MULTIDIMENSIONAL FREQUENCY DOMAIN RINGDOWN ANALYSIS FOR
POWER SYSTEMS USING SYNCHROPHASORS

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Wide-area implementations of synchrophasors enable real-time monitoring of power system dynamic responses during disturbances. These disturbances generally excite oscillatory modes of the system which can become problematic if the modes are either poorly damped or negatively damped. In light of the growing number of integrated wind farms with complex power electronics controls, dealing with the complex interactions of large interconnected power systems is becoming more challenging than ever. Therefore, an accurate on-line estimation of power systems oscillatory modes is important for power system operation. In this thesis, we introduce two new real-time monitoring algorithms for extracting power system oscillatory modes from a system response seen by multiple system-wide distributed phasor measurements units. The Multidimensional Fourier Ringdown Analyzer, or MFRA, uses Fourier analysis to extract dominant oscillatory modes from multiple synchrophasor measurements in real-time. The use of least square fitting in the proposed MFRA does not only give an accurate estimation of the damping ratio of the system oscillatory modes, but also provides a measure for detecting bad data and outlier signals. The proposed method is shown to be robust under noisy conditions
by testing with simulation data as well as real system data, and is able to extract multiple problematic oscillatory modes computationally fast. The Modal Energy Trending for Ringdown Analyzer, or METRA, estimates the system modes by tracking and analyzing the trend of modal oscillation energy seen in the Power Spectrum Density (PSD) of the measured ringdown response. Singular Value Decomposition of Power Spectrum Density matrix as in Frequency Domain Decomposition (FDD) algorithm is used to get overall energy measures for each dominant mode from multiple PMU signals in the ringdown response. The combination of frequency domain analysis and SVD enables the method to be robust under noisy conditions and makes it suitable for real-time oscillation detection and analysis. Both methods were tested with simulation data as well as real power system archived data, and are shown to accurately extract multiple oscillatory modes and their mode shapes from system measurements.
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Chapter 1

Introduction

Oscillatory stability has been an active area of power engineering research for a long time [3]. With the growing implementations of synchrophasor (or Phasor Measurement Unit (PMU)) devices across the power grid, it is now possible to observe and analyze system wide dynamic phenomena in real-time. Rapid introduction of diverse new generation facilities such as wind farms with complex power electronics controls is making the power grid behave in an unpredictable way and causing it to be more vulnerable towards oscillatory instability. Thus, it is necessary to monitor post disturbance ‘ringdown’ oscillations in real-time to detect poorly damped and negatively damped oscillations.

The western interconnection (WECC) has a long history of interarea oscillations problems. Most notable was the August 10, 1996 blackout. The blackout was caused by undamped oscillations of growing amplitude, which were seen across the entire western system prior to the system separation on August 10, 1996 [4, 5]. Poorly damped oscillatory modes if left uncorrected can affect system reliability and power quality. Low damped modes can also lead to generator rotor fatigue which reduces the lifespan of expensive equipment costing utilities millions of dollars. If oscillatory modes become negatively damped, the problem becomes more severe. Negatively damped modes can lead to tripping of major generating units potentially leading to system islanding and blackout. Therefore, robust oscillation monitoring algorithms are needed to accurately estimate the damping levels of the system modes and also track the changes in the
damping ratios in real-time.

Traditionally, model based modal analysis is most widely used for analyzing oscillatory stability of power systems. The approach requires linearizing the power system model around its equilibrium operating point and calculating the eigenvalues of the linearized system matrix. For a real power system, the operating point keeps changing due to frequent changes in system topology and load patterns, which present a major challenge when using the traditional modal analysis methods. This raises the need for robust measurement based oscillation detection and monitoring algorithms to accurately estimate the power system oscillatory modes, and track the changes in damping ratios of problematic modes.

Motivated by these constraints in using a model based modal analysis, measurement based modal analysis has recently become the focus of power systems engineers. With the growing implementation of phasor measurement units across the grid, real-time measurement based modal analysis has become a necessary tool for grid operators to securely operate the power grid. Such analysis can be performed using two types of PMU measurements; ambient measurements and ringdown measurements. Each one of which has its own mathematical model and analytical algorithms. Ambient data is obtained when the system is under normal conditions and is not experiencing any system disturbances, expect for small amplitude random load variations. Conversely, ringdown data occurs when the power system is experiencing a disturbance such as generator or line tripping, which excites the system oscillatory modes and causes an oscillatory ringdown response. The work in this thesis is focused on developing multi-dimensional ringdown analysers that carry out automatic analysis using multiple PMU signals, and provide an operator friendly results that can be used to take remedial action if needed.
Previously, an Oscillation Monitoring System (OMS) was developed at Washington State University that is intended to continuously monitor the system oscillatory modes and automatically detect poorly damped oscillations at early stages [15]. OMS consists of two complementary engines. A Damping Monitor engine that analyzes ambient PMU data using Frequency Domain Decomposition (FDD) to extract dominant system modes. The second is the Event Monitor engine, which detects power system events based on PMU data, and uses multiple time-domain modal analysis algorithms to extract the system oscillatory modes from the post event ringdown data, and provides a early warning when a consistent low damped oscillations were present in the data.

In this thesis, we propose two new robust multidimensional modal analyzers that accurately extract the power system oscillatory modes using frequency domain analysis. A Multidimensional Fourier Ringdown Analyzer (MFRA), and a Modal Energy Trending for Ringdown Analyzer (METRA). The proposed approaches carry out automatic ringdown analysis on post disturbance data and proved to be suitable for real-time oscillation monitoring. We also focus on the Event Monitor engine from the Oscillation Monitoring System as it provides a suitable benchmark for testing the newly developed modal analysis algorithms.

The basic concept in the proposed MFRA algorithm is to calculate the damping ratio by tracking the trend of oscillation energy for the mode of interest over time. The mode energy is calculated by observing the peak power magnitude associated with a resonant peak in the frequency domain, as seen in signal FFT. The mode energy will decay exponentially over time when the mode is positively damped [7], and the rate of decay is related to the damping ratio together with the mode frequency. Conversely, the mode energy will grow exponentially over time when the mode is negatively damped. In the
proposed MFRA, we carry out automatic analysis on post disturbance ringdown data, and track the rate of change in the oscillation energy by taking the Fourier transform of multiple analysis windows evenly spaced over time. MFRA has a superior advantage over existing methods as it can detect outlier signals that might introduce inaccuracy in the modal results. MFRA will be discussed in details in Chapter 3 of this thesis. Unlike MFRA, the proposed METRA algorithm calculates the damping ratio by tracking the trend of the oscillation energy seen in the power spectrum density for the mode of interest after applying Singular Value Decomposition (SVD) to the Power Spectrum Density (PSD) matrix which provides a cleaner spectrum especially when the noise levels present in the data are high. More detailed discussion on METRA will be presented in Chapter 4.

The content of this thesis is organized as follows; In Chapter 2 we provide a detailed description about the back-end algorithms used in the Oscillation Monitoring System for event analysis. We also present a couple of test cases from real power system events where the OMS detected consistent oscillations at an early stage. Chapter 3 is repeated from [1] where we discuss in details the theoretical derivation of the MFRA algorithm proposed. We present four test cases from simulation systems as well as real power systems that test the applicability of the proposed MFRA as a real-time oscillation monitoring application. A $\chi^2$ statistical test is presented as part of MFRA to detect outlier data within the signals before calculating the mode damping. Chapter 4 is repeated from [2] where we discuss in detail the proposed METRA. We present the mathematical derivation of the algorithm, and provide a step by step example on how the data is structured and processed for analysis. Events from simulation systems as well as real power systems are used to test the proposed method as a real-time oscillation
detection tool. Finally conclusions and suggested future work are presented in Chapter 5.
Chapter 2

Oscillation Monitoring System (OMS)

In this chapter, we revisit the Oscillation Monitoring System (OMS) previously developed at Washington State University, and specifically focus on the Event Analysis Engine (EAE) segment of OMS. Since the newly developed MFRA and METRA algorithms are aimed for analyzing event ”ringdown” data, therefore we will review the EAE of OMS as we compare our proposed algorithms with the results produced by the algorithms OMS.

The objective of Oscillatory Monitoring System is to provide an overview of the real-time operational reliability status of a power system in the context of oscillatory stability using wide-area synchrophasor measurements. OMS estimates the mode frequency, mode damping ratio and mode shape of dominant electromechanical oscillatory modes seen in PMU measurements. Persisting poorly damped oscillatory modes can lead to generator rotor fatigue; thus reducing the lifespan of expensive power system equipment in addition to affecting power quality. Negatively damped oscillatory modes can cause more severe problems leading to tripping of major generating units and loads from the power grid, potentially leading to system islanding and partial blackouts within a power system. Rapid integration of wind farms with complex power electronic controls, as well as continuing growth of system loads introduce operational uncertainty in terms of how the power system modes will evolve in the future. OMS provides a platform for continuously monitoring the oscillatory modes automatically from PMU measurements.
so that emerging problems can be detected in the early stages. OMS includes two types of oscillatory analysis engines. Prony type methods aimed for analysing post disturbance response of the power system and ambient noise methods aimed for analysing the data during normal system conditions. The next section will focus on the Prony type methods used in the OMS framework.

### 2.1 Event Analysis Engine

The event analysis engine provides two levels of oscillation detection by real-time analysis of wide-area measurements following any power system disturbance. That is, OMS is able to detect local modes as well as inter-area modes. For the local oscillation detection, we use multiple signals from the same PMU (or multiple PMUs located at the same substation). The signal groups used for local analysis are pre-specified. However, we form the inter-area mode signal groups automatically from those PMUs that participate in the specific inter-area oscillatory mode. All these tasks of local oscillation detection are executed in parallel by multi-threading in a powerful server exclusively for oscillation monitoring in the control center. Our approach is inherently multi-dimensional aimed at analyzing tens or hundreds of PMU measurements in real-time automatically.

The EAE uses three modal analysis algorithms namely, Prony, Matrix Pencil and Hankel Total Least Square [15, 16]. Each one of these algorithms can extract modal information, such as frequency, damping ratio and mode shape, from noisy PMU measurements by fitting a sum of exponentials to the data being analyzed. The modes from these three algorithms will then be passed through a predefined set of consistency
checks. If a dominant mode is consistent for 3 or 4 consecutive seconds, the engine will report a consistent oscillation detected and alarm will be triggered. A brief description of Prony, Matrix Pencil, and HTLS is presented in section 3, but a detailed theoretical background is described in [15,16]. The flowchart in Figure 1 shows the overall structure of the event monitor engine.

![Flowchart for the Event Analysis Engine in OMS](image)

Figure 1: Flowchart for the Event Analysis Engine in OMS

### 2.2 Test Cases

In this section we present two real power system test cases with problematic oscillations present in the system. The Event Analysis Engine was used to analyse the system events and the modal results obtained from the engine are presented. In both examples,
the engine was able to detect the problematic oscillatory inter area modes as well as subsynchronous modes in a short time.

2.2.1 Growing Oscillation Event

This example was taken from a recent event in the eastern interconnection. The disturbance resulted in oscillations from a local mode which were observed primarily by one PMU (PMU 6), as seen in Figure 2(a). Signals from each PMU are analyzed individually first as part of local PMU analysis. Accordingly, OMS issues a local estimate when moving window crosscheck reaches a consistent estimate. Then, signals from all PMUs that are reporting consistent estimates within a common frequency range are grouped together and inter-area analysis starts. When consistent estimates are seen from multiple PMU groups, inter-area estimates are issued and depending on the observed damping levels, operator alarms can be issued. Figure 2(b) shows the consistent local estimates from all PMUs and clearly PMU 6 shows the most consistent estimates across the three algorithms. The engine detected a consistent inter-area oscillation of 1.18 Hz frequency and 0.09% damping ratio at 352 seconds shown in Figures 2(c) and 2(d).

2.2.2 Sustained Subsynchronous Oscillation

The next example was also taken from the eastern interconnection where subsynchronous oscillations of 12.4 Hz were clearly visible in the PMU voltage and current measurements. These oscillations were caused by two different wind farms that share the same turbine model. The modes get excited when the wind farm experiences high winds and shows low energy during normal operation. OMS was able to detect this
Figure 2: Modal analysis results for the eastern interconnection event sustained subsynchronous oscillations of 12.4 Hz frequency and 0.08% damping ratio, 2 seconds into the data stream. Figure 3(a) and 3(b) shows the frequency and the damping ratio estimates produced by the engine.
Figure 3: Frequency and Damping Ratio estimates for the 12.4 Hz mode
Chapter 3

Multidimensional Fourier Ringdown Analysis (MFRA)

In this chapter we introduce a Multi-dimensional Fourier Ringdown Analyser (MFRA) that carries out automatic ringdown analysis in frequency domain using multiple PMU signals. The basic idea of the proposed MFRA algorithm is to calculate the damping ratio for the mode of interest by tracking the trend of oscillation energy over time. The proposed MFRA extends earlier work in [6–8] towards automatic modal analysis of multiple measurements in real-time. Here the energy is calculated by observing the peak power magnitude associated with a resonant peak in the frequency domain, as seen in signal FFT. The mode energy decays exponentially over time when the mode is positively damped, and the rate of decay is related to the damping ratio together with the mode frequency. Conversely, the mode energy will grow exponentially over time when the mode is negatively damped. This approach for estimating the damping ratio of oscillatory modes was first introduced in [6]. Poon and Lee in [6] also assumed some limitations on the window length and the time gap between the two windows that was then improved in [7] by removing the restrictions on the window length, and by providing a formula that calculates the optimal window length to be used to calculate the damping ratio of that specific mode. [8] extends the method in [7] by calculating the spectrum in more than two different windows, and then use the results to create a set of
simultaneous equations and solve for the mode amplitude, damping ratio and frequency by using a standard parametric estimation algorithm in [9] based on Linear Predictions (LP) and Singular Value Decomposition (SVD). Such algorithms are CPU intensive and may not be suitable for implementation within a relay or a PMU. The details will be discussed in Section 3.2 of this chapter. In this work, the proposed approach uses one big window to estimate the frequency, amplitude and the mode shape of each mode. This provides a better frequency and amplitude estimates since a longer window will contain most of the signal energy and therefore gives better estimates.

The major contributions of the proposed MFRA are as follows:

1) The proposed work is designed for Fourier based real-time automatic modal analysis for streaming multi-dimensional PMU measurement data with no human interactions during real-time implementation.

2) Automatic preprocessing is applied to the time domain signal prior to taking the Fourier Transform, such as by removing the center mean and by detrending. Detrending will be applied to the time domain signal to remove any drift that exists in the signal, and it is shown to result in better damping ratio estimates.

3) As for the damping ratio estimates, multiple smaller windows, evenly spaced, of the same length will be used to find the Fourier amplitude of the oscillatory mode in each window, and the best fit estimate in a least square sense will be used to calculate the slope at which the mode energy is changing and therefore calculate the damping ratio. The proposed work estimates the modal information directly from the FFT signal, as opposed to using a standard parametric estimation algorithm which is CPU intensive.

4) Least square estimation approach also enables handling of multiple signals simultaneously in ringdown analysis towards improved modal estimates and for automatic
detection of bad PMU signals using the proposed $\chi^2$ test.

The outline of this chapter is as follows. Section 3.1 provides an overview of the proposed MFRA algorithm followed by a flowchart that summarises the algorithm. A detailed comparison between the proposed MFRA algorithm and some existing modal extraction algorithms such as Prony, in terms of accuracy and computational speed is discussed in Section 3.2. Thereafter, the proposed MFRA algorithm is tested with simulation data as well as real system data in Section 3.3. Finally, the conclusions are drawn in Section 3.4.

## 3.1 Modal Parameter Estimation

The power system is a high order nonlinear system. For small disturbances, the system can be linearized around its equilibrium point, consequently, the system response following a small disturbance can be expressed as a linear combination of the system oscillatory mode responses. Therefore, the system post disturbance response can be modeled as a sum of exponential terms. Consider the noiseless signal, $y(t)$, which assumed to be the system’s post disturbance ringdown:

$$y(t) = \sum_{j=0}^{m} A_j e^{-\sigma_j t} \cos(\omega_j t + \phi_j) \tag{3.1}$$

where $\sigma_j$ and $\omega_j$ are the damping factor and the oscillatory frequency of the $j$-th mode. $A_j$ and $\phi_j$ is the amplitude and phase of mode $j$.

It is assumed that the signal $y(t)$ is sampled at $F_s$ samples per second. For a window of length $T$ samples that starts at $n_0$ and ends at $n_0 + T$, the Fourier Transform of the
signal over the window is:

$$F(\omega)_{n_0}^{n_0+T} = \sum_{n=n_0}^{n_0+T} y(n)e^{-j2\pi \frac{n}{T}}$$  \hspace{1cm} (3.2)

where $F(w)$ is the complex Fourier transform. The frequency spectrum will have $j$ peaks, one for every oscillation frequency component of the signal [3, 6]. Assuming the oscillation frequencies are far away from each other, (that is, the contribution from other frequency components on mode $j$ can be considered negligible), the Fourier Transform of each separated mode frequency $j$ becomes:

$$F(\omega)_{n_0}^{n_0+T} = \sum_{n=n_0}^{n_0+T} A_j e^{-\sigma j n} \cos(\omega_j n + \phi_j) e^{-j2\pi \frac{n}{T}}$$  \hspace{1cm} (3.3)

$T$ is assumed to be large enough to contain most of the oscillation energy of $\omega_j$. $F(\omega)$ can then be used to calculate the oscillatory frequency $f_j$, the amplitude $A_j$ and the phase $\phi_j$ of mode $j$ using equations (3.4), (3.5) and (3.6).

$$f_j = \arg \max_\omega |F(\omega)| \frac{2\pi}{T}$$  \hspace{1cm} (3.4)

$$A_j = \frac{2|F(w_j)|}{T}$$  \hspace{1cm} (3.5)

$$\phi_j = \text{atan2}(\text{Im}(F(\omega_j)), \text{Re}(F(\omega_j)))$$  \hspace{1cm} (3.6)

To calculate the damping factor $\sigma_j$ of mode $j$, consider a smaller window of $N$ samples, where $N < T$, within the big window $T$. That is, window $N$ starts at the beginning of window $T$, at $n_0$, and ends at $n_0 + N$. The Fourier transform of mode $j$ over the smaller window $N$ is:

$$F(\omega)_{n_0}^{n_0+N} = \sum_{n=n_0}^{n_0+N} A_j e^{-\sigma j n} \cos(\omega_j n + \phi_j) e^{-j2\pi \frac{n}{T}}$$  \hspace{1cm} (3.7)

The value of the Fourier Transform $F(\omega)_{n_0}^{n_0+N}$ at $\omega_j$ can be simplified according to [7]:

$$F(\omega_j)_{n_0}^{n_0+N} \approx A_j e^{j\phi_j} \sum_{n=n_0}^{n_0+N} e^{-\sigma j n}$$  \hspace{1cm} (3.8)
Similarly, taking the Fourier Transform of the signal \( y(n) \) over the same window length \( N \) but at later time, where \( N \) starts at \( n_0 + G \) and ends at \( n_0 + N + G \), where \( G \) is the step size between consecutive windows with \( n_0 + N + G < T \):

\[
F(\omega)|^{n_0+N+G}_{n_0+G} = \sum_{n=n_0+G}^{n_0+N+G} A_j e^{-\sigma_j n} \cos(\omega_j n + \phi_j) e^{-j2\pi \frac{n}{N}}
\]

\[
F(\omega_j)|^{n_0+N+G}_{n_0+G} \approx \frac{A_j e^{j\phi_j} n_0+N+G}{2} \sum_{n=n_0+G} e^{-\sigma_j n}
\]

Taking the ratio of the magnitude Fourier Transforms at two different time windows:

\[
\frac{F(\omega_j)|^{n_0+N+G}_{n_0+G}}{F(\omega_j)|^{n_0+N}_{n_0+G}} = \frac{A_j e^{j\phi_j} n_0+N+G}{2} \frac{\sum_{n=n_0+G} e^{-\sigma_j n}}{\sum_{n=n_0} e^{-\sigma_j n}}
\]

\[
= e^{-\sigma_j G}
\]

The damping \( \sigma_j \) of mode \( j \) can then be calculated by:

\[
\sigma_j = \frac{\ln(F(\omega_j)|^{n_0+N+G}_{n_0+G}) - \ln(F(\omega_j)|^{n_0+N}_{n_0+G})}{G}
\]

Similar approach can be used to calculate the magnitude Fourier transform of multiple \( K \) consecutive sliding windows of the same length \( N \) spaced \( G \) samples apart. The rate of change of the magnitude Fourier transform as the window slides is used to determine the damping factor of mode \( j \). The damping factor \( \sigma_j \) of mode \( j \) can then be attained from the slope of the best line fit (in a least square sense) of the Logarithmic magnitude Fourier transform of all the consecutive sliding windows. That is, if \( Y \) is a vector containing the magnitude Fourier transform of mode \( j \) at different sliding time windows and vector \( X \) contains the end time of each corresponding sliding window:

\[
Y = [F(\omega_j)|^{n_0+N}_{n_0}, F(\omega_j)|^{n_0+N+G}_{n_0+G}, F(\omega_j)|^{n_0+N+2G}_{n_0+2G}, ...]
\]

\[
X = [n_0 + N, n_0 + N + G, n_0 + N + 2G, ...]/Fs
\]
Assuming the original signal $y(t)$ is a noiseless signal, the Fourier transforms in $Y$ fit a straight line from equations (3.10)-(3.13) and therefore can be considered as an over determined system:

$$X_i\sigma_j + b = \ln(Y_i), i = 1, 2, 3, \ldots K$$

(3.16)

where $b$ is the magnitude Fourier transform of mode $j$ over the window $-T$ to 0 seconds.

To find the damping factor $\sigma_j$ that best fits the data in hand, we need to solve an optimization problem which minimizes the error $S(X,Y)$ in the fit:

$$S(X,Y) = \sum_{i=1}^{K} W_i^2 [\ln(Y_i) - (X_i\sigma_j + b)]^2$$

(3.17)

where $W_i$ denotes the relative weights in least square estimation. Finding the damping factor with the minimum error requires solving the system of equations (3.18):

$$\begin{bmatrix}
    \ln(Y_1) \\
    \vdots \\
    \ln(Y_K)
\end{bmatrix}
= 
\begin{bmatrix}
    X_1 & 1 \\
    \vdots & \vdots & \vdots \\
    X_K & 1
\end{bmatrix}
\begin{bmatrix}
    \sigma_j \\
    b
\end{bmatrix}$$

(3.18)

Then, the least square estimate $\hat{\beta}$ is given by

$$\hat{\beta} = (X^TW\bar{X})^{-1}X^TW\bar{Y}$$

(3.19)

where $W$ is a diagonal matrix of weights $W_i$. The damping ratio $\zeta_j$ can then be calculated using:

$$\zeta_j = \frac{-\sigma_j}{2\pi f_j}$$

(3.20)

Note that the window size $N$ should be large enough to contain at least one period of the mode of interest $f_j$. From our tests, $N$ is suggested to be two periods or $\frac{2}{f_j}$. The
choice of step size $G$ can be arbitrary in theory. Based on experiments, $G$ is set to be either half a period $\frac{1}{2f_j}$ or one period $\frac{1}{f_j}$. The overall window length $T$ has to be greater than $N + LG$ where $L = K - 1$ for getting $K$ entries in the least square equation (3.18). In this chapter, the window length $T$ is selected to be 15 seconds which covers typical power system electromechanical oscillations at frequencies higher than $f_j = 0.1667$ Hz. For the mode $0.1667$ Hz, one period for the mode is 6 seconds. Therefore, from the discussion above, $N = 12$ seconds, and step size $G$ is say at half a period 3 seconds. Then, the overall window of 15 seconds will give a total of 2 Fourier magnitude estimates. With $K = 2$ in equation (3.18), the damping ratio will be calculated by a direct fit for 0.1667 Hz mode. Whereas if the mode frequency is say 0.2 Hz, then $N = 10$ seconds, $G = 2.5$ seconds and $K = 3$ in equation (3.18) for the choice of $T = 15$ seconds. Accordingly, the window length $T$ can be chosen sufficiently long to accommodate whatever number of estimates $K$ are required in the least square estimation (3.18) for the lowest mode frequency of interest in the estimation.

### 3.1.1 Multi-dimensional Approach

All the previous research in [6–8] assume a single output model. The proposed work allows for multi-dimensional analysis. That is, multiple signals can be used to calculate one set of mode estimates. When performing the multi-signal fit, theoretically, the analysis is expected to provide more accurate modal estimates since richer information is provided to the analysis, assuming all signals are responding to the mode of interest. Conversely, if some signal is not showing a significant response for a specific mode as reflected in its Fourier magnitude at the mode frequency, such a signal will not be
included in the least square fit estimation of the damping ratio that is proposed in this section.

To analyze multiple signals using the proposed MFRA algorithm, we simply calculate the Fourier transform of multiple sliding windows of length $N$ spaced $G$ samples apart, as in equation (3.14), for each one of the signals and then stack vectors $y$ vertically while vector $X$ remains the same for all outputs:

$$
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_m
\end{bmatrix}
= 
\begin{bmatrix}
X & 1_K & 0_K & \ldots & 0_K \\
X & 0_K & 1_K & \ldots & 0_K \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
X & 0_K & 0_K & \ldots & 1_K
\end{bmatrix}
\begin{bmatrix}
\sigma_j \\
b_1 \\
b_2 \\
\vdots \\
b_m
\end{bmatrix}
$$

(3.21)

where each of $y_1$ through $y_m$ is a $(K \times 1)$-column vector from equation (3.18); $X$ is a $(K \times 1)$-column vector defined in equation (3.15); $1_K$ and $0_K$ denote $(K \times 1)$-column vectors of ones and zeros respectively; and $m$ is the number of signals analyzed. Now by solving equation (3.21), we perform a linear least square fit and calculate $\sigma_j$ which represents the damping factor of mode $j$ seen in all $m$ signals.

### 3.1.2 Example

Figure 4 shows a flowchart that summarizes the proposed algorithm for modal extraction of system modes. To further clarify the proposed MFRA approach, consider the 3 synthetic signals shown in Figure 20. Each one of the signals contains two low damped oscillatory modes plus 35dB random noise. First mode is a 1 Hz mode with 2% damping ratio and the second mode is 0.5 Hz with 5% damping ratio in signal (a), 0.25 Hz with 3% damping ratio in signal (b), and 0.35 Hz with 4% damping ratio in signal
(c). Using the multi-dimensional approach described in the previous section, all three signals will be used to extract the damping ratio of the 1 Hz oscillatory mode that exists in all three.

The sampling frequency $F_s$ used is 30 samples per second which is a common sampling frequency for PMUs. The large window $T$ is 15 seconds (450 samples) which is large enough to contain most of the oscillation energy. The smaller window $N$ is 10 seconds (300 samples). Different frequency components in the source signals can be distinguished when taking the Fourier Transform. In the frequency domain, different frequency components can be identified by different peak locations in the magnitude Fourier Transform. If any two frequency components are close to each other, say within

Figure 4: MFRA Algorithm Flowchart
±5%, the significant component will only be considered and the other mode will be neglected. For power system applications, this assumption is valid since most system modes are typically more than 5% apart. From the Fourier spectrums of the window $T$, a common mode was identified in all 3 signals at 1 Hz, thus the step size $G$ using $G = 1/f$ will be 1 second (30 samples). That is, the sliding window $N = 10$ will be moving every $G = 1$ seconds, giving a total of $K = 6$ windows total. Taking the Fourier transform of each one of these windows from all three signals, the logarithmic Fourier amplitude will be decaying at a rate of $\sigma$ the damping factor and that is shown in Figure 6. A similar method has previously been applied in the wavelet analysis of oscillatory modes [10] while the proposed method here is by directly using FFT analysis to calculate the damping.

To calculate the damping ratio $\zeta$, we take the logarithmic Fourier magnitude of all $K = 6$ windows for all 3 signals, and stack them vertically as in equation 3.21. Vector
Figure 6: The Change in Logarithmic Fourier Magnitude for different time windows

$X$ is simply constructed by stacking the end time of each window:

$$
\begin{bmatrix}
  y_1 \\
  y_2 \\
  y_3 
\end{bmatrix} =
\begin{bmatrix}
  4.466, 4.312, 4.214, 4.061, 3.962, 3.811, \\
  4.465, 4.328, 4.185, 4.075, 3.967, 3.872, \\
  4.468, 4.320, 4.192, 4.093, 3.937, 3.821
\end{bmatrix}^T
$$

$$X = [10, 11, 12, 13, 14, 15]^T$$

The final step will be calculating the least square fit of all the values in $[y_1 \ y_2 \ y_3]^T$ with respect to $X$ as in equation 3.21. Figure 22 shows the least square fit whose slope corresponds to damping factor $\sigma$. Similarly the formulation can be applied for the

Figure 7: The Change in Logarithmic Fourier Magnitude for different time windows

Slope = $\sigma = -0.125$

$$\zeta = -\frac{\sigma}{2\pi\times 1} = 2\%$$
estimation of the other three modes at 0.5 Hz, 0.25 Hz and 0.35 Hz in the test signals though the multi-dimensional formulation which will reduce to a single dimensional analysis of only one of the signals (a), (b) or (c) for each of 0.5 Hz, 0.25 Hz and 0.35 Hz modes respectively. The analysis is not repeated to save space.

### 3.2 Algorithm Performance

The proposed MFRA algorithm can accurately extract modal information of electromechanical oscillatory modes from noisy sampled measurements. The strengths of the proposed algorithm as compared to other available modal extraction approaches are a) speed of computation, b) ability to find multiple modes with various damping levels, and c) the ability to extract modal information even when nonlinearities, such as line switching and noise, are present in the system.

Prony analysis is one of the common methods for extracting modal information from evenly sampled data. The main steps of Prony analysis will be summarized in this section, however, refer to [11,15] for more detailed discussion on Prony analysis. Assuming signal $y(t)$ in equation 3.1 is evenly spaced by $\Delta t$, a Prony solution can be obtained by first constructing a discrete linear prediction model (LPM) that fits $y(t)$. Then find the roots of the $n^{th}$ order characteristic polynomial associated with the constructed LPM, which represents the complex modal frequencies of signal $y(t)$. Using the calculated roots, calculate the complex residues which determines the amplitude and the phase of each mode. Despite the fact that classical Prony algorithm is a common method for modal analysis, it is known to behave poorly when a signal is noisy. It yields parameter estimates with a large bias due to its sensitivity to measurement noise. It does not make
a separate estimate of the noise. It also fits exponentials to any additive noise present in the signal. When Prony analysis is applied to a signal embedded in noise, the damping and frequency terms are typically not close to their true values. The proposed algorithm has better performance under noisy signals, since the Fourier transform separates the signal modes from the noise and therefore the proposed algorithm has a better advantage over Prony analysis especially under noisy measurements.

Eigensystem Realization Algorithm (ERA) [12, 13] is another method also used for extracting modal information from measured system data. ERA is based on applying Singular Value Decomposition (SVD) to the Hankel matrix associated with the measured signals. The main steps of ERA can be summarised in the following steps. Build the Hankel matrices $H_0$ and $H_1$ whose entries are samples of the signal $y(t)$ assuming that the measured signal is the system's impulse response. Perform singular value decomposition of $H_0 = UΣV^H$ and retain the largest $N$ singular values in the diagonal matrix $Σ(Σ_N)$. Compute the discrete state matrix $F = Σ_N^{-1/2}U_N^TH_1Σ_N^{-1/2}$ and its corresponding continuous state matrix $A = \log_e(F\Delta t^{-1})$. The signal modes can then be calculated by finding the eigenvalues of the continuous state matrix $A$. Refer to [12, 13] for detailed derivation of the ERA algorithm. The major challenge in using ERA in a real-time application is the requirement of extensive processing time. SVD is computation intensive and requires a lot of CPU floating point processing time to perform which can be hard to implement within a PMU. On the other hand, the proposed FFT based approach does not require any extensive processing time and can easily be implemented in many PMU devices.

Matrix Pencil [14] and Hankel Total Least Square (HTLS) [15] have been previously applied to extract modal oscillatory information from measured power system response.
The idea of Matrix Pencil method comes from the pencil-of-function approach which is commonly used in system identification and spectrum estimation. The main steps of Matrix Pencil are described in this section. For more details on this algorithm refer to [14, 15]. First construct matrix $[Y]$ using the uniformly sampled signal $y(t)$ as described in [14, 15]. Apply SVD to $[Y] = U\Sigma V^T$ and use the largest $N$ singular values to reconstruct the original data matrix. From the unitary matrix $[V_N]$ whose column vectors correspond to the $N$ significant singular values of $\Sigma$, calculate $[V_N^1]$ and $[V_N^2]$ by deleting the last row and the first row of $[V_N]$ respectively. To find the modes of $y(t)$, calculate the eigenvalues of matrix $[V_N^2]^H([V_N^1]^H)^+$, where $+$ denotes pseudo-inverse and $H$ denotes conjugate transpose. HTLS is an improved version of the Matrix Pencil algorithm. The main steps of HTLS are summarised below, however, more detailed discussion of HTLS is described on [15]. Similar to ERA, construct the Hankel matrix $H$ which can be factorized as $H = SRT^T$, where $S$ and $T$ are Vandermonde matrices. Matrix $S$ is shift-invariant, that is $S_\downarrow Z = S_\uparrow$ where the up and down arrows stand for deleting the top and the bottom rows of matrix $S$, and $Z$ is a diagonal matrix whose entries are signal $y(t)$ poles. Now apply SVD to obtain $H = U\Sigma V^H$, where $U$ and $V$ are unitary matrices, and $\Sigma$ is a diagonal matrix of singular values. From the unitary matrix $U_N$ whose row vectors correspond to the $N$ most significant singular values of $\Sigma$, calculate matrices $U_N^\uparrow$ and $U_N^\downarrow$ by deleting the first and last row of matrix $U_N$ respectively. Matrices $U_N^\uparrow$ and $U_N^\downarrow$ are related by $U_N^\uparrow = U_N^\downarrow \hat{Z}$, where $\hat{Z}$ has the same eigenvalues as matrix $Z$. In noisy conditions, the relationship does not hold exactly and $\hat{Z}$ can be solved by total least square method. After calculating $\hat{Z}$, the signal poles are calculated by finding the eigenvalues of $\hat{Z}$. Matrix Pencil, HTLS, and ERA use singular value decomposition (SVD) to filter out the noise components of the system response.
Table 1: Most CPU intensive algorithm subtasks

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Intensive Subtasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFRA</td>
<td>Fourier Transform and Least Square fit</td>
</tr>
<tr>
<td>Prony</td>
<td>Finding the characteristic polynomial and its roots</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>SVD of $[Y]$ Matrix</td>
</tr>
<tr>
<td>HTLS</td>
<td>SVD of $[H]$ and $[U_N^T \cdot U_N]$ matrices</td>
</tr>
<tr>
<td>ERA</td>
<td>SVD of $[H_0]$ matrix</td>
</tr>
</tbody>
</table>

and accurately estimate the actual system modes. As mentioned earlier, SVD is computation intensive and can be difficult to implement in light computational platforms such as within PMUs. Table 1 summarises the most CPU intensive steps in each of the 5 modal analysis algorithms.

In order to evaluate the performance of the proposed MFRA algorithm, it is compared with other modal analysis algorithms in terms of accuracy and CPU processing time. Studying the processing time of these algorithms is important for ease of real-time implementation. This comparison will test the overall performance of the proposed MFRA compared to 4 different modal analysis algorithms, namely ERA, Prony, Matrix Pencil and HTLS. Every algorithm will process a standard signal, expressed in 3.22, containing two different modes, 0.5 Hz mode with 4% damping ratio and a 1 Hz mode with 1% damping ratio, plus randomly generated white noise.

$$y(t) = e^{-0.1257t} \cos(2\pi \times 0.5t) + e^{-0.0628t} \cos(2\pi \times 1.0t) + n(t)$$  \hspace{1cm} (3.22)

The algorithms are tested with two levels of noise, 25dB and 15dB. For each of these levels, 100 Monte Carlo simulations are performed, each with different random noise, but at the same noise level. The mean and the standard deviation for both the frequency
and the damping ratio estimates of both modes are calculated. The mean processing time and the standard deviation of the 100 Monte Carlo simulations are also calculated. Table 2 shows the results of the frequency and the damping ratio compared across all five algorithms under different noise levels.

From Table 2, we can observe that the proposed MFRA algorithm is comparable to ERA, Matrix Pencil and HTLS in terms of accuracy of damping estimates under high levels of noise. Prony had the worst performance under high levels of noise with the highest error in the damping estimate of the first mode with 11% error and 14% error for the second mode.

Table 3 shows the average processing time for each modal analysis algorithm. Looking at Table 3, the proposed MFRA algorithm clearly is much faster than the other four. It was able to extract the oscillatory modal information from noisy signals about 14 times faster than ERA and HTLS and about 35 times faster than Prony and Matrix Pencil. The main advantage of such a fast modal analysis algorithm comes clear when analysing large number of PMU signals. By design, the proposed MFRA algorithm is scalable which is a unique design feature that separates it from the other algorithms. That is, the CPU time to process larger number of PMU signals using the proposed MFRA algorithm will not be affected as much as the other algorithms. Since the Fourier Transform is calculated for every individual signal independently, analysing more signals will increase the CPU processing time in a linear sense. However, analysing larger number of signals using the other algorithms will increase the size of the data matrices exponentially, which in return will increase the CPU processing time exponentially. Also, the scalability feature of the proposed MFRA can be utilized using the multi-threading technologies of the newer CPUs, by calculating the Fourier Transform of each signal in
Table 2: Comparison between the proposed MFRA and four different algorithms

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5 Hz</td>
<td>4.0%</td>
<td>1.0 Hz</td>
<td>1.0%</td>
</tr>
<tr>
<td>MFRA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25dB</td>
<td>Mean</td>
<td>0.5000</td>
<td>3.9796</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0000</td>
<td>0.1451</td>
<td>0.0000</td>
</tr>
<tr>
<td>15dB</td>
<td>Mean</td>
<td>0.5008</td>
<td>3.8949</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0027</td>
<td>0.4396</td>
<td>0.0000</td>
</tr>
<tr>
<td>Prony</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25dB</td>
<td>Mean</td>
<td>0.4994</td>
<td>4.1018</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0009</td>
<td>0.1996</td>
<td>0.0006</td>
</tr>
<tr>
<td>15dB</td>
<td>Mean</td>
<td>0.4993</td>
<td>4.4435</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0025</td>
<td>0.4930</td>
<td>0.0019</td>
</tr>
<tr>
<td>HTLS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25dB</td>
<td>Mean</td>
<td>0.4995</td>
<td>3.9809</td>
<td>1.0001</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0007</td>
<td>0.1223</td>
<td>0.0005</td>
</tr>
<tr>
<td>15dB</td>
<td>Mean</td>
<td>0.4997</td>
<td>3.9044</td>
<td>0.9999</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0029</td>
<td>0.5894</td>
<td>0.0014</td>
</tr>
<tr>
<td>ERA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25dB</td>
<td>Mean</td>
<td>0.4995</td>
<td>3.9997</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0007</td>
<td>0.1515</td>
<td>0.0004</td>
</tr>
<tr>
<td>15dB</td>
<td>Mean</td>
<td>0.4998</td>
<td>4.0783</td>
<td>1.0003</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0024</td>
<td>0.4344</td>
<td>0.0014</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25dB</td>
<td>Mean</td>
<td>0.4994</td>
<td>3.9735</td>
<td>1.0001</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0006</td>
<td>0.1485</td>
<td>0.0004</td>
</tr>
<tr>
<td>15dB</td>
<td>Mean</td>
<td>0.4998</td>
<td>4.0061</td>
<td>1.0003</td>
</tr>
<tr>
<td></td>
<td>STD</td>
<td>0.0021</td>
<td>0.4455</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

parallel, which in return will cut down the CPU processing time even more.
Table 3: CPU time comparison between algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Processing Time (ms)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 dB</td>
<td>25 dB</td>
<td></td>
</tr>
<tr>
<td>MFRA</td>
<td>9.64</td>
<td>9.81</td>
<td></td>
</tr>
<tr>
<td>Prony</td>
<td>437.81</td>
<td>377.44</td>
<td></td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>365.93</td>
<td>359.66</td>
<td></td>
</tr>
<tr>
<td>HTLS</td>
<td>176.04</td>
<td>126.58</td>
<td></td>
</tr>
<tr>
<td>ERA</td>
<td>168.44</td>
<td>127.57</td>
<td></td>
</tr>
</tbody>
</table>

3.2.1 Automatic Real-time Framework

The proposed work is aimed for real-time automatic modal analysis for streaming multi-dimensional PMU measurements. That is, the proposed framework will extract the oscillatory modes seen in live PMU ringdown measurements automatically without any direct human interaction by extending the framework proposed earlier in [15]. Figure 8 presents the flowchart that shows different steps taken by the proposed framework while processing live PMU data before applying the MFRA algorithm. A sliding analysis window of $T$ seconds, say 15, is captured and processed every $S$ seconds, say 1 second. The analysis windows from multiple PMU channels will be first checked for bad data. Some channels might not be reporting any data, and will be discarded. Some channels might have some missing data points. In this case, if the number of missing points is less than a certain threshold, interpolation will be performed to replace those missing data points. If too many points are missing, the channel will be discarded.

Once all the data sanity checks and interpolation have been performed, the analysis windows are checked for events. The event detection routine can detect ringdowns,
Figure 8: Flowchart of the proposed framework

casted by sudden disturbances, and slow evolving oscillations. Sudden disturbances are detected by monitoring the change in voltage and/or current magnitudes, $dV/dt$ and/or $dI/dt$. On the other hand, slow evolving oscillations, which does not show a significant change in the $dV/dt$ or $dI/dt$, are detected by monitoring the Coefficient of Variance of the analysis window, similar to the standard deviation approach proposed by Tennessee Valley Authority engineers in [17]. Once the event flag is triggered, the signals that are
experiencing the event will then be selected for further processing.

The next step is to check if the event is slow evolving oscillations or a sudden disturbance. When a sudden disturbance occurs resulting in a ringdown oscillatory response, see Figure 9, the MFRA algorithm will see that mode as negatively damped for that analysis window for the time windows that have the event start time in the middle of the FFT analysis window. For example, if the mode is not excited in the first 12 seconds of the window and is excited in the last 3 seconds after the event, the peak of that mode will increase in magnitude over the period of that window. To solve this issue, the program skips the analysis windows where a high jump in the signal $dV/dt$ $df/dt$ occurs, and starts the analysis once that sharp jump leaves the analysis window. If the jump seen in the analysis window is small or medium in magnitude, and the value of $dV/dt$ or $df/dt$ is below threshold, the window will be processed since the proposed MFRA algorithm can handle switching and nonlinearities present in the analysis window, to a certain extent. This approach will also still work with events when the oscillations grow gradually, since the change in the respective signals will not show any sudden jumps. If the jump (or discontinuity) magnitude is above a preset threshold, the channel is discarded because FFT may be unreliable.

To further clarify the approach used to resolve the issue stated, consider the event shown in Figure 9. The dominant mode excited in this event is estimated to be 0.39 Hz with 9.3% damping ratio by averaging the results of the other four engines Prony, Matrix Pencil, ERA and HTLS (from Table 6 in 3.3.4). If windows with high $df/dt$ were processed, the program will see the event as negatively damped mode for the first part of the event and once the ringdown starts going through the analysis window, the damping estimates gets closer to the correct value and eventually will find the correct
damping ratio, as shown in the bottom left plot. But if the windows with high $df/dt$ were skipped, the estimates with negative damping will be skipped and the program will only report the correct damping ratio estimates, seen in the bottom right plot.

Figure 9: Handling events in proposed framework

Once the analysis windows passes the event start and signal jump checks, the data gets preprocessed before any analysis gets done. The mean value will be removed to get rid of the DC component in the signal and then normalized by dividing the windowed signal by its max value. Also, detrending is important before FFT analysis to remove any drift that exists in the signal. Detrending results in better damping ratio estimates. In the proposed approach, a best fit third order polynomial (in the least-squares sense) will be subtracted from the windowed signal to remove any drift in the signal. Figure 10 shows that the MFRA damping ratio estimates are more accurate after detrending the signal as compared to the MFRA estimates produced from the original signal. The windowed signals will then go through the MFRA algorithm, shown in Figure 4, to extract the oscillatory modes and their mode shapes. All PMU signals from across the system will be analyzed and the MFRA algorithm will automatically group the signals that are oscillating at the same frequency to calculate the mode frequency and damping
Figure 10: Effect of detrending on MFRA damping estimates

Note that MFRA does not require all the signals to have the same observability for the modes. As long as the mode is present and can be identified in the Fourier Transforms of the signals, it will be processed even if it is not the dominant mode. In other words, if the magnitude of the Fourier Transform for mode $j$ is higher than a predefined threshold, the magnitude Fourier Transform will be used in calculating the least square estimation of the damping ratio. If any of the signals is negatively affecting the precision
of the damping estimation, a $\chi^2$ test introduced in Section 3.3.4 will detect and discard that specific signal for that specific mode.

### 3.3 Test Cases

The main objective of this section is to test the proposed MFRA approach for real-time ringdown analysis. That is, the ability to detect and identify multiple time-varying modal parameters in a real-time environment in power systems is tested. To verify the applicability of the proposed MFRA approach, four test cases are adopted: 1) Kundur’s two-area network [3], 2) WECC real system event with low damping level, 3) IEEE First Benchmark for Subsynchronous oscillations [21], and 4) WECC real system event with a bad PMU signal. Four test cases are subsequently formulated and the results are presented in the upcoming subsections.

#### 3.3.1 Kundur’s Two-Area System

![Figure 11: Kundur two-area test system [3], [18]](image)

Referring to Figure 11, the Kundur system has two areas and 4 generators with tie-lines interconnecting the two regions. The operating condition and the system parameters are the same as in [3] and in [18]. TSAT [18] was used to simulate a case where
a system disturbance causes the inter-area mode to change to critically damped resulting in long sustained oscillations. Five seconds into the simulation, first disturbance occurs when a 300 MVAR capacitor bank switches out at load bus 9. This change causes the damping ratio of the inter-area mode to become nearly zero and results in sustained oscillations. After 20 seconds of sustained oscillations, because of some automatic control action or alert operator reaction, the 300 MVAR capacitor bank is switched back in and that brings the system back to small-signal stability.

The data from this simulation is analyzed using the proposed MFRA algorithm and the results are compared to the results obtained from 4 different modal analysis algorithms, namely Prony, Matrix Pencil, HTLS, and ERA in Table 4. The windows used to analyse the both events are highlighted in Figure 12. The window used for event 1 is from $t = 8$ sec. to $t = 23$ sec., and the window used for the second event is from $t = 29$ sec. to $t = 44$ sec. The system has an inter-area mode of 0.7 Hz whose damping ratio changes to an unacceptably low value nearly zero percent after the first disturbance, and then changes back to a somewhat better 1.6% after switching in the 300 MVAR capacitor bank at bus 9. Figure 12 below shows the system response of the described case measured at bus 1.

![Figure 12: Test Case 1: Kundur two-area system event](image-url)
Table 4: Modal results for the Kundur two-area system

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Event 1</th>
<th></th>
<th>Event 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
</tr>
<tr>
<td>MFRA</td>
<td>0.680</td>
<td>-0.008</td>
<td>0.690</td>
<td>1.593</td>
</tr>
<tr>
<td>Prony</td>
<td>0.682</td>
<td>-0.004</td>
<td>0.690</td>
<td>1.596</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>0.682</td>
<td>-0.010</td>
<td>0.689</td>
<td>1.570</td>
</tr>
<tr>
<td>HTLS</td>
<td>0.682</td>
<td>-0.010</td>
<td>0.689</td>
<td>1.570</td>
</tr>
<tr>
<td>ERA</td>
<td>0.682</td>
<td>-0.010</td>
<td>0.689</td>
<td>1.570</td>
</tr>
</tbody>
</table>

Looking at the results in Table 4, the proposed MFRA algorithm produced results with similar accuracy compared to all four engines. The mode shape identified by the proposed MFRA algorithm (Figure 13(b)) for the inter-area mode compares well with the average mode shape identified by all four algorithms shown in Figure 13(a).

Figure 13: Mode shapes for the interarea mode

To further demonstrate the proposed framework and how it processes real-time synchrophasor measurements, the simulation data shown in Figure 12 was processed through the proposed automatic real-time framework shown in Figure 8. The window size $T$ used in this analysis is 15 seconds. The program takes a moving window of 15
seconds and goes through the process of data sanity checks, event checks, and "jumps" checks. If the analysis window passes through all these checks, the results from the MFRA algorithm will be checked for consistency and will be reported as consistent estimates. These results are shown as green dots in Figure 14. Otherwise, if the analysis window does not pass any of these checks, the analysis window will be discarded. In the simulation case described above, when the capacitor banks switches in and out, these result in high jumps (high $dP/dt$) in the data which will cause any window containing this high jump to be rejected by the engine. The results shown in red in Figure 14 shows the results that would have been produced if the engine had not skipped those analysis windows. If the "jump" magnitude in the data signal is higher than a certain prespecified threshold, the FFT calculation will be unreliable and the data window should be discarded.

Figure 14: Damping ratio estimates during the switching event

3.3.2 WECC real system event

The western interconnection WECC has a long history of interarea oscillations. Inter-area power transfer capability from North to South has typically been limited by stability
concerns during summer months because the power has to travel over geographically long
transmission lines. Figure 25 shows a recent event recorded by PMUs in the western
interconnection. The event resulted in growing oscillations of 1 Hz. The oscillations
started as being negatively damped. After about 35 seconds of oscillations, some auto-
matic control action changed the mode oscillation damping to positive level and brought
the system back to small-signal stability. The whole event lasted for about two minutes
of oscillations. The proposed MFRA approach was used to analyze the event using a
sliding window of 15 seconds and a step size of 1 second.

![Figure 15: Test Case 2: A recent WECC event](image1)

![Figure 16: Damping ratio estimates using MFRA.](image2)

Figure 26 shows the frequency and damping ratio estimates for the dominant 1 Hz
mode throughout the event. The algorithm tracks the actual damping ratio of the
mode very well dynamically throughout the event. Specifically the mode damping ratio
changes from negative damping (168 seconds to 190 seconds) to zero damping (190 sec-
onds to 213 seconds) to positive damping (after 213 seconds). These results match very
well with the actual system response in Figure 25. Specifically, the proposed method-
ology is able to provide valuable feedback on the evolving status of the damping level
of the oscillatory mode to the operator or to automatic controls during this complex
event. Also, the proposed MFRA method handles the nonlinear switching events that
occur within the event (at 185 seconds and 204 seconds and 213 seconds which are likely
operator or automatic actions). Such nonlinear switching characteristics, nonsmooth in
nature, are particularly challenging for Prony class of algorithms. The final ringdown
section of the event when analyzed with the proposed MFRA algorithm find the modal
estimates at 1.07 Hz frequency and 2.61% damping ratio.

3.3.3 IEEE First Benchmark for Subsynchronous Resonance

Interaction of series compensators in transmission lines with the torsional mechanics
of synchronous generators can cause problematic oscillations in power systems in the
subsynchronous range. This problem is known as Sub-Synchronous Resonance (SSR).
Modern power systems are experiencing a rapid increase in renewable energy generation
from wind farms which can also introduce SSR issues [19]. Therefore, SSR oscillatory
modes and their damping levels need to be monitored continuously in real-time. In this
section, we apply the proposed MFRA algorithm to detect and analyze subsynchronous
oscillations. Typical electromechanical oscillatory modes of power systems are in the
range of 0.1 Hz to 2.0 Hz. These modes are well suited for synchrophasor based oscillation
monitoring. Since subsynchronous oscillations are in the 10-60 Hz range, a much higher
sampling rate is required to detect and analyze these oscillations. PSCAD/EMTDC [20]
was used to simulate the IEEE First Benchmark for Sybsynchronous Resonance [21]. The sampling frequency of the data produced by PSCAD was set to 125 s/sec which is enough to capture all of the IEEE First Benchmark subsynchronous modes. Note that a sampling time of 8 milliseconds (sampling frequency at 125 Hz) is used in the PSCAD simulation to avoid round-off errors in PSCAD simulation which would otherwise result say for 120 Hz sampling rate.

The proposed automatic framework is used to detect and analyse unstable oscillations following a system disturbance. The series compensation level was set to 74% at the start of the simulation which makes two of the subsynchronous modes to be unstable. Initially, the series capacitors are off-line. At t = 2 seconds, the series capacitors are switched in, which excites the electromechanical mode as well as SSR modes. The framework proposed in Figure 8 is applied to detect any unstable oscillatory modes following the disturbance using a moving window $T = 4$ seconds and the moving window is refreshed every 0.5 seconds. For each of the three modes, $N = \frac{2}{f_j}$ and $G = \frac{1}{2f_j}$ as discussed in Section 3.1. The output estimates of the engine throughout the event are shown in Figure 17. The proposed framework is able to detect a consistent unstable subsynchronous modes at t = 7.5 seconds, and the results from the engine are then used to trigger a bypass switch which takes the series capacitors off-line to bring the system back to stable condition. Figure 17 shows the real power output of the generator that was used by the proposed framework to detect unstable oscillations. The proposed framework detects two consistent subsynchronous oscillations, with frequencies of 15.907 Hz and 20.270 Hz and damping ratios of -0.079% and -0.278% at t = 7.5 seconds. A well damped local mode is detected at t = 7 seconds, with frequency of 1.633 Hz and damping ratio of 9.785%. This scenario illustrates the usefulness of the proposed framework for
application as an automatic real-time SSR detection tool.

![Figure 17: Real Power Output](image)

To show the effectiveness of the proposed MFRA algorithm to accurately extract subsynchronous oscillation modes from measured system response, all five modal analysis algorithms presented in section 3.2 were used to analyse 4 seconds of the measured real power output data highlighted in Figure 17 by the dotted box. The data contains two negatively damped sub-synchronous modes and a well damped local mode. The modal estimates for all three modes from all five modal analysis algorithms are tabulated in Table 5. Looking at the estimates in Table 5, it is clear that the proposed MFRA algorithm produced consistent results with the remaining four algorithms and was able to extract all three modes accurately.

### 3.3.4 $\chi^2$-test for detecting bad PMU signals

The purpose of this section is to demonstrate how the proposed framework can detect signals that would negatively impact on the accuracy of the damping ratio estimate using the standard $\chi^2$ statistical test automatically. Here it is assumed that the standard
Table 5: Modal analysis for the IEEE First Benchmark

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Electromech.</th>
<th>SSR 1</th>
<th>SSR 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
<td>$f$ (Hz)</td>
</tr>
<tr>
<td>MFRA</td>
<td>1.640</td>
<td>10.132</td>
<td>15.910</td>
</tr>
<tr>
<td>Prony</td>
<td>1.631</td>
<td>10.111</td>
<td>15.900</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>1.627</td>
<td>10.018</td>
<td>15.900</td>
</tr>
<tr>
<td>HTLS</td>
<td>1.627</td>
<td>9.982</td>
<td>15.900</td>
</tr>
<tr>
<td>ERA</td>
<td>1.627</td>
<td>10.018</td>
<td>15.900</td>
</tr>
</tbody>
</table>

assumptions for $\chi^2$-test are valid for the least square estimation problem within MFRA that was presented in Section 3.1. Referring to equation (3.18), after calculating the damping factor of mode $j$ and the vector $b$ for $m$ signals from the least square fit, a $\chi^2$ value will be calculated and checked to see which signals are outliers that are possibly contributing to bad least square estimate. These signals will get discarded iteratively as relevant and a new least square fit will be calculated until remaining signals pass the $\chi^2$ test.

The methodology is illustrated on the a recent WECC system event that was analyzed in Figure 9. Four signals from four different PMUs were used to calculate the mode frequency and damping ratio for a post event oscillatory ringdown. Modal estimation results for the four signals for the time interval between 15 to 30 seconds in Figure 9 are presented in Table 6 under the columns denoted "Actual data". The calculated damping ratio average $\zeta = 9.29\%$ from the results of all 5 algorithms will be used for this test as the reference value.

Figure 18(a) shows the voltage signal recorded by PMU1 during this event. For test
purposes, the actual signal in Figure 18(a) is modified to the signal shown in Figure 18(b) to emulate a PMU signal getting stuck at an arbitrary value because of an unknown device error. Such a signal will pass through all the data sanity checks discussed earlier in Section 3.2, and can potentially contribute to a bad damping estimate. The ringdown analysis is repeated in all five engines by replacing the actual PMU1 signal (Figure 18(a) with the modified PMU1 signal (Figure 18(b)) and the results are shown in Table 6. Prony like engines interpret the oscillations in such a bad signal as being highly damped. Therefore, the results from all the engines show higher damping compared to the reference value of 9.29%. The estimate 10.987 for MFRA is shown with a strike through in Table 6 because we will see next that this estimate fails the $\chi^2$ test.

The $\chi^2$ value calculated for the least square estimates when all PMU signals are "good signals" from the actual data is 0.6118. Five parameters $\sigma, b_1, b_2, b_3$ and $b_4$ are being estimated from twenty specified values in $y$ in equation (3.21). Therefore, the degrees of freedom is 15. Assuming a confidence level of 99%, the $\chi^2$ threshold is seen to be 30.58. Then, $\chi^2$ measure for the MFRA estimation using actual data (Figure 18(a)) at 0.6118 is well below the critical value $\chi^2_{0.01,15} = 30.58$. Since $\chi^2$ measure with the actual data 0.6118 is less than 30.58, it indicates that there are no bad data points in

Figure 18: Modified signal emulating a "stuck PMU"


Table 6: Modal results for different data quality

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Actual Data</th>
<th>Modified Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
</tr>
<tr>
<td>MFRA</td>
<td>0.390</td>
<td>9.154</td>
</tr>
<tr>
<td>Prony</td>
<td>0.381</td>
<td>9.410</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>0.388</td>
<td>9.309</td>
</tr>
<tr>
<td>HTLS</td>
<td>0.388</td>
<td>9.273</td>
</tr>
<tr>
<td>ERA</td>
<td>0.388</td>
<td>9.306</td>
</tr>
</tbody>
</table>

the least square estimation using actual data.

Next when we replace the good signal of PMU1 with the modified signal shown in Figure 18(b), the MFRA damping estimate changes to 10.987%. However, the corresponding $\chi^2$ value changes to 45.36 which is well above the critical value 30.58. Therefore, we can conclude with 99% confidence that at least one bad data exists in the least square fit. The $\chi^2$ measures are calculated for each PMU signal. PMU1 shows the highest $\chi^2$ value of 34.45 compared to 2.39, 4.30, and 4.22 for PMU2, PMU3, and PMU4 respectively. Accordingly, PMU1 is discarded and the least square fit will be recalculated using only PMU2, PMU3, and PMU4. The new MFRA estimates are shown in the second row below the struck through entries in Table 6. $\chi^2$ measure calculated for this new estimate using the remaining three PMU signals is 0.463 that is well below the critical value 30.58 and the estimation can be accepted as a valid estimate. The MFRA damping ratio estimate 9.066% after discarding the bad signal shown in Table 6 is clearly the closest to the reference value 9.29% compared with the other four algorithms.
Chapter 4

Modal Energy Trending for Ringdown Analysis
(METRA)

In this chapter we introduce a Modal Energy Trending for Ringdown Analyzer (METRA) which prolongs the approach proposed in MFRA in chapter 3. Like MFRA, the purpose of METRA is to automatically analyse oscillatory modal properties of a power system from ringdown PMU data in real-time. The idea behind the proposed METRA algorithm is to calculate the damping ratio by tracking the trend of the oscillation energy seen in the power spectrum density for the mode of interest after applying Singular Value Decomposition (SVD) to the Power Spectrum Density (PSD) matrix which provides a cleaner spectrum especially when the noise levels present in the data are high. The method proposed extends earlier work in [6–8] along with the proposed MFRA in chapter 3. In chapter 3 we proposed a Multi-dimensional Fourier Ringdown Analyzer (MFRA) which estimates the damping ratio of power system modes directly from the Fourier Transforms of the PMU data. The work proposed in this chapter extends the approach in MFRA by applying Singular Value Decomposition (SVD) to the Power Spectrum before calculating the damping estimates. The energy of the mode of interest is calculated by observing the peak associated with the resonant peak frequency in the frequency domain as seen in the power spectrum signal. The mode energy seen in the power spectrum will decay at a rate proportional to the mode damping ratio, if the
mode is positively damped. Contrarily, the mode energy will grow exponentially at a rate proportional to the mode damping if the mode of interest is negatively damped. The approach of estimating the damping ratio by tracking the change in the mode energy was first introduced in [6]. Poon and Lee in [6] assumed some limitations on the window length and time gap between two successive windows. [7] removed some of these restrictions and provided a formula for calculating the optimal window length. [8] extends the work in [7] by calculating the Fourier Transform in more than two different windows, and then use the results to create a set of simultaneous equations and solve for the mode amplitude, damping ratio, and frequency by using a standard parametric estimation algorithm in [9] based on linear predictions (LP) and SVD. In this chapter, the proposed method uses a large window to estimate the mode frequency, amplitude, and mode shape since a larger window will contain most of the signal energy, and multiple smaller sliding windows, within the larger window, to estimate the damping ratio.

The major contributions of the proposed METRA are as follows:

1) The proposed work is aimed at power spectrum based real-time automatic modal analysis for streaming multi-dimensional PMU measurement data.

2) Automatic preprocessing is applied to the time domain signal prior to calculating the power spectrum, such as by removing the center mean and by detrending.

3) SVD is applied to the power spectrum density matrix as in the FDD algorithm [22] to trend the energy changes for each mode in a decomposed fashion, which serves as an elegant method for combining the overall effect from multiple signals in the frequency domain analysis. This step is also helpful in filtering out noise effects especially when the PMU measurements have high levels of noise.

4) The damping ratio estimation in frequency domain is similar to the earlier work
in MFRA though the combination of power spectrum computations and SVD imply the least square estimation to be one-dimensional unlike the multi-dimensional least square method employed in MFRA.

The outline of this chapter is as follows. Section 4.1 provides an overview of the proposed METRA algorithm followed by a flowchart that summarises the algorithm. A detailed comparison between the proposed METRA algorithm and some existing modal extraction algorithms such as Prony, in terms of accuracy is discussed in Section 4.2. Thereafter, the proposed METRA algorithm is tested with simulation data as well as real system data in Section 4.3. The conclusions are drawn in Section 4.4.

4.1 Modal Parameter Estimation

Some of the theoretical derivation shown in this section is summarized from chapter 3. As mentioned earlier, the power system is a large-scale nonlinear system, that can be linearized around its equilibrium point for analyzing small disturbances. Making this assumption of small perturbations from an equilibrium condition, the system response to a small disturbance can be expressed as a linear combination of the system oscillatory modes. Thus, the system post disturbance response can be modeled as a sum of exponential terms. Consider the noiseless signal, $y(t)$, which is assumed to be the system’s post disturbance ringdown response:

$$y(t) = \sum_{j=0}^{p} A_j e^{-\sigma_j t} \cos(\omega_j t + \phi_j)$$ (4.1)

where $\sigma_j$ and $\omega_j$ are the damping factor and the oscillatory frequency of the $j$-th mode respectively. $A_j$ is the amplitude and $\phi_j$ is the phase of mode $j$.

Assuming $y(t)$ is evenly sampled every $\Delta t$, and the oscillation frequencies are far away
from each other, that is, the dominant mode frequencies are at least 0.1 Hz apart and the contribution of other frequency components on mode \( j \) can be considered negligible.

The Fourier Transform of the signal over the window of length \( T \) that starts at \( n_0 \) and ends at \( n_0 + T \):

\[
F(\omega)|_{n_0}^{n_0+T} = \sum_{n=n_0}^{n_0+T} \sum_{j=0}^{p} A_j e^{-\sigma_j n} \cos(\omega_j n + \phi_j) e^{-j2\pi \frac{n}{T}}
\]

(4.2)

where \( F(\omega) \) is the complex Fourier Transform. The frequency spectrum will have \( j \) peaks, one for every oscillation frequency component of the signal [3, 6]. It is assumed that \( T \) is large enough to contain most of the oscillation energy of \( \omega_j \). For a single-output (one dimensional) model, the Power Density Spectrum \( S(\omega) \) can then be calculated from \( F(\omega) \) by multiplying the complex Fourier Transform at each discrete frequency \( (\omega) \) by its complex conjugate:

\[
S(\omega) = F(\omega)F(\omega)^*
\]

(4.3)

where the symbol * denotes conjugate. The proposed work in this chapter allows for multi-dimensional analysis. That is, multiple signals can be used to calculate one set of mode estimates. When performing the multi-signal fit, theoretically, the analysis is expected to provide more accurate mode estimates since more and richer information are provided to the analysis, assuming the same mode is common to all the signals. For multi-dimensional analysis using the proposed METRA algorithm, the Power Spectrum Density (PSD) Matrix is constructed at each discrete frequency \( \omega_j \):

\[
S(\omega_j) = \begin{bmatrix}
F_1(\omega_j)F_1(\omega_j)^* & \cdots & F_1(\omega_j)F_m(\omega_j)^* \\
F_2(\omega_j)F_1(\omega_j)^* & \cdots & F_2(\omega_j)F_m(\omega_j)^* \\
\vdots & \cdots & \vdots \\
F_m(\omega_j)F_1(\omega_j)^* & \cdots & F_m(\omega_j)F_m(\omega_j)^*
\end{bmatrix}
\]

(4.4)
where $F_1$, $F_2$, and $F_m$ are the Fourier Transforms of the signal $y_1$, $y_2$, and $y_m$ respectively evaluated at $\omega = \omega_j$. The PSD matrix is then decomposed by taking the SVD as follows:

$$S(\omega_j) = U_j(\omega_j)S_j(\omega_j)V_j(\omega_j)^H \quad (4.5)$$

where matrix $U_j = [u_1, u_2, ..., u_m]$ is a unitary matrix of the left singular vectors and matrix $S$ is a diagonal matrix holding the singular values denoted $s_1(\omega_j)$ to $s_m(\omega_j)$. Assuming the PSD was evaluated near $\omega = \omega_j$, and there are no other dominant modes nearby, then it can be shown that the first singular value $s_1(\omega)$ (which is also the largest by properties of SVD) is an excellent measure of the overall energy content at the frequency $\omega_j$ from all the signals [22]. Therefore, the function $\hat{S}(\omega_j) = s_1(\omega_j)$ for the frequency variable $\omega_j$ compiled from the largest singular values of the SVD from power spectrum density matrices can be used as the Complex Mode Indication Function (CMIF) $\hat{S}$. The peaks in CMIF denote the dominant oscillatory modes [22]. For each CMIF peak frequency say $\omega_k$ that denotes a system mode, the corresponding first singular vector $u_1(\omega_k)$ is an estimate of the mode shape of the mode $\omega_k$ [22]. Representing $u_1(\omega_k)$ in the polar form, we get

$$u_1 = A_j \angle \phi_j \quad (4.6)$$

where $A_j$ and $\phi_j$ are the magnitude and phase of mode $\omega_k$. To calculate the damping factor $\sigma_k$ of mode $k$, consider a smaller window of $N$ samples, where $N < T$, within the big window $T$. That is, window $N$ starts at the beginning of window $T$, at $n_0$, and ends at $n_0 + N$. The Fourier transform of mode $k$ over the smaller window $N$ and its corresponding CMIF $\hat{S}(\omega_k)|_{n_0+N}$ can then be calculated using equations 4.3, 4.4, and 4.5. Similarly, taking the Fourier Transform and the corresponding power spectrum measure $\hat{S}(\omega_k)|_{n_0+G}$ of the signal $y(n)$ over the same window length $N$ but at later time, where
\( N \) starts at \( n_0 + G \) and ends at \( n_0 + N + G \), where \( G \) is the step size between consecutive windows and is determined by \( G = 1/2f_j \), where \( n_0 + N + G < T \). The same approach can be used to calculate the power spectrum of multiple \( K \) consecutive sliding windows of the same length \( N \) spaced \( G \) samples apart. The rate of change of the magnitude of the power spectrum measure \( \hat{S} \) evaluated at the peak of mode \( k \) as the window slides is used to determine the damping factor of mode \( k \). The damping factor \( \sigma_k \) of mode \( k \) can then be attained from the slope of the best line fit (in a least square sense) of the Logarithmic magnitude of the Power Spectrum of all the consecutive sliding windows [6]. That is, if \( Y \) is a vector containing the magnitudes of power spectrum measures of mode \( j \) for different sliding time windows and vector \( X \) contains the end time of each corresponding sliding window:

\[
Y = [\hat{S}(\omega_k)|_{n_0+N}, \hat{S}(\omega_k)|_{n_0+N+G}, \hat{S}(\omega_k)|_{n_0+N+2G}, ...] \\
X = [n_0 + N, n_0 + N + G, n_0 + N + 2G, ...]/Fs
\]  
(4.7)  
\( (4.8) \)

Assuming the original signal \( y(t) \) is a noiseless signal, the power spectrum measure magnitudes in \( Y \) theoretically fit a straight line [6] and therefore can be considered as an over-determined system:

\[
X_i\sigma_k + b = \ln(Y_i), i = 1, 2, 3, ...K
\]  
(4.9)

where \( b \) is the power spectrum measure magnitude of mode \( k \) over the window \(-T\) to \( 0 \) seconds. To find the damping factor \( \sigma_k \) that best fits the data in hand, we need to solve an optimization problem which minimizes the error \( E(X, Y) \) in the fit:

\[
E(X, Y) = \sum_{i=1}^{K} W_i[\ln(Y_i) - (X_i\sigma_k + b)]^2
\]  
(4.10)
where the weights $W_i$ give pre-specified emphasis on some spectrum measures, may be from good quality PMU data windows, over other windows. Finding the damping factor with the minimum error requires solving the system of equations (4.11):

$$
\begin{align*}
\begin{bmatrix}
\ln(Y_1) \\
\vdots \\
\ln(Y_K)
\end{bmatrix} &=
\begin{bmatrix}
X_1 & 1 \\
\vdots & \vdots \\
X_K & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_k \\
b
\end{bmatrix} \\
\hat{\beta} &= (\bar{X}^T W \bar{X})^{-1} \bar{X}^T W \bar{Y}
\end{align*}
$$

(4.11)

The damping ratio $\zeta_k$ can then be calculated using:

$$
\zeta_k = \frac{-\sigma_k}{4\pi f_k}
$$

(4.12)

4.1.1 Example

The flowchart in Figure 19 summarizes the main steps in METRA for extracting oscillatory modes from measured PMU data. As an example, let us consider three synthetic signals shown in Figure 20. Each of the three signals contains one common mode present in all three signal, plus a different second mode in each of the three signals. White Gaussian noise of 35 dB was added to each signal to make the example more realistic. The common mode present in all three signals is 0.5 Hz with 2% damping ratio. A second mode is added to each of the three signals; 0.75 Hz with 4% damping to the first signal, 1 Hz with 3% damping to the second signal, and 1.25 Hz with 2% damping to the third signal. Using the multi-dimensional approach described in Section 4.1, all three signals will be used to extract the damping ratio of the 0.5 Hz oscillatory
mode that is the common mode.

Figure 19: METRA Algorithm Flowchart

Figure 20: Three synthetic test signals
The sampling frequency $F_s$ used is 30 samples per second which is a common sampling frequency for PMUs in North America. The large window $T$ is 15 seconds (450 samples). After taking the Singular Value Decomposition of the Power Spectrum Density Matrix of the three signals and calculating the Power Spectrum, the mode of interest was identified at 0.5 Hz, thus the smaller window $N$ using $N = 1/f$ will be 2 seconds (60 samples) and the step size $G$ using $G = 1/2f$ will be 1 second (30 samples). That is, the sliding window $N = 2$ will be moving every $G = 1$ second, giving a total of $K = 14$ windows. The logarithmic Power Spectrum magnitude calculated using equations 4.4 and 4.5 will be decaying at a rate of $2\sigma$, where $\sigma$ the damping factor of the 0.5 Hz mode and that is shown in Figure 21. A similar method has previously been applied in the wavelet analysis of oscillatory modes [10] while the proposed method here is for calculating the mode damping using power spectrum density matrix and its SVD decomposition.

![Figure 21: The Change in power spectrum measure magnitude for different time windows](image)

To calculate the damping ratio $\zeta$, we take the logarithmic power spectrum measure magnitude of all $K = 14$ windows, and stack them vertically as in equation 4.11. Vector
$X$ is simply constructed by stacking the end time of each window:

$$
\bar{Y} = \begin{bmatrix}
5.833, & 5.824, & 5.790, & 5.440, & 5.344,
\end{bmatrix}^T
$$

$$
\bar{X} = [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]^T
$$

The final step will be the calculation of the least square fit of all the values in $\bar{Y}$ with respect to $\bar{X}$ as in equation 4.11. Figure 22 shows the least square fit whose slope corresponds to twice the damping factor $\sigma$.

![Figure 22: Least Square Fit of different moving windows](image)

### 4.2 Algorithm Performance

The proposed METRA algorithm can accurately extract modal oscillatory information from evenly sampled noisy PMU data. The major strengths of the proposed algorithm as compared to other available modal extraction approaches are its ability to extract modes of oscillation even when nonlinearities, such as noise and line switching, are present in the system, and its ability to find multiple modes of oscillation with different damping levels.
Prony analysis is one of the common methods used for estimating modal content from evenly sampled data. It has been used in power systems studies and applied to many Western Interconnection ringdown events [11, 15]. Although classical Prony method is a common power system modal analysis tool, it is known to behave poorly when nonlinearities, such as noise and switching events, are present in the data. Matrix Pencil and Hankel Total Least Square (HTLS) have been previously applied to extract modes of oscillations from measured power system response [14, 15]. Both algorithms use SVD to filter out the noise components of the system response and can accurately estimate the actual system modes. Refer to [15] for more detailed theoretical discussion of Prony, Matrix Pencil, and HTLS. Another method also used for extracting modal oscillatory information from measured system data is Eigensystem Realization Algorithm (ERA) [12, 13].

The performance of the proposed METRA algorithm is tested by comparing its modal estimates with the four modal analysis algorithms mentioned above (Prony, Matrix Pencil, HTLS, and ERA). Each one of the five algorithms will process the same signal produced by equation 4.13 which contains two modes of oscillations, one at 0.6 Hz with 4% damping, and one at 1.2 Hz with 2% damping plus random gaussian noise $n(t)$.

$$y(t) = e^{-0.1508t} \cos(2\pi \times 0.6t) + e^{-0.1508t} \cos(2\pi \times 1.2t) + n(t)$$  \hspace{1cm} (4.13)

Two levels of random white gaussian noise will be added to the signal to make the test more realistic and to compare their abilities to handle noisy measurements. Each algorithm was tested with two levels of noise, 15 dB and 25 dB. For every noise level 100 Monte Carlo simulations were performed, each with different white noise but at the same level. The mean and standard deviation of the frequency and the damping ratio
estimates for both modes were calculated and the results are presented in Figure 23 and 24. For Prony, a model order of 64 was used and for Matrix Pencil, ERA, and HTLS, the SVD threshold is set to 10% of the largest singular value. The sampling frequency of $y(t)$ is 30 samples per second.

![Figure 23: Frequency Estimates for 15 dB and 25 dB noise tests](image)

From Figure 23 and 24, the proposed METRA algorithm shows similar performance compared to the other existing modal analysis algorithms. With 25 dB white noise, the estimates for both modes were very close to their true values from all algorithms. Under 15 dB of noise, the estimates become more biased, since the noise gets more dominant.
over time as the modes damp out. Prony showed the most biased estimates, with an error of 4.5% in the mean damping estimate of the first mode and 3.75% error in the mean damping estimate of the second mode.

### 4.2.1 Automatic Real-time Framework

The work proposed in this chapter is aimed for automatic real-time modal analysis that automatically analyzes streaming multi-dimensional PMU measurements. The approach used in this chapter is adopted from the automatic framework proposed in [15] and chapter 3 where multi-level data sanity checks, event detection techniques, and modal estimates crosschecks are used to find consistent oscillations automatically without any direct human interaction. For a more detailed discussion of the automatic framework used in this chapter refer to [15] and chapter 3.

A pre-specified window length $T$ of streaming PMU data will be captured every step size of $S$ seconds. The rule of thumb here is that $T$ has to contain most of the oscillation energy of the lowest mode of interest. That is, for power system applications, the lowest mode of interest is about 0.15 Hz which needs a minimum of $T = 15$ seconds. The step size $S$ used is 1 second although any step size $S$ could also be used. The data is then passed through data sanity checks where missing data points will be checked and interpolated when necessary. The data then goes through an event detection stage where the analysis window is checked for event occurrence. The window’s Coefficient of Variance and the change in voltage $dV/dt$, current $dI/dt$, and frequency $df/dt$ will be monitored and if any of these values bypasses the predefined threshold, the analysis window will be flagged as ”experiencing an event” and will proceed to the next stage. The data is then checked for sudden jumps where the change in two consecutive data points is monitored
and checked. After passing through all the data sanity and event detection stages, the data will be preprocessed by removing the mean value of each window, normalizing the analysis window to a max value of 1, and detrending if necessary to remove any drift present in the signal. The data is then analyzed using the proposed METRA algorithm discussed in Section 4.1. The estimates of the 3 or 4 most recent runs are stored and crosschecked for consistency. If a consistent estimate is found across the last 3 or 4 runs, the mean frequency and mean damping ratio are then reported as consistent estimates of a system oscillation mode.

4.3 Test Cases

To test the ability of the proposed METRA approach to perform real-time automatic ringdown analysis, three power system related test cases were adopted. The aim of this section is to verify the applicability of the proposed METRA approach to detect power system modes from ringdown data seen in PMU measurements. The three test cases are 1) western American power system WECC simulation case, 2) WECC real power system event, and 3) an eastern real system event. The results are presented in the upcoming subsections.

4.3.1 WECC Simulation Case

As mentioned earlier, the western interconnection WECC has a long history of inter-area oscillations, caused by long tie-lines between the Pacific Northwest and Southern California. For this case, a WECC event was simulated using TSAT [18] where a system disturbance caused the system modes to get excited in an oscillatory response. Figure
Figure 25 shows the event seen in the bus voltage signals. The ringdown response contains 2 stable well damped inter-area modes, a 0.27 Hz dominant mode and a 0.7 Hz low magnitude mode. The proposed automatic framework was applied to the data shown in Figure 25 to detect and extract the modes of oscillations using a sliding window of $T = 15$ seconds and a step size $S = 1$ second. As mentioned in Section 4.2.1, the engine will skip the analysis windows that triggers the max $dV/dt$ threshold caused by the sharp voltage drop at $t = 60$ seconds. The engine will start analyzing the data right after the sharp voltage drop. Figure 26 shows the rejected damping ratio estimates which were discarded because of the sharp jump present in estimates of those specific analysis windows. The consistent estimates are those whose analysis windows passed all the sanity checks. The engine reported a consistent estimate at $t = 78$ seconds, with frequency of 0.27 Hz and damping ratio of 8.62% for the dominant mode, and 0.6933 Hz with 3.81% for the low magnitude inter-area mode.

To test the effectiveness of the proposed METRA algorithm to accurately extract the modes of oscillations from measured system response, all five modal analysis algorithms presented in Section 4.2 were used to analyse 15 seconds of bus voltage data, measured
Figure 26: Damping ratio estimates for 0.27 Hz and 0.7 Hz modes

at 30 samples per second, highlighted in Figure 25 by the dotted box. The modal estimates for the dominant 0.27 Hz interarea mode from all five modal analysis algorithms are presented in Table 7. The model order for Prony was set to 125, and the SVD threshold was set to 10% for ERA, Matrix Pencil, and HTLS. The mode shape calculated by the proposed METRA algorithm compared with the average mode shape from the other four different modal analysis algorithms is also shown in Figure 27. Table 7 shows that

Figure 27: Mode shape for 0.27 Hz interarea mode
Table 7: Modal results for the Simulated WECC event

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\zeta$ (%)</td>
</tr>
<tr>
<td>METRA</td>
<td>0.2700</td>
<td>9.0366</td>
</tr>
<tr>
<td>Prony</td>
<td>0.2708</td>
<td>9.0057</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td>0.2695</td>
<td>9.2175</td>
</tr>
<tr>
<td>HTLS</td>
<td>0.2696</td>
<td>9.2511</td>
</tr>
<tr>
<td>ERA</td>
<td>0.2695</td>
<td>9.2163</td>
</tr>
</tbody>
</table>

the proposed METRA algorithm produced consistent estimates compared with the different algorithms for both modes. The mode shape produced by the proposed METRA in Figure 27(a) matches the average mode shape produced by the rest of the algorithms shown in Figure 27(b).

### 4.3.2 WECC Real System event

The following test case was recorded recently by 3 PMUs located in the western interconnection in San Leandro, San Diego, and Denver. Figure 28 shows the frequency signals recorded for the event. The ringdown data in Figure 28 shows one dominant but well damped inter-area mode of oscillation. The event excited a dominant inter-area mode with a frequency of 0.37 Hz, and the data was tested using the proposed METRA algorithm. Figure 29 shows the event along with the damping estimates for the 0.37 Hz mode, as the window of $T = 15$ seconds slides through the event at a step size of $S = 1$ second. The engine detects a high $df/dt$, or "jump", in the data and skips all
Figure 28: Event recorded in the western interconnection

Figure 29: Damping ratio estimates for the 0.37 Hz mode

analysis windows that contains a jump. The red colored estimates shown in Figure 29 are the rejected estimates from the proposed METRA algorithm. Once the analysis window passes through the "jump", the rest of the data will pass the sanity checks and the modal analysis results will be crosschecked for consistency. The engine detected a consistent modal estimate at $t = 49$ seconds with a frequency of 0.3767 Hz and a damping ratio of 8.32%.

The proposed METRA algorithm was also tested with the event shown in Figure 28 by processing one single 15 seconds window, sampled at 30 samples/sec, and highlighted by the dotted box. The results were compared with the modal analysis results produced
by the four different algorithms for the same analysis window. Table 8 shows the modal analysis results for the event from all 5 algorithms. The model order for Prony was set to 138 and the SVD threshold was set to 10% for Matrix Pencil, ERA, and HTLS. The results are relatively consistent, except for Prony which has a slightly inconsistent frequency and damping ratio estimates compared with the rest. The mode shape was also calculated for the window in Figure 28 using the proposed METRA, shown in Figure 30(a), and the average mode shape from all 4 algorithms, shown in Figure 30(b). The plot shows the ability of the proposed METRA algorithm to accurately extract the mode shape of the 0.37 Hz mode as it is consistent with the average mode shape from all 4 algorithms.

4.3.3 Eastern system event

The following case was recorded by PMUs located in the eastern American power system. A 500 kV line trip caused sustained oscillations seen across the system. Figure
Figure 30: Mode shape for the 0.37 Hz mode

Figure 31 shows the bus voltage magnitude recorded by 4 different PMUs that experienced these oscillations. The plot shows that the oscillations are most dominant in PMU 2 which indicates that these oscillations were likely caused by a local mode. Signals from all 4 PMUs were processed through the proposed automatic framework for detecting consistent oscillations.

Figure 32: Bus voltage magnitude time-plots from an eastern system event

Figure 32 shows the event aligned with the damping ratio estimates for the local mode throughout the event using a window size $T = 15$ seconds and a step size $S = 1$ second. The engine rejected the estimates where a high $dV/dt$ was present in the analysis window.
at $t = 342$ seconds, and the estimates produced by the algorithm after the analysis window passed through the sudden change in voltage ($dV/dt$) were crosschecked for consistency. From Figure 32 the first estimate considered for consistency crosscheck was at $t = 356$ seconds, and was crosschecked with the next 2 or 3 estimates. The engine detected a sustained local oscillation of 1.1743 Hz and 0.04% damping ratio and an alert was given at $t = 358$ seconds.

The event was also tested with other modal analysis algorithms and the results were compared with METRA algorithm. The sampling frequency of the data was 30 s/sec and the analysis window used is highlighted by the dotted box in Figure 31. The frequency and damping ratio estimates for the 1.17 Hz local mode from all 5 algorithms are compared in Table 9. The mode shape for the event was also calculated and compared across the different algorithms. Figure 33(a) shows the mode shape for the 1.17 Hz mode produced by the proposed METRA algorithm and Figure 33(b) shows the average mode shape from the rest of the 4 algorithms. The mode shape plots were consistent and shows that PMU 2 was experiencing the 1.17 Hz mode the most and the rest of the
Table 9: Modal analysis results for the eastern system event

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Local Mode</th>
<th>$f$ (Hz)</th>
<th>$\zeta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>METRA</td>
<td></td>
<td>1.1792</td>
<td>0.0249</td>
</tr>
<tr>
<td>Prony</td>
<td></td>
<td>1.1773</td>
<td>0.0024</td>
</tr>
<tr>
<td>Matrix Pencil</td>
<td></td>
<td>1.1767</td>
<td>0.0335</td>
</tr>
<tr>
<td>HTLS</td>
<td></td>
<td>1.1767</td>
<td>0.0326</td>
</tr>
<tr>
<td>ERA</td>
<td></td>
<td>1.1767</td>
<td>0.0328</td>
</tr>
</tbody>
</table>

system is oscillating with PMU 2 at the same phase. In this test case, the model order for Prony was set to 125, and the SVD threshold for HTLS, ERA, and Matrix Pencil was set to 10%.

Figure 33: Mode shape for the 1.17 Hz local mode
Chapter 5

Conclusions

This thesis presented an automatic oscillation detection framework based on newly developed modal analysis algorithms specially designed for real-time PMU applications. We revisited the previously developed Oscillation Monitoring System and focused on the Event Analysis Engine segment of OMS. The algorithms in OMS provided a solid benchmark for testing the newly developed modal analysis algorithms as they proved to be a suitable benchmark for monitoring power system oscillations.

We introduced a Multi-dimensional Fourier based ringdown analysis algorithm, MFRA. The algorithm was proposed and formulated to extract multiple poorly damped oscillatory modes from synchrophasor measurements of ringdown responses in power systems. The proposed MFRA algorithm was compared with existing modal analysis algorithms and shown to be accurate, fast, and robust under noisy conditions, with ability to find reliable damping ratio estimates during switching events. Least square fit framework is introduced for multi-dimensional ringdown analysis that enables $\chi^2$ test for rejecting bad PMU signals automatically. Multiple test cases demonstrate the performance and the robustness of the proposed MFRA algorithm. A new frequency domain algorithm, METRA, for extracting the dominant modal properties of power system ringdown responses has also been proposed that analyzes the modal energy trends for calculating the damping levels of the dominant modes. The method combines the modal effects of multiple PMU measurements into a single energy measure for each mode by using the
CMIF SVD singular values as in FDD algorithm. The method is shown to be effective in extracting the dominant mode frequency, mode damping ratio, and its mode shape in simulated responses as well as for recorded power system event measurements.

Power Systems are continuously evolving and becoming more complex. Having said that, some of the assumptions made in this research might become weak, which raises the need for continued work and future improvements. We assumed for a given system event, the ringdown oscillatory mode frequencies are well separated. Future research can focus on this limitation and make the algorithms more robust in case of closely spaced oscillatory modes.

Another area that can be improved in future research is frequency resolution in the FFT spectrum. Since a finite short time window is used for analysis, that limits the frequency resolution in the FFT, therefore, interpolation in the frequency domain can be applied to provide a more accurate mode frequency estimates.

The proposed work is aimed for analysis of synchrophasor data with high sampling frequency. If lightly sampled data is used in the analysis, the accuracy of the results can be affected, therefore, future improvements on the dependency on sampling frequency can be addressed to provide a more robust algorithms that can handle any sampling.
Bibliography


