# EXPERIMENTAL CHARACTERIZATION OF ICE HOCKEY STICKS AND PUCKS

By

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To the Faculty of Washington State University:

The members of the Committee appointed to examine the thesis of Rosanna Leah Anderson find it satisfactory and recommend that it be accepted.

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Chair

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Abstract

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The characteristics of hockey pucks are integral to the dynamic behavior of hockey sticks. Manufacturers use quasi-static to qualify pucks and drop tower testing at freezing and room temperatures has been previously performed. No data exists in the literature characterizing pucks at game speeds, which often reach 70 – 100 miles per hour in elite – level play. A high speed impact test was developed to simultaneously measure the puck hardness and coefficient of restitution. Puck brand, temperature, and speed were all shown to have significant effects on the impact properties of hockey pucks.

The ice hockey stick has been an evolving piece of equipment over the last 20 years, with the introduction of aluminum and composite materials. Finer control over the manufacturing processes have also led to advances in stick features, including tapered shafts, finely tuned flex, and variable shaft geometry. It has been unclear what effect, if any, these modifications have on the performance of hockey sticks in the field of play.

The current study examined laboratory measures of ice hockey stick feel and performance. Seventeen sticks were considered to compare wood (6 sticks) and composite (11 sticks) sticks, including different shaft tapers for each group. Modal analysis was performed to quantify vibration characteristics that play a role in perceived player feel. The first two bending modes were compared for all sticks. While differences were noted in the natural frequencies for

the first two bending modes of different sticks, these differences were small. Tapered composite sticks showed lower first bending mode node locations than wood sticks.

A high speed laboratory performance test was developed and used to compare the 17 sticks. Pucks were fired from a high speed air cannon at a stationary pivoted stick. The frame of reference was then changed to a moving stick and a stationary puck to derive a performance value for puck speed resulting from a slap shot. A field study was also conducted to correlate laboratory performance results to on-ice performance.

The test was found to be repeatable within 3% of the peak performance (3 sticks). The composite group had an average peak performance that was 18% higher than the wood group. No effect was found on lab performance from the shaft taper. Field study data provided useful information for the derivation of the stick performance metric.

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# CHAPTER ONE

# INTRODUCTION

The game of ice hockey began as an adaptation of field hockey that was played in the winter on frozen lakes. Although the true origin of the game is a topic of dispute, there is a general consensus that early forms of the game originated in northern Europe around the 17$^{th}$ century [1.1]. When the lakes froze during winter, field hockey players began playing a form of the game on ice using a wood or cork ball. With the arrival of the modern skate in the early 1800's people combined ice skating with the early forms of ice hockey, creating the modern game that is known today [1.2]. Ice Hockey made its North American debut in Canada in the 1870's [1.3].

Ice hockey was first played with wood balls and lacrosse balls. Players (and rink owners) found that these balls were hard to control, often flying high through the air and breaking windows and teeth [1.1]. The ball was cut into three slices and the middle section was kept to play with [1.2]. This gave rise to the cylindrical rubber disc, or puck that is used today in ice hockey.

Regulation ice hockey pucks are made of vulcanized rubber (treated with heat and sulfur). Treating the rubber makes it harder and more resistant to cold temperatures and more durable [1.2]. Hockey pucks are 3 inches in diameter, 1 inch thick, and weigh 5.5 to 6 ounces [1.2]. Pucks are often frozen prior to use to increase their hardness and reduce bouncing of the puck, therefore increasing the controllability for players.

One of the most important pieces of hockey equipment in terms of player performance is the hockey stick [1.4]. The word *hockey* came from the French word *hoquet*, which means

“shepherd’s crook” [1.2]. The first sticks used in early ice hockey were made of bent willow branches, like the one shown in Figure 1.1. In the late 1800’s, people began making sticks out of solid hardwood, such as elm, birch, aspen, or ash [1.1]. Hardness, flexibility, and wear resistance are all crucial properties, making hardwoods good materials for hockey sticks [1.2].

Figure 1.1: Old hockey stick made of a bent branch – Copied from [1.8]

The first hockey sticks used in the NHL were solid wood with a straight blade on the end (no curvature). In the late 1950’s, a player for the New York Rangers named Andy Bathgate began experimenting with curving the blade of his stick [1.3]. Adding a curve to the blade allowed better control of the puck during handling and shooting. The curved blade also made it easier to lift the puck off of the ice during a pass or shot. During a slap shot, curved blades saw less deflection than straight blades, resulting in a more consistent and accurate shot trajectory [1.2]. It wasn’t long before other people took notice of the curved blade and most NHL players began using different curves that suited their playing style. Figure 1.2 shows some common blade curve patterns currently available on hockey sticks.


Figure 1.2: Common blade curve patterns – Copied from [1.9]

A player's perceived feel of the puck on the stick is important in handling the puck down the ice. Player's prefer to feel small vibrations imparted in the stick while handling the puck so that they don't have to look directly at the puck to know its location on the blade. In addition, a poor feeling stick can cause received passes to bounce off the blade, losing control of the puck. Many players believe that wood sticks offer the best feel for the puck. Figure 1.3 shows some modern wood sticks.


Figure 1.3: Wood hockey sticks with composite blade wraps – Copied from [1.10]

Players also began wrapping the blade of the stick with plastic materials to increase the durability in the 1970's to 1990's [1.5]. This led to composite reinforced wood hockey sticks, which are the only kind of wood sticks available today. Composite reinforced sticks provide a good feel for the puck, but are among the heaviest sticks. In addition, wood has a tendency to degrade or wear over time because wood is a natural material, and the stiffness of the stick can vary by up to 40 % for sticks of the same model [1.2].

After the introduction of the curved blade and reinforced wood sticks, the hockey stick market became stagnant for many years. The fairly recent outbreak of Dutch elm disease has devastated the elm tree population, eliminating it as a commercially viable source of wood [1.2]. This sent manufacturers looking for new stick materials.

The first new material was aluminum. These sticks consist of a hollow aluminum shaft with a wooden blade that is inserted in the bottom end of the shaft. Aluminum sticks are lighter and more consistent than wood and also have good durability. Players found that aluminum sticks have a cold, metallic feel and do not offer a good feel for the puck on the stick [1.2]. Aluminum shafts also tend to develop a permanent bend after repeated use. Some aluminum hockey sticks with wood blade inserts are shown in Figure 1.4.


Figure 1.4: Aluminum hockey sticks with wood blade inserts – Copied from [1.10]

In the 1990's, the price of composite materials, which were previously reserved mainly for military applications, became more competitive, making them widely available to civilian markets [1.6]. Hockey stick manufacturers have been among the list of sporting equipment producers to take advantage of the benefits of composite materials.

The newest types of sticks, made entirely of composite materials, have revolutionized the hockey stick market. These sticks are made of combinations of fiberglass, Kevlar, graphite, resin, and/or carbon fiber [1.2]. A composite ice hockey stick is shown in Figure 1.5. There are three different types of composite sticks: two-piece, one-piece, and true one-piece. Two-piece sticks have a hollow composite shaft with a removable blade that is inserted into the bottom end of the shaft, allowing players to use any blade pattern they prefer and the option to replace blades if they break. One-piece sticks have the blade permanently bonded into the shaft at the junction, or hosel. True one-piece composite sticks are the newest addition and are molded out of a single piece of composite material, reducing overall weight and the stiffness in the hosel.


Figure 1.5: One-piece composite ice hockey stick – Copied from [1.9]

The first fully composite sticks were only marginally lighter than wood sticks and offered players a very poor feel for the puck. In addition, they were very expensive and not very durable. Further development of composite materials in sporting goods has led to a new kind of composite stick that is manufactured to meet a number of player needs and offers many advantages over wood sticks.

Composite sticks are the lightest on the market and degrade less than wood over time. The stiffness, or flex, of the stick can be controlled during manufacturing to provide greater consistency from one stick to another. In addition, the strength of composite materials enables the production of more compliant sticks without sacrificing durability, which is preferable among many players. Better control in the manufacturing of composite sticks has led to the design of a shaft that is tapered toward the bottom. This is intended to reduce overall weight and increase the amount of energy that can be transferred from the stick to the puck [1.2]. The cross-sectional shape of the shaft can also be altered to accommodate player feel. One of the main drawbacks to composite hockey sticks is their high cost, sometimes as much as $200 compared to wood sticks that cost around $25.

Manufacturers and players claim that composite sticks perform better than wood sticks, but little independent experimental research has been conducted to confirm or deny these claims. This study sought to quantify hockey stick performance and vibration characteristics, and experimentally compare the performance of several wood and composite hockey sticks. Experimental methods were implemented to determine the effect of new technology on ice hockey stick performance and vibration characteristics.

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[1.8] Image copied from: placenamehere.com/objects/blog/oldblade.jpg

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# CHAPTER TWO

# BACKGROUND

## 2.1 Ice Hockey Pucks

Hockey pucks are made of a combination of approximately a dozen materials. Natural rubber and a filler, usually carbon black or coal dust for the black color, make up about 90% of the puck [2.1]. Additives such as sulfur and anti-oxidants make up the rest and help in the curing process and contribute to the strength and hardness of the pucks [2.2]. Carbon also helps improve wear resistance of the rubber in hockey pucks.

The ingredients are poured into a large, automatic mixer called a Banbury and then extruded into soft rubber logs that are approximately three feet long [2.2]. These logs are sliced into slugs that are roughly the size of pucks. Curing the slugs at 150 degrees C for about 22 minutes hardens the rubber [2.2]. The edges are knurled during the curing process and then excess rubber left over from the mold is trimmed off, leaving a finished puck. Pucks used in the NHL are made by a more expensive injection process in which the rubber is liquefied, injected into a mold, and allowed to set, allowing the puck maker better control over the final product [2.2].

Previous research has examined the behavior of hockey pucks at low speeds, but no literature exists to characterize them at game speeds, which can reach 100 miles per hour in professional play [2.1]. Drop tower tests, in which the puck is dropped from a known height and the rebound height is measured have been performed on pucks to examine their elasticity. This test is described in more detail in Section 2.1.1. Manufacturers test the rubber compound before it is cured by placing a sample in a rheometer, which analyzes the curing curve and hardness of

the rubber. The curing curve is then compared to a pre-defined quality curve, which determines if the batch passes inspection. In this study, pucks were characterized by their impact properties at game speeds. The coefficient of restitution, or e, provided a measure of the puck's elasticity, while the dynamic stiffness, or $k_d$, provided a measure of its dynamic hardness.

Pucks are often frozen before use in play in order to reduce their bounciness, allowing for better control by the players. In addition, freezing pucks to the temperature of the ice before play ensures uniform response throughout a game by keeping the puck temperature uniform. During play, pucks are moved up and down the ice mainly by the sticks of the players. The speed of the puck during play varies greatly depending on whether a player is passing, handling, or shooting the puck. Previous studies have found that shot speeds can easily achieve 80 miles per hour in elite play, with the fastest shooters in the 100 mile per hour range [2.1, 2.3, 2.4]. Several different brands of pucks are available to players in the commercial market. Pucks in this study were characterized for different brands, speeds, and temperatures.

### 2.1.1 Coefficient of Restitution

When two solid bodies undergo a collinear collision, reaction forces act in opposite directions on the two bodies, changing the incident speed of the bodies. These reaction or contact forces cause compression in the small area of the bodies that are in contact, or the contact area [2.5]. During this compression, kinetic energy is transferred to stored elastic strain energy in the bodies. At some point, the contact forces bring the approach speeds of the bodies to zero and the stored elastic energy in the solids is returned to kinetic energy, forcing the bodies apart until they separate with some resulting velocity [2.5]. This portion of the collision is termed restitution. In the compression and restitution phase, some energy is always lost, typically due to permanent deformation of the layers of atoms at the colliding surfaces [2.6].

The coefficient of restitution (e), is a measure of the elasticity of a collision. For two colliding objects, it is defined as a ratio of the relative normal velocity of the objects after an impact to their relative normal velocities before impact. More specifically,

$$e = -\left(\frac{v_1 - v_2}{V_1 - V_2}\right) \tag{2.1}$$

where $v_1$ and $v_2$ are the post-impact rebound speeds and $V_1$ and $V_2$ are the incident speeds of objects 1 and 2, respectively. The negative sign in Equation 2.1 is due to the change in direction of the velocities of the objects after a collision. In this case, the velocity can be considered positive when the objects are moving toward each other and negative when they are moving away from each other. Throughout the duration of this paper, a capital V denotes incident speed, while a lowercase v denotes the post-impact speed.

The coefficient of restitution is a measure of the amount of kinetic energy that is lost in a collision [2.7]. A perfectly elastic collision would have e = 1, while a perfectly inelastic collision would have e = 0 [2.8]. A typical collision will have a coefficient of restitution value lower than e = 1 due to energy losses from permanent deformation, internal modes of oscillation, or a slow recovery to the original shape [2.9]. Some researchers attempt to predict e using models that account for factors that include material stiffness, energy losses, or vibration [2.5]. Developing a coefficient of restitution model is out of the scope of this work, but Equation 2.1 is still an accurate equation to measure e.

The idea of coefficient of restitution began with Isaac Newton examining collisions of identical spheres. He recognized that only normal velocity components are important when determining the resulting velocity of two objects after a collision [2.6]. Hodgkinson (1834) examined collisions between dissimilar spheres and determined that the e is not a constant, but is

a function of the relative velocity of the colliding objects as well as the stiffness of the material of each object [2.6].

In general, e for two colliding bodies depends on the elastic properties of both bodies, but under some conditions may depend almost entirely on the elastic properties of only one body. For example, if a very elastic ball, such as a tennis ball, is impacted on a very rigid wall, then the resulting value of e provides a measure of the elasticity of the ball as long as there is no significant deformation in the wall [2.8]. In this case, it would be appropriate to assume that the coefficient of restitution is an inherent property of the ball, regardless of the rigid surface it is impacted against [2.8]. If one of the objects in the collision is rigid and stationary, then Equation (2.1) reduces to:

$$e = -\frac{v_1}{V_1} \tag{2.2}$$

Values for e are often used in sports as a measure of ball liveliness. For various sports balls e is usually determined by shooting the ball at a rigid wall and measuring the inbound and rebound speeds [2.10]. The coefficient of restitution has been investigated for a variety of sporting balls and spherical objects, including golf balls, cricket balls, tennis balls, baseballs, and softballs.

If an object is dropped onto a surface that has a much larger mass than the object, then e can be determined from the square root of the ratio of the rebound height to the drop height [2.8]. This procedure is referred to as a standard drop tower test, or drop test. The theory comes from balancing the kinetic energy of a moving ball immediately before and after impact to the potential energy of the ball in the air at both the drop height and final bounce height. That is, for a ball with mass m that is dropped from a known height, $h_1$ and rebounds to a measured height of $h_2$:

$$\frac{mgh_2}{mgh_1} = \frac{\frac{1}{2}mv_r^2}{\frac{1}{2}mV_i^2} = e^2 \tag{2.3}$$

where $V_i$ and $v_r$ are the velocities at the instant before and after contact, respectively, and g is the gravitational constant.

Vincent (1898-1900) and Raman (1918) expanded upon Hodgkinson's work by examining the repeatability of a collision of spheres over a range of approach speeds. They found that as speed approaches infinity, e approaches zero (perfectly inelastic) and that as the speed approaches zero, e approaches one (perfectly elastic) [2.6]. More recent studies have shown that e decreases linearly with increasing incident speed for golf balls, baseballs, softballs, and tennis balls over a finite speed interval ranging from 50 – 110 mph [2.11, 2.12, 2.13].

The decrease in e with increasing incident speed for a ball dropped on a rigid block can be attributed to three major energy loss mechanisms [2.13]:

1. Increased excitation of internal waves or vibration modes in the block and/or the ball
2. Increased plastic deformation or the rigid wall or block
3. Viscoelastic behavior of the rigid block or ball.

Rayleigh (1906) estimated the fraction of energy stored in the fundamental vibration mode for two slowly colliding spheres is about $0.02V_i/c$, where c is the velocity of sound in the ball material and $V_i$ is the incident speed [2.7]. While this is quite small for many realistic scenarios, it does identify the dependence of e on speed due to an increase in vibration with increasing speed.

Limited testing has been done to determine e of hockey pucks at slow speeds. Standard drop tests on a rigid surface have found the e of hockey pucks to range between 0.45 – 0.55 for room temperature pucks and 0.12 – 0.27 for frozen pucks [2.1]. These tests were conducted at

low speeds, far from the speeds representative of game play. Gerl and Zippelius (1998) studied the coefficient of restitution for elastic disks and found, like spheres, that it is a function of initial relative velocity and the elastic properties of the disks [2.7]. Collisions were found to be elastic for vanishing relative velocity (the quasistatic case) and increasingly inelastic for increasing relative velocity [2.7].

As previously mentioned, pucks are often frozen prior to use in play in order to reduce bouncing of the puck and increase the controllability for players. Freezing them prior to their contact with the ice also prevents puck cooling during play, ensuring uniform response throughout the duration of play. Previous slow speed puck testing shows that e decreases by over half when pucks are frozen [2.1]. The e of softballs and baseballs has also been shown to decrease with decreasing temperature [2.14, 2.15]. As the ball is heated, the material becomes softer. A softer ball will deform more, retaining less kinetic energy and therefore having a lower e than a hard ball [2.16].

ASTM Standard F1887-02 describes a method for determining the coefficient of restitution of baseballs and softballs and is used in ball certification processes [2.17]. This standard describes the coefficient of restitution as "a numerical value determined by the exit speed of the ball after contact divided by the incoming speed of the ball before contact with a massive, rigid, flat wall." The e of hockey pucks was found in an analogous manner, and therefore, all e values determined in this study are calculated using equation (2.2) unless otherwise stated.

### 2.1.2 Dynamic Stiffness

Hockey pucks made by Viceroy are tested for their tensile strength and hardness in a quasi-static manner [2.2]. These measurements to not account for the fact that the dynamic

properties of a puck may differ from its static properties [2.9]. Currently, no data exists in the literature about the dynamic hardness of hockey pucks.

Previous studies have suggested that the performance of sports balls can be more completely characterized by using a dynamic hardness method [2.9, 2.18, 2.26, 2.10]. One important factor in making a good puck is achieving the right hardness [2.2]. If a puck is too hard, it can cause excessive injury and break the glass surrounding the ice rink. If a puck is too soft, it will wear quickly and deaden when it hits the boards. During a collision, localized compression occurs in the contact area. This compressed area between the bodies acts like a short, stiff spring that is compressed between the two bodies [2.5]. How stiff this "spring" is depends on the hardness of the objects.

The hardness of an object can be viewed in terms of its peak impact force during a collision with a rigid surface. A force measurement also provides useful information on the behavior of the objects during the collision and the elastic properties of the colliding bodies [2.9]. Force vs. time data for a puck can be measured by impacting it against a strike plate with rigidly mounted load cells. This has been done for a variety of balls, including golf balls, tennis balls, superballs, baseballs, and softballs [2.10, 2.12, 2.9, 2.16, 2.18, 2.19].

The method for calculating the dynamic stiffness of an object originates from an energy balance between the kinetic energy of the moving puck and the spring energy stored in the compressed region of the puck at its maximum displacement. During the collision between a puck and a rigid strike plate, the reaction forces in the contact area do work on the system, compressing the puck during impact until the inbound velocity of the puck is zero (compression phase) [2.5]. The compressed region then recoils, forcing the puck away from the strike plate with some rebound velocity (restitution phase).

For a puck in constant motion, the kinetic energy, $K_p$ is described by

$$K_p = \frac{1}{2} m_p V^2 \tag{2.4}$$

where $m_p$ is the mass of the puck and V is the velocity of the puck. Potential energy is stored in the compressed region of the puck in the form of elastic strain energy [2.5]. This stored energy can be described by a linear spring as

$$U_p = \frac{1}{2} k_d x^2 \tag{2.5}$$

where $U_p$ is the potential energy, x is the displacement of the compressed region, and $k_d$ is analogous to the spring constant. For a linear spring, the applied force can be described by Hooke's Law as equivalent to the product of the spring stiffness and the displacement, x. Or, rearranging gives

$$x = \frac{F}{k_d} \tag{2.6}$$

A brief instant occurs between the compression and restitution phases in which the velocity of the puck is zero [2.5]. If frictional heating and vibration effects are small, most of the kinetic energy of the puck before impact is converted to elastic strain energy at the point of maximum puck compression. This also corresponds to point in which the peak force is exerted by the puck. Substituting Equation (2.6) into Equation (2.5) and setting the kinetic and potential energies equal to each other gives:

$$\frac{1}{2} m V^2 = \frac{1}{2} \frac{F^2}{k_d} \tag{2.7}$$

where F is the peak force exerted by the puck on the load cells. Rearranging the energy balance gives the following equation for the dynamic stiffness

$$k_d = \frac{1}{m_p}\left(\frac{F}{V_i}\right)^2 \tag{2.8}$$

Equation (2.8) is currently used in a proposed ASTM standard for measuring the dynamic stiffness of softballs and baseballs, and was also used in this study.

Additional insight into the dynamics of a collision can be obtained by plotting a force-displacement, or hysteresis curve. An object impacting a surface experiences an impulsive force described by

$$F = m\frac{dV}{dt} \tag{2.9}$$

where V = dx/dt. For a given force vs. time waveform, the displacement of the center of mass of the object can be found by solving the equation

$$\frac{d^2x}{dt^2} = \frac{F}{m_p} \tag{2.10}$$

with the initial conditions x = 0 at time, t = 0 and dx/dt = $V_p$ at t = 0 [2.9, 2.18]. Regardless of the compression and shape of the colliding object, the area enclosed by the force vs. displacement hysteresis loop represents the net energy loss in the collision [2.9].

The correlation between static and dynamic compression has been studied for various types of balls for safety and dynamic modeling purposes [2.9, 2.12, 2.16, 2.18, 2.19]. For softballs and baseballs, static compression is defined as the maximum force required to compress the ball 0.25 inches between flat plates over a 15 second time interval. As one would expect, the impact force from an object increases with increasing incident speed and static compression, while the amount of deformation decreases with increasing static compression [2.18]. The results regarding the relationship between static and dynamic compression for various types of balls do not always agree.

Hendee [2.12] found a linear correlation between static and dynamic compression for baseballs at three different impact velocities. Later work by Chauvin [2.19] compared the static compression of various baseballs and softballs to the dynamic compression obtained by firing balls at a rigid wall with a pressure sensitive film. He found virtually no correlation between static and dynamic compression values.

Cross [2.9] examined static and dynamic hysteresis curves for a tennis ball, a superball, a golf ball, and a baseball. For all cases, he found that the area enclosed by the dynamic curve is greater than the area enclosed by the static curve, indicating a greater energy loss under dynamic conditions. The golf ball and the superball both showed fairly linear compression behavior under static and dynamic conditions, while compression of the tennis ball and baseball was nonlinear for both cases. All balls exhibited nonlinear behavior during restitution. Cross noted that compression and restitution are nonlinear and frequency dependent, but a specific relationship between static and dynamic behavior was not determined.

The effect of temperature on $k_d$ of sporting balls has not been studied in detail. Duris [2.15] showed that as the temperature of softballs increased, $k_d$ decreased. As the temperature of the material increases, it becomes softer. Following Chauvin's logic [2.16], a softer (warmer) ball will deform more easily than a hard (cold) ball, exerting a smaller force upon impact.

Thus far, compression and restitution behavior has been considered in terms of a material that acts as a linear spring. The same treatment can also be considered in terms of a nonlinear spring, which is described by the relationship

$$F = k_n x^n \tag{2.11}$$

where $k_n$ is again analogous to the nonlinear spring constant and n is an exponent representing the degree of nonlinearity. Substituting the nonlinear relationship into Equation (2.7), the Equation (2.8) for dynamic stiffness becomes

$$k_n = \left[ \frac{2}{m_p(n+1)} \right]^n \frac{F^{n+1}}{V_i^{2n}} \tag{2.12}$$

For the problem of typical Hertzian contact for small deformation of spheres, n = 1.5 and large deformation effects tend to increase the exponent [2.5]. Smith [2.18] found for softballs that response becomes approximately linear when n = 1.25, which was surprising since softballs undergo relatively large deformations. Sources of nonlinearity can include Hertzian contact, which is nonlinear, geometric effects, internal vibration, and material effects [2.5, 2.18]. In the case of softballs, material effects, which tend to decrease n, are large enough to overcome geometric and large deformation effects, which tend to increase n [2.18].

Giacobbe [2.10] used another method, called the Scarton Dynamic Hardness test to characterize baseballs, softballs, golf balls, racquetballs, marbles, a hockey puck, ping pong, bowling, lacrosse, cricket, bocce, squash, tennis, and billiard balls. In his study, balls were modeled either as a spring-mass model with damping, or as a two mass system with damping for composite sports balls. Each type of ball was dropped onto a force platform from a known height and characterized in terms of a damping coefficient, $\zeta$ and Scarton Dynamic Hardness (SDH). Correlations between SDH and injury potential were made and recommendations for incorporating SDH into the design of baseball bats and tennis rackets were also made.

## 2.2 Ice Hockey Sticks

### 2.2.1 General Trends in Ice Hockey Sticks

Hockey sticks are fabricated from wood, aluminum, or most recently composite materials. The stick consists of the shaft, or the straight, long, handle portion, and the blade, or the curved portion on the bottom of the stick used for moving the puck (Figure 2.1). Most players wrap cloth tape around the top of the shaft for better grip and around the blade to cushion the puck, making it easier to control.

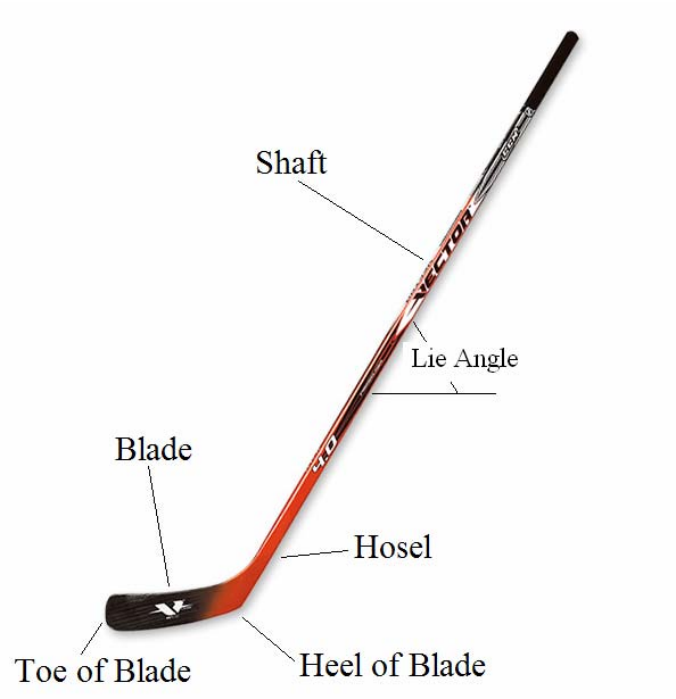


Figure 2.1: Hockey stick components

The National Hockey League (NHL) regulates the dimensions of legal hockey sticks [2.20]. These regulations mandate that sticks shall not exceed 63 inches in length from the heel of the blade to the end of the shaft, and the blade shall not exceed 12.5 inches from the heel to the toe. Blades must be between 2-3 inches in height and have beveled edges. In addition, the NHL requires that "the curvature of the stick shall be restricted in such a manner that the

distance of a perpendicular line measured from a straight line drawn from any point at the heel to the end of the blade to the point of maximum curvature shall not exceed three-quarters of an inch," as illustrated in Figure 2.3 [2.20]. While allowable dimensions are specified, there are no regulations concerning the materials used in hockey stick construction.

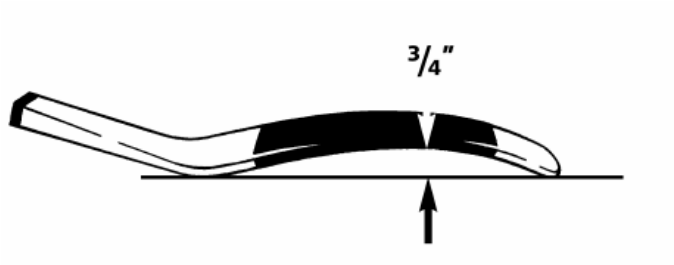


Figure 2.2: Measurement of blade curvature – Copied from [2.20]

When selecting a stick, players are most concerned with durability, weight, the feel of the puck on the stick, blade curve pattern, and stiffness of the shaft. There are many different blade patterns available to suit a variety of player preferences. Blade patterns are chosen to suit different styles of handling and shooting the puck. The general shape of the curve is classified as either a heel, mid, or toe curve. This classification corresponds to where the majority of the curve is located. Blades are then classified according to their curve depth as slight, moderate, or deep. Finally, the face angle of the blade is rated as closed, slightly open, or very open [2.21].

Sticks are also rated by their stiffness (or flex), and their lie. The stiffness is typically printed on the shaft of the stick and corresponds to the amount of force (usually in pounds) required to deflect or bend the shaft one inch [2.1]. The span used to measure the stiffness of hockey sticks is not defined in the literature. Adult sticks usually range from 75 – 115 stiffness ratings. During shooting, the stick is deflected (loaded) and the energy stored in the stick is transferred to the puck as it is released. Previous studies have measured the stiffness by supporting the ends of the stick and deflecting the middle, as in a three point bend test [2.3, 2.22]. The concept of stick loading is discussed in greater detail in the next section. The amount

of loading a player can achieve during a shot is determined by both the player strength and the stiffness of the stick.

The lie of the stick is a designation that refers to the angle between the shaft and the blade and usually ranges from 5 to 7. A lie of 5 corresponds to a 135° angle between the blade and the shaft [2.21]. Each additional lie value corresponds to a decrease of 2° in lie angle [2.21]. The lie angle is shown previously in Figure 2.1. Different lie angles are chosen according to player preference and skating style.

The geometry of the shaft cross-section is another factor that can be altered to enhance how the stick feels in a player's hands. The standard shaft has a basic rectangular shape. Some sticks have rounded corners while others have a concave shape on the faces of the shaft. Little research has been done to determine the effect of shaft geometry on the performance of hockey sticks.

Most ice hockey sticks are constructed either of wood with composite laminates and wraps on the shaft and blade, entirely from composite materials, or a combination of a composite shaft with a reinforced wood blade. Fiberglass, Kevlar, graphite, resin, aramid, and/or carbon fiber are composite materials commonly used in ice hockey sticks. Each manufacturer uses its own proprietary combinations of materials.

Composite hockey sticks are fabricated through a molding process. This process, combined with the consistency of materials allows for fine control over the properties of the end product, which was not previously available for wood sticks. Manufacturers can tune the flex of the stick to a desired value and have been able to experiment with different features of a stick's geometry that affect how it flexes during a shot. Shaft tapers can also be better controlled to

lower the kick point. The kick point is the portion of the shaft that deflects the most during shaft loading.

To date, the effects of these changes in stick characteristics on shot performance is not fully understood [2.23]. It is generally agreed that factors such as physical attributes and skill level of the player, mechanical properties of the stick, and the environment can all play a role in determining shot performance. The effect of specific stick characteristics on shot speed is still not well understood [2.23, 2.24, 2.4].

### 2.2.2 Stick and puck interactions

The hockey stick is used mainly to move the puck around on the ice. Stick handling refers to the method of maneuvering the puck along the ice in front of a player. Stick handling is also used to quickly move the puck around another player or stick. During stick handling, players prefer to have a good feel for vibrations in the stick resulting from small puck impacts. This allows them to know where the puck is relative to the stick without having to look down.

The stick is also used to pass the puck to another player and to shoot the puck at the net. Four methods are used to shoot the puck: the wrist shot, the slap shot, the snap shot, and the backhand shot. In any type of shot, the stick contacts the ice ahead of the puck and the shaft is deflected, or loaded. The stick then comes in contact with the puck, and the shaft recoils, propelling the puck in addition to the initial swing speed as it leaves the blade. The amount of shaft loading that occurs is dependent on the player strength, stick stiffness, and the type of shot.

In a wrist shot, the puck is placed on the heel of the blade slightly behind the player. The player makes a forward sweeping motion with the stick, pushing the puck along the ice and eventually releasing it from the toe of the blade. The wrist shot is favored for its quick release and highest accuracy of all types of shots.

The backhand shot is a variation of the wrist shot. In this case, the puck is placed toward the heel on the backhand side of the blade. A sweeping motion is performed as the puck rolls along the blade toward the toe. The puck is then released from the toe of the blade in a smooth, sweeping motion. The backhand shot is advantageous when the player is not facing the goal and provides fair shot accuracy with reduced power.

The snap shot is an abbreviated form of the slap shot. It employs a short backswing in which the stick is raised off the ice and behind the puck. The player then swings the stick toward the puck (downswing phase), contacting the ice shortly ahead of the puck and preloading the shaft (preloading phase). The blade then contacts the puck and the shaft recoils before releasing the puck. The snap shot has a faster release time and better accuracy than the slap shot.

The slap shot is the fastest method of moving the puck and can be completed either as a stationary shot or while moving [2.25]. The slap shot involves six phases: backswing, downswing, preloading, loading, release, and follow through [2.23, 2.25]. During the preloading phase, the stick contacts the ice six to twelve inches in front of the puck, causing bending in the shaft [2.23]. In this phase, kinetic energy is translated to elastic strain energy that is stored in the shaft [2.25]. The deflected stick then contacts the puck during the loading phase, which lasts approximately 30-40 ms [2.25]. During this contact time the stick recoils and elastic strain energy from the recoiling shaft is transmitted to the puck in addition to the kinetic energy of the swinging stick. The puck is subsequently released from the stick toward the target at the end of the contact time or loading phase. The slap shot is the fastest, but also least accurate method of shooting the puck.

Previous studies have found puck velocities up to 110 miles per hour during game play [2.3]. Average puck velocities resulting from slap shots for elite and collegiate players have

been found in the range of 65-80 mph [2.23, 2.24, 2.25]. Most of the previous studies to investigate slap shot characteristics have focused on technical aspects of skill execution with little regard for stick characteristics [2.23]. The hockey stick performance metric developed in section 2.3 was based on a model of a slap shot. The slap shot will now be discussed in greater detail.

### 2.2.3 The Slap Shot

The slap shot is the fastest and most dramatic method of shooting the puck [2.25]. It is employed 26% of the time by forwards and 54% of the time by defensive players in game play [2.24]. Previous studies on the slap shot have investigated the biomechanics of shooting, the effects of bending stiffness of the shaft [2.23], geometry of the shaft [2.22], and player skill level [2.24, 2.25]. These studies found that the predominant factors in shot performance were player skill level and the movement patterns of elite players compared to recreational players [2.25]. Little is understood about the effect of the stick's physical features or the mechanical factors regarding movement and contact time on the resulting velocity of the slap shot [2.22, 2.23, 2.25]. Several factors are generally believed to be important in shooting, including: velocity of the lower end of the stick prior to contact with the ice, preloading of the stick, elastic stiffness characteristics of the stick, and contact time with the puck [2.23, 2.25].

Early researchers in the 1960s and 1970s sought an understanding of the kinematics of a slap shot and its injury potential in hockey shots [2.3]. Such studies were concerned with the technical aspects of shooting regardless of the stick type [2.23]. In 1978 Sim and Chao [2.26] used cinematographic analysis to conduct an in-depth biomechanical analysis of hockey shots. The main goals were to measure the speeds of a skating player, the stick, and the puck during shooting. Skater velocities were found in the range of 20-30 miles per hour and relative angular

velocity of the stick ranged from 20 - 30 radians per second. They found puck velocities up to 90 miles per hour for high school hockey players and up to 120 miles per hour for college and professional players.

A more in-depth biomechanical analysis was performed by Marino in 1998 [2.4]. Key results highlighted four mechanical factors as being important in the performance of shooting. These factors were: (1) velocity of the lower end of the shaft prior to contact with the ice, (2) preloading of the stick, (3) elastic stiffness characteristics of the stick, and (4) contact time with the puck. Still, the direct relationship between mechanical properties of the stick and shot velocity was not determined.

Hoerner [2.3] summarized the results of several investigations into wood stick durability that were completed in the late 1970's-late 1980's. He found the following five failure modes for hockey sticks during slap shots: junction of blade and shaft or hosel (44%), mid-shaft break (16%) or delamination (15%), and blade break (14%) or delamination (11%). Examining broken sticks, he determined some main variables are directly related to the durability and longevity of a hockey stick. These variables included width of the handle, thickness of the handle, and the mode of rigidity of the shaft to a plane parallel to the blade.

Also summarized by Hoerner [2.3] were several studies that determined the forces at different points on the stick and the deflection of the shaft during wrist and slap shots. Deflections in the shaft were 13° for a stiff stick and 15° for a flexible stick, which were twice the deflections seen for wrist shots. The most important force in shooting was found to be at the location of the lower hand. These studies also confirmed that the speed of shooting is directly related to the acceleration impact of the stick in the forward, downward phase of the shot

movement. This demonstrated that a more flexible stick required a smaller force at the time of impact than a more rigid one to obtain the same puck speed.

Few new developments in the research of hockey slap shots were made in the 1990's. The advent of new materials and greater selection in hockey stick properties inspired new research into the effect of the characteristics of the stick itself on the performance of hockey shots. Wider availability of high speed analysis techniques also contributed to the renewed interest in hockey stick research [2.27].

Pearsall, et al. (1999) investigated the effect of stick stiffness on the performance of ice hockey slap shots [2.23]. This study attempted to verify the belief by players and coaches that a stiffer stick will permit greater force to be applied to the puck and thus yield a higher shot velocity. Six elite male ice hockey players performed slap shots with composite hockey sticks of four different stiffnesses that are commonly used by players. The stiffnesses used were rated at 13, 16, 17, and 19 KN/m. High speed video and force platforms were used to analyze the speed and reaction forces in three dimensions for each shot.

Results showed significant differences in maximum puck velocity for shaft stiffness, subject, and the interaction of subject and stiffness. The only statistically significant differences according to shaft stiffness were in the most flexible and the second stiffest sticks. The most flexible stick had the highest puck velocity and the lowest vertical peak reaction force. The second stiffest stick had the lowest puck velocity and the highest vertical peak reaction force; however, the greater reaction force did not translate to greater shot velocities. In addition, peak shaft deflection showed an inverse relationship with shaft stiffness, with peak deflections in order from most flexible to stiffest of 20.4°, 18.7°, 18.4°, and 17.9°. The interaction between subjects and stiffness was found to account for 67% of the variation in peak shaft deflection. In

general, the stiffness of the stick used in a slap shot seemed unimportant in determining the kinematic behavior of the stick. Subjects appeared to be more important in determining the velocity of the shot than the stick characteristics.

Further research by Moreno, Wood, and Thompson (2004) studied the effects of technique and stick dynamics in the performance of standing snap shots [2.22]. One elite ice hockey player performed snap shots with two different sticks. The first was a standard composite stick with a rectangular cross-section (stick 1) and the second was a composite stick with wood veneer siding and a double concave shaft shape (stick 2). A quasi-static characterization of force vs. deflection was found by a three point bend test for both sticks. An essentially identical linear relationship was found for each stick.

During shooting trials the two sticks exhibited different loading diagrams. In all trials, stick 2 reached a higher peak force in a shorter amount of time than stick 1, but recorded puck velocities were very similar for both sticks (stick 1 – 23.6 +/- 0.6 m/s and stick 2 – 23.3 +/- 0.6 m/s). The angular velocity of the hosel after the blade leaves the ice was found to be drastically higher for stick 2 (896 deg/s) than for stick 1 (281 deg/s), indicating that stick 2 recovers from being deflected much faster than stick 1. They concluded that changing the shape of the shaft from standard rectangular to a double concave design increases the recovery rate and maximum deflection, yet the overall velocity of the puck stays the same. No comment was made regarding the effect of the added wood veneers to the double concave stick design. It was postulated that the faster recovery time of stick 2 allowed the puck to reach its end velocity faster than for stick 1, giving the player a quicker release and the goalie less time to react to the shot.

More recent studies have focused on the mechanical parameters of shooting, such as the recoil effect (unloading of the deflected shaft) of the hockey stick [2.25] and the effect of player

skill on blade contact and deformation in three dimensions during slap shots [2.24]. Villaseñor found that both recreational and elite players applied the same magnitude of force to the puck during slap shots, but the elite group produced greater stick recoil. Elite players had a longer blade-puck contact time during the recoil phase of the shot, resulting in greater puck velocities than recreational players [2.25]. Findings suggested that elite players were also able to generate a lower kick point during shaft loading and unloading, therefore utilizing the recoil effect with greater efficiency than recreational players [2.24, 2.25].

Lomond et al. (2007) recognized that little was known about the timing parameters between the stick and puck within the ice hockey slap shot even though proper timing and movements have been identified as an essential component in successful striking tasks [2.24]. They sought to quantify the influence of player skill on stationary slap shot performance during the critical period of blade-ground contact.

A unique recoil phase was identified within the duration of the slap shot for both elite and recreational players, during which the deflected shaft of the stick straightened, transferring energy to the puck. This phase was longer for elite players and at times relatively nonexistent for recreational players. Greater puck velocity was found in the elite players due to timing adjustments that increased linear displacement, velocity, and acceleration of the stick and puck during the rocker phase compared to recreational players who utilized greater rotational variables. Contrary to popular industry opinion, the different construction parameters of blades currently on the market did not affect the blade's global position or orientation during the slap shot, suggesting that greater shot performance came from player movements and/or parameters of the stick.

In previous research regarding the execution and performance of ice hockey slap shots, subjects performed shots that were then analyzed in a lab setting. Performance has either been a function of player skill or subject variability has shown significant interference in the performance results. The current study developed a stick performance measure that is independent of player movements or skill level. Such a performance measure would allow direct comparison between sticks and stick characteristics, regardless of player interference or skill level.

## 2.3 Hockey Stick Performance Metrics

It is desirable to have a standard measure of performance, regardless of the player using the stick, as an objective measure that can be used to compare the efficacy of one stick to another. This measure should relate the performance of a stick at game speeds, regardless of player skill level or strength.

The concept of a high speed performance measure that describes a collision in the field of play began with the bat-ball sports of softball and baseball. Various measures such as Bat Performance Factor (BPF), Bat-Ball Coefficient of Restitution (BBCOR), Ball Exit Speed Ratio (BESR), and Batted-Ball Speed (BBS) are used to describe the efficiency of a high speed bat-ball collision. These bat performance metrics have been used by regulatory agencies such as the National Collegiate Athletic Association (NCAA), the Amateur Softball Association (ASA), and United States Specialty Sports Association (USSSA) to regulate the performance of bats allowed on the field of play.

ASTM F 2219-04 [2.28] describes a standard method for measuring and calculating these quantities. The basis for these performance metrics is a momentum balance between a ball and a

bat. A ball is fired from a high speed cannon at a stationary pivoted bat with a known velocity. The rebound speed of the ball after hitting the bat is measured and performance measures are calculated. Analogous tests have been performed for cricket, tennis, and golf [2.29, 2.30]. The same reasoning was applied here to a collision between a stick and a puck.

During a stick-puck collision momentum is conserved. A stick-puck collision momentum balance is of the form

$$m_p V_1 Q + I\omega_1 = m_p v_2 Q + I\omega_2 \tag{2.13}$$

where $V_1$ and $\omega_1$ are the puck linear velocity (in/sec) and the stick angular velocity (rad/sec), respectively, before impact, $v_2$ and $\omega_2$ are the puck linear velocity and the stick angular velocity after impact, $m_p$ is the mass of the puck (oz), Q is the impact location measured from the pivot point (in), and I is the stick moment of inertia, or MOI (oz-in$^2$) measured about the same pivot location. Figure 2.3 illustrates the sign convention used for determining the momentum of the stick and puck.

For softball and baseball bats, ASTM F 2398-04 [2.31] describes a method for determining the moment of inertia. In this standard the bat is modeled as a physical pendulum, and the MOI is determined by measuring its period of oscillation, mass, and distance between the pivot point and center of mass. A similar method was used for determining the MOI of hockey sticks in this study. The axis of rotation was taken as perpendicular to the length of the shaft and in a plane parallel to the blade. This can be pictured as coming out of the page in Figure 2.3.

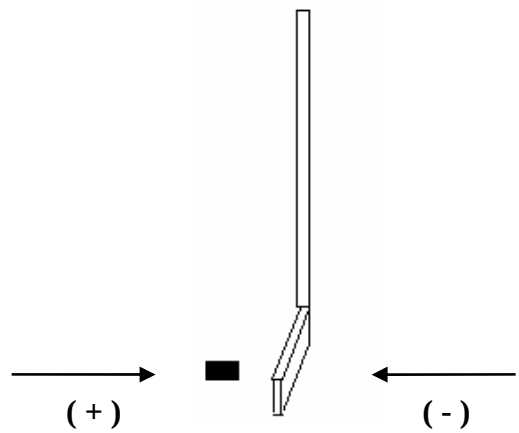


Figure 2.3: Sign convention definition

The inertia of a physical pendulum can be found from the equation

$$I = \frac{\eta^2 g W_t a}{4\pi^2} \tag{2.14}$$

where η is the period of oscillation of the stick (Hz), g is gravitational acceleration (in/sec$^2$), $W_t$ is the total weight of the stick, and a is the distance (in) between the center of mass and the pivot point. The period, η is an average value found by measuring the time for the stick to oscillate through fifteen cycles.

The center of mass, or balance point (BP) can be found by measuring the mass at distances of six inches and 42 inches from the handle end of the shaft. The BP is then calculated as a weighted average, or

$$BP = \frac{6W_6 + 42W_{42}}{W_t} \tag{2.15}$$

where $W_6$ and $W_{42}$ are the weights of the stick at six inches and 42 inches from the end of the stick. A schematic of the fixture used to measure the weight at six and 42 inches is shown in Figure 2.4.

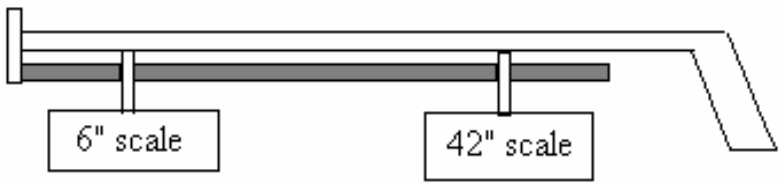


Figure 2.4: Schematic of the balance point fixture

The distance a between the pivot point and the balance point is then

$$a = BP - l \tag{2.16}$$

where l is the distance measured from the end of the stick handle to the pivot point. The pivot point was taken as 35 inches along the shaft from the handle end of the stick.

A stick with a higher MOI will have more momentum, and therefore more kinetic energy available to transfer when it hits the puck. On the contrary, more energy is required from the player to swing a stick of higher MOI at the same speed as one of lower MOI. The speed at which a player can swing a bat, racket, or club depends on the MOI rather than the mass of the striking implement [2.32]. The MOI itself is dependent on the mass, length, and mass distribution of the stick. Two sticks can have the same mass and length, but different MOI if the weight is distributed differently in one compared to the other.

Previous studies on the effects of MOI on swing speed of a striking implement have found an inverse relationship between swing speed and MOI [2.32, 2.33, 2.34, 2.35]. These studies examined baseball and softball bats, golf clubs, and simple rods. The effects on swing speed due to MOI have been expressed in terms of a ball speed coming off the implement for softball and baseball bats [2.33, 2.34] and tennis rackets [2.32]. In both cases curves of ball speed vs. MOI were exponential in form, and Bahill [2.34] claimed that an optimum MOI value exists for a given player to achieve the maximum ball speed. This optimum value is a trade-off

between how fast a player can swing an implement and the amount of momentum generated from the inertia of the implement.

For a puck fired at a stationary pivoted stick, the mass of the puck, $m_p$, the MOI of the stick, I, the impact location, Q, and the inbound velocity $V_1$ are all known quantities. For an initially stationary stick, Equation 2.13 reduces to

$$m_p V_1 Q = m_p v_2 Q + I\omega_2 \tag{2.17}$$

For the same collision, the puck does not actually rebound, but rather continues in the positive direction after impact. The speed of the puck $v_2$ after impact is difficult to measure, but the angular velocity of the stick, $\omega_2$ can be easily measured. Knowing these values, Equation 2.17 can be rearranged to calculate the speed of the puck after the collision.

$$v_2 = V_1 - \frac{I}{m_p Q}\omega_2 \tag{2.18}$$

The collision efficiency, $e_a$ is a model independent relationship that is derived from conservation laws [2.33] and is found by

$$e_a = \frac{-v_2}{V_1} \tag{2.19}$$

where the negative sign comes from the sign convention and the expectation that the fired projectile rebounds in the negative direction after impact, as is the case for sporting balls like softballs, baseballs, cricket balls, and tennis balls [2.29]. In this case, the puck does not rebound, but continues in the positive direction, resulting in a negative collision efficiency. The collision efficiency has been found to depend on the relative velocity of the objects, but only weakly [2.33]. Nevertheless, it is still strongly desirable in a laboratory setting to achieve relative velocities close to those seen in actual play.

Because the collision efficiency is model independent, it can be used to predict the speed of an initially stationary puck that is impacted by the blade of a stick, such as in a slap shot. The following equation describes this relation [2.33]:

$$v_p = V_1 e_a + v_s (1 + e_a) \tag{2.20}$$

where $v_s$ is the linear velocity of the blade of the stick when it strikes the puck. The linear velocity of the stick in this case is not known, but previous studies have shown the linear velocity of a striking implement to be dependent on the impact location and the MOI of the implement [2.32, 2.33, 2.34, 2.35]. With this knowledge, a nominal stick speed can be scaled by factors for MOI and impact location to estimate the speed for a given stick and impact location.

For a shaft that is rotating about a point with a known angular velocity, ω, the linear velocity, v of a point on the shaft can be found by the relation v = ωr, where r is the distance from the center of rotation to the point of interest. This indicates that a linear relationship exists between angular and linear velocity, and therefore a linear scaling factor for impact location can be used in scaling stick swing speed. In this case, a nominal impact location of 35 inches was chosen and all other impacts were scaled from that location. The determination of impact location is discussed in more detail in Chapter 4.

Previous studies have developed relationships between MOI and swing speed for various striking implements [2.32, 2.33, 2.34]. In all cases, the relationship was nonlinear. The degree of nonlinearity appeared to depend on the value of the MOI of the striking implement [2.32]. For golf clubs, softball bats, baseball bats, and cricket bats varying the swing speed proportional to MOI raised to the power n = 0.25 provided a good model [2.32, 2,33]. The MOI for an average hockey stick with a 35 inch pivot distance (10,000 oz-in$^4$) was slightly higher than that

of a typical slow pitch softball bat (9000 oz-in$^4$), but still within the range that is successfully estimated when n = 0.25.

With the scaling factors for impact location and MOI, the linear velocity of the stick at the time of contact can be estimated by

$$v_s = v_n \left( \frac{Q}{35} \right) \left( \frac{10{,}000}{I} \right)^{1/4} \tag{2.21}$$

where $v_n$ is a nominal value representing the linear velocity at an impact location of 35 inches for a stick that has an MOI of 10,000 oz-in$^4$.

Finally, a value for the nominal stick swing speed must be developed. Several previous hockey studies provide measurements of puck speeds resulting from a slap shot [2.1, 2.3, 2.23, 2.24, 2.25], but none give estimates of the linear velocity of the stick at the time of impact. Hoerner found the angular velocity of a hockey stick during a slap shot to be 20 rad/s, but did not provide the location of the center of rotation of the stick. If the center of rotation is considered to be the handle end of a 60 inch stick, then the linear velocity of the blade would be 1200 in/sec, or approximately 68 mph. For this study, $v_n$ was chosen as 60 mph. The selection of $v_n$ is discussed in greater detail in Chapter 4.

## 2.4 Modal Analysis

Players are concerned with how a stick feels in their hands during play. When a player is receiving a pass or stick handling the puck down the ice, they prefer to feel the vibrations from the puck transmitted to the stick. This gives them more control over the puck and a better idea of the puck's location without having to look down. Player perception of feel is determined by a combination of vibration and human response. Hand transmitted vibrations are felt in the range of 8 – 1000 Hz [2.27]. Above 1000 Hz, human response to hand-transmitted vibrations

diminishes. Sports equipment manufacturers are concerned with optimizing the feel of their equipment compared to their competitor's.

Vibration patterns of an object or structure appear to be an unpredictable phenomenon, but they are in fact very measurable. When a structural object is excited, it vibrates at specific frequencies, called natural or resonant frequencies, and with specific deformation patterns. At a resonant frequency, the response is maximized when compared to the stimulus [2.27]. Resonant vibration is caused by an interaction between the inertial and elastic properties of the materials in the structure [2.36].

The deformed shape of a vibrating object is termed its mode shape. Each resonant frequency has exactly one mode shape for a given structural object. Under normal operating conditions, a structure typically vibrates in complex combinations of several or all of its mode shapes. Damping is a term that describes how quickly vibrations will decay in an object. Modal analysis is a method of dynamically characterizing an object in terms of its frequency, damping, and mode shapes.

Modal parameters are extracted by mechanically exciting a structure and measuring its operating deflection shapes. Two types of modal testing are commonly used in practice: shaker testing and impact hammer testing. In shaker testing an exciter, or shaker is fixed to the object of interest and provides either a random or sinusoidal excitation to the object. The vibrational response of the object is then observed at one or more locations. In impact hammer testing, an impact hammer is used to excite the object with a measured input impulse at as many locations as desired to define the mode shapes of the structure. The response is measured by a fixed accelerometer on the object for each input excitation. A modal test in which the accelerometer is

fixed and the hammer impacts different locations is known as a roving hammer test [2.36]. Roving impact hammer tests were used in this work and will be considered in more detail.

A force transducer in the tip of the impact hammer can be used to measure the magnitude of the impulse as a function of time. The responding motion of a fixed accelerometer can be used to measure the response at a point as a function of time. By varying the frequency of the input loading cycle, the amplitude of the response varies as the frequency of the input loading changes [2.37]. This is seen by plotting the response in the time domain. Viewing the response in the time domain can provide useful information about the damping of the system.

Mode shapes and resonant frequencies can be obtained by viewing the input and response functions in the frequency domain. This data can be transformed from the time domain to the frequency domain using the Fast Fourier Transform (FFT). The Frequency Response Function (FRF) describes the input-output relationship between two points on a structure as a function of frequency. It is defined as [2.36]:

$$FRF = \frac{FFT(Output)}{FFT(Input)} \tag{2.22}$$

The FRF is a complex function with both real and imaginary parts that relates the output per unit of input for each frequency. Peaks occur in the FRF at resonant or natural frequencies where the time response was observed to have maximum response corresponding to the rate of oscillation of the input excitation [2.37]. The amplitude of the peak describes the mode shape, while the steepness of the peak describes the damping of that mode.

The simplest modal model is a single-degree-of-freedom (SDOF) system, which corresponds to a single input (impact hammer) with a single output (accelerometer). This model consists of three elements: a spring, a mass, and a damper. Energy enters the system through the

input excitation, causing oscillation, and is diminished through damping. The equation of motion for a SDOF system is [2.38]:

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{2.23}$$

where m is the mass, c is the damping coefficient, k is the spring stiffness, and f(t) is the sum of the applied forces as a function of time. The displacement of the system is x and each dot represents a time derivative of displacement. The equation (2.23) shows that vibration consists of a balance between inertia, damping, spring, and excitation forces [2.39].

A SDOF system will have a single peak in the FRF, located at the natural frequency at which it resonates. A more complex system will have many natural frequencies, each corresponding to a different vibration mode. The response will appear more complex than that of a SDOF system, but this system can be simplified by viewing it in terms of combinations of SDOF systems [2.38]. The FRF for a SDOF system is defined as a function of frequency, ω by the relation

$$H(\omega) = \frac{1}{k}\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2 + i2\xi\left(\frac{\omega}{\omega_n}\right)} \tag{2.24}$$

where $\omega_n$ is the natural frequency and ξ is the critical damping factor [2.40]. Solving this equation for the critical damping factor yields the damping rate, $\sigma_n$ of the system:

$$\sigma_n = \sqrt{\frac{\omega_n^2 \xi^2}{1-\xi^2}} \tag{2.25}$$

Let us now consider a modal test for a hockey stick. Figure 2.5 illustrates the test setup to characterize the bending modes of a hockey stick. In this case, bending is examined in the plane of the stick that exhibits the most bending when the shaft is loaded during a shot. In order to obtain mode shapes via a roving hammer test, the FRF must be obtained for several locations

along the length of the stick. This is done by impacting the stick at one inch intervals along its length and measuring the response with an accelerometer that is fixed to a single location, in this case the middle of the shaft.

Figure 2.5: Experimental setup for modal analysis of a hockey stick

As earlier mentioned, the FRF is a complex quantity with both real and imaginary parts, or magnitude and phase. One method of analyzing FRF data is to plot the imaginary part of the function. Quadrature picking describes a method in which the imaginary part of the FRF is analyzed to find peaks. Each resonant frequency appears as a peak in the imaginary part of the FRF. The phase of the signal is indicated by either a positive or negative peak [2.36]. Quadrature picking is a simple method to determine mode shapes of an object.

The imaginary portion of each FRF can be plotted as a function of location along the length of the stick to show the deformation patterns, or mode shapes, for each resonant frequency. Plotting the FRFs along the length of the stick produces a waterfall plot. A sample waterfall plot showing the first four modes is below in Figure 2.6.

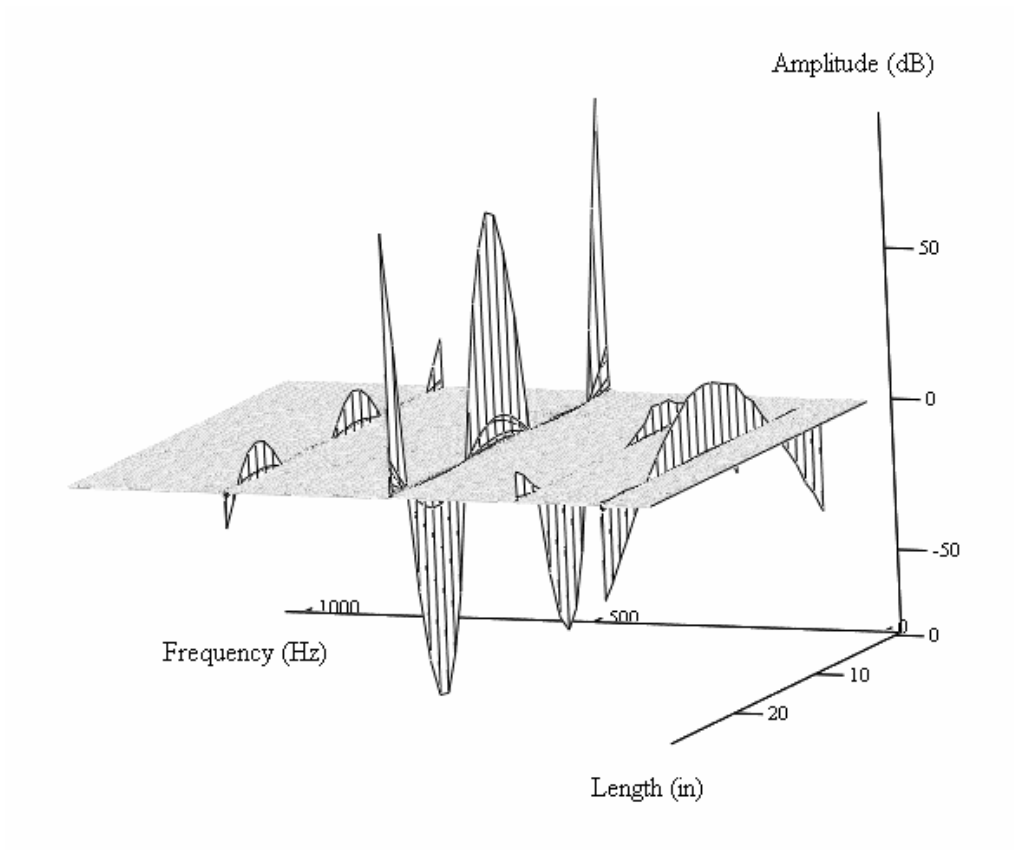


Figure 2.6: Sample waterfall plot for a hockey stick

Each mode is given a number, n, that corresponds to the number of peaks seen in the mode shape. The mode number increases with increasing frequency. For each n-mode, n+1 nodes, or locations of zero movement during vibration will exist within the structure. These node points remain stationary during oscillation and are important in determining the dynamic behavior of a structure [2.39].

The range of frequencies that are excited in a modal test is determined by the type of tip on the impact hammer that is used. A harder tip will excite a higher range of frequencies than a softer tip [2.37]. This occurs because the contact time is shorter for a hard tip than for a soft tip. In this work, a medium hardness tip was used because the frequencies of interest in testing hockey sticks were found to be relatively low (below 1000 Hz).

The boundary conditions that are imposed during a modal test can have a significant effect on the dynamic behavior of a system, and therefore must be carefully considered [2.40]. Typically boundary conditions that approximate service conditions are desired in vibration testing. A free-free setup was used in this case by supporting the hockey stick on elastic supports that allow the stick to vibrate freely when excited.

In an impact modal test, the desired location is impacted several times and the data is averaged over the number of impacts. The coherence is a parameter that indicates how accurately the dynamic characteristics of a structure are being characterized. The coherence provides a measure of how much of the response is due to the input excitation vs. external noise and/or nonlinearities in the system [2.40]. It is defined from the FRF data as:

$$C = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{2.26}$$

where $\sigma_{xy}$ is the cross-correlation coefficient, $\sigma_x$ is the input signal's variance, and $\sigma_y$ is the output signal's variance [2.39]. Derivation of the cross-correlation coefficient and signal variance is outside the scope of this paper, and the reader is referred to [2.39, 2.40] for further information. Coherence values are a function of frequency and range between zero and one. For a coherence value of one, the tester can say with confidence that the input and output response are directly linked. A coherence of zero indicates that they are not linked and the response is due to external noise [2.40]. FRFs are unique for each combination of impact and accelerometer location, so coherence cannot be used to compare data for multiple impact locations [2.40].

In a simple, hollow, one-dimensional system vibration modes can be categorized as either flexural bending, or hoop modes. Flexural modes describe bending of the stick, while hoop modes describe oscillation in the radial direction along the length of the stick. The hoop frequencies are dependent on the wall thickness of the stick. Because a hockey stick is long and

slender, hoop modes occur at much higher frequencies than bending modes and were not considered in this study. Figure 2.7 illustrates various bending modes for a hockey stick.

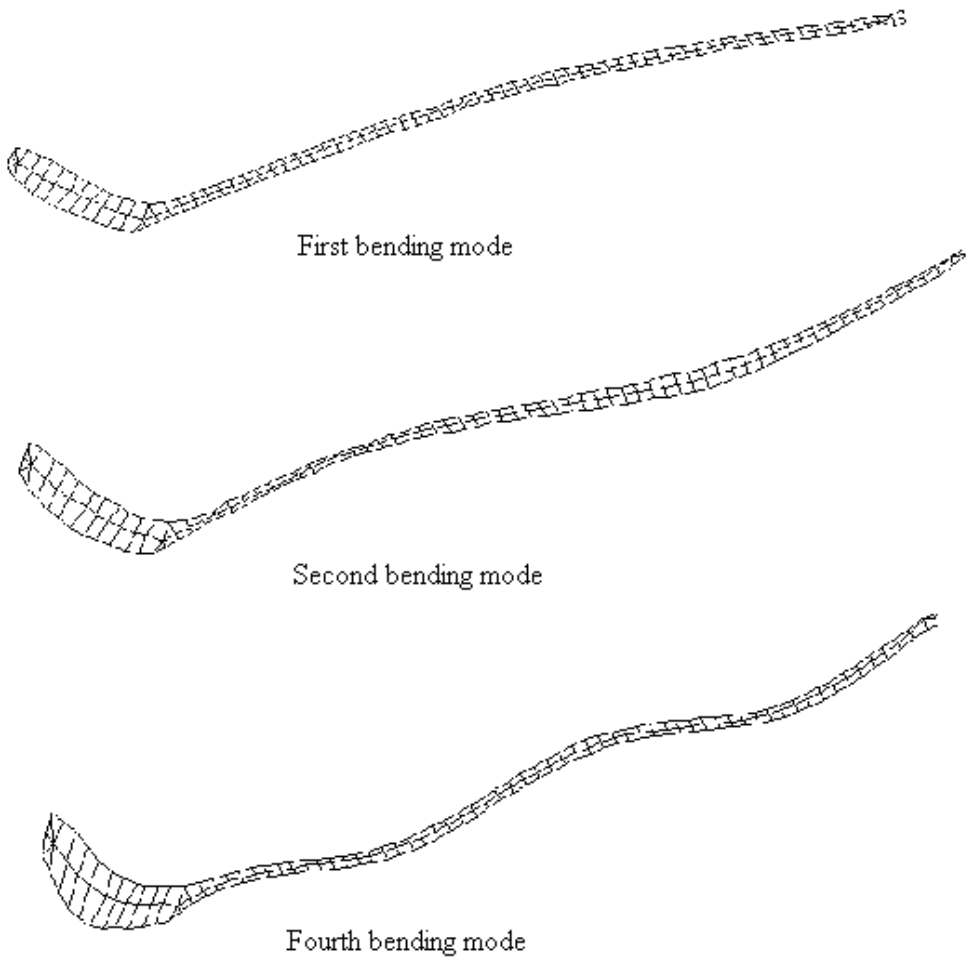


Figure 2.7: Flexural bending modes 1, 2, and 4 for a hockey stick (courtesy of Dr. Dan Russell, Kettering University)

## 2.5 Summary

This chapter provided the background information necessary to introduce the fundamental principles that were utilized for research in this study. In the first section, the

characteristics and manufacturing processes of ice hockey pucks were discussed. The mechanics of a collision were also discussed in some detail. The concept of coefficient of restitution has been introduced as a means of characterizing the elasticity of a puck. Little data exists regarding e of hockey pucks, aside from slow speed e values for both room temperature and frozen pucks. Coefficient of restitution has been shown to decrease with increasing speed for a variety of sporting balls. It has also been shown to decrease with increasing temperature.

Another impact property, dynamic stiffness, was also introduced in this section. The $k_d$ is a measure of the hardness of an object. It can be calculated by assuming the object behaves as a linear spring and equating the kinetic energy before impact to the potential energy stored in elastic deformation at the time of greatest deflection (peak impact force). Additional information about the dynamics of a collision can be obtained from the force-deflection, or hysteresis curve, which is found from force-time data during an impact.

Previous studies have shown hysteresis curves for different types of balls. Several comparisons have been made between static and dynamic behavior, but the results are conflicting. It has been shown that as speed increases, the peak force and the amount of energy lost in the collision also increase. The peak force, and therefore $k_d$, has also been shown to decrease with increasing temperature for softballs and baseballs.

The next section of this chapter discussed ice hockey sticks and stick – puck interactions. The impact of new technologies on the manufacturing and end characteristics of hockey sticks was discussed. The slap shot was presented in detail, and the results of previous research regarding the performance of hockey sticks and shots was discussed. Previous studies have all employed test subjects performing various hockey shots. No previous studies have examined the performance of ice hockey sticks regardless of the player using them.

The inertia of a swinging implement and its effects on swing speed were introduced. A concept of a high speed performance measure that describes a collision was also presented. Momentum concepts were used to examine the collision of a stationary stick and a moving puck and develop a performance measure that described a moving stick and a stationary puck. Similar test methods have been used to describe collisions for softball, baseball, and cricket.

Finally, the concepts of vibration and modal analysis were presented. The vibration behavior of an object can be described in terms of its natural frequencies, mode shapes, and damping. Modal analysis is a dynamic analysis tool that is used to determine these vibration characteristics. It employs an impact hammer and a force transducer that measures the excitation input and an accelerometer to measure the output response of the object. The input excitation and output can be viewed in the frequency domain to determine the modal characteristics of an object.

The concepts discussed in this chapter will be applied in subsequent chapters involving the characterization of ice hockey sticks and pucks. The impact properties of hockey pucks will be discussed in Chapter 3. The principles relating to stick characterization and performance testing will be discussed in Chapter 4.

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# CHAPTER THREE

# EXPERIMENTAL PUCK RESULTS

## 3.1 Introduction

Hockey pucks are three inches in diameter, one inch thick, and weigh between five and a half to six ounces. They are made of vulcanized rubber. Little is known about the impact properties of hockey pucks. Standard drop tests have been performed on a small number of hockey pucks from which coefficient of restitution values in the range of 0.45-0.55 at room temperature and 0.12-0.27 for frozen pucks have been found [3.1]. These tests were performed at low speeds, far from the 70 - 100 mph slap shots often seen in elite-level play. The response of viscoelastic materials is affected by the rate of loading, indicating that dynamic behavior may not be described accurately by static testing.

High speed impact data from ice hockey pucks could prove beneficial to hockey stick and equipment manufacturers. By understanding the behavior of pucks at game-like speeds, manufacturers could design more effective protective equipment. In addition, hockey stick design could benefit from a firm understanding of the dynamic properties of pucks. Dynamic impact properties of softballs and baseballs have been used to evaluate the injury potential of a certain type of ball [3.2]. Hockey pucks travel at high speeds during play, often striking players or spectators. While no implications regarding injury potential are made in the current study, a future extension of this work could be to evaluate the dynamic properties of pucks in terms of their injury potential.

The following study considers the coefficient of restitution (e) and dynamic stiffness ($k_d$) for various ice hockey pucks commonly used in play. Two different brands of pucks were used

in this study, denoted brand PA and brand PB. The effects of speed and temperature on each impact property were also investigated for both brands of puck.

## 3.2 Test Apparatus

A high speed air cannon was used to propel the pucks with repeatable and accurate speed and impact location. Pucks were fired at a strike plate that was attached to an array of three rigidly mounted load cells (PCB Model 208C05). The load cells were arranged in an equilateral triangle with two inch spacing between each cell. The load cells were summed and sampled at 100 kHz, from which a force vs. time profile was measured for each impact, shown in Figure 3.1. Three pairs of ADC iBeam light gates (Automated Design) were used to measure the incident speed and rebound speeds of the puck. A schematic of the test apparatus is shown in Figure 3.2.

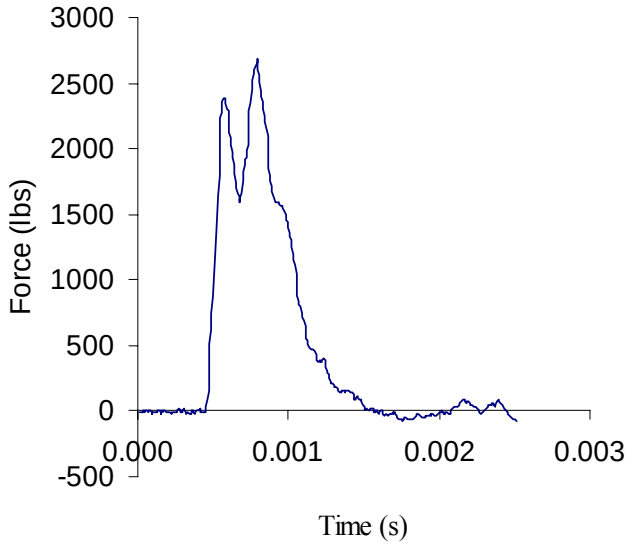


Figure 3.1: Force vs. time profile from the load cells during a room temperature puck impact

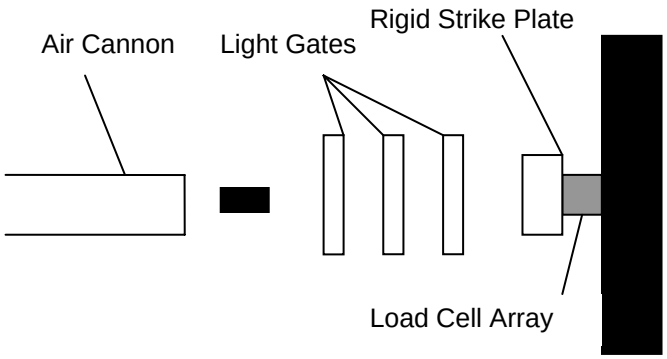


Figure 3.2: Puck testing schematic

In order to achieve consistent impacts, the puck was propelled in a manner that it was consistently released on the same trajectory without spin. For this purpose, a sabot was developed that carried the puck along the barrel of the cannon and released it at the end of the barrel, illustrated in Figure 3.3. The sabot was made of polyethylene and allowed pucks to be fired with either a vertical or horizontal orientation.

Figure 3.3: Ice hockey puck sabot

For each impact, the puck was randomly placed into the sabot and the puck/sabot combination was manually loaded into the cannon. The cannon was fired by opening the valve to the air accumulator tank using computer control. The puck/sabot traveled down the barrel

until the sabot struck the arrestor plate. Figure 3.4 shows the cannon's air accumulator tank, breach plate, and barrel.

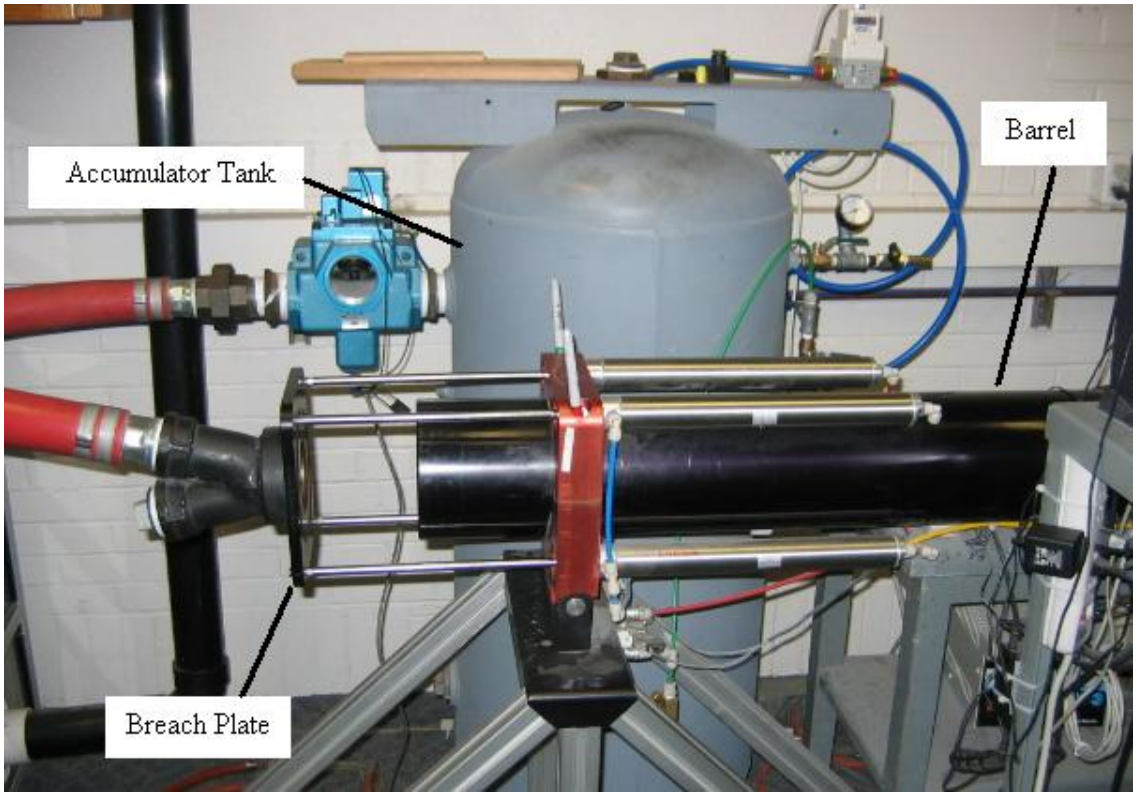


Figure 3.4: Cannon accumulator tank, breach plate, and barrel

Upon impact with the arrestor plate, the puck left the sabot, continuing through the hole in the arrestor plate. The puck then traveled into the light gate box where it passed through three pairs of light gates that measured puck speed. The puck impacted the strike plate and rebounded back through the three light gates, which again measured puck speed. The firing speed of the cannon was adjusted by manipulating the pressure inside the air accumulator tank. LabView 7.1 was used control the cannon settings and collect data from the light gates and load cells. Figure 3.5 shows the arrestor plate, light gates, and strike plate with load cell array. Figure 3.6 shows the load cell array on the strike plate in more detail.

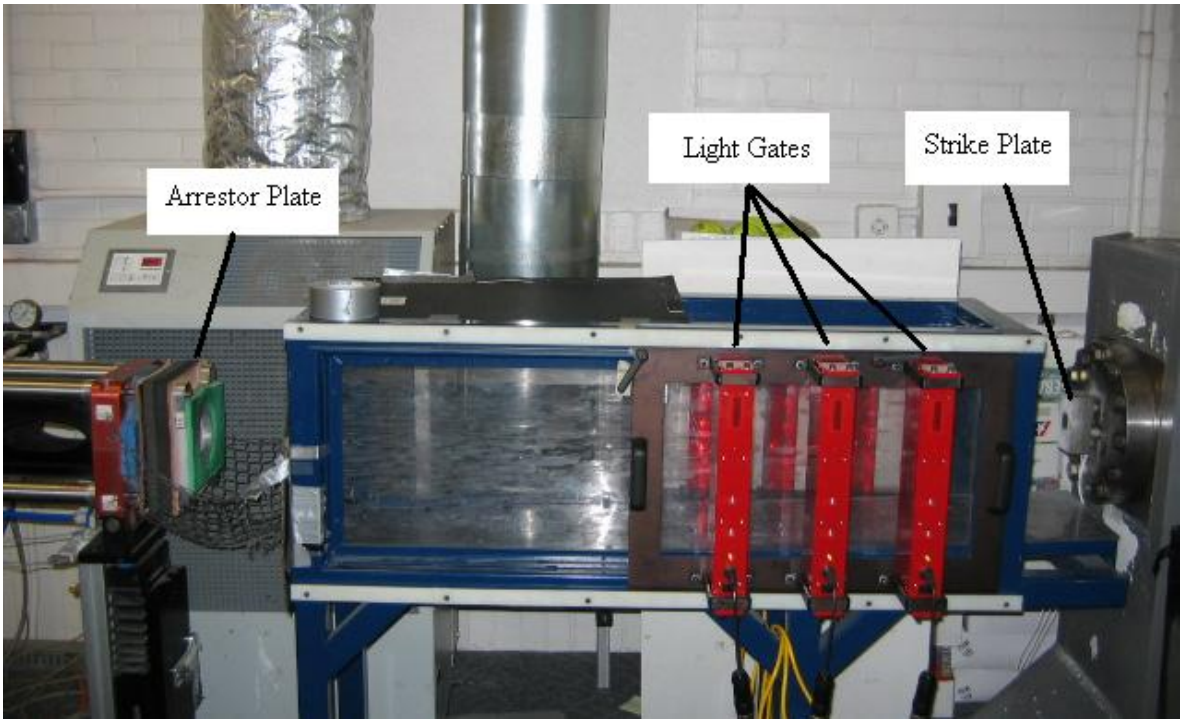


Figure 3.5: Cannon arrestor plate, light gates, and strike plate

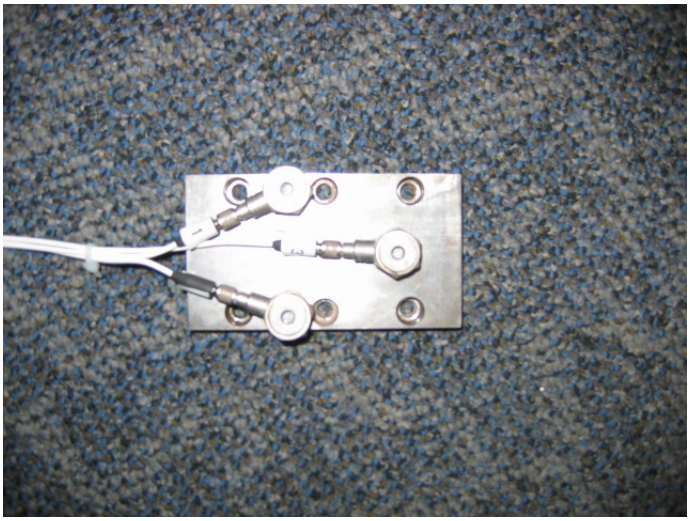

Figure 3.6: Load cell array

During initial puck testing, it was found that the puck was not releasing flatly from the sabot, and was wobbling on its inbound trajectory. This indicated that there was sufficient slop in the puck carriage of the sabot to allow it to wobble while riding down the barrel of the cannon. To alleviate this problem, small cardboard shims were placed inside the sabot to create a tighter

fit for the puck. Figure 3.7 shows an illustration of the shims used to stabilize the puck inside the sabot. In addition, firing the pucks with a vertical orientation reduced wobbling and made the puck more visible in the high speed video viewframe.

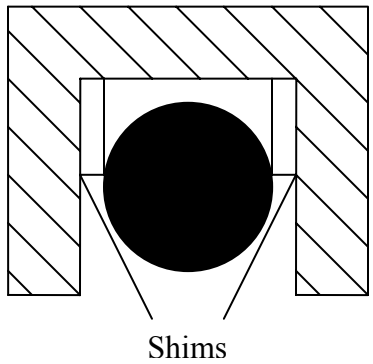


Figure 3.7: Illustration of shims inside the puck sabot

In all puck testing, a good impact occurred when the puck speed was within 1 mph of the target, the puck passed through all three light gates without bouncing or wobbling, and impacted the strike plate. In addition, the rebound was required to pass through all three sets of light gates at an angle less than ± 5° from the inbound trajectory without bouncing, wobbling, or spinning. High speed video and powder spray on the strike plate were used to verify good impacts.

Pucks were conditioned to ensure uniform properties for each test. For room temperature testing, pucks were kept in a controlled environment at 72 ± 2 ° F and relative humidity of 50 ± 5%. For room temperature tests, pucks were impacted twice, then allowed to rest for one hour to cool down to a uniform temperature. Temperature increases of less than 2° F were noted after one impact, with the temperature increasing to 75° F or more after two impacts. For frozen tests, pucks were stored in a controlled freezer. They were impacted once and then underwent a minimum one hour cool down period in the freezer before further testing. A non-contact infrared

temperature sensor was used to verify the temperature of the puck prior to firing. All testing was conducted in a controlled environment at 72 ± 2 ° F and relative humidity of 50 ± 5%.

### 3.3 Coefficient of Restitution

The coefficient of restitution, or e, was compared for two different brands of pucks that are made by two different manufacturers. The dependence of e on temperature and speed was also investigated for both brands of pucks. ASTM 1887 provides a standard test method for determining e of softballs and baseballs [3.3]. The same method was used in this study. The coefficient of restitution for each puck was measured by firing the puck at a rigid strike plate and measuring the inbound and rebound speeds. The value e is then given by Equation (2.2). Reported e values are averaged from six good impacts for each puck.

For softballs, e decreases linearly with increasing incident velocity [3.4]. Hendee, Greenwald, and Crisco [3.2] showed a similar linear correlation for baseballs, and found that e was independent of ball mass. Duris [3.4] and Drane [3.5] have examined the relationship between softball and baseball temperature and ball coefficient of restitution and found that e increases nonlinearly with increasing temperature. It is unclear how puck e changes with temperature and speed.

### 3.4 Dynamic Stiffness

The dynamic stiffness, or $k_d$ provided a measure of the dynamic hardness of pucks. A proposed ASTM standard provides a method for determining the dynamic stiffness of softballs and baseballs. The present study employed the same method for finding $k_d$ of hockey pucks. In

this method, pucks were fired at an array of three load cells. The load cells produced a force vs. time profile, like the one shown in Figure 3.1.

The dynamic stiffness equation was derived in detail in Chapter 2, section 2.1.2. The concept of hysteresis, or force-displacement curves was also discussed in the same section. A sample hysteresis plot for a cricket ball impacting a flat plate at 70 mph is shown in Figure 3.8 below. Testing of softballs and cricket balls has shown a linear loading phase, followed by a nonlinear restitution phase. Hysteresis curves were generated for each brand of puck, as well as for different temperatures and impact speeds. Characteristics of hysteresis plots will be discussed in greater detail in the following sections.

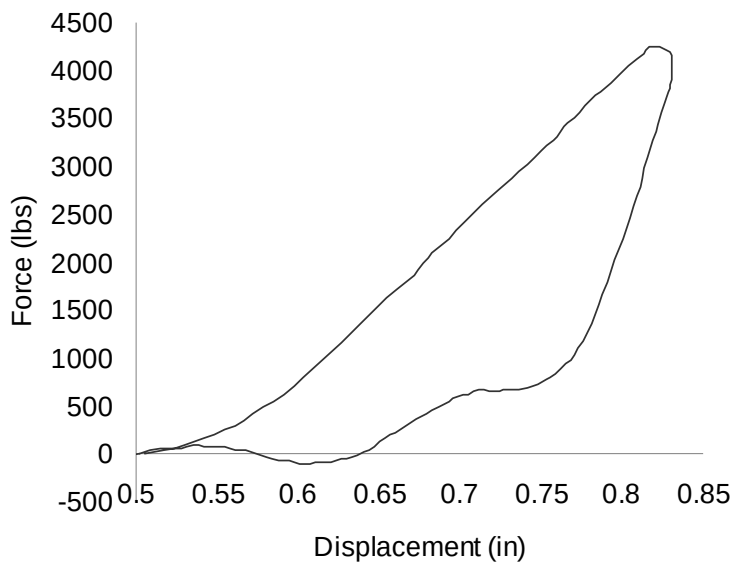


Figure 3.8: Sample hysteresis plot for a cricket ball impacting a flat plate

## 3.5 Test Speed

It is highly desirable to conduct rigid-wall lab testing at speeds that induce response similar to that seen in actual play. The primary goal is to replicate the amount of deformation of

the puck in a collision in play during the lab test. The laboratory force and coefficient of restitution should then represent play conditions.

To determine the laboratory speed, consider an energy balance for an impact between a moving puck (point mass) and a freely recoiling stick (rotating mass). A stick recoil factor, $k_s$ may be defined as

$$k_s = \frac{m_p a^2}{I} \tag{3.1}$$

where $m_p$ is the mass of the puck, a is the distance between the impact location and the pivot point of the stick, and I is the inertia of the stick [3.6].

The relationship between the laboratory speed of the puck and the swing speed of the stick can then be found by considering impacts between a puck and rigid blocks with different masses $m_1$ and $m_2$. Let $m_p$ be the mass of a puck with stiffness k. The puck impacts the initially stationary blocks 1 and 2 with an initial velocity of $V_1$ and $V_2$, respectively. At the instant of maximum displacement, $x_p$, the puck and block move with the same speed, v. The energy balance is then

$$\frac{1}{2} m_p V_i^2 = \frac{1}{2} k x_i^2 + \frac{1}{2}(m_p + m_i) v_i^2 \tag{3.2}$$

where i = 1 or 2 for a collision with block 1 or 2, respectively.

Momentum is conserved during the impact according to

$$m_p V_i = (m_p + m_i) v_i \tag{3.3}$$

Equations (3.2) and (3.3) may be combined to eliminate $v_i$ as

$$m_p V_i^2 = k x_p^2 + \frac{m_p^2 V_i^2}{(m_p + m_i)} \tag{3.4}$$

It is desirable to obtain the same maximum puck deformation, $x_p$ in both collisions. Equation (3.4) may then be evaluated for blocks 1 and 2, and combined to eliminate $x_p$ . Solving for the speed of block 1 gives

$$V_1 = V_2 \left( \frac{1 + m_p / m_1}{1 + m_p / m_2} \right)^{1/2} \tag{3.5}$$

For the case where block 1 is a rigid wall, $m_1$ approaches infinity, so the ratio $m_p/m_1$ approaches zero.

For the case of a swinging object, $m_i$ can be viewed as the equivalent swing-weight, or $I/a^2$ where I is the moment of inertia and a is the distance between the pivot point and the impact location. Substituting Equation (3.1) into Equation (3.5) and letting $V_2$ be the velocity of the incoming puck, $v_p$ and $V_1$ be the resulting velocity of the stick and puck, $V_s$ gives:

$$V_s = V_p (1 + k_s)^{1/2} \tag{3.6}$$

[3.6]. This equation provides a means for calculating the firing speed of a puck, $V_p$ that will produce the same deformation for known swing speed and $k_s$ values. The nominal stick conditions that were used in developing the stick swing speed formula in Equation (2.21) can be applied here to get a puck firing speed. That is, for a stick of 10,000 oz-in$^2$ MOI at a pivot distance of 35 inches that is swinging at 60 mph and a puck weighing 5.8 oz, the equivalent laboratory speed from solving equation 3.6 is 56.9 mph.

While an ideal test speed for hockey pucks was found to be 56.9 mph, lab and test restrictions also limit the range of speed that can be used in this study. At speeds below 50 mph, pucks do not rebound fast enough to pass through all light gates, particularly at lower temperatures. In addition, at lower temperatures the peak impact force of the puck exceeded the

load cell capacity in the test apparatus at speeds above 55 mph. Given these restrictions, a speed of 55 mph was used for brand comparison and temperature characterization.

### 3.6 Brand Comparison

Many of the pucks used in play are manufactured by a Canadian company called Viceroy and distributed under different puck brands. Two brands of pucks made by different manufacturers that are commonly seen in play were employed in this study. One brand was produced in Canada and the other in the Czech Republic. The brands will be referred to as brand PA and brand PB. One dozen of each brand of pucks was tested at 55 mph in this study.

The 55 mph brand comparison showed significant differences in both the coefficient of restitution and dynamic stiffness for each type of puck. On average, PA pucks were higher in e by 10.1% and higher in $k_d$ by 33.2%. Previous room temperature values for e of hockey pucks have been found from drop tower tests in the range of e = 0.45 – 0.55 [3.1]. Results here indicated an average of e = 0.383 for PA and e = 0.349 for PB, both of which are lower than the results previously found from slow-speed tests.

Average values are summarized in Figure 3.9. Figure 3.10 shows the room temperature results for dynamic stiffness and e for each puck tested. For the brand comparison at room temperature, pucks with lower coefficient of restitution also had lower dynamic stiffness. The differences in both e and $k_d$ do not show any overlap of the error bars representing one standard deviation of the average value. Error bars indicate a stronger difference between $k_d$ values than e in this case.

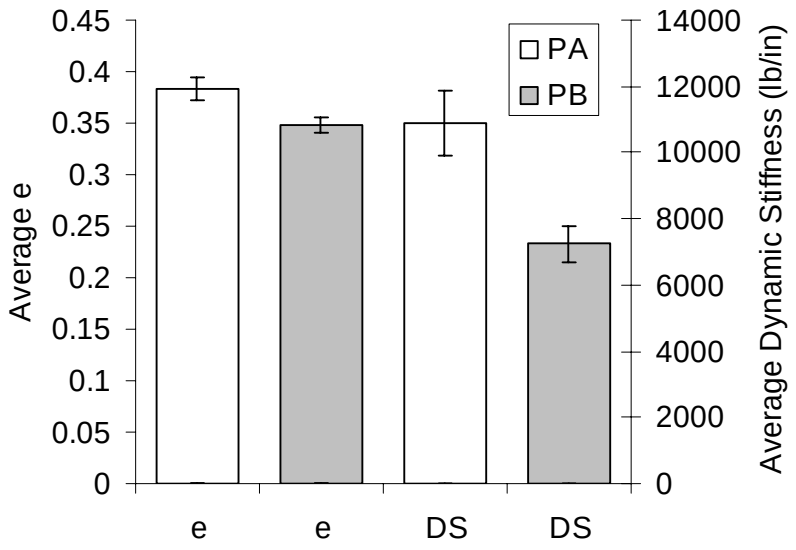


Figure 3.9: Average 55 mph e and $k_d$ for pucks tested at room temperature

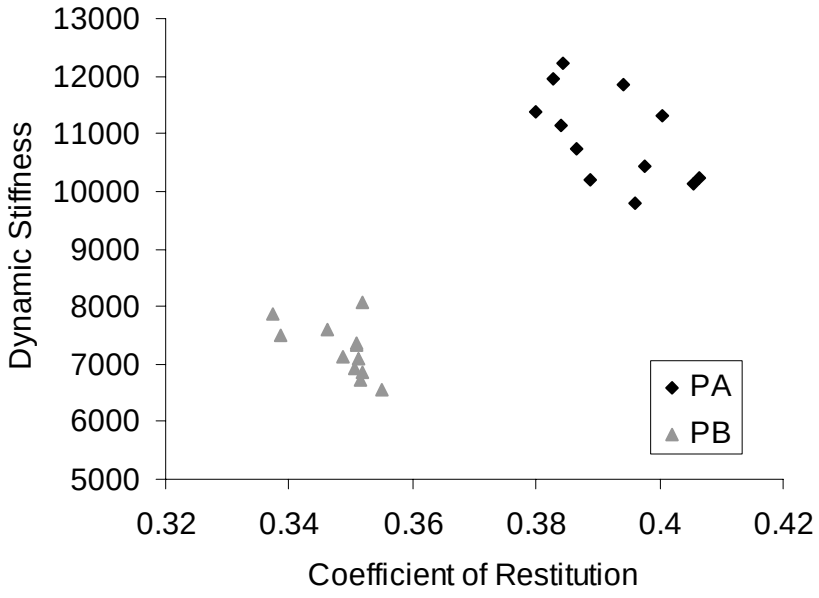


Figure 3.10: 55 mph room temperature e and $k_d$ for each puck

Further insight into the differences between the two types of pucks can be found by examining the hysteresis curves for impacts from each brand, shown in Figure 3.11. The center

of mass displacement was calculated during the contact time for a given impact by solving Equation (2.10), as previously discussed in section 2.1.2. In each hysteresis curve, a positive displacement indicates compression of the puck.

A double peak in the force-time and force-displacement curves occurs for both types of pucks. This double peak can also be seen in Figure 3.1. The double peak is likely due to an elastic wave traveling in the puck during the contact time. The double peak is not due to tooling of the test fixture, as the post-impact vibration is small. It is suspected that the wave travels in the transverse direction, or through the thickness of the puck. Because pucks are roughly three times larger in diameter than they are thick, a wave in the transverse direction would travel three times faster than a wave across the diameter of the puck. This provides a reasonable explanation for the double-peak observed in these tests. For brand PA, the first peak was always higher than the second peak, while the opposite was true for all impacts with brand PB at room temperature.

For both brands, the loading phase was essentially linear up to the first peak displacement, while the unloading phase was nonlinear. The slope of the loading phase on the hysteresis curves was also different between the two brands of pucks, with brand PA having a steeper slope. The brand PA puck used in this comparison reached a peak force that was 20% higher than the brand PB puck. Brand PB peak displacement was 9% higher than brand PA for the comparison between the two impacts shown in Figure 3.11. Similar results were found when comparing other pucks from the two brands.

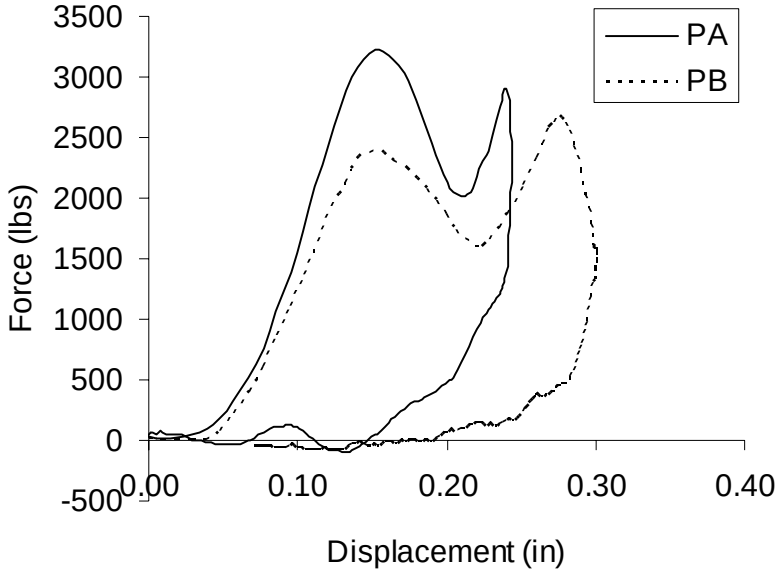


Figure 3.11: 55 mph hysteresis plots for a puck of each brand

### 3.7 Temperature Dependence

An infrared temperature sensor was used to measure the temperature of pucks used in play at two different ice rinks and on four different occasions. Frozen puck temperatures were observed to range from 12 - 25° F. At temperatures below 25° F in the lab setting, the peak impact force was high enough to exceed the load cell capacity. For this reason, frozen pucks were tested at 25° ± 2° F. All impacts for temperature testing were done at 55 ± 1 mph.

One dozen of each brand of puck was tested at both room temperature and at frozen temperatures. In addition, three pucks of brand PA were tested at a range of temperatures from 25 - 72° F. Average results from 12 pucks are shown below in Figure 3.12 for each brand. This data can be compared with the room temperature results in Figure 3.9. Brand PA shows a 626% increase in dynamic stiffness and a 32 % decrease in coefficient of restitution when going from

72° to 25° F. Similarly, brand PB showed a 488 % increase in dynamic stiffness and a 26 % decrease in coefficient of restitution over the same temperature range.

At room temperature the brands had error bars representing one standard deviation of e that did not overlap, but at frozen temperatures the error bars do overlap, suggesting that differences in e became less significant at lower temperatures. Drop tower tests found frozen e values in the range of 0.12 – 0.27 [3.1]. Averages found here were e = 0.267 for PA and e = 0.256 for PB, which are within the range previously found from slow speed tests.

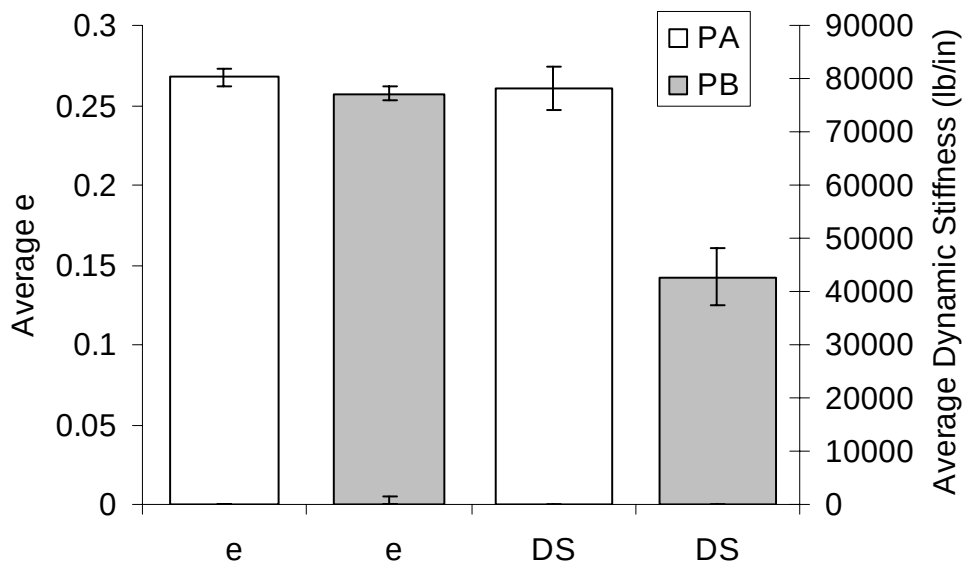


Figure 3.12: Average 55 mph e and $k_d$ for pucks tested at 25° F

Figure 3.13 shows the results for each puck at room temperature (72° F) and frozen (25° F). Brand PA underwent larger changes in impact properties than brand PB for the same temperature interval. For both the frozen and the room temperature cases, brand PB had lower dynamic stiffness and coefficient of restitution than brand PA.

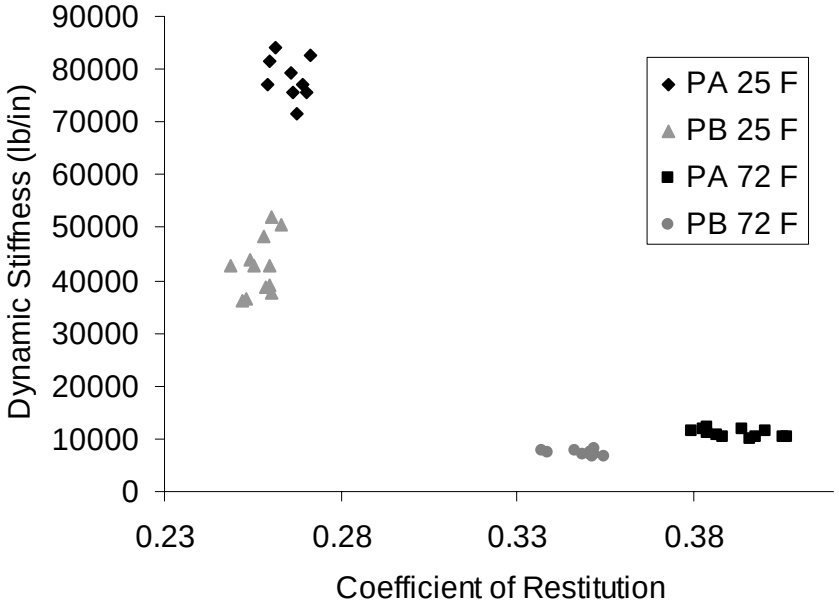


Figure 3.13: 55 mph room temperature and frozen results for each puck

Three pucks of brand PA were tested at a range of temperatures between 25° to 72° F. Results of this temperature characterization are shown in Figure 3.14. As expected, the coefficient of restitution decreases with decreasing temperature, while the dynamic stiffness increases with decreasing temperature. Both changes are nonlinear. It is interesting to note that the coefficient of restitution appears to plateau at lower temperatures, while the dynamic stiffness appears to plateau at higher temperatures.

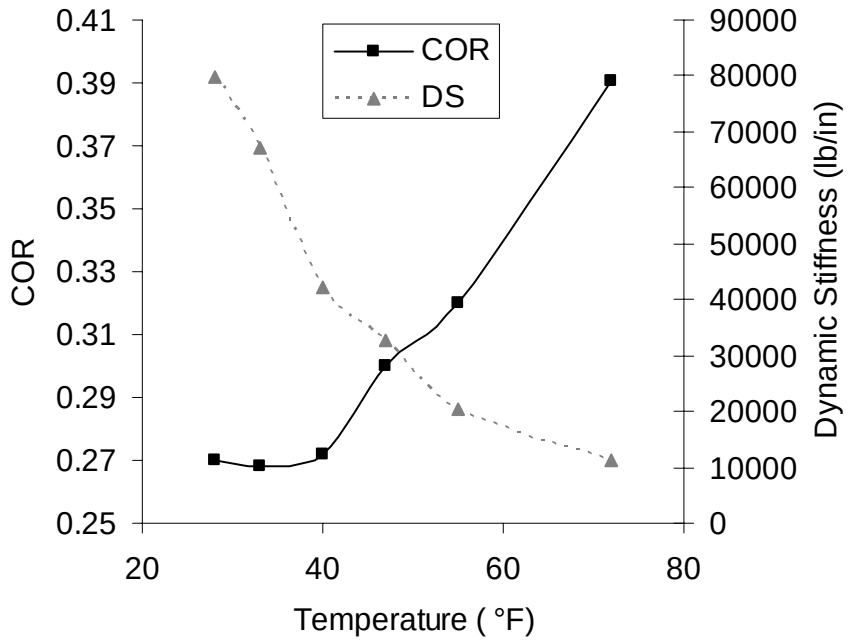


Figure 3.14: Temperature characterization results

The force – time profile for each temperature was used to produce force - displacement curves. The curves for select temperatures for one puck are displayed in Figure 3.15. The slope of the loading portion of the hysteresis curve depended strongly on temperature. Lower temperatures produced much steeper loading curves than higher temperatures did. In addition, the double peak that was seen in room temperature impacts did not occur at lower temperatures. Impacts that were conducted at 25° F had average contact times that were half that of impacts at room temperature (0.41 ms compared to 0.81 ms, respectively), which may explain why the double peak was not observed for frozen pucks. The elastic wave would travel faster in a cold puck than a warm one, but evidence shows that the speed increase is not sufficient enough to overcome the decrease in contact time.

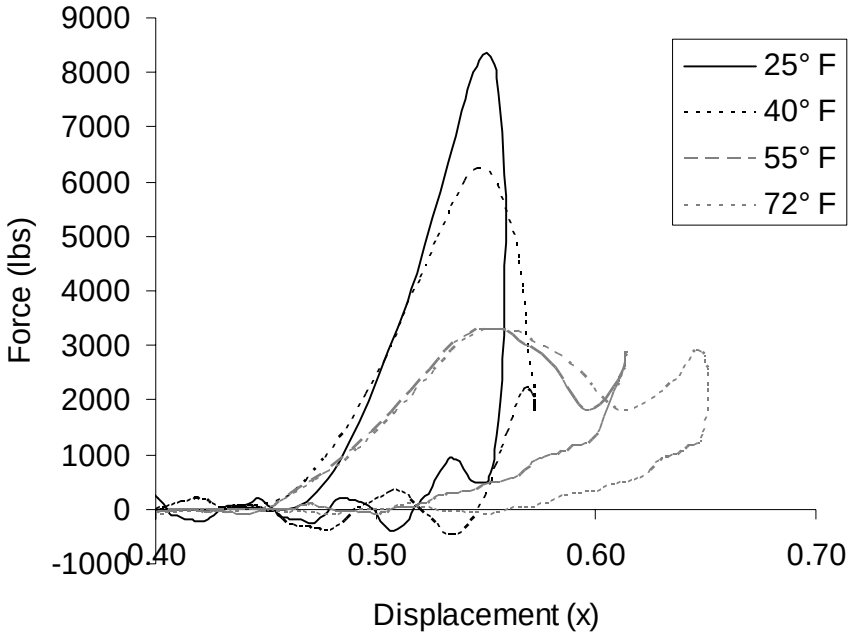


Figure 3.15: Hysteresis curves for a puck impacted at 55 mph at various temperatures

Figure 3.16 shows the averaged peak force and displacements that were found for each impact temperature. Both peak force and peak displacement changed nonlinearly with temperature. At lower temperatures the displacement appeared to plateau, similar to the results found for coefficient of restitution at lower temperatures.

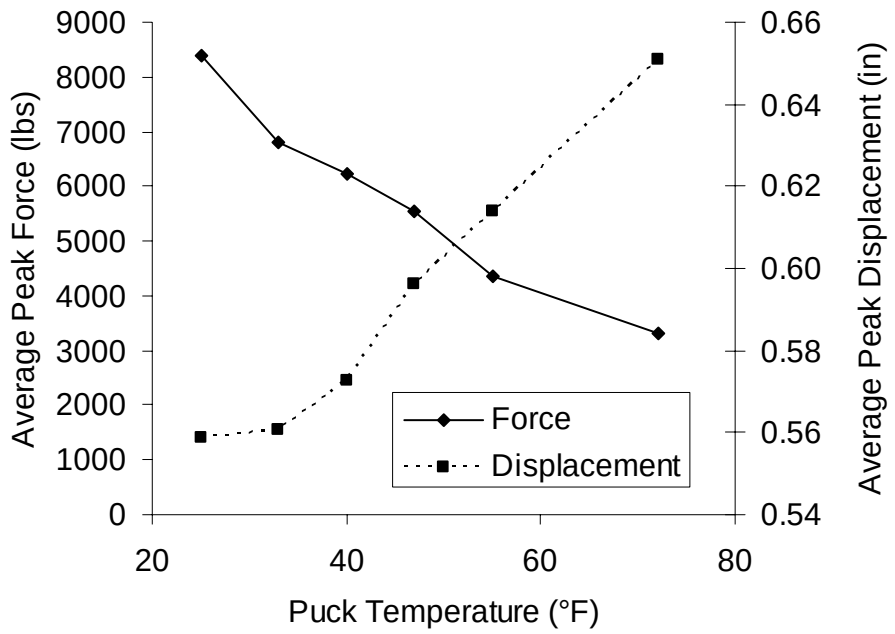


Figure 3.16: Peak force and displacement for pucks impacted at 55 mph at various temperatures

### 3.8 Rate Dependence

The response of viscoelastic materials depends on the applied strain rate [3.7]. While the term rate dependence typically refers to the rate of loading, this study is more concerned with how e and $k_d$ change with impact speed. Furthermore, rate dependence in this study refers to the speed of the impact for PA brand pucks. Golf balls, softballs, baseballs, and cricket balls have all been shown to have varying degrees of rate dependence [3.8]. The rate dependence of hockey pucks has not been previously studied.

Brand PA pucks were tested for rate dependence at room temperature (72 $\pm$ 2°F). Impact speeds ranged from 55 $\pm$ 1 mph to 85 $\pm$ 1 mph in 10 mph intervals. Figure 3.17 shows how the dynamic stiffness and coefficient of restitution varied with impact speed. The coefficient of restitution decreased linearly with increasing speed. Drop test results [3.1] have been included

for comparison, with a linear trend between the high speed and drop test results. A reasonable drop height for a drop tower test is 20 feet. A puck that weighs 5.80 ounces would be traveling at approximately 24.5 mph, which is the value used in Figure 3.17. The dynamic stiffness increased with speed, suggesting the puck behaves as a non-linear spring.

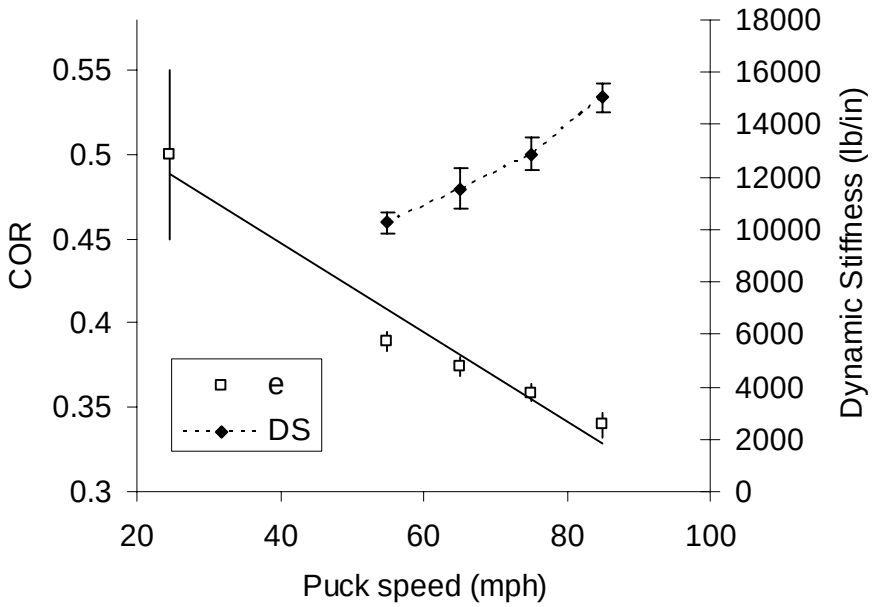


Figure 3.17: Rate dependence of $k_d$ and e

Thus far, dynamic stiffness has been considered in terms of a material that acts as a linear spring. The same treatment can also be considered in terms of a nonlinear spring, which is described by the relationship previously discussed in Section 2.1.2, Equations (2.11) and (2.12). To determine the degree of nonlinearity of hockey pucks, the exponent n in Equation (2.12) was manipulated until the stiffness became constant with speed. For brand PA pucks in this study, $k_n$ was nearly constant with speed when n = 5.5. This is significantly higher than the exponent found for softballs (1.25) and classical Hertzian contact (1.5) [3.8], indicating a high degree of nonlinearity for hockey pucks. Both the linear and nonlinear results with n = 5.5 are presented

below in Figure 3.18. For the nonlinear results, a bend in the curve appears at 75 mph that prevents $k_n$ from becoming entirely constant with speed.

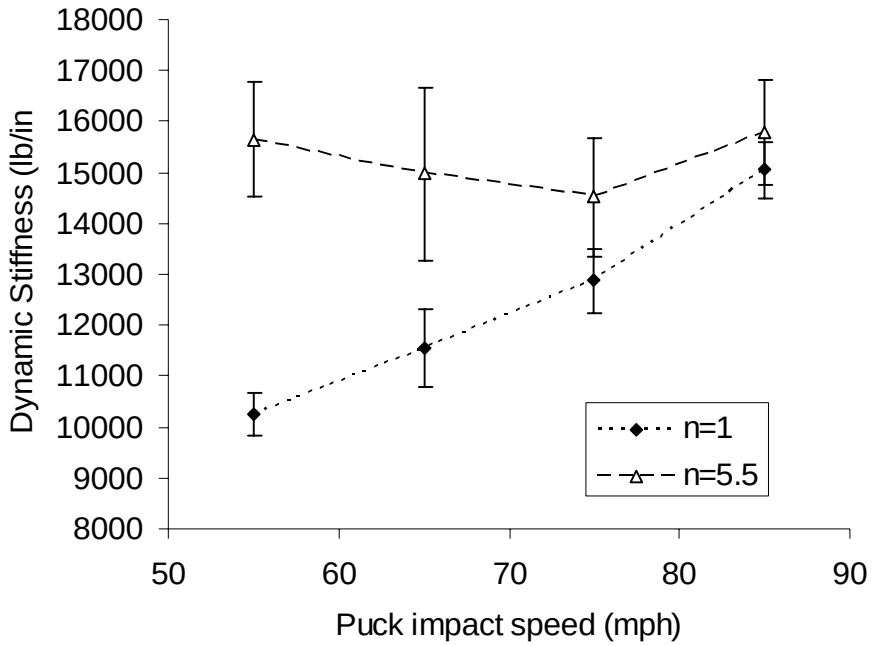


Figure 3.18: Linear and nonlinear stiffness for pucks impacted at different speeds

The hysteresis curves were plotted for each impact speed, shown in Figure 3.19. The relationship between force and displacement during the loading phase appeared constant, regardless of impact speed. The rate of unloading appears to be constant until the onset of the second peak for each impact speed also. Pucks underwent greater deformations at higher impact speeds. As the impact speed increased, the pucks showed a longer relaxation phase in which the force decreased under constant displacement, seen as a vertical portion of the unloading curve. It is unclear what caused the nearly vertical unloading section seen in each curve.

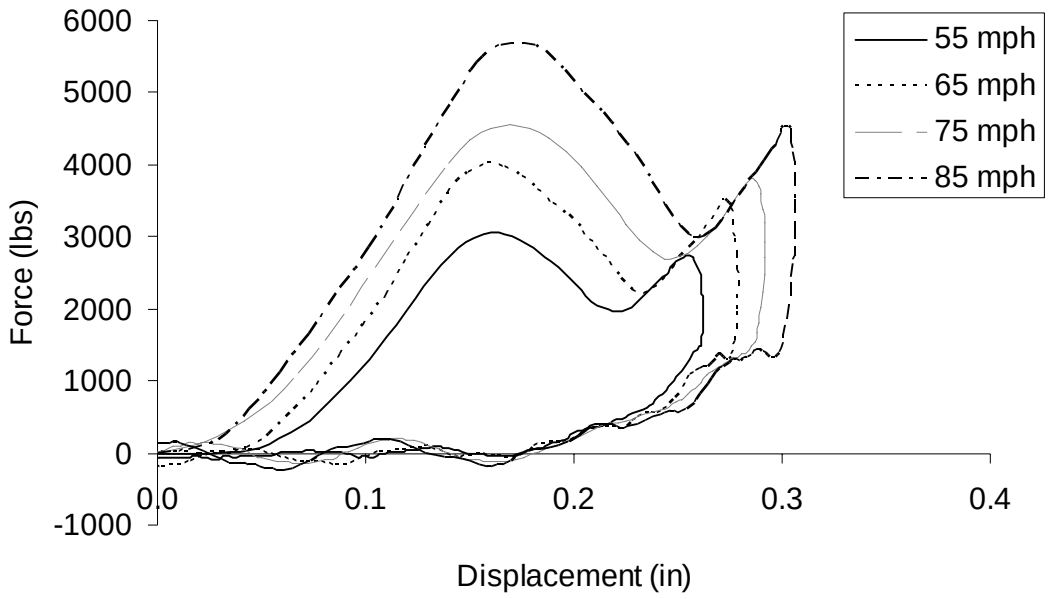


Figure 3.19: Hysteresis plots for pucks impacted at room temperature at various speeds

The force – time profile for each impact speed is shown in Figure 3.20. The contact time was the same for each impact speed (0.84 ms). The pucks showed a double peak at all impact speeds. This reinforces the idea that the double peak is not due to tooling in the test fixture. The second peak appears to be proportional to the first peak for each impact speed. Peak impact force and peak deflection both increased with increasing speed and are shown in Figure 3.21. The displacement and peak force both increased linearly with speed.

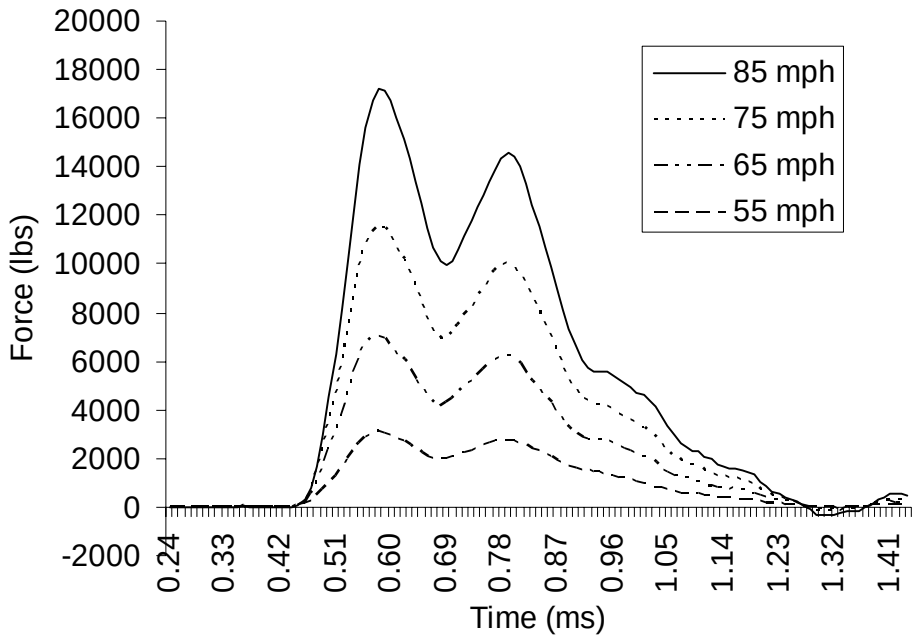


Figure 3.20: Force – time profile for each puck impact speed at room temperature

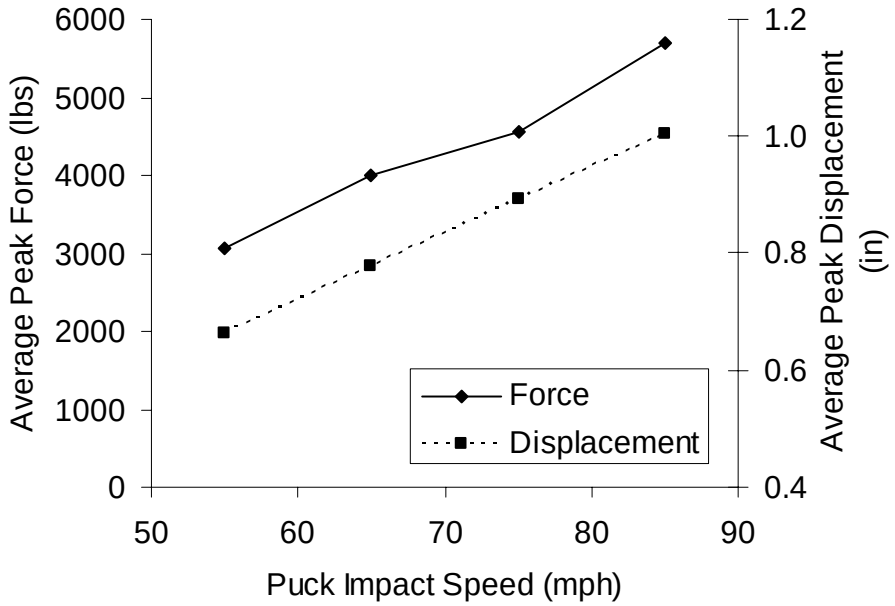


Figure 3.21: Peak force and displacement for pucks impacted at various speeds

## 3.9 Quasi-Static Stiffness

Softballs are characterized according to their quasi-static compression, described by ASTM F1888-04 [3.10]. To determine the difference between quasi-static and dynamic puck stiffness, pucks were compressed quasi-statically using the same procedure at room temperature. Pucks were placed between two flat plates in a load frame (MTS Systems Corporation, Eden Prairie, MN), shown in Figure 3.22. A preload of 4 ± 1 pounds was first applied to the puck. The puck was then compressed 0.25 inches over a 15 second time period and the maximum force was recorded. The puck was then rotated 90° in the radial direction and compressed again. The reported stiffness value for each puck was the average stiffness from the two locations.

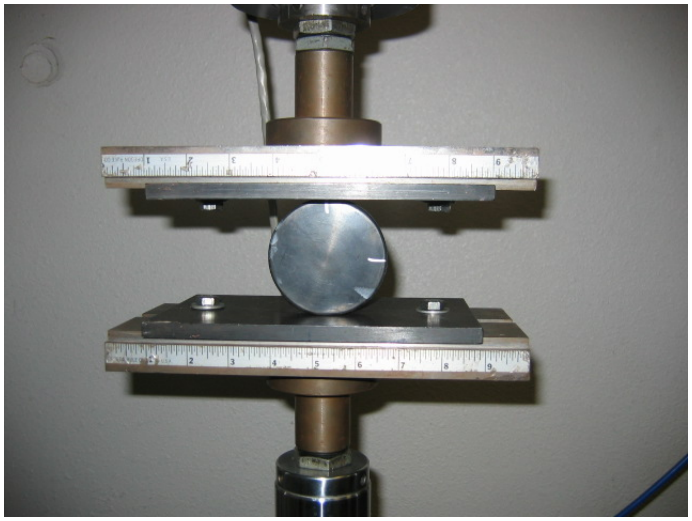

Figure 3.22: Quasi-static puck compression testing setup

Average compression results for the 12 pucks of each brand are presented in Figure 3.23. Dynamic stiffness results at 72° F are also included for comparison. As expected, the dynamic stiffness is much larger than the quasi-static stiffness due to the larger impact force that is exerted by a faster traveling puck in the dynamic case. Brand PA pucks were 49.4% stiffer than PB pucks when stiffness was measured quasi-statically. This is a larger difference than was seen

for dynamic stiffness measured at 55 mph, in which PA pucks were 33.2% stiffer than PB. This indicates that quasi-static testing does not describe the dynamic behavior of hockey pucks.

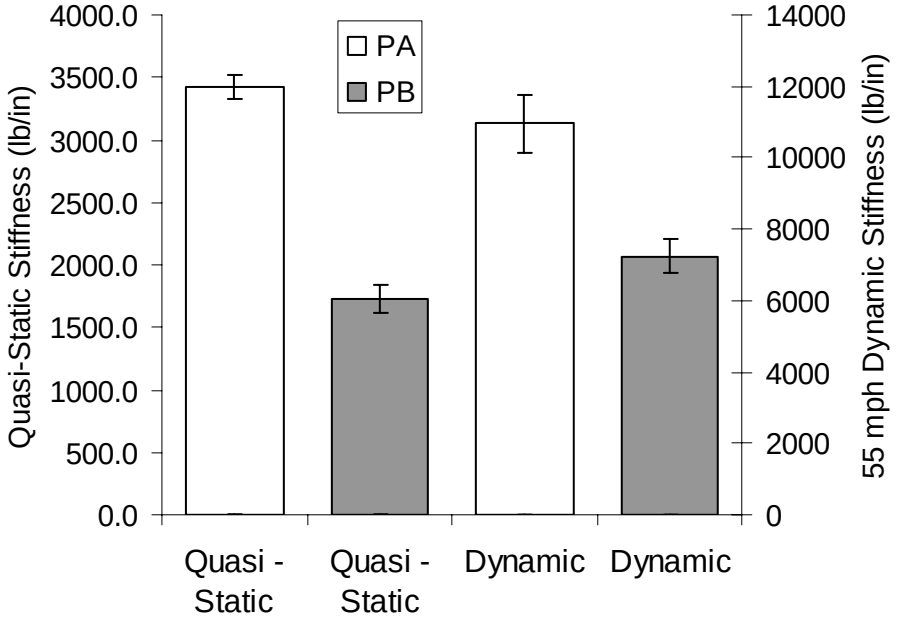


Figure 3.23: Quasi-static and dynamic puck stiffness results at room temperature

### 3.10 Summary

Prior to this study, no data existed in the literature regarding the high speed properties of hockey pucks. A high speed test method was successfully developed and implemented to characterize ice hockey pucks by their impact properties. This chapter details the methods and results of the high speed puck characterization. Pucks were tested with an air cannon at a speed representative of game play and characterized by brand, temperature, and impact speed. The impact properties of dynamic stiffness ($k_d$) and coefficient of restitution (e) were used to describe the behavior of hockey pucks in this study. Notable differences were observed in all three cases.

Results showed that different brands of pucks do have different impact properties. A difference of 10% in coefficient of restitution and 33% in dynamic stiffness were noted between

two brands. In addition, a difference of 20% in the peak force and 9% in peak deflection were found. These results indicate that measurable differences in puck properties do exist from one brand to another which is not apparent by physical examination. It is possible that these differences were noted in the large ranges observed in e for drop tower tests.

Pucks were tested at a variety of temperatures ranging from 25° – 72° F and at 55 ± 1 mph. The behavior of both brands of pucks appeared to be highly dependent on temperature. In general, e increased with increasing temperature (by 32% and 26%) while $k_d$ decreased drastically with increasing temperature (626% and 488%). Freezing pucks before play makes them significantly stiffer and less elastic, therefore improving the controllability for players.

Freezing pucks decreased the contact time of the collision. On average, the contact time at 25° F was half that at 72° F. This is a likely explanation for why the double peak from an elastic wave in the puck was not seen at low temperatures. Examination of the hysteresis curves showed that the puck temperature has a significant effect on the relationship between force and displacement for both the loading and unloading phases. Peak deflection increased nonlinearly with increasing temperature.

Pucks were tested at room temperature (72 ± 2° F) at speeds ranging from 55 – 85 mph in 10 mph intervals. The coefficient of restitution was shown to decrease while the dynamic stiffness increased with increasing impact speed. Significant changes were observed, showing that quasi-static testing (like drop tower tests) may not be indicative of dynamic behavior. The increase in dynamic stiffness with speed indicates that hockey pucks are more nonlinear than softballs. The contact time of the collision was shown to be constant for changing impact speed. The peak displacement and peak force both increased linearly with increasing speed. Quasi-static stiffness testing showed larger differences between the two brands than the stiffness values

that were found at 55 mph. In addition, puck impact properties appear to be more dependent on temperature than speed, as multiple differences were observed in temperature tests that were not seen in speed testing.

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# CHAPTER FOUR

# EXPERIMENTAL STICK RESULTS

## 4.1 Introduction

The ice hockey stick market has been rapidly evolving in recent years. New materials and features have been introduced to hockey sticks, but the effects of these modifications on the performance of the stick are not currently understood. Previous studies regarding the performance and mechanics of ice hockey sticks and shots have mainly concentrated on identifying player movement patterns that are important in developing a good shot, regardless of the stick type being used [4.1, 4.2, 4.3].

The following compares laboratory measures of stick performance. A performance test using a high speed cannon was developed and implemented to test different types of sticks in terms of a modeled slap shot. Modal analysis was used to investigate the vibration characteristics of hockey sticks, with the hope of gaining some insight into player feel. Finally, lab performance data was compared to slap shots seen in the field of play.

Two main stick variables were investigated in this study: stick material and shaft taper. The sticks used in this study were classified by their material with a W# corresponding to a wood stick and a C# corresponding to a composite stick. Furthermore, sticks were classified by the amount of taper in the shaft. Sticks with no taper were classified as straight-shaft composite. Intermediate tapered sticks indicate some tapering in the shaft, and long tapered elite sticks that are among the best sticks on the market, reportedly with the longest shaft tapers of any sticks currently available. Seventeen hockey sticks were compared in this study, including 6 wood and

11 composite sticks. They were a selection from four different hockey stick manufacturers. The sticks are summarized in Table 4.1.

Table 4.1: Summary of sticks tested

| Stick Number | Material | Geometry |
|---|---|---|
| W1 | Wood | Straight shaft |
| W2 | Wood | Straight shaft |
| W3 | Wood | Straight shaft |
| W4 | Wood with composite blade | Straight shaft |
| W5 | Wood | Tapered shaft |
| W6 | Wood with composite blade | Tapered shaft |
| C1 | Composite | Straight shaft |
| C2 | Composite | Straight shaft |
| C3 | Composite | Tapered shaft - low |
| C4 | Composite | Tapered shaft - mid |
| C5 | Composite | Tapered shaft - mid |
| C6 | Composite | Tapered shaft - mid/high |
| C7 | Composite | Tapered shaft - high |
| C8 | Composite | Elite composite |
| C9 | Composite | Elite composite |
| C10 | Composite | Elite composite |
| C11 | Composite | Elite composite |

## 4.2 Modal Analysis

One of the primary player concerns when choosing a hockey stick is the perceived feel of the stick. Players want to be able to feel small impacts between the stick and the puck due to handling so they have some perception of where the puck is in relation to the stick without having to look down. In addition, feel is important in catching a pass with the stick. If a player cannot feel the puck impacting the stick, it is harder to cushion the pass, often sending the puck careening away from the player.

The analysis of feel is complicated by its highly subjective nature [4.15]. It was hoped that modal analysis could be used to characterize stick vibration, offering some insight into the issue of perceived feel of a stick. Mode shapes, modal frequencies, and damping were compared

for different types of sticks. In addition, the location of the lower first bending node provides some insight into the kick point of the stick. A stick with a lower node location will have a lower kick point, providing a qualitative basis for comparison of the kick point location between different sticks.

### 4.2.1 Mode Shapes and Natural Frequencies

A one-dimensional modal analysis was used for this study. A roving impact hammer (PCB Piezotronics, Model # 350B23, Depew, NY) test was done on each stick. A 0.5 g uniaxial accelerometer (PCB Piezotronics, Model # 352C22, Depew, NY) was fixed in the middle of the top of the shaft and the shaft was impacted at one inch intervals along the length. The modal test setup is shown in Figure 4.1 below. The stick was tested for bending modes in the plane of the stick that exhibits the most bending when the shaft is loaded. Modal data was averaged from four impacts at each location.

Figure 4.1: Modal test setup

Free-free boundary conditions were implemented in this test by supporting the shaft by elastic bands and the blade by a small piece of light foam. The free-free boundary conditions used assume that the hands are not important to the behavior of a stick. This is true for light handling down the ice, when player feel is most important.

For a single degree of freedom (SDOF) system, the characteristic equation of motion is:

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{4.1}$$

where m is the mass, c is the damping coefficient, k is the spring stiffness, and f(t) is the sum of the applied forces as a function of time. The displacement of the system is x and each dot represents a time derivative of displacement. Vibration consists of a combination of inertial, damping, spring, and excitation forces.

The Frequency Response Function (FRF) describes the input-output relationship between two points on a structure as a function of frequency. It is defined as [4.13]:

$$FRF = \frac{FFT(Output)}{FFT(Input)} \tag{4.2}$$

where the FFT is the Fast Fourier Transform of the input or output function. The FRF is a complex function with both real and imaginary parts that relates the output per unit of input for each frequency. Peaks occur in the FRF at resonant or natural frequencies where the time response was observed to have maximum response corresponding to the rate of oscillation of the input excitation [4.13]. The amplitude of the peak describes the mode shape, while the steepness of the peak in the time domain describes the damping of that mode.

The method of quadrature picking was used to analyze modal data and extract mode shapes and natural frequencies. This method involves plotting the imaginary part of the FRF as a function of position along the length of the stick. The resulting three-dimensional plot is referred to as a waterfall plot. A sample waterfall plot is shown in Chapter 2, Figure 2.6 and another is shown in Figure 4.2. The waterfall plot shows the various mode shapes along the length of the stick.

In this study, a one-dimensional bending analysis was performed on each hockey stick. In reality, the vibration of a hockey stick is more complex. The blade acted as a point mass that oscillated in torsion around the shaft, causing more complex vibration patterns. Torsional modes

appeared as mirrors of bending modes at frequencies higher than the second bending mode natural frequency. Figure 4.2 shows the first torsional mode appearing as a mirror of the third bending mode for a composite hockey stick.

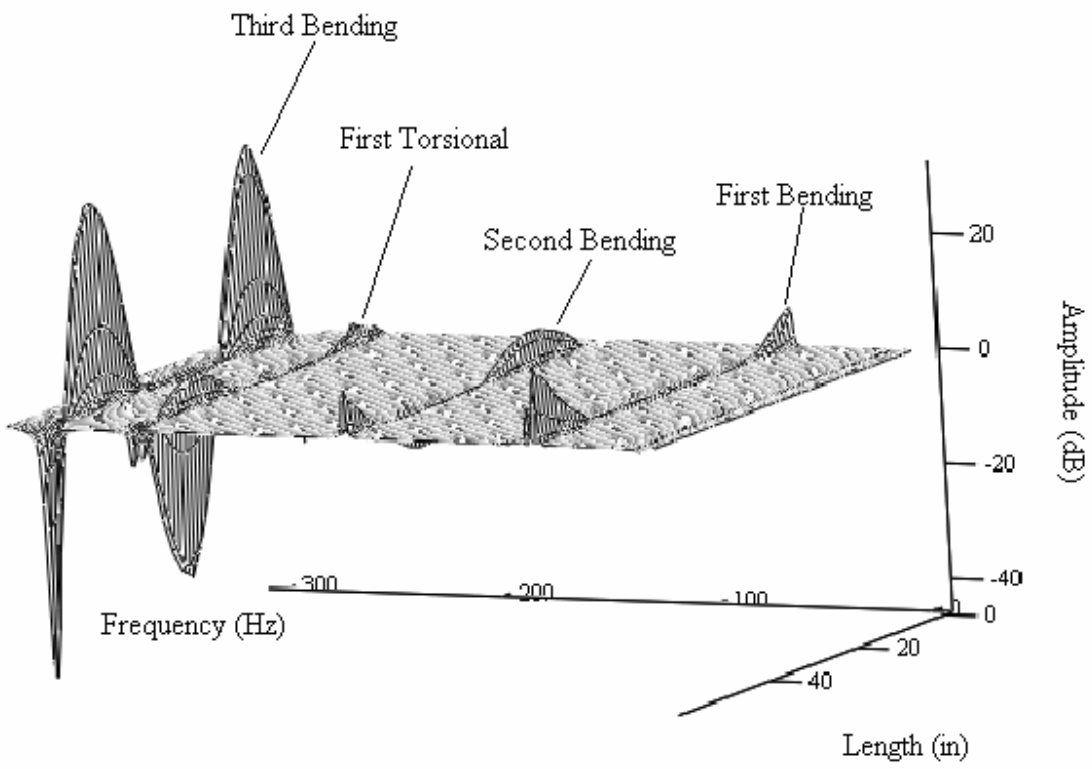



Figure 4.2: Waterfall plot with torsional mode appearing as mirror of bending mode

More robust analysis techniques were needed to distinguish between torsional and bending modes, which were outside the scope of this study. One mid-tapered composite hockey stick was sent to Dr. Dan Russell at Kettering University for further analysis. By conducting a two-dimensional modal analysis and using more complex analysis software, he was able to determine which modes were bending and torsional modes. The accelerometer was fixed to the blade of the stick a total of 213 impact locations on the handle and blade were used. Dr. Russell's modal setup is shown in Figure 4.3 and the resulting modal frequencies up through 1000 Hz in Table 4.2.

Figure 4.3: Dr. Dan Russell's 2-dimensional modal analysis setup

Table 4.2: Summary of modal frequencies found by Dr. Dan Russell

| Frequency (Hz) | Mode Shape |
|---|---|
| 56 | First Bending |
| 144 | Second Bending |
| 243 | First Torsional |
| 284 | Third Bending |
| 424 | Fourth Bending |
| 478 | Second Torsional |
| 575 | Fifth Bending |
| 749 | Sixth Bending |
| 966 | Seventh Bending |

The remainder of the modal comparison focused on the first two bending modes for each of the 16 sticks tested. Also of interest was the location of the first bending node. The location provided some insight into the location of the kick point of the stick. Knowing the location of the lower first bending node allows for qualitative comparison of the kick point location between different hockey sticks.

Results showed that natural frequencies for modes 1 and 2 were fairly similar for all sticks. Mode 1 ranged from 46 – 60 Hz, while mode 2 showed a wider range, from 111 – 161 Hz. All of the wood sticks had lower natural frequencies than the composite sticks. Wood sticks are reported to offer a better feel for the puck than composite, but no drastic difference between the first two bending modes for wood and composite sticks was seen. Mode 1 and 2 frequencies are shown in Figure 4.4 for all sticks.

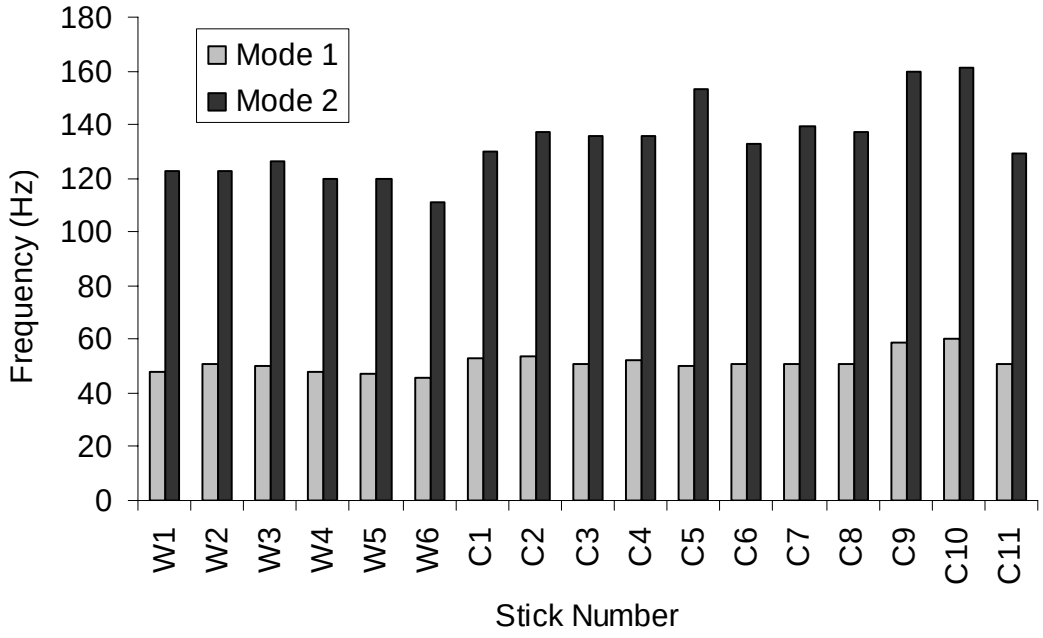



Figure 4.4: Mode 1 and 2 natural frequencies for all sticks

It is postulated here that higher frequencies may contribute more to the player's perception of feel. Examining higher modal frequencies was determined to be outside the scope of this study. Hocknell, et al. [4.15] compared the subjective feel and vibration characteristics for wood and aluminum golf club shafts. They found that vibration in the range of 0 – 2.5 kHz was believed to be of greater importance to the sensation in the golfer's hands. They concluded that desirable sensation in the hands can be achieved by exciting modes of vibration in the

frequency range of 500 Hz – 2.5 kHz more strongly than modes in the region of 100 Hz. It is unclear at this time how a hockey player's thick leather gloves would affect perception of vibrations in these ranges.

Each bending mode, m has m + 1 nodes, meaning that mode 1 has two nodes. The nodes are shown below in Figure 4.5, which contains a plot of the first two bending modes for stick C5. The 0 inch location corresponds to the bottom of the hosel, where the blade ends and the shaft begins. An array was created with the imaginary part of the FRF for each impact location along the length of the stick. The natural frequencies were then identified and the FRF value (amplitude) for each impact location at that frequency was plotted. The natural frequencies are identified in the legend of the plot. The nodes are the location in which the mode shape has zero amplitude. Locations on the stick were measured up from the bottom of the shaft, where the heel of the blade begins to curve. Of particular interest in this study was the location of the lower first bending node. The The lower node location is shown in Figure 4.6 for each stick.

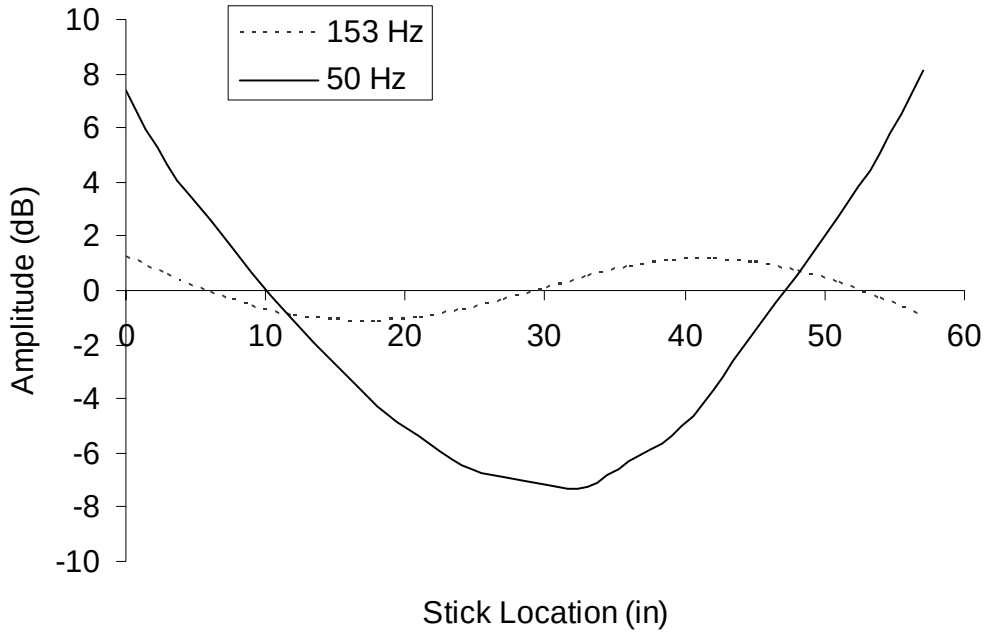



Figure 4.5: Bending modes 1 and 2 for stick C5

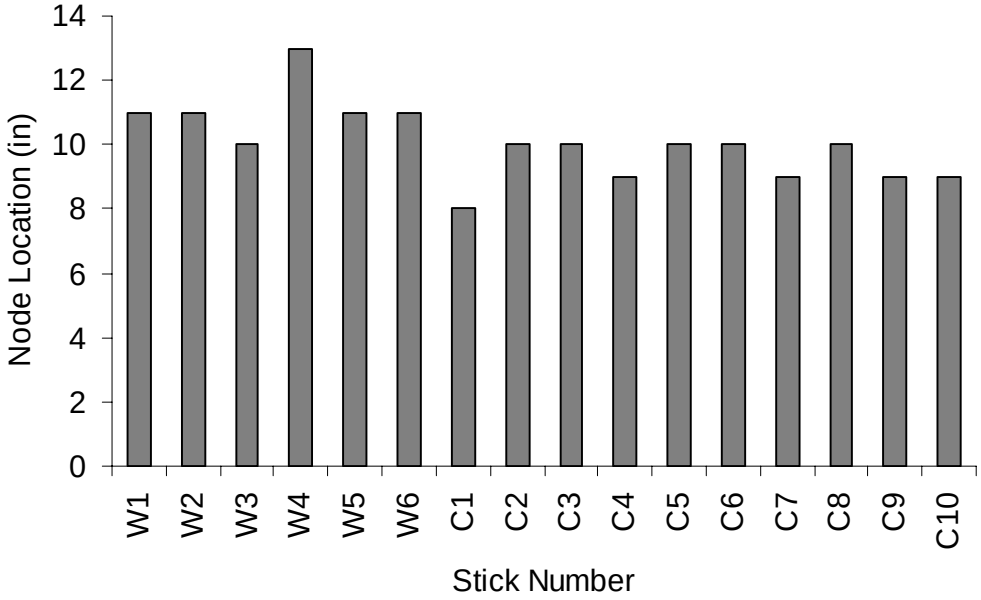



Figure 4.6: Lower mode 1 node location for each stick

In general, the wood sticks had higher node locations than the composite, with the exception of stick W3. The stick with the lowest node location was stick C1, a low end straight shaft composite stick. The next lowest node locations were found in two of the elite composite sticks (C9 and C10), the high end tapered composite (C7) and a mid tapered composite (C4). Lower node locations were expected in the higher end composite sticks because they have longer tapers in the shaft.

It was surprising, however, to find the lowest node location in the low end straight shaft composite stick. In this case, stick C1 was performance and field tested before modal analysis. A noticeable rattle existed in the lower portion of the stick during modal analysis, which could indicate material damage in the hosel area. Such damage could cause a decrease in the stiffness of the portion of the stick with damage, which would in turn lower the node location. Further examination of the stick would be required to confirm the existence of internal damage.

### 4.2.2 Damping

Damping refers to how quickly vibration decays in a structure, or the amount of energy dissipation occurring in vibration. This concept is illustrated in Figure 4.7, which shows the damped response of a vibrating system. It was hoped in this study that damping could provide some insight into the issue of player feel. The critical damping factor was discussed previously in Section 2.4, Equation (2.25).

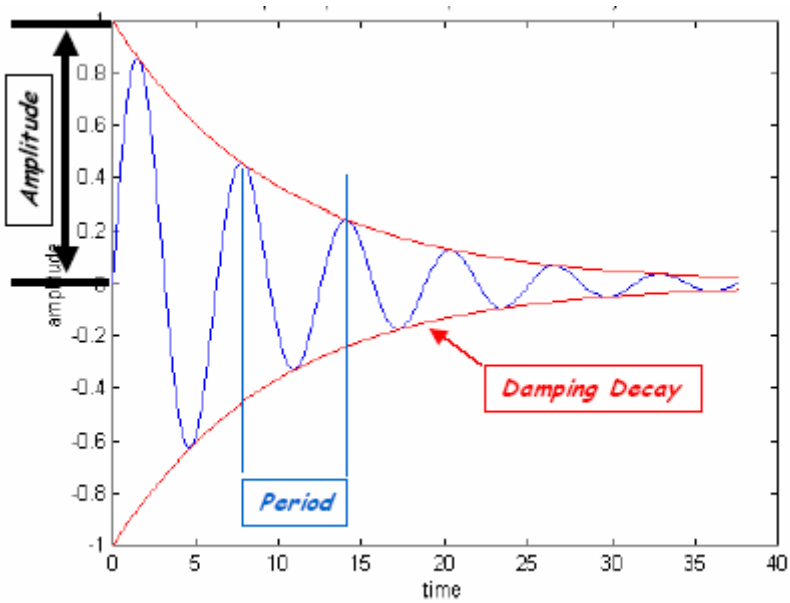



Figure 4.7: Damped response of a vibrating system - copied from [4.14]

Damping is dependent on both the stiffness of the structure and the amplitude of vibration input. For this reason, a method of consistently applying an input impulse with the same amplitude was needed to test for damping. The impact hammer was set up as a pendulum in front of the hockey stick. The hammer was pulled back to the same distance for each impact, and released to impact the stick. LabView 7.1 was used to analyze the input and response and fit a curve to the response, similar to the damping decay depicted in Figure 4.7. The accelerometer

was placed at a distance of 28.5 inches from the heel of the blade and the stick was impacted at a distance of 24 inches from the heel of the blade. The test setup is shown in Figure 4.8.

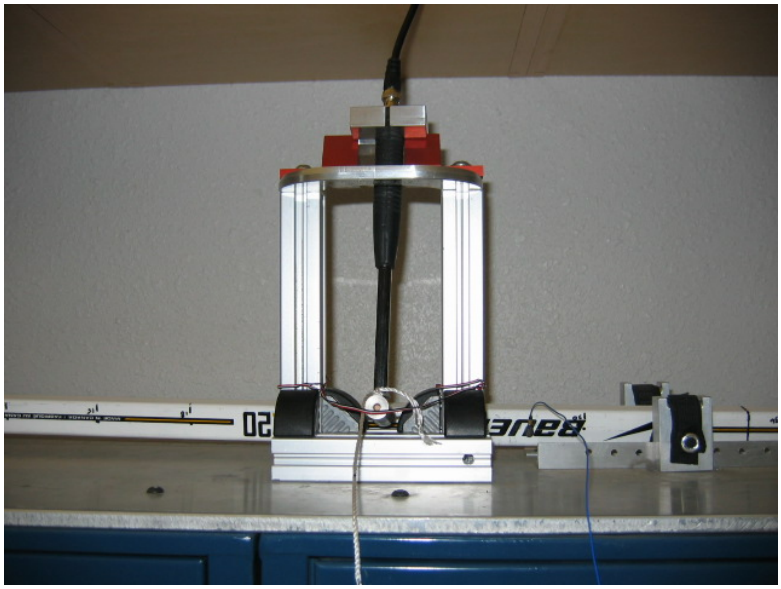


Figure 4.8: Modal damping test apparatus

Modal damping results were largely inconclusive. Critical damping factors were found for 5 wood sticks and 3 composite sticks. Results are shown in Figure 4.9. For the other sticks, an error likely occurred in the curve fitting process, resulting in erroneous results. Damping depends on the stiffness of the stick, which was different for each stick used in this study. To successfully determine the damping of each stick, the bending stiffness needed to be measured and inputted into the LabView program. For the few sticks that were tested with a fixed bending stiffness value, all of the wood sticks had higher damping factors than the composite. This study was restricted to a small sample size, however, and little insight has been shown into the affect of damping on perceived player feel of a hockey stick.

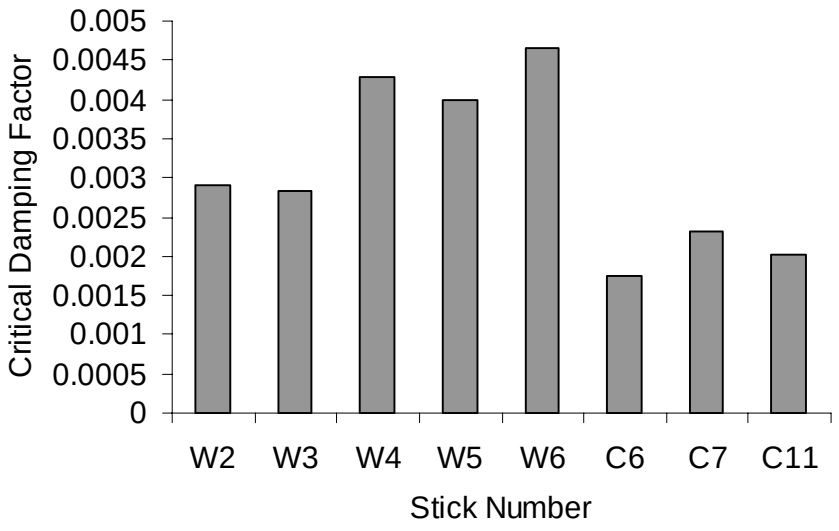



Figure 4.9: Critical damping factor for 8 sticks

## 4.3 Field Study

A field study was conducted that utilized three players shooting pucks with various sticks on ice. The primary goals of this field study were to determine stick swing speeds seen in actual play and to correlate lab performance to data taken from actual play. Correlation between lab performance data and field study is presented in section 4.4.8 below, while speed findings are presented here. Subjects used six different hockey sticks to perform stationary slap shots on the ice toward a net. A high speed video camera (Photron FASTCAM 1024 PCI) was used to capture the stationary slap shots at 1000 frames/second while a radar gun (JUGS INC.) was used to measure resulting puck speeds.

The field study was conducted at the Rotary Veteran's Memorial Pavilion Ice Rink in Moscow, Idaho. Subjects utilized in this study were three local recreational hockey players. Six sticks were selected for subjects to shoot with: a wood stick, a straight shaft composite stick, a low-end tapered shaft composite stick, an intermediate tapered stick, and two long tapered elite

composite sticks. Subjects completed a minimum number of ten stationary slap shots with each stick. Sticks were selected in random order for each subject. Acceptable slap shots were those in which the subject indicated maximum effort, expressed satisfaction with the quality of the shot, and in which the puck hit the net.

Six sticks from the overall study were selected for use in the field study and are summarized below in Table 4.3. The selection of sticks represented a good survey of stick types currently available on the market. Five of the sticks were composite, ranging from low end straight-shaft to elite tapered, and the sixth stick was wood. The wood stick broke early in the study before any usable high speed video was collected. The "sister" designation after sticks C11 and W1 indicates that these sticks broke before a full performance scan was completed, and another stick of the same model was used in the performance testing and modal analysis.

Table 4.3: Summary of sticks used in the field study

| Stick Number | Stick Type |
|---|---|
| C11 sister | Elite tapered composite |
| C8 | Elite tapered composite |
| C2 | Low end tapered composite |
| C1 | Low end straight-shaft composite |
| W1 sister | Wood |
| C4 | Mid-level tapered composite |

Results showed distinct differences in shot speeds for each player and for each stick, as shown in the radar data in Figure 4.10. One subject produced high recreational level speeds (up to 70 mph), while the other two subjects were in the lower recreational end (45-55 mph). Because the puck speeds for subjects 2 and 3 were relatively slow, data from these subjects was used only in comparison with tracking data. This helped verify the accuracy of the tracking software to gain confidence in the speeds obtained from subject 1.

The stick that produced the highest shot speed was different for each subject. The average shot speeds for each subject and stick are shown below in Figure 4.10 with error bars representing one standard deviation of the sample data. Reported averages and standard deviations are the result of a minimum of ten trials for all sticks except for stick W1 Sister, which had a minimum of five trials before breaking. For subjects 1 and 3, stick W1 Sister produced the fastest average shot speed, while stick C11 Sister produced the fastest average shot speed for subject 2.

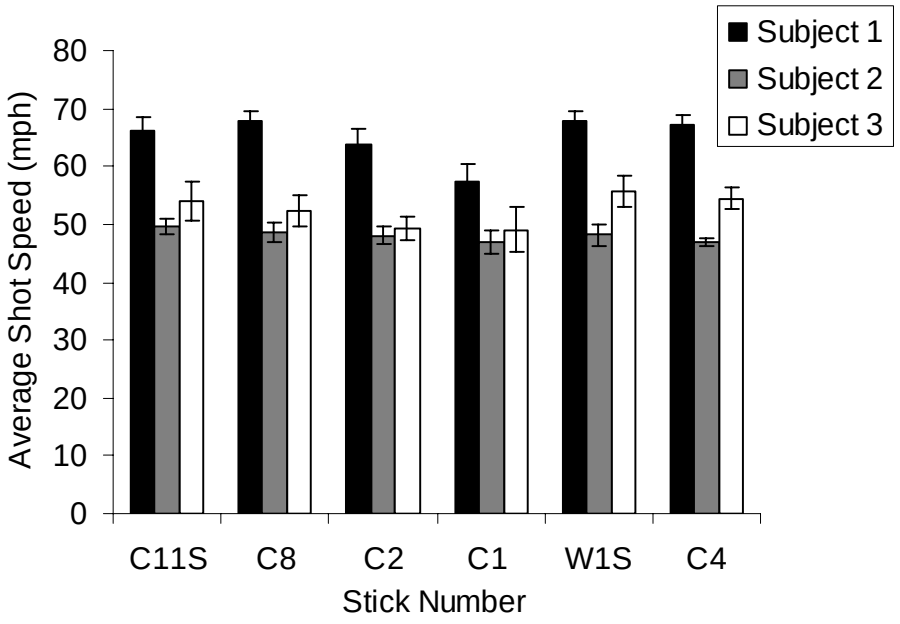



Figure 4.10: Field study radar data for each subject and stick

After conducting the field study, high speed video of subjects taking slap shots was analyzed using two-dimensional tracking software (Photron Motion Tools v1.06). Tracking points included the puck and several locations along the length of the stick. Figure 4.11 shows the tracking points along the length of the stick. The tracking software was calibrated using a tape in the image that was marked in 12 inch intervals, seen at the bottom of the image in Figure 4.11. The puck was placed 24 inches behind the tape for each shot, and the visual distortion

caused by projecting the camera at an angle was corrected for in the tracking software. Tracking software provided the coordinates of each tracking point for each frame of video. The instantaneous linear velocity of each point was then calculated frame-by-frame over the shot duration.

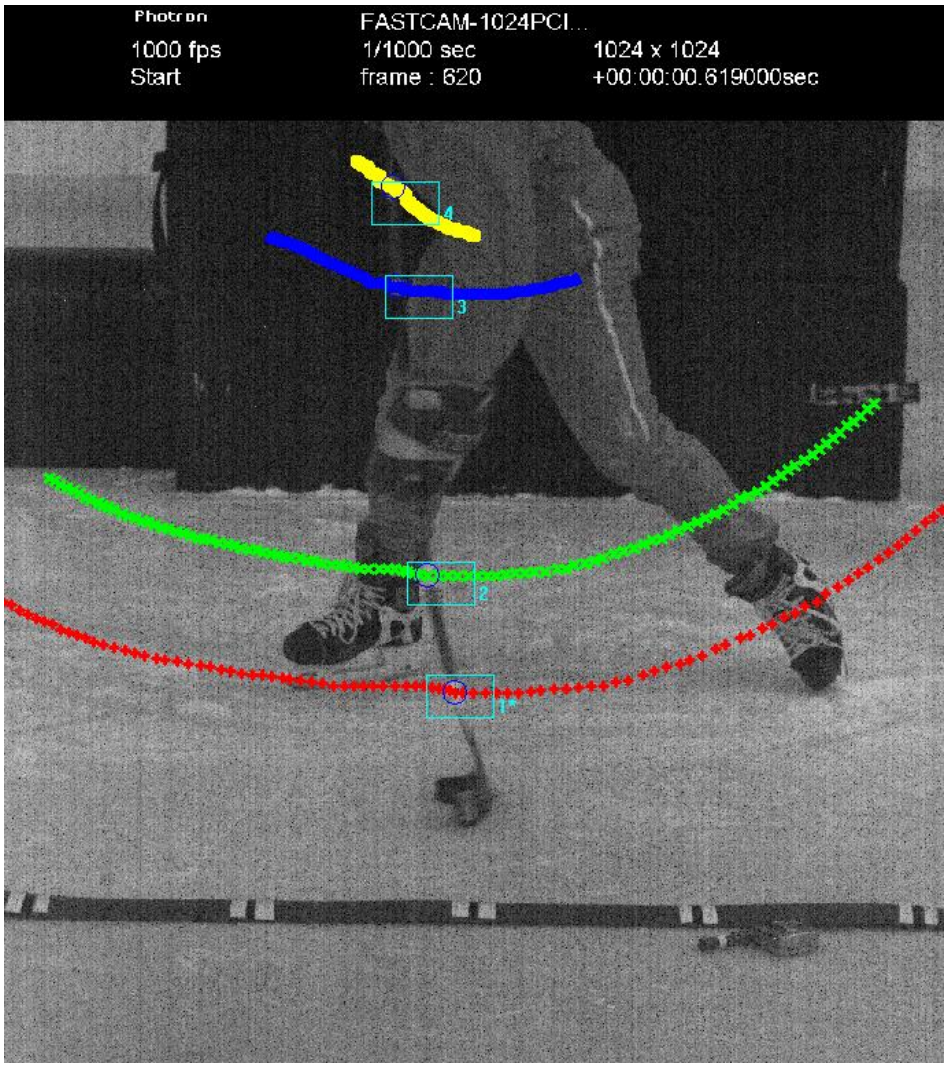



Figure 4.11: Tracked high speed video image of a stationary slap shot

The high speed video camera utilized in this study was not well suited for the cold temperatures of an ice rink. The resulting video was grainy and provided poor contrast. This caused the tracking software to not recognize the tracking points for auto-tracking, so all tracking had to be completed manually frame by frame. Speed data showed significant noise due to the

lack of contrast in the tracking image and resulting manual tracking. Figure 4.12 shows the velocity of the hosel of stick C1 throughout the duration of a slap shot obtained from manual tracking data. The velocity was calculated frame–by–frame from the coordinates that were found from manual tracking. Similar noise was found in calculated puck speeds over the shot time.

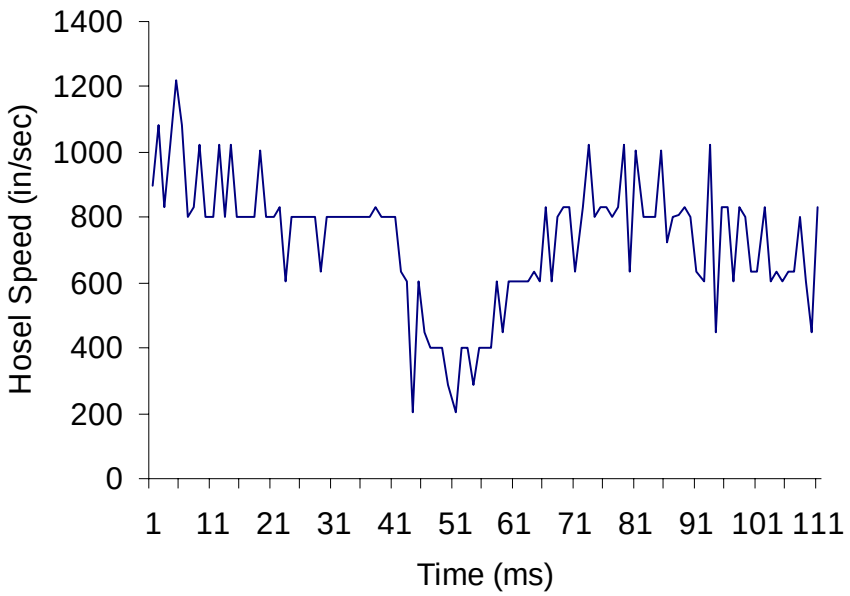



Figure 4.12: Hosel velocity during a slap shot obtained from manual tracking data

Puck speeds found from radar data were compared to those found from the tracking software. Average puck speeds found from radar and tracking data for all three subjects are presented in Figure 4.13. Puck speed values found from tracking were averaged over the last 20 frames before the puck was released from the stick. In most cases, there was good agreement between the radar and tracking puck speeds. The exception occurred with stick C4, in which a difference of 6 mph, or approximately 10% was observed. For the first 4 sticks, the differences between the average radar and tracking speeds ranged from 0.32% – 4%. These results provided fair confidence in the accuracy of the tracking software.

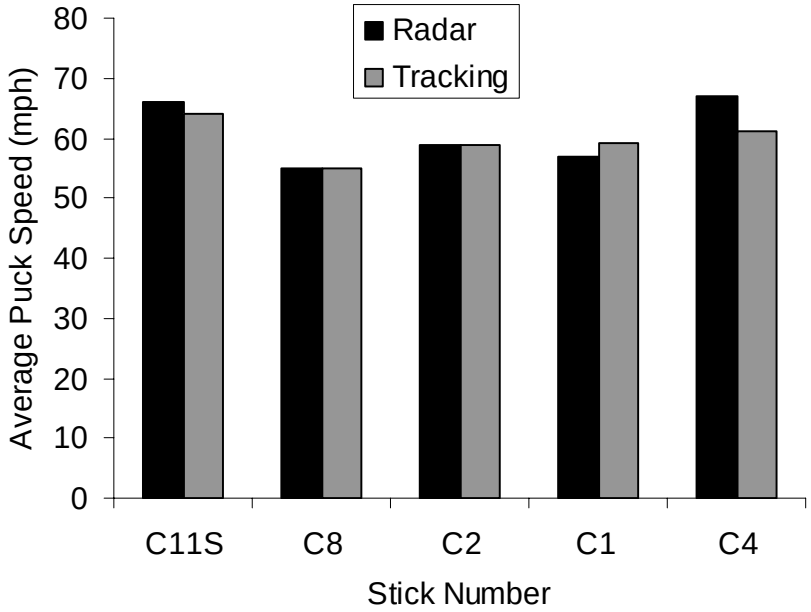



Figure 4.13: Comparison of average puck speeds found from radar data and tracking software

The first primary goal of the field study was to measure the swing speed of the stick during a slap shot at the time of stick-puck contact. This measurement was important in confirming the value chosen for the nominal stick swing speed, $v_n$ used to calculate the swing speed in performance testing, $v_s$ discussed previously in Section 2.3, Equation (2.21).

In Figure 4.12 above, three distinct phases of the shot can be observed in the velocity data. The first 40 ms show the downswing phase of the slap shot, in which the stick is approaching the ice and the puck. At approximately 40 ms, the shaft and puck loading began, showing a drastic decrease in the linear velocity of the stick as the shaft was deflected and the puck began to move. The final phase was the release phase, from roughly 75 ms onward, in which the stick and puck travel together until the puck was released from the blade toward the target. These characteristic phases were observed in all of the slap shots with all sticks.

In most shots, stick speed during the release phase was slower than the speed during the downswing phase. This is consistent with the findings of Lomond, et al. [4.2] for recreational players, in which the downswing phase had an average stick linear velocity of $v_s$ = 45.7 mph and the release phase had an average $v_s$ = 33.9 mph. The opposite was found to be true for elite players, in which the downswing phase had an average $v_s$ = 46.4 mph and the release phase had an average $v_s$ = 61.5 mph [4.2].

Tracking data from subject 1 was used to determine the average linear velocity of the hosel end of the stick during the slap shot in this study. Tracking data from the hosel was averaged over the time interval after the stick had contacted the puck and before it was released from the blade. Eight shots from subject 1 were analyzed and an average hosel velocity of 41.0 $\pm$ 1.8 mph was found.

It was also desirable to compare results to the 20 – 30 rad/sec reported by Sim and Chao [4.9]. The center of rotation was not reported in their results. In order to directly compare the results from the current study to those of Sim and Chao, the instantaneous center of rotation of the stick at the time of puck release needed to be determined. This could be done by determining the velocity vectors of two points along the length of the stick and determining their point of intersection. It was determined in this study, however, that too much variation existed in the velocity vectors to accurately determine a center of rotation. During the short stick – puck contact time, the shaft deflects and then recoils, causing the lower portion of the stick to rotate about the kick point, further complicating the determination of the instantaneous center of rotation.

Overall, stick linear velocity values for subject 1 were found to lie within the range previously reported in the literature [4.2]. Data obtained from this field study was consistent

with recreational player abilities. The performance study considered in the next section was concerned with high recreational to elite player abilities. Field study findings for the linear velocity of the stick for a recreational player were in the range of 41 mph. Findings in the literature report up to 61.5 mph for elite players and 41 mph for recreational players [4.2]. It can therefore be assumed that an estimate of 60 mph swing speed for an elite player is a reasonable estimate for this study.

## 4.4 Performance Testing

There were two main goals in the development of the hockey stick performance test. The first was to determine if a hockey stick could be tested in a lab setting in a manner similar to those currently used for testing softball, baseball, and cricket bats [4.4]. The second was to determine if noticeable differences exist in the lab performance of different types of hockey sticks.

In this study, a method of testing performance of ice hockey sticks was developed and implemented to test 17 different sticks. A test fixture was designed to grip the hockey stick and pivot it freely at the end of a high speed air cannon. The performance test was developed as a model for a slap shot. A derived value for puck speed after it was shot from the stick was developed and applied to test data for each hockey stick.

### 4.4.1 Moment of Inertia

The method for determining the moment of inertia, or I, of hockey sticks was analogous to the method described in ASTM F2398-04 for softball and baseball bats [4.5]. The stick was considered as a physical pendulum with the blade acting as a point mass on the end. The stick was pivoted at a distance of 35 inches from the heel of the blade. The moment of inertia was

taken about an axis perpendicular to the length of the stick and in the same plane as the blade. The period was found by timing the stick as it was swung freely through 15 oscillations and finding the average period. The moment of inertia was then calculated using Equation (2.14).

The setup for measuring the period is shown in Figure 4.14. Hockey sticks in this study that were pivoted about a 35 inch pivot point varied in moment of inertia from approximately 8700-12,000 oz-in$^2$. Mass properties of each stick were tabulated and are presented in Appendix A.


Figure 4.14: Setup for determining the period of oscillation of a hockey stick

### 4.4.2 Test Setup

An air cannon similar to the one used for high speed puck testing was used in this study for testing hockey sticks. The rigid array of load cells was replaced with a pivoted stick. Figure 4.15 shows the test setup. A fixture was designed and fabricated to hold the stick with the blade positioned at the end of the barrel, shown in Figure 4.16. This fixture allowed the stick to pivot freely about an axis that was 35 inches from the bottom of the shaft. It was fabricated out of steel and bolted directly onto the mill base at the end of the cannon.


Figure 4.15: Performance test setup

Figure 4.16: Hockey stick performance test fixture

For an actual slap shot in the field of play, the instantaneous center of rotation of the stick is toward the handle end of the shaft. For this study, the stick was pivoted at a distance of 35 inches from the bottom of the shaft, or near where the lower hand would be placed. It is assumed here that the stick is relatively flexible, and therefore the exact center of rotation or pivot distance was not important in determining the dynamic behavior of the stick. This also assumes that the player hands are not important in the resulting shot, which is not the case in play. It is unclear what role the hands do play in shot performance, so it was assumed that for comparing one stick to another the player hands were not important.

Pucks were fired at the stick at 50 ± 1 mph and impacted on the lower inch of the blade of the stick. The speed of 50 mph was chosen in an attempt to ensure that the stick would withstand an entire scan without breaking. In preliminary testing, sticks that were impacted at higher speeds showed significant damage before a scan was completed.

Three pairs of ADC iBeam light gates (Automated Design) were used to measure the inbound speed of the puck. A digital encoder (CUI Inc., Tulatin, OR) attached to the bottom of the pivot shaft was used to measure the swing speed of the stick after impact. The first 15 degrees of travel of the stick were neglected, and the encoder measured the speed of the stick over the next 45 degrees. LabView fit a line to the encoder data, reporting the slope as the average swing speed of the stick.

High speed video and powder spray on the blade of the stick were both used to visually ensure a good impact. A good impact was defined as one in which the puck passed through all three light gates within the acceptable speed range, did not wobble or spin, impacted the stick with the flat edge of the puck, and impacted the stick only once. During the test, stick and puck velocity were considered positive in the direction that the puck was fired, or to the right in Figure 4.15.

For each stick, a scan across the blade to find the location of maximum performance was completed. Impact locations started at the heel of the blade and moved horizontally across the blade in one inch intervals. Eight impact locations were utilized for each stick, covering the area of the blade where the puck would typically impact in a slap shot during play. Each location was impacted with six different pucks, and reported performance values were the averages from all six pucks. Pucks were rotated and impacted every sixth fire in an attempt to regulate puck

warming. An infrared temperature sensor was used to ensure that puck temperatures stayed within 72 ± 2° F during testing.

It is important to discuss the concept of shaft loading when considering this test method. In an actual slap shot, the resulting puck speed is due to a combination of shaft loading and swing speed of the stick. The small effect of stick stiffness on puck velocity [4.1] suggests that stick speed and deflection increase with player ability [4.3]. Thus, the amount of energy available for a player to swing and deflect the shaft can be viewed as constant for a given player, regardless of the stiffness of the stick.

Energy stored by deflecting the stick to a distance of x can be viewed in terms of a linear spring, or

$$U = \frac{1}{2}kx^2 \tag{4.3}$$

for a shaft of stiffness k. For the energy to be constant, a stick with a lower stiffness value must deflect more than a stick with a higher stiffness value. Stiffer sticks have been found to deflect less during shots than more compliant sticks in previous research [4.1]. Shaft loading may be viewed as a constant increase in stick speed (independent of shaft stiffness) that may be accounted for in the stick swing speed.

### 4.4.3 Data Reduction

The performance measure for puck speed ($v_p$) is a derived value to quantify ideal stick performance. First, a slap shot with an initially stationary puck and a moving stick was considered. The frame of reference for the test was then changed to a stationary stick and a moving puck. Changing the frame of reference aligns the data reduction process with the test method of firing a puck at a stationary pivoted stick.

The puck was fired through three sets of light gates that measured the inbound speed, $V_1$. The swing speed of the stick, $\omega_2$ (rad/s) after impact was then measured using a digital encoder. After impacting the blade, the puck continued moving in the positive direction, making its velocity after impact difficult to measure. The derivation of performance calculations is detailed in Section 2.3, Equations (2.13) – (2.21).

The impact location, Q was the perpendicular distance from the pivot axis to the location on the blade where the puck impacted, shown in Figure 4.17. Knowing both the horizontal and vertical distances from the pivot axis to the impact location, the value Q was calculated using the Pythagorean Theorem.

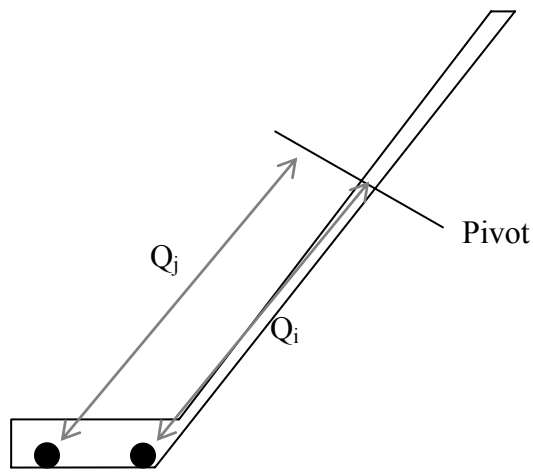



Figure 4.17: Calculation of impact location

The calculated swing speed is the result of a nominal swing speed, $v_n$, that is scaled for stick inertia, I and impact location, shown previously in Equation (2.21). Nominal swing speed, inertia, and impact location values were needed. The nominal impact location chosen was 35 inches, which corresponds to an impact at the heel of the blade. This was selected in order to keep performance calculations consistent with the measured and observed swing speeds used to

determine the nominal stick swing speed, $v_n$. Stick swing speeds calculated in the field study and from angular velocities found in the literature were calculated at the heel of the blade. The linear velocity of the stick at the time of impact then increased as the impact location moves across the blade toward the toe. The inertia of the sticks tested in this study ranged from roughly 8000 – 12,000 oz-in$^2$. A median value of 10,000 oz-in$^2$ was chosen for this study.

Finally, a nominal stick swing speed of $v_n$ = 60 mph (1056 in/sec) was chosen, making Equation (2.21):

$$v_s = 60\left(\frac{Q}{35}\right)\left(\frac{10{,}000}{I}\right)^{1/4} \tag{4.4}$$

This value was selected based on field study calculations, and previous research that measured the angular velocity of hockey sticks during slap shots.

The swing speed was used to calculate a puck speed based on a moving stick and a stationary puck. For this case, Equation (2.20) reduces to

$$v_p = v_s\left(1 + e_a\right) \tag{4.5}$$

Once the swing speed was calculated for each impact, $v_s$ was substituted into Equation (4.5), yielding a puck velocity that represents a modeled speed of a puck coming off the stick in a slap shot. The values for puck speed from six pucks were averaged at each impact location and plotted along the length of the blade. Impact locations were numbered 1 – 8, with 1 corresponding to an impact at the heel of the blade and each successive number representing a one inch horizontal interval toward the toe of the blade.

### 4.4.4 Stick Comparison

Seventeen sticks were performance tested in this study, summarized in Table 4.1. Two stick variables were investigated for performance: stick material and shaft taper. Wood and

composite sticks were compared, and for both material groups different tapers were compared. Peak $v_p$ values are shown for each stick below in Figure 4.18 with error bars representing one standard deviation of the average for the six pucks impacted at that location. Complete stick test results are also detailed in the table in Appendix B.

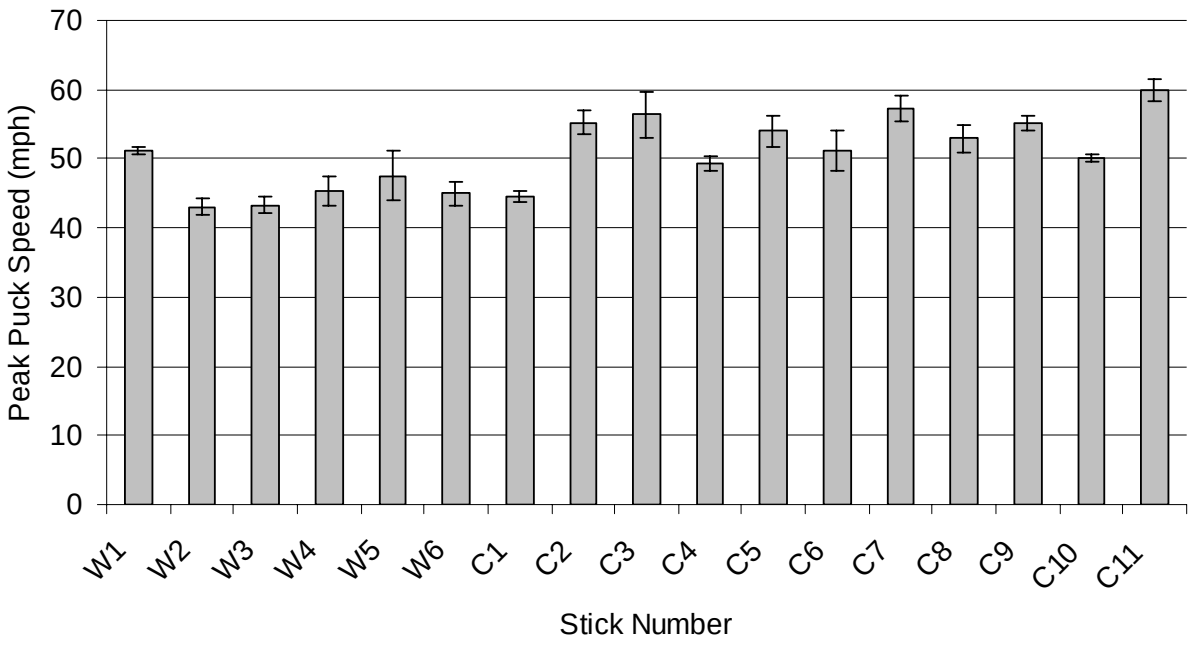



Figure 4.18: Peak $v_p$ results for 17 sticks tested

Noticeable differences do exist between the performance values found for the different sticks. The highest performing stick was C11, with $v_p$ = 59.9 ± 1.6 mph. The lowest performing stick was W2 with $v_p$ = 43 ± 1.2 mph. Overall, the 6 wood sticks had an average performance of $v_p$ = 45.9 ± 3.1 mph and the 11 composite sticks had an average performance of $v_p$ = 54.1 ± 4.3 mph. Significant differences do exist between the performance of wood and composite sticks in this test, with composite sticks having higher peak performance values.

It was also interesting to compare the shape of the performance curves for wood and composite sticks. Some sticks showed a smooth performance curve with a distinct "sweet spot"

along the blade. Others exhibited more erratic performance curves, sometimes increasing in performance until the very end of the blade, in which case a definite peak was not shown. Performance curves for three sticks are shown below in Figure 4.19 to illustrate this point. Stick C5 showed a definite sweet spot and performance peak. The other two sticks had erratically changing performance across the blade and peak performance values occurred either at the heel or toe end of the blade. Of the wood sticks tested, all except stick W6 exhibited erratic performance curves with no definite sweet spot. Of the composite sticks tested, all except sticks C1 and C3 showed smooth performance curves with definite sweet spots.

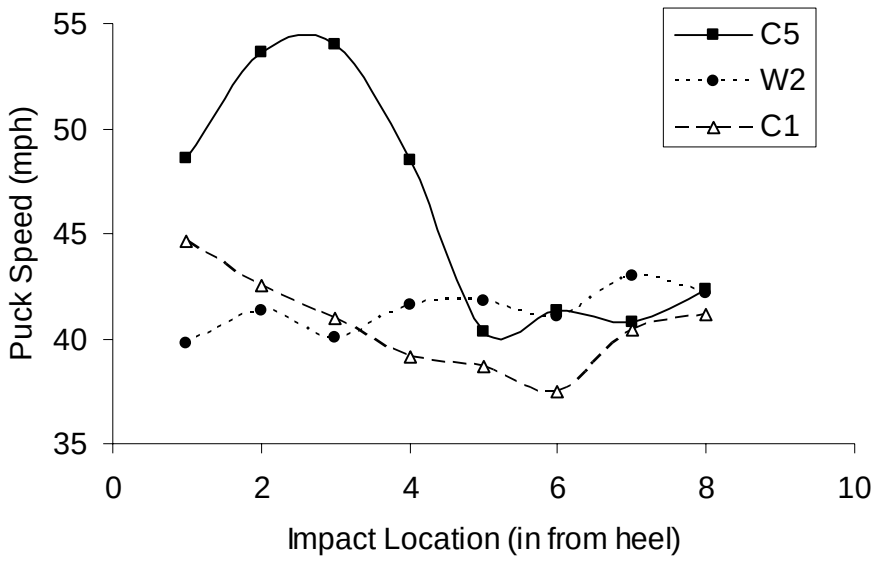



Figure 4.19: Sample performance curves for three sticks

Shaft taper appeared to have a less definite effect on peak performance results. For the wood sticks, W5 and W6 have tapered shafts, while the rest have straight shafts. For wood, tapered shafts do not appear to result in a significant increase in peak performance. The performance of W1 was significantly higher than that of the other wood sticks, but performance differences between the other 5 wood sticks do not appear to be significant.

Composite sticks showed a wider range of peak performance values, ranging from $v_p$ = 44.6 ± 0.9 (C1) mph to $v_p$ = 59.9 ± 1.6 mph (C11). Sticks C2 and C3 showed surprising performance results, with $v_p$ = 55.3 ± 1.8 mph and $v_p$ = 56.4 ± 3.3 mph, respectively. These two sticks are a straight – shaft composite and a very low end tapered composite, which were not expected to perform as well as or better than the elite composite sticks. Although the elite composite sticks have longer tapers than the other composites, this increased taper does not result necessarily in increased performance according to this test. It is possible that subtle material differences in the makeup of the 11 composite sticks that are not apparent from visual inspection could result in performance differences.

### 4.4.5 Performance Test Evaluation

An important aspect of developing a performance test was to investigate the repeatability or consistency of the test. To do this, three sticks were performance tested three times each and the results were compared for each stick. The sticks tested for repeatability were sticks W4, C5, and C11, representing one stick from the wood group, one from the mid-range tapered composite group, and one from the elite composite group. The second and third tests for each stick showed the peak by impacting five locations with six pucks.

Peak average $v_p$ values for the three tests are shown for each stick in Figure 4.20. Performance curves for each stick are shown in Figure 4.21. Stick C5 had the highest variation with 3 % (54 and 52.3 mph), followed by C11 with 2.5 % (61.1 and 51.5 mph), and W4 with 2.4 % difference (46.5 and 45.4 mph) between the three tests. The differences are relatively small, although they are higher than the differences seen in similar tests for softball, roughly 1 mph or 1 % for successive tests on the same bat. In all cases, the peak $v_p$ shifted horizontally one inch between the tests, but the peak values remained relatively similar. Although results do

show some differences, all results lie within one standard deviation for each stick, indicating that the differences are not significant.

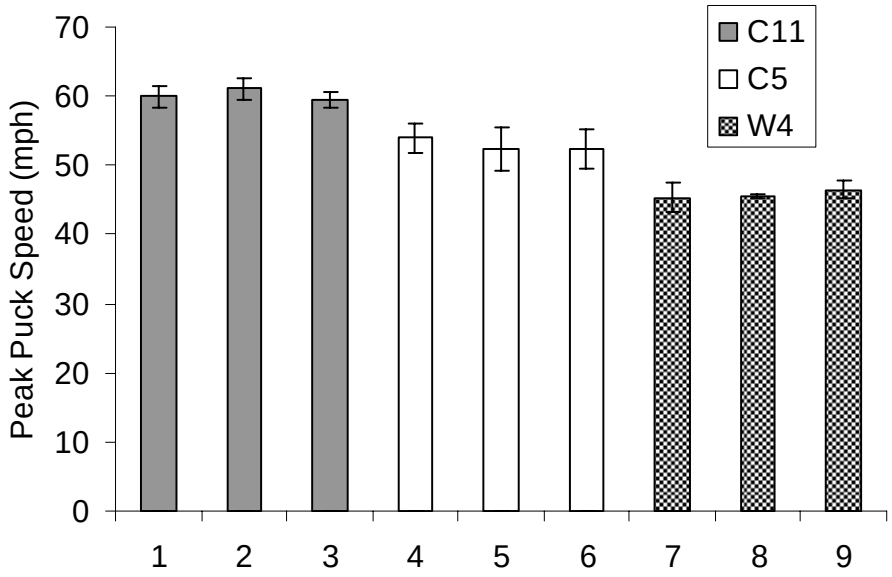
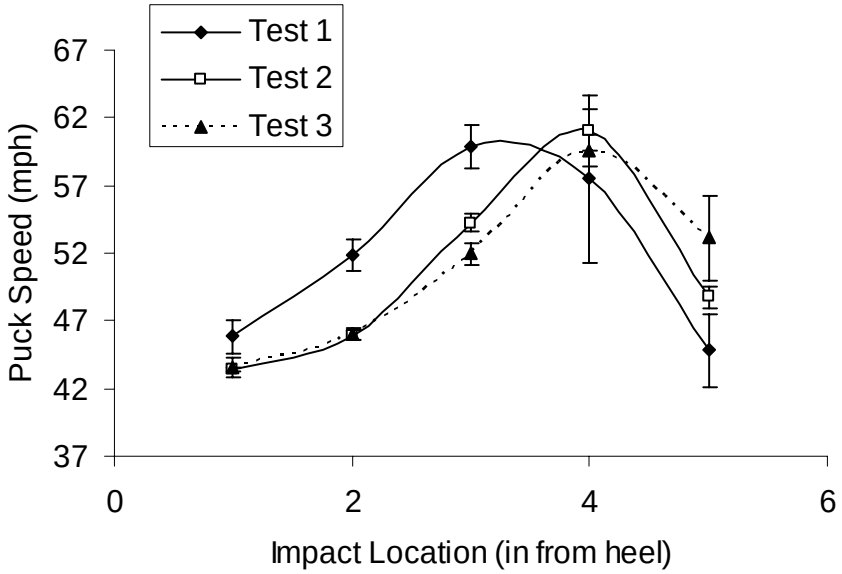
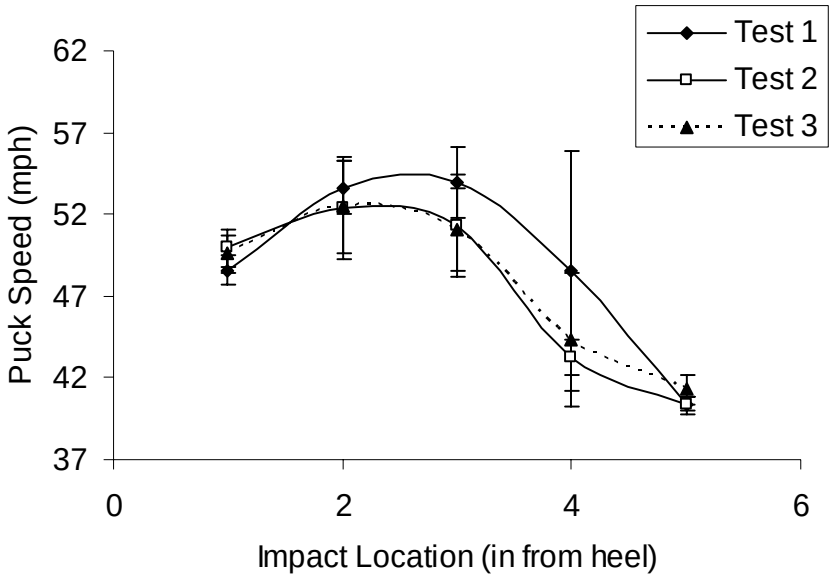


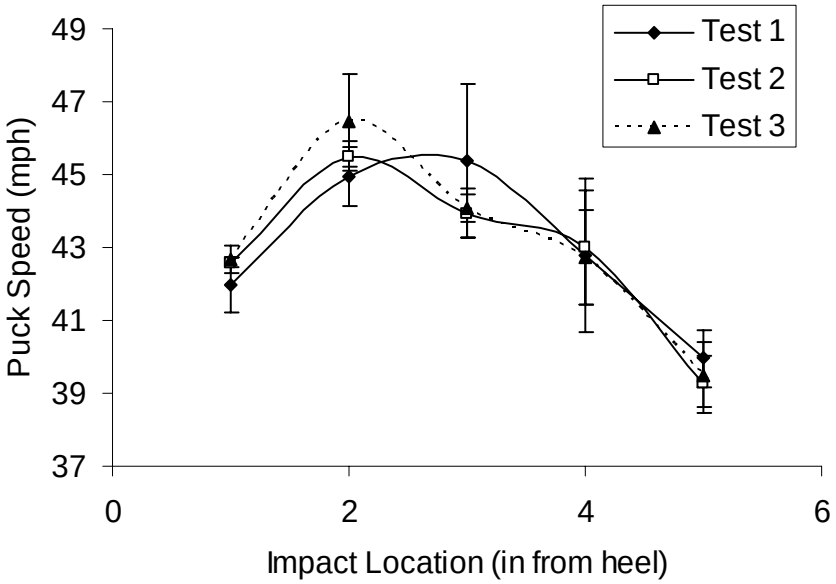

Figure 4.20: Repeatability of peak $v_p$ for three sticks




(a)




(b)




(c)

Figure 4.21: Repeated performance curves for (a) C11, (b) C5, and (c) W4

Sticks in this study were impacted along the lower inch of the blade during performance testing. It was desirable to determine the effect, if any, of the vertical impact location on performance results. A scan was completed on one hockey stick by fixing the horizontal pivot

distance at the middle of the blade and impacting the blade in vertical intervals of 0.2 inches. The stick was impacted with 6 pucks at each location and averages from the 6 impacts are reported as $v_p$ values here. The first impact location corresponds to the bottom edge of the puck impacting the bottom of the blade. Results are presented below in Figure 4.22.

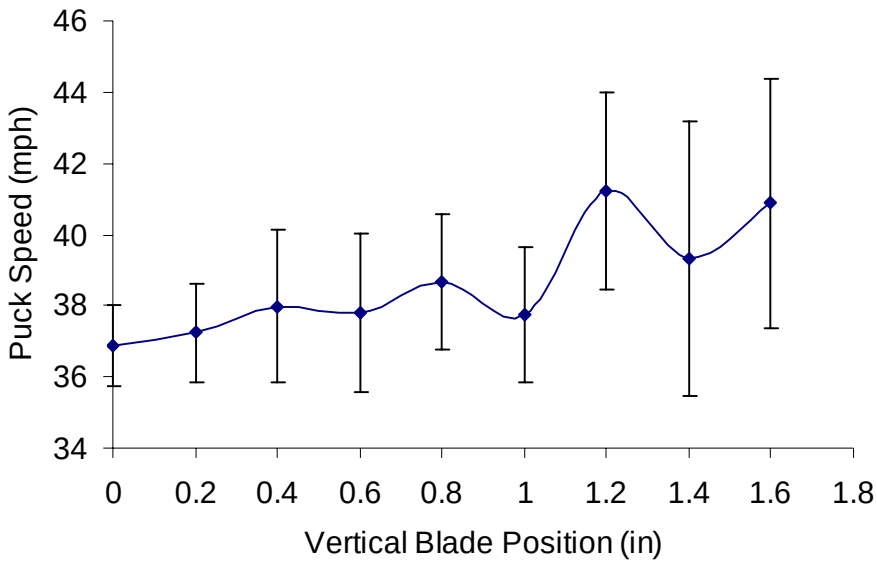



Figure 4.22: Vertical blade position test results

Performance values remained essentially constant along the lower half of the blade, and then showed larger changes. At the 1.2 inch impact location, $v_p$ results were 10.7% higher ($v_p$ = 41.2 ± 3.8 mph) than at the first location at the bottom of the blade ($v_p$ = 36.9 ± 1.1 mph). As the impact location moved toward the top of the blade, more scatter was seen in the results, as is evident by the increasingly large error bars in Figure 4.22. It is postulated that impacting the blade toward the top caused increased vibration in the shaft and blade of the stick, therefore resulting in less consistent data at those locations. Stick vibration during performance testing is discussed further below.

Further testing examined the effect of puck temperature on performance test results. One stick (C1) was tested first with six pucks that were at room temperature, or 72 ± 2° F. It was then tested again at six locations with three frozen pucks at 25 ± 2° F. A non-contact infrared temperature sensor was used to verify puck temperature before firing. Performance curves for both tests are shown in Figure 4.23. No significant difference in performance was observed when different temperature pucks were used. Although puck temperature has significant effects on the elasticity and controllability of a puck, these results implied that puck temperature is not important in determining the dynamic behavior of a hockey stick. This also implies that the stiffness of the puck is not important in determining the dynamic behavior of the stick.

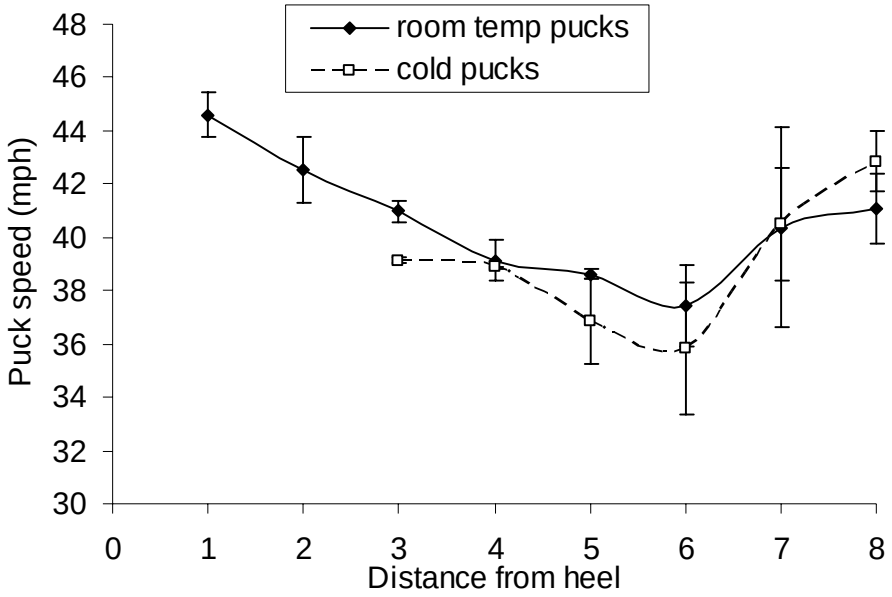



Figure 4.23: Performance results for a stick tested with warm and frozen pucks

It is highly desirable in sports equipment testing to show a correlation between lab performance and performance in the field of play. Radar data from subject 1 in the field study was compared to the cannon performance test data in an attempt to find a correlation. Results are compared in Figure 4.24. In both cases, stick C1 had the slowest puck speed. The small sample size of the field study prevented any other correlations between lab and on-ice

performance. Previous studies have shown a high interaction of test subjects and skill level in performance results [4.1, 4.3], suggesting that a substantial sample size is needed to obtain meaningful correlations between lab and field performance.

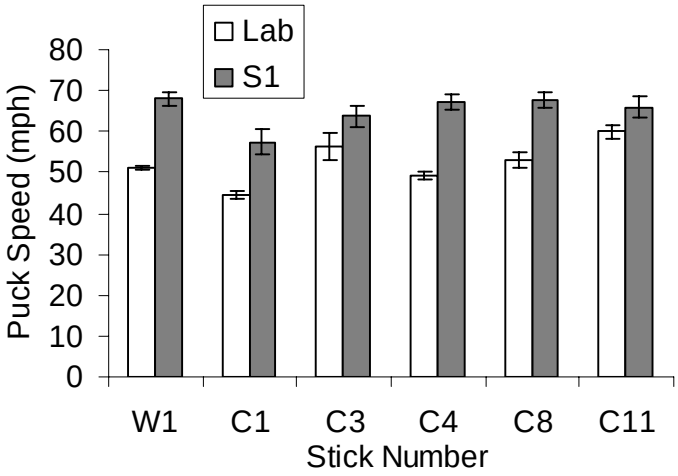



Figure 4.24: Comparison of lab and field study stick performance

Some concerns regarding the performance test do exist. The first concern is the amount of vibration seen in the stick during an impact in the lab setting. Hockey sticks are long and slender, making them very flexible. In play, the deflections of a hockey stick during a slap shot are very complex. Sticks undergo bending in two planes, torsion, and the blade flattens regardless of its curve pattern. A result of the stick's flexibility is evident in the performance test by the amount of vibration observed in the stick following an impact.

The digital encoder mapped the angular velocity of the stick as it traveled through 45 degrees of rotation. LabView analyzed the encoder signal, applying a linear fit and outputting the slope as the average angular velocity. Figure 4.25 shows the encoder output for several different impact locations along the length of the blade. The linear trendline represents the swing speed that was recorded by the program for each impact.

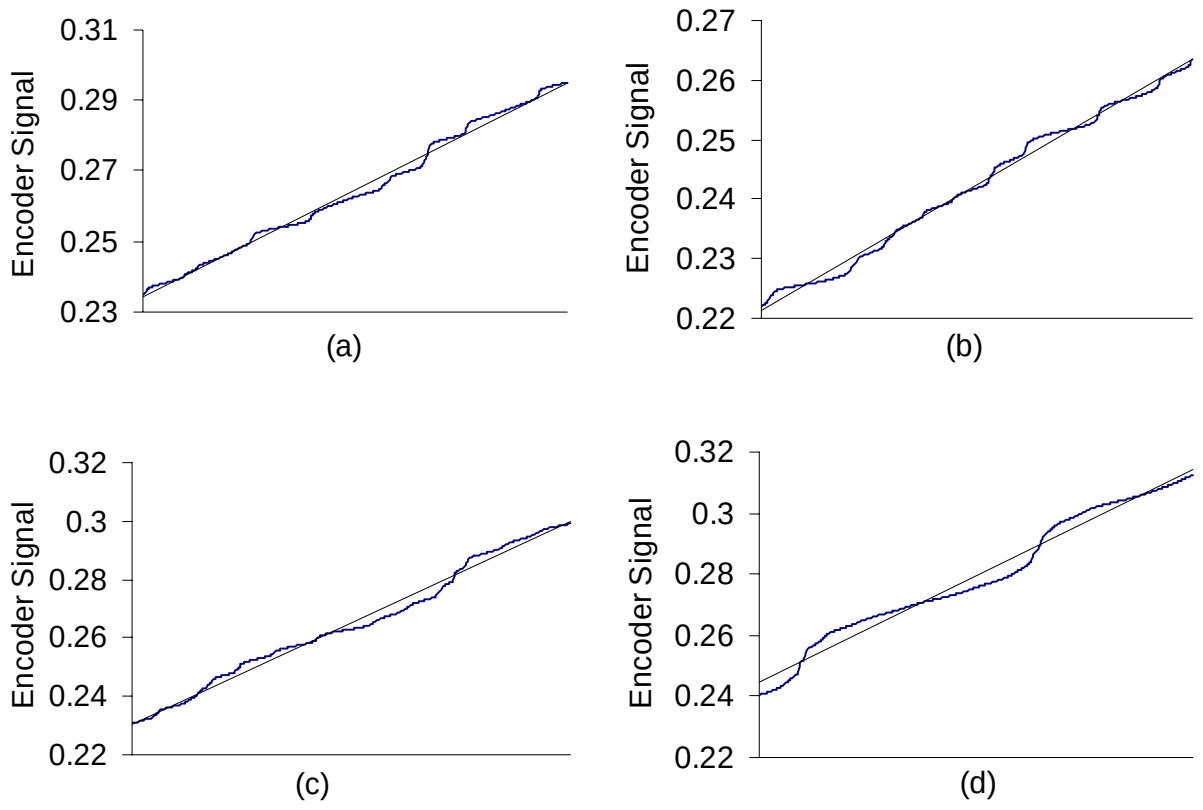



Figure 4.25: Encoder output for a heel (a), a mid-heel (b), a mid-toe (c), and a toe (d) impact with sticks C11 (a, b) and W4 (c, d)

Similar vibration behavior was observed in all types of sticks during testing. Such vibration caused noise in the encoder signal, which could account for much of the differences seen in the repeatability study and standard deviations of performance testing. It was interesting to note that vibration patterns became smoother as the impact location moved toward the toe of the stick, shown in Figure 4.25 (c) and (d).

Significant bending and torsion in the stick were observed in high speed video recordings of performance testing impacts. These deformations oscillated as the stick traveled after the impact. Torsion resulted from the puck impacting the outer portion of the blade, which created a torque about the axis of the shaft. Bending resulted from a moment about the pivot axis that was introduced by the puck impacting the blade. These oscillations could be responsible for

introducing unpredictable variations in measured swing speed data, causing higher standard deviations in test data.

In elite-level play, puck speeds resulting from slap shots often reach speeds of 80 – 100 mph [4.1, 4.2, 4.11]. In the field study conducted for this research, top puck speeds were approximately 70 mph. The performance test utilized a faster stick swing speed than that found in the field study, but puck speeds were found to be much slower, in the range of 44.4 – 59.9 mph. It is suspected that this discrepancy can be explained in terms of stick – puck contact time and shaft loading.

It is suspected that the stick and puck exhibit an extended trampoline effect in an actual slap shot, in which the flex stored in the shaft is releasing at approximately the same time the puck leaves the stick. Previous research has found the stick-puck contact time for slap shots to be in the range of 38 $\pm$ 9 ms for elite players and 27 $\pm$ 5 ms for recreational players [4.3]. Field study data from recreational players in this study found contact times for recreational players in the range of 32 $\pm$ 5 ms. In the cannon impact testing, the stick-puck contact time was found between 5 – 10 ms. It is postulated here that the shorter contact duration in the lab testing prevents the full effects of shaft loading on the final puck speed than actually occurs on ice, where contact times are roughly four times longer. The shorter contact time in lab testing does not allow sufficient transfer of bending energy from the shaft to the puck during the impact.

This theory can be examined in more detail by considering puck speed as a result of both inertia and potential energy stored in the shaft of the stick. First, let us consider a stick with a lab performance rating of $v_p$ = 50 mph, which is roughly the median of sticks tested in this study. For the moment, it is assumed that shaft loading produces no effect on lab performance. Assume that during an actual slap shot, the puck has reached this speed at the time of maximum shaft

deflection during the shot. The kinetic energy of the puck after it leaves the stick must be equivalent to the total energy of the moving stick/puck and the potential energy of the deflected shaft at its peak deflection. Or mathematically,

$$\frac{1}{2}m_p v_p^2 + \frac{1}{2}kx^2 = \frac{1}{2}m_p v_f^2 \tag{4.6}$$

where $m_p$ is the mass of the puck, $v_p$ is the puck speed found from lab performance testing, k is the stiffness of the stick (lb/in), x is the displacement of the shaft (in), and $v_f$ is the final puck speed due to both inertial and potential energy effects. This equation can be solved for the final puck speed, $v_f$, yielding:

$$v_f = \left[\frac{m_p v_p^2 + kx^2}{m_p}\right]^{1/2} \tag{4.7}$$

Three point bend tests were performed on the 17 sticks tested in this study. A 50 inch span was used to deflect the stick in a load frame (MTS Systems Corporation, Eden Prairie, MN), and maximum force values were recorded every 0.2 inches to develop a force – displacement curve. The test setup for measuring bending stiffness is shown in Figure 4.26. In this case, the stiffness of the shaft was considered, while the stiffness of the blade was neglected. Figure 4.27 shows some of the force – displacement curves that were found. All sticks showed a linear force – displacement profile and stiffness values were found to range from 34.2 – 49.0 lb/in. Stiffness values for each stick are given in Appendix B, Table B.3. This value represented the 3-point bending stiffness of the sticks, or k in Equation (4.7).


Figure 4.26: Stick bending stiffness test setup

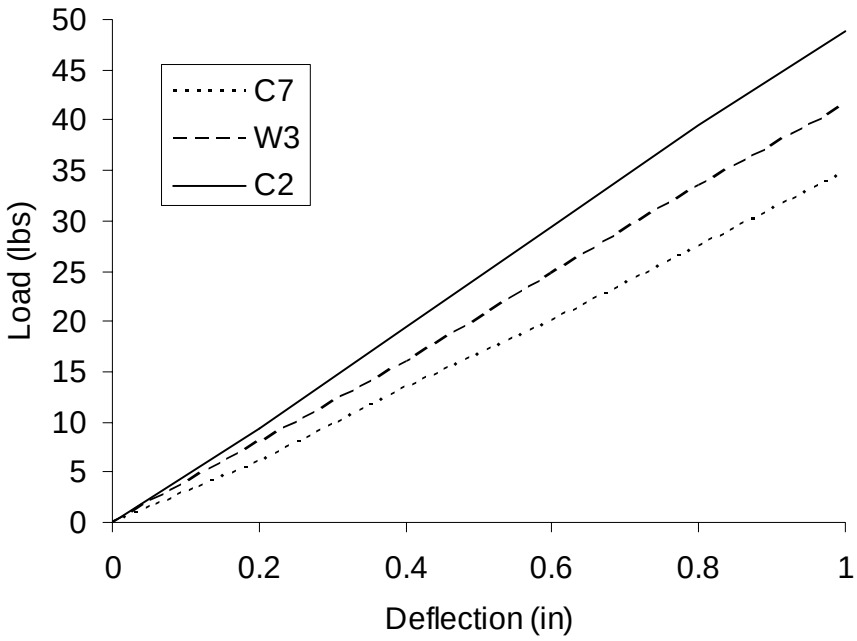



Figure 4.27: Stick stiffness results from 3-point bend tests

By taking $m_p$ to be an average puck mass of 5.8 oz, Equation (4.7) was plotted as a function of shaft displacement, x in Figure 4.28 for a range of shaft stiffness from k = 0 (no shaft loading) to k = 40 lb/in. To achieve a final puck speed of 70 mph with the 40 lb/in stick, a shaft

deflection of approximately 4 inches is required. With a shaft deflection of 6 inches, a final puck speed of approximately 85 mph can be obtained. These deflections are within the range commonly seen in high level player slap shot, in which bending in the shaft can be as much as 16 - 30° [4.1, 4.12].

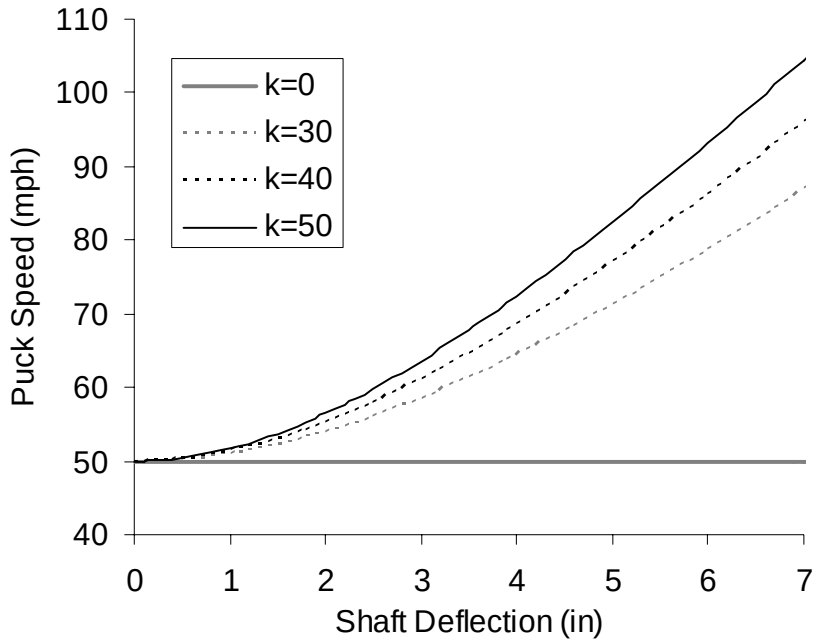



Figure 4.28: Final puck speed as a function of shaft deflection

It is believed that shaft loading does play some role in the lab performance results because no correlation between the inertia of the stick and stick performance was noted in this study. Still, the preceding argument provides a plausible explanation for the slower puck speeds seen in lab performance compared to on-ice performance. For a stick that is more compliant than the one described here, the decreased stiffness would result in an increased deflection for the same final puck speed.

Measured stick stiffness was also used to evaluate the validity of the assumption that stick stiffness can be neglected for the lab test. For each stick tested, a shaft loading factor was

added to the final performance of the stick to obtain a new peak performance value. For this case, a constant force was applied to each stick that produced a corresponding displacement, x, dictated by the stick stiffness. The force used was 100 pounds, which corresponds to approximately a 2.5 inch deflection for the average stick stiffness. A new peak performance value was calculated using Equation (4.7) and compared to the peak performance obtained directly from lab testing. Results are shown in Figure 4.29 and are also summarized in Appendix B, Table B.3.

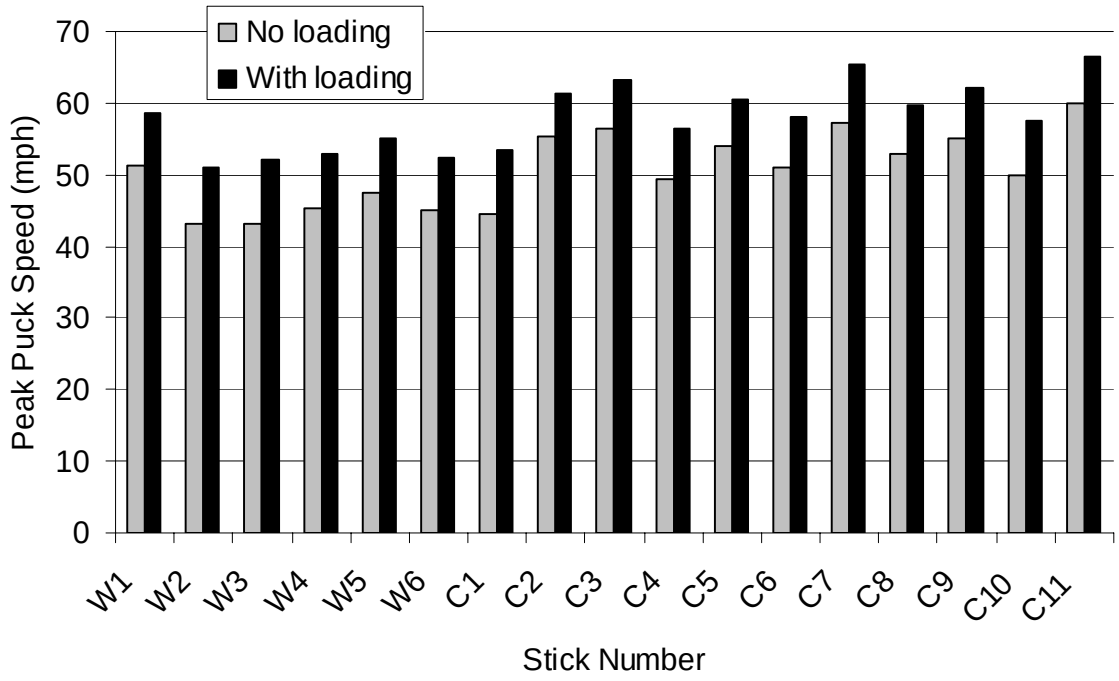



Figure 4.29: Peak $v_p$ results for 17 sticks with a shaft loading factor

Including the shaft loading factor produced an increase in $v_p$ ranging from 9.8% - 16.8% of the original peak performance. The overall trends, however, remained the same. The wood group had a new average peak $v_p$ of 53.6 ± 2.8 mph, while the composite group had a new average peak vp of 60.4 ± 3.9 mph with shaft loading. The relative performance of each stick stayed the same, with the exception of sticks W6 and C1. With no shaft loading, W6 had slightly

higher performance than C1, while the opposite was true with shaft loading. These findings show that a shaft loading factor produces an increase in lab performance, but does not change the relative performance of one stick to another. The assumption of neglecting stick stiffness in the performance test can therefore be considered valid.

Several sticks broke during the performance test. It is significant here to note the mode of failure in these cases. In all cases, sticks failed in a manner that is consistent with failure seen in the field of play. The major methods of failure were: shaft breaking at the hosel or blade-shaft junction, shaft breaking in the middle, shaft delamination, and blade delamination. Some failed sticks are shown below in Figure 4.30.

All of these failure modes were identified by Hoerner [4.11] as common failures that occur during play. In order of most frequent to least frequent, he found: hosel broke (44 %), the shaft broke (16 %), shaft delaminated (15 %), and blade delamination (11 %). Hoerner also identified fracture of the blade as another failure mechanism (15 % frequency), but this type of failure was not observed in the current study.

Figure 4.30: Stick failures that occurred during performance testing

## 4.5 Summary

This chapter has described test methods and results for characterizing ice hockey sticks in a laboratory setting, independent of any test subjects using the stick. Sticks were characterized in terms of their vibration by modal analysis and in terms of a high speed laboratory performance test. An on-ice field study was also conducted in to determine a correlation between lab performance and field performance.

The vibration patterns of a hockey stick are complex and require more robust analysis techniques to distinguish vibration modes above the second bending mode. The first two bending modes were compared for each type of stick, as well as the location of the lower first bending node. Natural frequencies for the first two bending modes showed differences between

the sticks. The wood sticks had lower frequencies than the composite sticks, though differences were very small. It is suspected that higher frequencies are responsible for characterizing player perceived feel of a stick. Composite sticks exhibited lower node locations than wood sticks, indicating a lower kick point in composite sticks.

A high speed laboratory performance test was developed to compare 17 sticks. Notable differences were found in the performance results of wood and composite sticks. Composite sticks performed better than wood sticks with an average peak performance of $v_p = 54.1 \pm 4.3$ mph, compared to $v_p = 45.9 \pm 3.1$ mph for the wood group. The test was shown to have repeatability within 2.4 – 3 % for three sticks that were tested three times each. Sticks were impacted along the lower inch of the blade in this study to emulate stick-puck contact seen in play. The vertical impact location on the blade was shown to have a small effect (9%) on the performance of the stick, but impacting the blade at higher locations increased the variation of the results. Puck temperature and dynamic stiffness was shown here to be unimportant in determining the dynamic behavior of hockey sticks.

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# CHAPTER FIVE

# SUMMARY AND FUTURE WORK

## 5.1 Summary

### 5.1.1 Hockey Pucks

Hockey pucks were characterized by their impact properties of coefficient of restitution (e) and dynamic stiffness ($k_d$). A high speed air cannon was used to fire pucks from two brands at a rigidly mounted array of load cells. Pucks were tested at speeds ranging from 55 – 85 mph and at temperatures ranging from 25 - 72° F.

A two-brand comparison at room temperature showed small differences in e, but rather large differences in $k_d$, 33%. The brand of puck with a higher $k_d$ also reached a peak force that was 20% higher than the other. Both brands were also tested at 25° F and showed 26% and 32% decreases in e and 488% and 626% increases in $k_d$. Differences in e between the two brands that were found at room temperatures diminished at lower temperatures. Testing at several intermediate temperatures showed nonlinear changes in both e and $k_d$. The coefficient of restitution also decreased fairly linearly with increasing speed. The dynamic stiffness increased with speed, suggesting that pucks have a relatively high degree of nonlinearity.

### 5.1.2 Hockey Sticks

Seventeen different stick models were used in this study, comprised of 6 wood and 11 composite. A one-dimensional bending analysis was performed on all sticks to compare natural frequencies and the location of the lower first bending node. The vibration of hockey sticks is complex, with torsional modes appearing as mirrors of bending modes above the second bending mode. Small differences were noted in the first two bending mode frequencies for each stick.

Overall, wood sticks had the lowest natural frequencies for modes 1 and 2 and the highest node locations.

A fixture was designed and fabricated to pivot a hockey stick at the end of a high speed air cannon. Pucks were fired at a stationary pivoted stick, and momentum principles were used to calculate a performance measure of puck speed as if it were shot from the stick, or $v_p$. Results showed significant differences for the different types of sticks. Overall, the composite sticks performed better than the wood sticks, with an average performance of $v_p = 54.1 \pm 4.3$ mph for composite and $v_p = 45.9 \pm 3.1$ mph for wood. Shaft taper was shown to be less important than stick material in determining the stick performance. Three sticks were tested three times each and showed variations in peak performance of 2.4 – 3%. While puck temperature was previously shown to be important for determining the dynamic behavior of the puck, it was not important in determining the dynamic behavior of the stick in this performance test.

Significant vibrations in the stick during a stick – puck impact were noted, which could cause noise in the results, reducing the repeatability of the test. In addition, laboratory performance was notably lower than speeds found on the ice. This was explained in terms of contact time and a shaft loading factor that increased performance. An on-ice field study was conducted that utilized three recreational level test subjects and 6 sticks. Stick and puck speeds obtained in the field study correlated to those seen in play.

## 5.2 Future Work

### 5.2.1 Hockey Pucks

This study has provided a firm foundation for the characterization of ice hockey pucks at speeds closer to those seen in game play. Pucks were compared for different brands, speeds, and

temperatures, and each was shown to have differences in impact properties. Further work on this topic could include determining the combined effect of changing speed and temperature on puck properties. In addition, it is known that higher stiffness and lower coefficient of restitution are desirable for players handling the puck. It is unclear, however, what impact these properties have on other tasks like passing and shooting.

The differences found in impact properties could be useful for both equipment manufacturers and injury analysis. Understanding the impact properties of pucks at game speeds is important in determining how to best design protective equipment for players. Implications could also be made regarding injury mechanisms and likelihood for pucks traveling at different speeds or temperatures.

### 5.2.2 Hockey Sticks

A suggestion for future work would include examining higher frequency vibration patterns of hockey sticks. More robust analysis techniques would be needed to do so in order to examine bending in two dimensions. It is suspected that it may be possible to synchronize different vibration patterns of the stick in order to maximize energy transfer to the puck. Bending in the blade, torsion of the shaft, and bending in the shaft all affect the power of a shot. It is unclear what frequencies of vibration produce desirable affects on shot speed. In addition, further damping studies should be investigated to better characterize the vibration of sticks.

A larger scale field study is needed that utilizes players of high skill level to effectively correlate laboratory and on-ice performance of hockey sticks. Instrumenting hockey sticks to determine in situ vibration patterns during a field study could also provide useful information that may be correlated with player perception of feel. Additionally, the movement and deformation of a hockey stick in a shot is very complex. Three-dimensional tracking software

would be much better suited to analyze high speed video files than the 2-dimensional software used in the present study.

A high speed laboratory test that showed statistical differences and repeatable results for different hockey sticks was developed and implemented in this study. Because this research did not receive outside funding, test samples were limited mainly to donations from manufacturers. Sticks tested were all sticks that are manufactured for player use, and not specifically tailored to meet the needs of this study. This likely led to some interaction affects between different stick tapers and different composite composition for the composite group. It would be beneficial to obtain test samples that are specifically tailored to vary only one factor at a time to better see the results of one stick factor compared to another.

For the performance test, it is assumed that the stick is relatively flexible and that the hands play little role in the execution of a slap shot. In reality, it has been shown that sticks to not behave as a free – free system and that the hands are important. While it is clear that the hands are important in shooting tasks, it is unclear if they are needed to compare the lab performance of one stick to another. This factor should be investigated in more detail, with numerical modeling to determine what impact the hands may have on the performance and execution of slap shots.

Finally, much more could be learned about laboratory vs. on-ice performance by conducting numerical modeling of both slap shots and on-ice performance. Numerical modeling could also be used to compare the behavior of a hockey stick in the laboratory performance test to an actual slap shot in greater detail. The topics previously discussed could also be investigated through numerical models.

# APPENDIX A

## MASS PROPERTIES OF EACH STICK

Table A.1: Mass properties of each stick

| Stick Number | Length (in) | 6 in weight (oz) | 42 in weight (oz) | Total Weight (oz) | Balance Point (in) | MOI (oz-in^2) |
|---|---|---|---|---|---|---|
| W1 | 57.25 | 5.79 | 15.10 | 20.88 | 32.03 | 9370.27 |
| W2 | 56 | 6.75 | 17.09 | 23.83 | 31.81 | 10841.60 |
| W3 | 56 | 6.26 | 15.64 | 21.90 | 31.72 | 9937.42 |
| W4 | 58.5 | 5.94 | 18.12 | 24.06 | 33.12 | 11296.65 |
| W5 | 59.5 | 4.62 | 17.84 | 22.46 | 34.59 | 11619.04 |
| W6 | 59 | 5.28 | 19.77 | 25.04 | 34.42 | 12545.97 |
| C1 | 55.5 | 3.15 | 15.68 | 18.82 | 35.98 | 11673.60 |
| C2 | 59.25 | 2.53 | 17.64 | 20.17 | 37.48 | 11114.15 |
| C3 | 55.75 | 2.90 | 15.24 | 18.14 | 36.24 | 11060.94 |
| C4 | 59.375 | 2.45 | 15.38 | 17.83 | 37.05 | 10485.13 |
| C5 | 58.875 | 3.51 | 15.15 | 18.66 | 35.24 | 10462.14 |
| C6 | 60 | 2.03 | 14.86 | 16.89 | 37.67 | 9762.85 |
| C7 | 58.5 | 1.91 | 14.48 | 16.39 | 37.80 | 9782.74 |
| C8 | 55.375 | 3.40 | 12.52 | 15.92 | 34.31 | 8866.02 |
| C9 | 56.25 | 2.11 | 13.20 | 15.30 | 37.05 | 9752.38 |
| C10 | 55.75 | 2.63 | 13.32 | 15.94 | 36.07 | 8753.07 |
| C11 | 60.25 | 0.91 | 15.40 | 16.31 | 40.00 | 10025.38 |

APPENDIX B

COMPLETE STICK TESTING RESULTS

Table B.1: Modal analysis results for each stick

| Stick Number | Material | Shaft Geometry | MOI (oz- in$^2$) | Mode 1 Node (in) | Mode 1 Frequency (Hz) | Mode 2 Frequency (Hz) | Damping Ratio |
|---|---|---|---|---|---|---|---|
| W1 | Wood | Straight shaft | 9370.27 | 11 | 48 | 123 | |
| W2 | Wood | Straight shaft | 10841.60 | 11 | 51 | 123 | 0.00291348 |
| W3 | Wood | Straight shaft | 9937.42 | 10 | 50 | 126 | 0.00282303 |
| W4 | Wood with composite blade | Straight shaft | 11296.65 | 13 | 48 | 120 | 0.00429556 |
| W5 | Wood | Tapered shaft | 11619.04 | 11 | 47 | 120 | 0.00398702 |
| W6 | Wood with composite blade | Tapered shaft | 12545.97 | 11 | 46 | 111 | 0.00466692 |
| C1 | Composite | Straight shaft | 11673.60 | 8 | 53 | 130 | |
| C2 | Composite | Straight shaft | 11114.15 | 10 | 54 | 137 | |
| C3 | Composite | Tapered shaft - low | 10444.96 | 10 | 51 | 136 | |
| C4 | Composite | Tapered shaft - mid | 10485.13 | 9 | 52 | 136 | |
| C5 | Composite | Tapered shaft - mid | 10462.14 | 10 | 50 | 153 | |
| C6 | Composite | Tapered shaft - mid/high | 9762.85 | 10 | 51 | 133 | 0.00175093 |
| C7 | Composite | Tapered shaft - high | 9782.74 | 9 | 51 | 139 | 0.00230624 |
| C8 | Composite | Elite composite | 9491.08 | 10 | 51 | 137 | |
| C9 | Composite | Elite composite | 9752.38 | 9 | 59 | 160 | |
| C10 | Composite | Elite composite | 8753.07 | 9 | 60 | 161 | |
| C11 | Composite | Elite composite | 10025.38 | 9 | 51 | 129 | 0.00202254 |

Table B.2: Performance testing results for each stick

| Stick Number | Peak $v_p$ (mph) | Standard Deviation (mph) | Peak $v_p$ (mph) | Standard Deviation (mph) | Peak $v_p$ (mph) | Standard Deviation (mph) |
|---|---|---|---|---|---|---|
| W1 | 51.15 | 0.43 | | | | |
| W2 | 43.03 | 1.19 | | | | |
| W3 | 43.27 | 1.18 | | | | |
| W4 | 45.36 | 2.11 | 45.51 | 0.38 | 46.48 | 1.27 |
| W5 | 47.57 | 3.59 | | | | |
| W6 | 45.01 | 1.69 | | | | |
| C1 | 44.6 | 0.85 | | | | |
| C2 | 55.25 | 1.80 | | | | |
| C3 | 56.39 | 3.28 | | | | |
| C4 | 49.3 | 1.04 | | | | |
| C5 | 53.98 | 2.19 | 52.34 | 3.12 | 52.45 | 2.82 |
| C6 | 51.12 | 2.85 | | | | |
| C7 | 57.34 | 1.81 | | | | |
| C8 | 52.9 | 1.91 | | | | |
| C9 | 55.09 | 1.09 | | | | |
| C10 | 50.02 | 0.55 | | | | |
| C11 | 59.92 | 1.59 | 61.08 | 1.56 | 59.54 | 1.18 |

Table B.3: Performance testing results with stick loading

| Stick Number | Peak $v_p$ without loading (mph) | Bending Stiffness (lb/in) | Peak $v_p$ with loading (mph) | Peak $v_p$ increase (mph) | Percent Increase |
|---|---|---|---|---|---|
| W1 | 51.15 | 42.27 | 58.56 | 7.41 | 12.66 |
| W2 | 43.03 | 46.36 | 50.92 | 7.89 | 15.50 |
| W3 | 43.27 | 41.38 | 51.99 | 8.72 | 16.77 |
| W4 | 45.36 | 46.72 | 52.85 | 7.49 | 14.18 |
| W5 | 47.57 | 44.28 | 55.13 | 7.56 | 13.71 |
| W6 | 45.01 | 48.34 | 52.32 | 7.31 | 13.97 |
| C1 | 44.6 | 40.05 | 53.36 | 8.76 | 16.42 |
| C2 | 55.25 | 48.95 | 61.28 | 6.03 | 9.84 |
| C3 | 56.39 | 42.55 | 63.15 | 6.76 | 10.70 |
| C4 | 49.3 | 44.79 | 56.55 | 7.25 | 12.82 |
| C5 | 53.98 | 46.11 | 60.49 | 6.51 | 10.77 |
| C6 | 51.12 | 45.39 | 58.06 | 6.94 | 11.95 |
| C7 | 57.34 | 34.25 | 65.51 | 8.17 | 12.47 |
| C8 | 52.9 | 44.47 | 59.76 | 6.86 | 11.48 |
| C9 | 55.09 | 42.15 | 62.05 | 6.96 | 11.22 |
| C10 | 50.02 | 43.18 | 57.43 | 7.41 | 12.90 |
| C11 | 59.92 | 41.64 | 66.45 | 6.53 | 9.83 |