr>
<td>70</td>
<td>53.43</td>
<td>60.93</td>
</tr>
<tr>
<td>80</td>
<td>52.57</td>
<td>67.50</td>
</tr>
<tr>
<td>90</td>
<td>52.10</td>
<td>73.97</td>
</tr>
<tr>
<td>100</td>
<td>51.63</td>
<td>80.57</td>
</tr>
<tr>
<td>110</td>
<td>50.97</td>
<td>87.80</td>
</tr>
<tr>
<td>121</td>
<td>50.33</td>
<td>95.27</td>
</tr>
</tbody>
</table>

Table 1. Dielectric properties of Whey-protein gel with temperature

Figure 17: The comparison of temperature band plots on the top face of food packet, for heating of whey protein gel in microwave horn applicator, obtained by (c) Transient computation (Max: 375.65K; Min: 312.67K) (d) Infrared image obtained from experiments.
Figure 17: The comparison of temperature band plots on the middle face of food packet, for heating of whey protein gel in microwave horn applicator, obtained by (e) Transient computation (Max: 383.53K; Min: 312.67K) (f) Infrared image obtained from experiments.

Figure 18(a): The temperature contour profile after heating with power of 5KW for 1 minute using coupled analysis (Max Temp = 384.82K, Min Temp = 310.28K)

Figure 18(b): Variation of temperatures with time at hot and cold spots in comparison with lumped average temperature with the inset of lumped average temperature with time till the steady state is reached.
The temperatures predicted by transient and coupled analysis are close because the dielectric properties at 60°C are used for entire range of temperatures. Even though the coupled analysis is more accurate prediction than pure transient analysis, the transient analysis is more time efficient once the properties that should be used for entire range are approximated.

Figure 19: The mesh used for simulation of food heating in single mode applicator.
Figure 20(a): The front and top views of the single mode applicator used for heating food.

Figure 20(b): The schematic of the single mode applicator.
4.5 Microwave heating of food in single mode cavity applicator.

The microwave heating of food in single mode applicator, a waveguide terminated in an oversized cavity, fed with TE\textsubscript{10} is simulated. The front, top views and schematic of the applicator are shown in Fig 20. The mesh used for the applicator is shown in Fig 19. The steady state temperature profiles for food being heated (1) when in air, (2) when immersed in water, along with experimental results are shown in Fig 21(a-e) and Fig 22(a-e) respectively. The temperature profiles are matching well with the experiments. The transient analysis of heating the food packet with power of 6KW for 30s is done for the case of food packet kept in air. The contour profile of the temperature is shown in Fig 21(f) and the line plot of temperature of hot and cold spots with time is shown in Fig 21(g). The temperature band plot comparison for both experiment and computation is shown in Fig 21(h). From the figures it can deduced that the experiments and theory compare well. The transient analysis of heating the food packet with power of 6KW for 2 minutes is done for the case of food packet immersed in water. The contour profile of the temperature is shown in Fig 22(f) and the line plot of temperature of hot and cold spots with time is shown in Fig 22(g). The temperature band plot comparison for both experiment and computation is shown in Fig 22(h). From the figures it can deduced that for this case the steady state temperature profile matches well with experiments but the profile obtained form the transient analysis shows hot spots on the edges in Y direction as opposed to hot spot in the center shown by experiments.
Figure 21(a): Temperature band plot at the center of food packet for food packet placed in air.
Figure 21(b): Temperature contour plot at the center of food packet for food packet placed in air.
Figure 21(c): Temperature band plot for entire food packet for food packet placed in air.
Figure 21(d): Temperature contour plot for entire food packet for food packet placed in air.
Figure 21(e): Temperature band plot for top face of food packet for food packet placed in air (experimental).
Figure 21(f): Temperature contour plot of the top face of food packet after heating for 30s with a power of 6KW (Max: 339.28K, Min: 291.21K)

Figure 21(g): Variation of temperatures with time at hot and cold spots in comparison with lumped average temperature with the inset of lumped average temperature with time till the steady state is reached.
Figure 22(a): Temperature band plot at the center of food packet for food packet placed in water.
Figure 22(b): Temperature contour plot at the center of food packet for food packet placed in water.
Figure 22(c): Temperature band plot for entire food packet for food packet placed in water.
Figure 22(d): Temperature contour plot for entire food packet for food packet placed in water.
Figure 22(e): Temperature band plot for top face of food packet for food packet placed in water. (experimental)
Figure 22(f): Temperature contour plot of the top face of food packet immersed in water after heating for 30s with a power of 6KW (Max: 376.91K, Min: 313.64K)

Figure 22(g): Variation of temperatures with time at hot and cold spots in comparison with lumped average temperature with the inset of lumped average temperature with time till the steady state is reached.
4.6 Advantages of Coupled Modeling for Microwave Heating Systems

Perhaps, one of the greatest advantages of computer modeling is that once developed and validated, the computer model can be used to explore design variations and new design concepts for microwave thermal processing systems in order to improve the system performance and to develop guidelines for process optimization. Some of the thermal results obtained by varying various conditions for microwave thermal processing are comparatively illustrated in Figure 15. Examination of these results indicates that as the package moves from the symmetry position to right, the heating pattern changes significantly. First of all, the symmetry of the heating pattern is destroyed by such a move, as expected. Also, as the center of the package moves towards right from the geometric center of the microwave horn, the power absorption increases first and then starts decreasing. The temperature remains high at the end that is the one near the geometric center. The calculated results seem to suggest that the high energy absorption occurs when the package is moved at 10 cm away towards right, which is evident from lowest and highest temperatures.

4.7 Summary

In this chapter principle of microwave heating is described and simulation of microwave heating of whey protein gel in microwave horn applicator has been presented. From the results and discussion the importance of coupled finite element model, once validated, in designing and optimizing the microwave applicators is evident.
5. LASERS AND ELECTRO-MAGNETIC ANALYSIS OF LASER BEAMS

In this chapter a discussion on coherent and monochromatic high frequency electromagnetic waves, LASER, is presented. Basic principle behind laser production, different types of lasers, electromagnetic field modes of lasers, laser beam propagation and its interaction with materials is presented.

5.1 Lasers and their Types [13]

A laser is a device that controls the way that energized atoms release photons. "Laser" is an acronym for light amplification by stimulated emission of radiation, which describes very succinctly how a laser works.

Although there are many types of lasers, all have certain essential features. In a laser, the lasing medium is “pumped” to get the atoms into an excited state. Typically, very intense flashes of light or electrical discharges pump the lasing medium and create a large collection of excited-state atoms (atoms with higher-energy electrons). It is necessary to have a large collection of atoms in the excited state for the laser to work efficiently. In general, the atoms are excited to a level that is two or three levels above the ground state. This increases the degree of population inversion. The population inversion is the number of atoms in the excited state versus the number in ground state.

Once the lasing medium is pumped, it contains a collection of atoms with some electrons sitting in excited levels. The excited electrons have energies greater than the more relaxed electrons. Just as the electron absorbed some amount of energy to reach this excited level, it can also release this energy. As the Fig 23 below illustrates, the electron can simply relax, and in turn rid
itself of some energy. This **emitted energy** comes in the form of **photons** (light energy). The photon emitted has a very specific wavelength (color) that depends on the state of the electron's energy when the photon is released. Two identical atoms with electrons in identical states will release photons with identical wavelengths.

![Diagram of electron relaxing to lower state](image.png)

**Figure 23: Electron relaxing to lower state**

Laser light is very different from normal light. Laser light has the following properties:

- The light released is **monochromatic**. It contains one specific wavelength of light (one specific color). The wavelength of light is determined by the amount of energy released when the electron drops to a lower orbit.
- The light released is **coherent**. It is “organized” -- each photon moves in step with the others. This means that all of the photons have wave fronts that launch in unison.
- The light is very **directional**. A laser light has a very tight beam and is very strong and concentrated. A flashlight, on the other hand, releases light in many directions, and the light is very weak and diffuse.

To make these three properties occur takes something called **stimulated emission**. This does not occur in your ordinary flashlight -- in a flashlight, all of the atoms release their photons randomly. In stimulated emission, photon emission is organized.
The photon that any atom releases has a certain wavelength that is dependent on the energy
difference between the excited state and the ground state. If this photon (possessing a certain
energy and phase) should encounter another atom that has an electron in the same excited state,
stimulated emission can occur. The first photon can stimulate or induce atomic emission such
that the subsequent emitted photon (from the second atom) vibrates with the same frequency and
direction as the incoming photon.

The other key to a laser is a pair of mirrors, one at each end of the lasing medium. Photons, with
a very specific wavelength and phase, reflect off the mirrors to travel back and forth through the
lasing medium. In the process, they stimulate other electrons to make the downward energy jump
and can cause the emission of more photons of the same wavelength and phase. A cascade effect
occurs, and soon many, many photons of the same wavelength and phase propagate. The mirror
at one end of the laser is "half-silvered," meaning it reflects some light and lets some light
through. The light that makes it through is the laser light.

Fig 24 illustrates how a simple ruby laser works. The laser consists of a flash tube, a ruby rod
and two mirrors (one half-silvered). The ruby rod is the lasing medium and the flash tube pumps
it.
The laser in its non-lasing state

The flash tube fires and injects light into the ruby rod. The light excites atoms in the ruby.

Some of these atoms emit photons.

Some of these photons run in a direction parallel to the ruby's axis, so they bounce back and forth off the mirrors. As they pass through the crystal, they stimulate emission in other atoms.

Monochromatic, single-phase, collimated light leaves the ruby through the half-silvered mirror

Figure 24: The working of simple ruby laser.

There are many different types of lasers. The laser medium can be a solid, gas, liquid or semiconductor. Lasers are commonly designated by the type of lasing material employed:

- **Solid-state lasers** have lasing material distributed in a solid matrix (such as the ruby or neodymium:yttrium-aluminum garnet "Yag" lasers). The neodymium-Yag laser emits infrared light at 1,064 nanometers (nm). A nanometer is $1 \times 10^{-9}$ meters.
- **Gas lasers** (helium and helium-neon, HeNe, are the most common gas lasers) have a primary output of visible red light. CO2 lasers emit energy in the far-infrared, and are used for cutting hard materials.

- **Excimer lasers** (the name is derived from the terms *excited* and *dimers*) use reactive gases, such as chlorine and fluorine, mixed with inert gases such as argon, krypton or xenon. When electrically stimulated, a pseudo molecule (dimer) is produced. When lased, the dimer produces light in the ultraviolet range.

- **Dye lasers** use complex organic dyes, such as rhodamine 6G, in liquid solution or suspension as lasing media. They are tunable over a broad range of wavelengths.

- **Semiconductor lasers**, sometimes called diode lasers, are not solid-state lasers. These electronic devices are generally very small and use low power. They may be built into larger arrays, such as the writing source in some laser printers or CD players.

A **ruby laser** (depicted on the previous page) is a solid-state laser and emits at a wavelength of 694 nm. Other lasing mediums can be selected based on the desired emission wavelength (see table below), power needed, and pulse duration. Some lasers are very powerful, such as the CO2 laser, which can cut through steel. The reason that the CO2 laser is so dangerous is because it emits laser light in the infrared and microwave region of the spectrum. Infrared radiation is heat, and this laser basically melts through whatever it is focused upon.

Other lasers, such as diode lasers, are very weak and are used in today’s pocket laser pointers. These lasers typically emit a red beam of light that has a wavelength between 630 nm and 680 nm. Lasers are utilized in industry and research to do many things, including using intense laser light to excite other molecules to observe what happens to them.
Here are some typical lasers and their emission wavelengths:

<table>
<thead>
<tr>
<th>Laser Type</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon fluoride (UV)</td>
<td>193</td>
</tr>
<tr>
<td>Krypton fluoride (UV)</td>
<td>248</td>
</tr>
<tr>
<td>Nitrogen (UV)</td>
<td>337</td>
</tr>
<tr>
<td>Argon (blue)</td>
<td>488</td>
</tr>
<tr>
<td>Argon (green)</td>
<td>514</td>
</tr>
<tr>
<td>Helium neon (green)</td>
<td>543</td>
</tr>
<tr>
<td>Helium neon (red)</td>
<td>633</td>
</tr>
<tr>
<td>Rhodamine 6G dye (tunable)</td>
<td>570-650</td>
</tr>
<tr>
<td>Ruby (CrAlO$_3$) (red)</td>
<td>694</td>
</tr>
<tr>
<td>Nd:Yag (NIR)</td>
<td>1064</td>
</tr>
<tr>
<td>Carbon dioxide (FIR)</td>
<td>10600</td>
</tr>
</tbody>
</table>

5.2 Electromagnetic Field Modes of Laser

The laser electromagnetic waves are generally transverse electromagnetic (TEM) waves where both electric and magnetic field are perpendicular to the direction of propagation. The TEM waves are generally in different radial symmetric modes denoted by TEM$_{mn}$. The mode numbers $m$ and $n$ denote the variation pattern of the electric and magnetic fields in transverse plane. The
lowest of the TEM modes is $\text{TEM}_{00}$ mode. The variation of electric field for some TEM modes is shown in Fig 25.

![Figure 25: The electric field variation of different TEM modes.](image)

### 5.3 Laser Beam Propagation and Interaction with Materials

The Laser beams propagate similar to light rays. The electric and magnetic field of the laser are perpendicular to the direction of propagation. This is the reason why Laser waves are said to propagate in Transverse Electromagnetic Modes (TEM). The basic mode in which laser can propagate is $\text{TEM}_{00}$ mode. Diffraction causes Laser waves to spread transversely as they propagate, and it is therefore impossible to have a perfectly collimated beam. The spreading of a
Laser beam is in precise accord with the predictions of pure diffraction theory. Under quite ordinary circumstances, the beam spreading can be so small it can go unnoticed. The following formulae accurately describe beam spreading, making it easy to see the capabilities and limitations of laser beams. Even if a Gaussian TEM_{00} laser-beam wavefront were made perfectly flat at some plane, it would quickly acquire curvature and begin spreading in accordance with

\[
R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]^{1/2}
\]

and

\[
w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}
\]

where \( z \) is the distance propagated from the plane where the wavefront is flat, \( \lambda \) is the wavelength of light, \( w_0 \) is the radius of the 1/e^2 irradiance contour at the plane where the wavefront is flat, \( w(z) \) is the radius of the 1/e^2 contour after the wave has propagated a distance \( z \), and \( R(z) \) is the wavefront radius of curvature after propagating a distance \( z \). \( R(z) \) is infinite at \( z = 0 \), passes through a minimum at some finite \( z \), and rises again toward infinity as \( z \) is further increased, asymptotically approaching the value of \( z \) itself. The plane \( z = 0 \) marks the location of a Gaussian waist, or a place where the wavefront is flat, and \( w_0 \) is called the beam waist radius.

The irradiance of the Gaussian beam is given by

\[
I(r) = I_0 e^{-\frac{2r^2}{w^2}} = \frac{2P}{\pi w^2} e^{-\frac{2r^2}{w^2}}
\]

where \( P \) is the power of the Laser beam and \( r \) is the distance from center of beam.
Whenever a laser beam interacts with material some part of the beam is reflected while some other part is transmitted and the rest is absorbed. This interaction of the laser beam with material mainly depends on its dielectric constant at that frequency. Depending on the complex dielectric constant of the material the absorptivity of the material is calculated and based on the absorptivity of the material the amount of heat generated in the material is estimated. This estimation is important while analyzing laser-manufacturing processes.

5.4 Summary
The basic of Lasers, their types, propagation, and interaction with materials is dealt with.
A integrated finite element model has been developed for predicting temperatures, groove profiles, stresses and fracture probabilities during laser cutting of ceramics. In this chapter complete finite element formulation for thermal, stress and fracture calculations are presented.

6.1 Governing Equations

The evolution of temperature and stress states during the laser cutting operation is governed by the energy and mechanical balance equations. The governing equations are as follows,

\[ \nabla (k \nabla T) = \rho \frac{\partial H}{\partial T} \]  \hspace{1cm} (6.1)

\[ \nabla \cdot (\sigma - \sigma_0) = f_b \]  \hspace{1cm} (6.2)

where,

- \( k \) is the thermal conductivity
- \( \rho \) is density
- \( T \) is temperature
- \( H \) is enthalpy
- \( \sigma \) is stress tensor
- \( \sigma_0 \) is initial stress tensor due to temperatures
- \( f_b \) is body force

The heat transfer during cutting is dominated by the laser heating and radiation to the environment. The contribution to temperature change due to the gas jet is relatively a small
fraction (less than 0.01%) in comparison with laser heating, and is thus neglected to simplify the calculations. The laser heating has been input as a flux assuming a Gaussian distribution as given in Eq. (6.3)

$$- k \mathbf{n} \cdot \nabla T = \frac{2 I_0 a}{\pi r_b^2} e^2 \left( \frac{(x-u t)^2 + (y-v t)^2}{r_b^2} \right) \in R_L^2$$

(6.3)

where,

$I_0$ is the peak power of the laser

$a$ is absorptivity factor

$u, v$ are X and Y components of velocity

$r_b$ is the beam radius

$R_L$ is the region where laser falls

The radiative and convective boundary conditions are summarized in Eq. (6.4)

$$q = \varepsilon \sigma_s (T^4 - T_\infty^4) + h_c (T - T_\infty) = h (T - T_\infty) \in \partial \Omega$$

(6.4)

where,

$q$ is flux

$\sigma_s$ is Stefan Boltzmann constant

$\varepsilon$ is emissivity

$h_c$ is convective heat transfer coefficient

$T_\infty$ is the ambient temperature.

6.2 Finite Element Formulation

The above equations along with appropriate boundary conditions can be readily solved using the Galerkin finite element method. The finite element procedures for the solution of
multi-physics problems are given elsewhere [14-17], and thus only an outline is given below for the thermal and stress calculations. In essence, the computational domain is first discretized into an ensemble of small finite elements. Within each element, the dependent variables $u_t$ and $T$ are interpolated by the shape functions $\theta$ and $\phi$,

$$T(x,t) = \theta^T(x)T(t) = \sum_{j=1}^{N_e} \theta_j(x)T_j(t) \quad (6.5)$$

$$u^i(x,t) = \phi^T(x)U^i(t) = \sum_{j=1}^{N_e} \phi_j(x)U^i_j(t) \quad (6.6)$$

where, $U^i$ and $T$ are the column vectors of element nodal point displacement and temperature unknowns. Substituting these functions into the governing equations, followed by integration by parts, one has the integral expression for the thermal and stress balance equations, viz.,

$$\left( \int_{\Omega_1} \rho C_p \theta \theta^T dV \right) \frac{dT}{dt} + \left( \int_{\Omega_1} k \nabla \theta \cdot \nabla \theta^T dV \right) T = -\int_{\partial\Omega_1} q_T \theta ds \quad (6.7)$$

$$\left( \int_{\Omega_1} B^T D B dV \right) U^j = \int_{\partial\Omega_1} n \cdot \bar{\tau} \phi ds + \int_{\Omega_1} f_b \phi dV + \int_{\Omega_1} B^T \alpha \Delta T dV \quad (6.8)$$

Once the form of shape functions are specified, integration is carried out over each discretized element and the results may be assembled to obtain the final global matrix equation,

$$\begin{bmatrix} N_T^T(T) & 0 & T \\ 0 & 0 & U \end{bmatrix} + \begin{bmatrix} K_T(T) & 0 & T \\ 0 & K_E & U \end{bmatrix} \begin{bmatrix} T \\ U \end{bmatrix} = \begin{bmatrix} G_T \\ F \end{bmatrix} \quad (6.9)$$
where, the coefficient matrices in the above equation are calculated by

\[
\begin{align*}
\mathbf{N}_T &= \int_{\Omega_i} \rho C_{p} \theta \mathbf{\nabla}^{T} dV \\
\mathbf{K}_T &= \int_{\Omega_i} k\mathbf{\nabla} \cdot \mathbf{\nabla}^{T} dV \\
\mathbf{K}_E &= \int_{\Omega_i} \mathbf{B}^{T} \mathbf{D} \mathbf{B} dV \\
\mathbf{G}_T &= -\int_{\partial \Omega_i} q_T \theta \, ds \\
\mathbf{F} &= \int_{\partial \Omega_i} \phi^{T} \cdot \mathbf{n} \, ds + \int_{\Omega_i} \phi^{T} \cdot f_k \, d\Omega + \int_{\Omega_i} \mathbf{B}^{T} \mathbf{D} \mathbf{e}_a \, d\Omega
\end{align*}
\]  

(6.10)

and the strain displacement matrix \( \mathbf{B} \) and constitutive matrix \( \mathbf{D} \) for elastic materials are calculated as follows,

\[
\mathbf{B} = \begin{bmatrix}
\frac{\partial \phi^T}{\partial x} & 0 & 0 \\
0 & \frac{\partial \phi^T}{\partial y} & 0 \\
0 & 0 & \frac{\partial \phi^T}{\partial z} \\
\frac{\partial \phi^T}{\partial y} & \frac{\partial \phi^T}{\partial x} & 0 \\
0 & \frac{\partial \phi^T}{\partial z} & \frac{\partial \phi^T}{\partial y} \\
\frac{\partial \phi^T}{\partial z} & 0 & \frac{\partial \phi^T}{\partial x}
\end{bmatrix}
\]  

(6.11)

\[
\mathbf{D} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix}
1-v & v & v & 0 & 0 & 0 \\
v & 1-v & v & 0 & 0 & 0 \\
v & v & 1-v & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1-2v}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1-2v}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2}
\end{bmatrix}
\]  

(6.12)

\[
\mathbf{e}_a^T = \alpha \Delta T \begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix} 
\]  

(6.13)

In the preceding formulations, \( \mathbf{U} \) is a global vector containing all nodal values of displacement components \( u, v \) and \( w \). The assembled global matrix equations are stored in the
skyline form and solved using the Gaussian elimination method. To model the phase change of the problem due to melting, the enthalpy method is applied. For the present model, the enthalpy is defined with respect to a reference temperature $T_{\text{ref}}$ as

$$
H = \begin{cases} 
\int_{T_{\text{ref}}}^{T} C_p \,dT & T < T_m \\
\int_{T_{\text{ref}}}^{T} C_p \,dT + L & T \geq T_m 
\end{cases}
$$

(6.14)

where the effective specific heat $C_{p}^*$ is calculated by the following expression,

$$
C_{p}^* = \left( \frac{\nabla H \cdot \nabla H}{\nabla T \cdot \nabla T} \right)^{1/2}
$$

(6.15)

This formulation causes the specific heat to be a strong function of the temperature field and thus, the equations are clearly nonlinear. The successive substitution method is applied for nonlinear iterations, and the time derivatives are approximated using the implicit finite difference scheme, with automatic time step control.

### 6.3 Treatment of Ablation

While the finite element formulations are well established, the procedure for numerical modeling of ablation processes where material is continuously removed during the calculations needs to be developed. As would be expected, the dual laser configuration produces a complex melting front and moving boundaries during operation. For such complex moving boundaries, the algorithms based on the deforming finite elements can be difficult to apply [18]. On the
other hand, the fixed grid method can be very useful alternative [19]. The central idea of this algorithm is such that the elements being ablated are deactivated from further calculations. When doing so, the relevant nodes and boundary conditions associated with the elements deactivated must be carefully assigned to the adjacent elements that are still active in calculations as illustrated in Figure 3. After each time step, the averaged temperature is calculated for all elements and the ones with average temperature higher than melting point are deactivated. The deactivated elements have no contribution in coming time steps. Once the elements are deactivated, the natural boundary conditions are passed on to corresponding neighboring elements (depicted in Fig. 28). The nodes are floated once all the elements sharing the node are deactivated. The field variables such as temperature and displacements at these floated nodes remain unchanged for all future time intervals.

Figure 26: Illustration of procedures used to deactivate the elements and re-assigning the boundary conditions during deactivation of ablated elements.

The errors involved in the analyses are of the order of the length of the element. With this type of approach, the size of the element should be chosen such that the temperature gradients are relatively small within each element. Compared with the front tracking method by which the ablation moving interface is precisely tracked in time (useful for simple geometries),
the present method offers an advantage of modeling more complex, time evolving ablation geometries such as those induced by a dual laser power source.

**6.4 Stress calculation and smoothing**

Once the temperatures are calculated from thermal model those temperatures are fed into mechanical model and the model is solved for displacements under given boundary conditions. After displacements are calculated strains and consequently stresses are calculated at integration points of a typical element. These stresses obtained at the integration points are smoothed out to obtain nodal stress. The smoothing is done as described.

Eq (6.16) gives the stress at any interior point in the element.

$$\{\sigma\} N = \sigma_{\text{int}} \tag{6.16}$$

where

- $\{\sigma\}$ is the vector of nodal values of any stress component.
- $\{N\}$ is vector of nodal shape functions.
- $\sigma_{\text{int}}$ is stress at any interior point of the element.

The Eq (6.16) is multiplied by shape function and integrated over the volume of each element and added to obtain Eq. (6.17).

$$\left[ \sum_{n=1}^{N} \int_{V_e} \{N\} \{N^T\} dV \right] \{\sigma\} = \left[ \sum_{n=1}^{N} \int_{V_e} \{N\} \sigma_{\text{int}} \right] dV \tag{6.17}$$

The equation (6.17) is solved to obtain the nodal stresses.

**6.5 Fast fracture reliability analysis**

The use of advanced ceramic materials in structural applications requiring hinge component integrity has led to the development of a time dependent probabilistic design methodology. This method combines three major elements: (1) Linear elastic fracture mechanics theory that relates
the strength of ceramics to the size, shape, orientation and growth of critical flaws; (2) extreme value statistics to obtain the characteristic flaw size distribution function, which is a material property; and (3) material microstructure. Inherent to this design procedure is that the requirement of total safety must be relaxed and that an acceptable failure probability must be specified.

The statistical nature of fracture in engineering materials can be viewed from the two distinct models [20]. The first was presented by Weibull and used the weakest link theory as originally proposed by Pierce [21]. The second model was analyzed by Pierce [21] and in addition also by Daniels [22]. The weakest link model assumes that the structure is analogous to a chain with n links. Each link may have a different limiting strength. When a load is applied to the structure such that the weakest link fails, then the structure fails. Observations show that advanced monolithic ceramics closely follow the weakest link theory (WLT). A component fails when as equivalent stress at a flaw reaches a critical value, which depends on the fracture mechanics criterion, crack configuration, crack orientation and the crack density function of the material. In comparison with the bundle model, WLT is, in most cases, more conservative.

Weibull’s WLT model does not consider failure caused by purely compressive stress states. When a principal compressive stress exceeds three times the maximum principal tensile stress in a given element, the compressive stress state predominates and the corresponding reliability is set equal to unity. Classical WLT does not predict behavior in a multiaxial stress state.

6.5.1 Volume And Surface Flaw Reliability Analysis

Consider a stressed component containing many flaws and assume that failure is due to any number of independent and mutually exclusive mechanisms (links). Each link involves as
infinitiesmal probability of failure $\Delta P_{iv}$. Discretize the component into $n$ incremental links. The probability of survival $P_{sv}$ of the $i^{th}$ link is:

$$(P_{sv})_i = [1 - \Delta P_{iv}]_i$$  \hspace{1cm} (6.18)$$

where, the subscript $V$ denotes the volume dependent terms. The resultant probability of survival of the whole structure is the product of the individual probabilities of survival and is given by:

$$P_{sv} = \exp\left[\sum_{i=1}^{n} (\Delta P_{iv})_i \right]$$  \hspace{1cm} (6.19)$$

Assume the existence of a function $N_V(\sigma)$, referred to as crack density function, representing the number of flaws per unit volume having a strength equal to or less than $\sigma$. Under a uniform tensile stress, $\sigma$, the probability of failure of the $i^{th}$ link representing the incremental volume $\Delta V_i$ is

$$(\Delta P_{iv})_i = \left[ N_V(\sigma) \Delta V \right]_i$$  \hspace{1cm} (6.20)$$

and substituting into equation Eq (6.19) the resultant probability of survival is:

$$P_{sv} = \exp[-N_V(\sigma)\Delta V]$$  \hspace{1cm} (6.21)$$

and the probability of failure is

$$P_{iv} = 1 - \exp[-N_V(\sigma)\Delta V]$$  \hspace{1cm} (6.22)$$

where, $V$ is the volume. If the stress is a function of location then
A term called the risk of rupture by Weibull and denoted by the symbol \(-B_v\) is commonly used in the reliability analysis. Equations similar to (6.22) and (6.23) are applicable to the surface – distributed flaws where the surface area replaces volume and surface crack density replaces the volume crack density.

Weibull introduced a three-parameter power function for the crack density function \(N_v(\sigma)\),

\[
N_v(\sigma) = \left[ \frac{\sigma - \sigma_{uv}}{\sigma_{ov}} \right]^{m_v}
\]

where, \(\sigma_{uv}\) is the threshold stress parameter, which is usually taken as zero for ceramics. This parameter is the value of the applied stress below which the failure probability is zero. When this parameter is zero, the two parameter Weibull model is obtained. The scale parameter \(\sigma_{ov}\) then corresponds to the stress level where 63.2 \% of specimens with unit volume would fracture. The scale parameter has dimensions of ‘stress x (volume)\(1/m_v\)’, where ‘\(m_v\)’ is the shape parameter (Weibull modulus), a dimensionless parameter that measure the degree of strength variability. As \(m_v\) increases, the dispersion is reduced. For large values of \(m_v\), such as those in ductile metals the magnitude of the scale parameter corresponds to the material ultimate strength. These three statistical parameters are material properties, and they are temperature and processing dependent. Three–Parameter behavior is rarely observed in as-processed monolithic ceramics and statistical estimations of the three material parameters is very involved. So the two-parameter model is used in this work. The subsequent reliability predictions are more conservative than for the three-parameter model since the minimum strength of the material is taken as zero.
The two-parameter crack density functions is expressed as

\[
N_v(\sigma) = \left( \frac{\sigma}{\sigma_{0v}} \right)^{m_v} = K_{wv} \sigma^{m_v}
\]  \hspace{1cm} (6.25)

and when Eq. (6.25) is substituted into Eq. (6.21) the failure probability becomes

\[
P_{IV} = 1 - \exp\left[ -K_{wv} \int V \sigma^{m_v} \, dV \right]
\]  \hspace{1cm} (6.26)

where \( K_{wv} = (\sigma_{0v})^{-m_v} \) is the uniaxial Weibull crack density coefficient. Various methods have been developed to calculate \( \sigma_{0v} \) and \( m_v \) for a given material by using fracture strength data from simple uniaxial specimen tests [23].

The two most common techniques for using the uniaxial data to calculate \( P_{IV} \) in a polyaxial stress states are the PIA method [24, 25] and the Weibull normal tensile averaging method [26, 27]. In the PIA model the principal stresses \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \) are assumed to act independently. If all the principal stresses are tensile the probability of the failure according to this approach is

\[
P_{IV} = 1 - \exp\left[ -K_{wv} \int V (\sigma_1^{m_v} + \sigma_2^{m_v} + \sigma_3^{m_v}) \, dV \right]
\]  \hspace{1cm} (6.27)

Compressive principal stresses are assumed not to contribute to the failure probability.

In the similar grounds the surface flaw is given as:

\[
P_{IS} = 1 - \exp\left[ -K_{ws} \int S (\sigma_1^{ms} + \sigma_2^{ms} + \sigma_3^{ms}) \, dS \right]
\]  \hspace{1cm} (6.28)

The total flaw probability is given as:

\[
P_f = P_{IV} + P_{IS}
\]  \hspace{1cm} (6.29)
The integrals in Equations (6.27) and (6.28) are calculated by converting all the stresses at the integration points into principle stresses and performing gaussian integration over each element and summing it up over entire domain.

6.6 Summary

In this chapter all required theoretical formulations for integrated finite element model capable of modeling ablation, stress and fracture are dealt with.
7. RESULTS OF LASER CUTTING SIMULATIONS

The laser cutting operation of thin ceramic plates and all pertinent geometric information used for model development and analyses are shown in Figures 27 and 28, respectively. Referring to these figures, the cutting laser heats and melts the material as it moves along a pre-designated cutting line. The melted layer, which is very thin [28], is removed by the jet of gas blown from the nozzle. The laser heat is partially absorbed, with a fraction used to melt the material and the remainder being conducted through the material. The heat transfer process occurring in laser cutting operation is classified as an ablation problem, where materials are removed immediately after it is melted. Because of concentrated heating due to the laser, thermal stresses develop in the ceramic plate and are largely concentrated in the heat-affected zone. Experiments show that such thermal stresses strongly depend on the temperature distribution, which in turn is dependent on how the laser is delivered to the work piece. These thermally induced stresses can be detrimental in that they may cause fractures in the material if not controlled [29, 30]. However, only the tensile stresses are cause for concern since the compressive strengths for most ceramics tend to be 3-6 times greater than the tensile strength.

The thermal and stress evolution model developed above enables the prediction of time-varying temperature and stress distributions during the cutting or drilling of materials by single, dual, or multiple lasers. Extensive numerical simulations were carried for a variety of laser machining conditions including both continuous and pulsed laser operations. A selection of the computed results is presented below for single and dual laser cutting. The parameters used for the finite element simulations of laser machining operations are given in Table 2.
7.1 Comparison with Existing Data

Before discussing the results obtained for laser cutting, the algorithm and its computational implementation were first tested against data and models already published in the literature. To test the ablation algorithm, a simple 1-D ablation problem was considered, which has been studied extensively in literature [14] using both the moving grid and fixed grid methods.

Figure 27: Geometric arrangement of lasers and work piece during laser ablation or cutting operations.

Figure 28: Schematic of dual laser beam cutting, along with coordinate system used for numerical analyses.
Figure 29 shows a comparison of results obtained using the current finite element algorithm and those obtained using an enthalpy method which artificially changes the pertinent properties of the materials as opposed to deleting them from further calculations for the ablated elements [14]. The finite element meshes used for the calculations are given in the inset of the figure. To simulate the 1-D heat transfer effects, all the sides of the 3-D bar were thermally insulated so that heat was restricted to the longitudinal direction. Figure 29 shows that the current method compares very favorably with the published data. This simple 1-D problem also allows us to study the mesh sensitivity associated with the ablation problems. Testing with various meshes shows that a mesh of fifty 8-node elements gives reasonably good accuracy and further refinement of the mesh produces an error of less than 0.1%.

Figure 29: Normalized temperature Vs Normalized length in comparison with Standard values from [14].
With the thermal code verified, the thermal stress model was integrated into the procedure by feeding the temperature distributions obtained from thermal calculations to the mechanical model for stress calculations. The computer code for the stress calculations is also based on the finite element method and is capable of predicting both linear and nonlinear elastic and plastic deformations in small and large strains using both the Eulerian and Lagrangian formulations [15]. The code was extensively tested against many classical textbook examples in many different geometric configurations to ensure the correctness of the code as well as the numerical accuracy [16, 17, 31-33]. The testing procedures and problems are fairly standard as elaborated in the relevant textbooks and thus omitted here.

With the integrated thermal and stress model tested against known solutions, extensive simulations were undertaken to investigate the thermal and stress developments during laser cutting operations. For the results presented below, the finite element meshes used for the simulations are given in Figure 30, which consist of 1260 8-node elements. Since the cutting operation reaches a quasi-steady state in a relatively short time interval, only a small region of dense mesh was required with the calculations stopped once the thermal field reached a quasi-steady state. The averaged size of the element used in the dense mesh region was approximately (100 x 50 x 62.5) cubic microns.
7.2 Single and Dual Laser Cutting Results

The results for single laser operations are plotted in Figures 31. This figure depicts the thermal transient development and the approximate morphology of the ablation front during laser cutting. The higher temperature region represents the materials that are being removed during the time interval. As shown by the figure, the temperatures near the ablation front experiences rapid changes and quickly drop to near-ambient due to thermal radiation to the environment. Cooling by the gas jets used to remove the ablated materials seems to be minimal (less than 1.5%). The groove shapes or ablation fronts that evolve as the cutting operation continues are shown in Figure 32. The predicted shapes are similar to the results obtained by others researchers using different techniques [28]. For the particular laser cutting operations under consideration, it takes approximately 0.09s to cut through the ceramic 0.625mm thick plate. It is important to note that the present model presents no difficulty in modeling the material removal through the thickness, which would require re-meshing and restarting calculations if a deforming element method was applied.
Thermal stresses arise as a result of the sharp changes in temperature near the ablation front. If not appropriately controlled, any thermally induced tensile-stresses can be detrimental, and cause the fracture or cracking in the ceramics during cutting. The thermal stress development along the (laser cutting line) ahead of the cutting laser is shown in Figure 33. Examination of the results indicates that a large thermal stress exists near the cutting front and fades down quickly away from the front. In fact, after approximately 1mm away from the cutting front, the thermal stresses reduce to an almost negligible level. Also, the stresses in other (y and z) directions are an order of magnitude smaller and should be less harmful than the stress component in the x-direction. It was also noticed that during the early time of laser heating, compressive stresses developed near the cutting front whereas stresses tend to be compressive near the ablation front during the latter portion of the cutting interval. This is consistent with the temperature distribution as shown in Figure 33(d).

Analysis of thermal stress development and temperature variations at various depths ahead of the cutting laser further shows that for these locations, the stress components in other directions are much smaller and that the stress level decreases deeper inside the plate. This is consistent with the temperature distribution at the same locations.

For laser cutting processes, the laser beam diameter represents a crucial parameter. Most measured data of laser beam refers to the focused diameter (see Figure 27). In practice, the actual beam diameter on the surface of the work-piece is in fact, larger than the measured beam diameter. A mathematical model can be of valuable tool in determining the sensitivity of the laser beam diameter on the ablation operations.
Figure 31: Computed results for temperature, morphology of ablation fronts during a single laser beam cutting operation showing temperature distribution (Top) and morphology of ablation fronts (Bottom) at time steps 0.045s, 0.09s and 0.135s.

CD—cutting direction; LB—Laser beam

Figure 32: Evolution of cutting groove shapes during a single laser beam cutting.
Using the mathematical model, simulations were conducted to study the effect of beam diameter while keeping the total laser power fixed. The computed results are shown in Figure 34, where the depth of laser ablation vs. the laser input power appears. Clearly, the laser beam diameter can have a critical effect on the cutting operation and a correct input of the laser beam diameter is required if accurate results are obtained. From the figure it is apparent that the required power to ablate through the ceramic is approximately 17W with a beam diameter of 0.2 mm, and a value of ~25W with a beam diameter of 0.25 mm.

Figure 33. Computed thermal stresses and temperature distribution along the x-direction (Y = 0.1mm, Z = 0.625mm) ahead of the cutting laser during ablation at time steps (0.045s, 0.09s, 0.0135s): (a) $S_{xx}$, (b) $S_{yy}$, (c) $S_{zz}$, and (d) temperature for Single Laser case.
This variation in thermal field as a function of power input parameters may be explained by the fact that a larger diameter absorbs and dissipates more energy and as a result, high laser input power is required to cut through the same thickness of the work piece.

Experience shows that cutting by a single laser induces strongly non-uniform temperature field and thus unfavorable stress level during cutting. In addition, the increasingly severe mechanical stresses resulting from the loss of supporting section ultimately results in premature fractures and damage.

To study ways to alleviate the premature fracture problem, a dual laser arrangement was also investigated. In this arrangement, a front laser is used to preheat or cut a small groove line (to dictate and control the fracture path) and a main cutting laser behind to completely cut through along the groove. The temperature and morphology evolution for dual laser beam cutting is
given in figure 35. The numerical results of detailed groove shapes created by both the lead and main lasers during dual laser cutting are shown in Figure 36. In comparison with the single laser results, several important features are observed. First, dual lasers can cut the work piece more quickly, once the cutting laser reaches the groove created by the lead laser as evidenced by the temperature contours. In this particular dual laser arrangement, the lead laser is positioned 0.5 mm ahead of the main cutting laser. The operation starts with both lasers turned on at the same time and moving at the same speed. The power input to the cutting laser is the same as for the single laser. Examination of the groove shapes for both lasers indicates that the lead laser reaches steady state at $t=0.09s$ and the shape remains the same afterwards. As for the main cutting laser, the groove shapes are similar to those produced by single laser for $t=0$ to $0.9s$ and afterwards the shape differs from that of the single laser. These groove profiles are reasonable in that during $t=0$ to $0.9s$, the lead laser effect has not yet been felt.

The evolution of the stress distributions in the ceramic plate is shown in Figure 37 for the dual laser cutting operations. To compare with the single laser case, the stress distributions at the same locations are plotted. As in the dual laser case, material at these locations at $t=0.135s$ are already ablated and thus, the values are available only at $t=0.045s$ and $t=0.09s$. Comparison of stress and temperature distributions shown in Figures 33 and 37 illustrates the difference between the single and dual laser cutting operations. In the dual laser operation, the temperature is higher, which is attributed to the higher power input. The temperature rise around $x=0.6$ at $t=0.045s$ and $x=0.7$ at $t=0.09s$ clearly comes from the lead laser heating. This change in the temperature distribution results in the variation of the thermal stresses as shown. Again, the thermal stresses have a predominant component in the x-direction and a sharp drop in thermal stress right in front of the cutting laser, which is in compression. The results obtained from both
single and dual laser simulations show that the thermal stresses in these two types of operations have comparable stress level near the cutting laser, but differ afterwards with the stress developing following the temperature variation resulting from the dual laser.

Figure 35: Computed results for temperature, morphology of ablation fronts during a dual laser beam cutting operation showing temperature distribution (Top) and morphology of ablation fronts (Bottom) at time steps 0.045s, 0.09s and 0.135s.

CD—cutting direction; LB—Laser beam
Figure 36: Time development of groove shapes ablated (a) by the lead laser and (b) by both the lead and cutting lasers (0.045s, 0.09s, 0.135s)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>10.04 W/mK</td>
</tr>
<tr>
<td>Specific heat</td>
<td>1338.9 J/Kg.K</td>
</tr>
<tr>
<td>Density</td>
<td>3900 Kg/m³</td>
</tr>
<tr>
<td>Heat of removal</td>
<td>1025 KJ/Kg</td>
</tr>
<tr>
<td>Absorptivity factor</td>
<td>0.99</td>
</tr>
<tr>
<td>Melting point</td>
<td>2025K</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>3.72E011 N/m²</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.19</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>6.8E-08 K⁻¹</td>
</tr>
<tr>
<td>Laser Power for single pulsed laser cutting</td>
<td>30W</td>
</tr>
<tr>
<td>Laser frequency</td>
<td>200 Hz</td>
</tr>
<tr>
<td>Beam Diameter</td>
<td>0.25mm</td>
</tr>
<tr>
<td>Distance between two beams</td>
<td>0.5mm (2 laser diameters)</td>
</tr>
<tr>
<td>Laser Power for main cutting laser</td>
<td>30W</td>
</tr>
<tr>
<td>Power of lead beam</td>
<td>15W</td>
</tr>
<tr>
<td>Velocity of the beams</td>
<td>3.18E-03m/s</td>
</tr>
<tr>
<td>Pulsed Laser details</td>
<td>5ms cycle with 1ms ON cycle and 4ms OFF cycle;200HZ</td>
</tr>
</tbody>
</table>

Table 2. Properties of parameters for laser cutting simulation
7.3 Single Laser Drilling

Here a single laser drilling operation by which a through-hole is machined in the plate by laser ablation is considered. The finite element mesh used for numerical simulations is given in Figure 38 and a laser beam is located at the center of the plate. Because of symmetry, only a
quarter of the plate is needed for simulation and again the dense mesh is used in the laser-affected area. Some snapshots of the transient development of the hole morphology in 3-D views during laser drilling are given in Figure 39. The evolution of the groove shapes is lineated in figure 40 at several time steps.

The temperature distributions and stress development at various locations are given in Figure 41. These results illustrate that the temperature changes rapidly from the edge of the hole into the plate and that as a result of a rapid change in temperature distribution large compression stresses develop near the edge. It is noteworthy here that numerical simulation of laser-drilling of through-holes can be very difficult for the moving grid-based approached, because of the geometric discontinuity caused by the continuous removal of materials by ablation.

No difficulty has been encountered with the method presented in this paper, however. Also, the model can be used to simulate more complex laser machining operations such as laser writing and laser surgery.
Figure 39: The hole morphology at (a) 0.017s, (b) 0.0258s, (c) 0.0429s. Depth given in (mm).

Figure 40: The cross section of grooves shape (axisymmetric) during lasers drilling at different time steps.
Figure 41: (a) Temperature and (b) stress (Sxx) distribution with radial distance (r) from the center at different depths of the plate. The stress and temperature distribution are axisymmetric.

7.4 Fracture Probability Predictions

The numerical predictions of temperatures and corresponding thermal stress are done during the single and dual laser cutting of ceramics. Fracture probability prediction coupled as a postprocessor after evaluating both thermal and bending or twisting stresses will be helpful in estimating the fracture probability during laser cutting due to given cutting parameters. Before predicting fracture during laser cutting our fracture prediction post processing module is tested with examples given in [34].

7.4.1 Transversely Loaded Circular Disk

A transversely loaded circular disk is analyzed and stresses calculated are fed into the fracture predicting post processor program. The volume flaw probabilities ($m_v = 28.53$ and $\sigma_{0V} = 169.70 \text{MPa}$) with increasing pressure are plotted and shown in the figure 42. Axisymmetric analysis had been done to calculate the stresses.
Figure 42: Volume Flaw Probability of Transversely Loaded Circular Disk

The loading and the boundary conditions of the disk are pictorially represented in the figure 40. The mesh required for the analysis should be such a way that the stress volume integral converges. The results predicted from the code compare well with once predicted with CARES [34].

7.4.2 Thermal Fracture Prediction for Laser Cutting of Alumina

The thermal fracture probability can be predicted after calculating the thermal stresses for particular time step from the temperatures. The mesh used for temperature stress and fracture calculations is shown in Fig 43.
As shown in the above mesh the laser (power = 4W; beam diameter = 96 microns; velocity = 4.32E-03 m/s; pulse duration = 1ms; pulse ON duration = 0.08 ms) starts from point A and moves in the direction of arrow. So the elements in the region bounded by black lines are more prone to thermal fracture. In the present simulations the elements in the top layer are only prone to have some fracture probability. So the fracture probability of these elements for different time steps is depicted in Fig 44. This fracture prediction was during the very start of laser cutting.

7.4.3 Thermal Fracture Prediction during Laser Scanning of Alumina with CW Laser.

The thermal fracture analysis has been done for continuous wave laser scanning process over a ceramic plate (0.08m X 0.04m X 0.089m) with a laser of Power = 900W moving at 0.05m/s having a beam diameter of 0.003m. The laser is scanned along X direction. The thermal fracture probabilities are captured after laser moves 0.0125m along X direction. The mesh used for laser scanning process is shown in Figure 45 and the fracture probabilities are shown in Figure 46. The alumina plates fractures in this case showing that a laser of larger beam diameter and high power causes fracture.
Figure 44: The thermal fracture probabilities of top layer elements during the laser cutting. The number in the box gives fracture probability (%) with effective stress of element in GPa and gray shaded cell means zero fracture probability and ones hatched are deactivated. The values in ovals are temperatures at nodal points.

Figure 45: The mesh used for CW laser scanning of ceramic. The elements shown in box are prone to fracture through depth.
7.5 Summary

The finite element model formulated in last chapter is first validated and used for predicting temperatures, grooves and thermal stresses for both dual and single laser cutting. Post processing module for fracture probability prediction is also validated using published results. This module can further be coupled with stresses predicted during laser cutting to estimate fracture.
8. CONCLUSIONS AND FUTURE WORK

In this work an integrated electromagnetic and thermo-mechanical finite element model has been developed. The electromagnetic model development is based on the edge finite element formulation to solve Maxwell’s equations in 3D space for the time harmonic electromagnetic fields. Tetrahedral elements are used to discretize the domain. This finite element model is validated by testing with the following cases having well established analytical solutions:

1. Electric field in semi infinite dielectric piece
2. Magnetic field in semi infinite metal induction case
3. Electric field in lossless shorted rectangular wave guide
4. Electric field in lossy-shorted rectangular wave guide
5. Electric field in shorted rectangular waveguide having both lossy and lossless media.

The sensitivity of electric field prediction due to the shape of tetrahedral elements is presented. It is found that the tetrahedral edge elements are quite shape sensitive and it is recommended to use tetrahedral elements with approximately equal sides. The electromagnetic model is then coupled with the thermal module of the thermo-mechanical model to predict heating in microwave systems. The heating of whey-protein gel in microwave horn applicators using the coupled model is studied for the following case of food being surrounded by air in the rectangular cavity. For this case, the temperature profile and temperature ranges compare quite well with the experimental values.

Heating of whey-protein gel in single mode applicator is studied for the following cases:

1. Food surrounded by air in the rectangular cavity.
2. Food immersed in water in the rectangular cavity.
For the first case temperature profile and values match well with experiments. In the second case steady state temperature profile matches well but the transient temperature profile, after heating for 2 minutes, doesn’t match with that of experiments. The computed results show hot spots at the edges in Y direction where as experiments show hot spot in the center.

The thermo-mechanical model is used to develop a 3-D numerical model for the ablation phenomena and thermal stress evolution during single and dual laser cutting of ceramic plates. The numerical model development is based on the finite element solution of thermal conduction with ablation resulting from an applied surface laser source. To model the removal of materials due to ablation, a fixed grid finite element method is employed by which material elements at temperature above the melting point are deleted from further calculations.

Compared with the front tracking method by which the moving ablation interface is precisely tracked in time and which is useful for simple geometries, the present method has an advantage of modeling more complex ablation geometries such as those induced by a dual laser power source. The thermal model is integrated with a stress model to predict the evolution of thermal stresses, which are developed during laser cutting as a result of strong temperature gradient near the laser source. The model predictions compared well with those available in literature. The model is capable of predicting the groove shapes, the temperature distributions and thermal stresses generated by the single and dual lasers operating in both continuous and pulsing modes.

Numerical results show that the groove shapes, and temperature and stress distributions are similar in front of the cutting laser for both single and dual lasers, but they differ in the region when the lead laser becomes effective.

The stresses obtained are post processed with a probabilistic fracture module to obtain volume and surface probabilities of fracture. This probabilistic fracture module is first tested with an
example problem of fracture of transversely loaded disk published in NASA’s CARES manual. Then this module is used to predict thermal fracture for pulsed laser cutting and laser scanning of alumina with continuous wave laser. All of these calculations will be used to develop and optimize dual laser cutting arrangements to reduce premature fractures in ceramics.

For the future work the following things can be done:

- Since the present mesh generation technique is not quite good to predict accurate temperatures in complex microwave heating systems an unstructured mesh generation technique with local refinement capability has to be incorporated.

- The mesh required for simulating microwave systems is quite large and requires lots of memory and computation time if only Finite Element Method is used. Therefore Boundary Integral Method has to be coupled with finite element method to decrease mesh size and make computations faster.

- The groove depth predictions have to be validated with experiments to make the thermo-mechanical model more reliable.

- It is very useful if adaptive remeshing is coupled with the thermo-mechanical model to make the model faster and more useful for predicting laser cutting for varied laser scan paths.
BIBLIOGRAPHY


Akademien Handlinger, 151(1939)

Handlinger, 153(1939)

28. M.F.Modest, “Three-Dimensional, Transient Model for Laser Machining of

29. K. Li and P. Sheng, “Plane Stress Model For Fracture of Ceramics During Laser
1506.

30. M.F.Modest, “Transient elastic thermal stress development during laser scribing of


33. D.R.J. Owen and E. Hinton, Finite Elements In Plasticity, Pineridge Press, Swansea,
1980.


35. Dorre, E., and H. Hubner. Alumina-Processing, Properties, and Applications, Berlin:
Springer-Verlag, 1984

36. Gitzen, Walter H. Alumina as a Ceramic Material, Columbus: American
Ceramic Society 1970.


APPENDIX A

EDGE SHAPE FUNCTIONS FOR TETRAHEDRAL ELEMENT

In this appendix edge shape functions for the tetrahedral element are derived. These are used in evaluating the basic integrals used in the finite element matrices.

A.1 Edge Shape Functions

Any unknown function \( \phi \) can be interpolated in a tetrahedron, using its values at the nodes, as

\[
\phi = \sum_{i=1}^{4} L_i \phi_i \quad (A.1)
\]

where \( L_i \) are the volume coordinates defined as

\[
L_i^e(x, y, z) = \frac{1}{6V^e} \left( a_i^e + b_i^e x + c_i^e y + d_i^e z \right) \quad (A.2)
\]

where

\[
a^e = \frac{1}{6V^e} \begin{vmatrix}
\phi_1^e & \phi_2^e & \phi_3^e & \phi_4^e \\
x_1^e & x_2^e & x_3^e & x_4^e \\
y_1^e & y_2^e & y_3^e & y_4^e \\
z_1^e & z_2^e & z_3^e & z_4^e \\
\end{vmatrix} = \frac{1}{6V^e} \left( a_1^e \phi_1^e + a_2^e \phi_2^e + a_3^e \phi_3^e + a_4^e \phi_4^e \right) \quad (A.3)
\]

\[
b^e = \frac{1}{6V^e} \begin{vmatrix}
\phi_1^e & \phi_2^e & \phi_3^e & \phi_4^e \\
y_1^e & y_2^e & y_3^e & y_4^e \\
z_1^e & z_2^e & z_3^e & z_4^e \\
1 & 1 & 1 & 1 \\
\end{vmatrix} = \frac{1}{6V^e} \left( b_1^e \phi_1^e + b_2^e \phi_2^e + b_3^e \phi_3^e + b_4^e \phi_4^e \right) \quad (A.4)
\]

\[
c^e = \frac{1}{6V^e} \begin{vmatrix}
\phi_1^e & \phi_2^e & \phi_3^e & \phi_4^e \\
x_1^e & x_2^e & x_3^e & x_4^e \\
z_1^e & z_2^e & z_3^e & z_4^e \\
1 & 1 & 1 & 1 \\
\end{vmatrix} = \frac{1}{6V^e} \left( c_1^e \phi_1^e + c_2^e \phi_2^e + c_3^e \phi_3^e + c_4^e \phi_4^e \right) \quad (A.5)
\]
\[ V^e = \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1^e & x_2^e & x_3^e & x_4^e \\ y_1^e & y_2^e & y_3^e & y_4^e \\ z_1^e & z_2^e & z_3^e & z_4^e \end{vmatrix} = \text{Volume of element} \] (A.6)

\[ x_i^e, y_i^e, z_i^e, \phi_i^e \] are the X,Y,Z coordinates and the nodal values of the tetrahedron.

Now consider the tetrahedral edge element shown in the figure 47.

![Figure 47: Tetrahedral edge element with edge and node numbering.](image)

After examining for the function with zero divergence the edge shape functions take the form

\[ N_j^e = (L_{j1}^e \nabla L_{j2}^e - L_{j2}^e \nabla L_{j1}^e) L_{e} L_j^e \] (A.7)

where \( j \) is the edge number and \( j_1 \) and \( j_2 \) are the starting and the ending nodes of the edge and \( Le \) is the length of the edge.

Alternative expression shown in chapter 2 is derived in [44].

**A.2 Elemental Matrices**

When the vector basis functions mentioned above are employed for three dimensional finite element discretization of a vector wave equation, the resulting matrices contain following two integrals,
These two integrals can be evaluated analytically for tetrahedral element, as discussed below.

For tetrahedral elements, if we adopt the expression in \( (A.7) \) we have

\[
\nabla \times N_i^e = 2L_i^e \nabla L_i^e \times \nabla L_i^e
\]

\[
= \frac{L_i^e}{(6V)^2} \left[ (c_{ii}d_{ii}^e - d_{ii}^ec_{ii})k + (d_{ii}^e b_{ii} - b_{ii}^e d_{ii})y + (b_{ii}^e c_{ii} - c_{ii}^e b_{ii})z \right]
\]

\[(A.10)\]

Substituting \((A.10)\) into \((A.9)\) we obtain

\[
E_{ij}^e = \frac{L_i^e L_j^e V^e}{(6V)^3} \left[ (c_{ii}d_{ii}^e - d_{ii}^ec_{ii})c_{ij}^e d_{ij}^e - d_{ij}^ec_{ij}^e \right] + \left[ (d_{ii}^e b_{ii} - b_{ii}^e d_{ii})d_{ij}^e b_{ij}^e - b_{ij}^e d_{ij}^e \right] + \left[ (b_{ii}^e c_{ii} - c_{ii}^e b_{ii})b_{ij}^e c_{ij}^e - c_{ij}^e b_{ij}^e \right]
\]

\[(A.11)\]

For the integral in \((A.9)\) using \((A.7)\) integrand becomes,

\[
N_i^e \cdot N_j^e = \frac{L_i^e L_j^e}{(6V)^2} \left[ L_i^e L_j^e f_{i2j2} - L_i^e L_j^e f_{i2j2} - L_i^e L_j^e f_{ij2} + L_i^e L_j^e f_{ij2} \right]
\]

\[(A.12)\]

where \( f_{ij} = b_{i}^e b_{j}^e + c_{i}^e c_{j}^e + d_{i}^e d_{j}^e \)

Using the equation

\[
\int_\varepsilon \int \int \left( L_1^e \right)^k \left( L_2^e \right)^l \left( L_3^e \right)^m \left( L_4^e \right)^n = \frac{k!l!m!n!}{(k+1+m+n+3)!} 6V^e
\]

\[(A.13)\]

and integrating \((A.12)\) over the volume of the element we get

\[
F_{ij}^e = \frac{L_i^e L_j^e}{720V^e} \left[ f_{i2j2} - pf_{i2j2} - qf_{ij2} + f_{ijl} \right]
\]

\[(A.14)\]

where \( p = 1 \) if \( i2 \neq j2 \) else \( p=2 \) and \( q = 1 \) if \( i1 \neq j2 \) else \( q=2 \).