DEMONSTRATION OF ACOUSTIC INSTABILITY
IN RESONATORS WITH HEAT ADDITION
AND MEAN FLOW

BY

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The members of the Committee appointed to examine the thesis of Rafael Hernandez find it satisfactory and recommend that it be accepted.

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To my parents Rosa and Eustorgio Hernandez, muchas gracias por el sacrificio que hicieron hace muchos años en venir a los Estados Unidos. Es ese sacrificio que me dio la oportunidad a llegar donde estoy ahora.
Thermoacoustic instabilities can arise in systems where unsteady heat release is favorably coupled with acoustic pressure oscillations. A modified Rijke tube with segments having different sections is found to have lower threshold levels of heat addition rate needed for exciting the fundamental acoustic mode of the tube. Experiments indicate that about 115 W of heating power is required to produce sound in the 90 cm long segmented tube, while over 230 W is needed in 60 cm long and 90 cm long tubes with constant-area cross sections. One of the causes for lowering the threshold of heat addition rate is due to a significant reduction of the resonator natural frequency. The presented results suggest an importance of including the system geometry details into analysis of practical devices prone to thermoacoustic instabilities.
Two types of acoustic energy harvesters are also tested and demonstrated in this study. Tonal sound is excited by heat addition or vortex shedding/impinging in the presence of mean flow in the resonator. The sound generated within the resonator is partially converted into electrical energy by using a piezoelectric disk with a brass back plate. One of the systems, a thermoacoustic engine, generated a maximum electric power of 0.446 mW at a resistance of 14.8 kΩ. The baffled tube with mean flow produced more than 0.5 mW of electric power at a resistance of 10 kΩ and mean velocity of 2.6 m/s. Optimization of the system geometry and piezoelement are required in order to increase the power output.
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a$</td>
<td>average speed of sound</td>
</tr>
<tr>
<td>$B$</td>
<td>beam of plate</td>
</tr>
<tr>
<td>$c_p$</td>
<td>gas specific heat at constant pressure</td>
</tr>
<tr>
<td>$d$</td>
<td>inner diameter of baffle</td>
</tr>
<tr>
<td>$D$</td>
<td>outer diameter of baffle</td>
</tr>
<tr>
<td>$\dot{E}_{TA}$</td>
<td>time averaged thermoacoustic power</td>
</tr>
<tr>
<td>$f_n$</td>
<td>natural frequency of a straight pipe</td>
</tr>
<tr>
<td>$k$</td>
<td>wavenumber</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the resonator or length of beam plate</td>
</tr>
<tr>
<td>$L_e$</td>
<td>length correction</td>
</tr>
<tr>
<td>$n$</td>
<td>number of acoustic mode in the pipe</td>
</tr>
<tr>
<td>$n_2$</td>
<td>integer number of vortices between baffles</td>
</tr>
<tr>
<td>$p$</td>
<td>acoustic pressure amplitude</td>
</tr>
<tr>
<td>$p_m$</td>
<td>mean pressure ($= P_m$)</td>
</tr>
<tr>
<td>$p_1$</td>
<td>acoustic pressure waveform</td>
</tr>
<tr>
<td>$p'$</td>
<td>fluctuation of pressure.</td>
</tr>
<tr>
<td>$P_E$</td>
<td>electric power</td>
</tr>
<tr>
<td>$P_A$</td>
<td>maximum pressure amplitude</td>
</tr>
<tr>
<td>$q'$</td>
<td>fluctuation of heat</td>
</tr>
<tr>
<td>$\dot{Q}_0$</td>
<td>convective heat transfer rate</td>
</tr>
<tr>
<td>$R$</td>
<td>electric resistance or radius of the resonator</td>
</tr>
<tr>
<td>$s$</td>
<td>heater location</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T$</td>
<td>acoustic period</td>
</tr>
<tr>
<td>$</td>
<td>Tr</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>mean temperature gradient</td>
</tr>
<tr>
<td>$\Delta T_{cr,id}$</td>
<td>ideal critical temperature gradient</td>
</tr>
<tr>
<td>$u'$</td>
<td>acoustic velocity fluctuation</td>
</tr>
<tr>
<td>$u_0$</td>
<td>incident flow mean velocity</td>
</tr>
<tr>
<td>$u_1$</td>
<td>acoustic amplitude of gas</td>
</tr>
<tr>
<td>$u_v$</td>
<td>effective vortex velocity</td>
</tr>
<tr>
<td>$V_{RMS}$</td>
<td>RMS voltage</td>
</tr>
<tr>
<td>$V$</td>
<td>chamber volume</td>
</tr>
<tr>
<td>$\dot{W}$</td>
<td>acoustic power generation</td>
</tr>
</tbody>
</table>
$x$ coordinate along the resonator

$\varphi$ phase delay

$\gamma$ specific heat ratio

$\kappa$ fluid thermal diffusivity

$\omega$ radian frequency of acoustic oscillations

$\tau$ time lag due to thermal inertia

$\rho$ mean air density ($\rho_m$)

$\delta_k$ thermal penetration depth of the fluid
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Figure 20: Harvested electric power with varying loads

Figure 21:(a) Acoustic pressure amplitudes at measured at three locations: \( \nabla, 1; \circ, 2; +, 3 \). (b) Frequency of excited tone
Chapter 1

Introduction
1. Introduction

1.1. Background

Acoustic instabilities in the form of tonal sound excitation in resonators with mean flow and/or heat addition represent a class of phenomena important in engineering. Sound waves in a gas are usually regarded as consisting of coupled pressure and motion oscillations, but temperature oscillations are present within the sound wave as well. As sound travels in a channel, heat is also transferred to and from the channel walls. The combination of all oscillations produces a variety of thermoacoustic effects.

These types of acoustic instabilities in practical engineering systems may be both harmful and useful. Unstable combustion coupled with acoustic oscillations in rocket motors and noise generation in ducts with mean flows are two examples of undesirable phenomena (Harris, 1998; Culick, 2006). In contrast, there are some devices that may benefit from the acoustic oscillations coupled with unsteady heat transfer. Examples include thermoacoustic prime movers and heat pumps (Garrett and Backhouse, 2000; Swift, 2002), pulse combustors and engines (Putnam, 1986) and advanced musical instruments, such as a fire organ (Stodola, 2005). Generation of tonal sound in some traditional musical instruments and whistles involves isothermal mean flows in resonators. Even when undesirable, excited tonal sound can also be harvested to produce electricity for powering sensors and other devices (Liu et al., 2008).

In the 19th century, Lord Rayleigh understood the qualitative features of heat-driven acoustic oscillations and proposed a criterion:
“If heat be given to the air at the moment of greatest condensation [i.e., greatest density] or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged.” (Rayleigh, 1945)

Several systems are studied in this work with the purpose to further explain the acoustic instability in resonators with mean flow and heat addition. The systems include a modified Rijke tube, a standing-wave thermoacoustic engine, and a baffled tube with mean flow.

1.2. Modified Rijke tube

Thermoacoustic instabilities are a complex phenomenon, to better understand its nature simplified models must be used. For convenient studies of thermoacoustic instabilities, both experimentally and theoretically, a modified Rijke tube is used. The first tube of this kind (Rijke, 1859) comprised of a vertical pipe with gauze which was located in the lower half of the tube. The gauze was then heated by means of a flame (Figure 1). Due to natural convection, a mean flow appears within the tube. A high intensity tone is generated when the gauze temperature reaches a sufficiently high value.
Although the vertical Rijke tube is quite simple, it still does not permit independent variation of the mean flow and the power supplied to the gauze. A horizontally positioned Rijke tube with the addition of mean flow and electric heater overcomes this deficiency (Figure 2). Another advantage of an electric Rijke tube is the possibility for precise control of the main system parameters in wide ranges: heater location, air flow rate, and power to the heater. For these reasons, this type of Rijke tube is implemented in our experiments.

Figure 1: Original Rijke tube configuration
Certain systems, where thermoacoustic instability is of a concern, have more complex geometries than the single-segment tube. Different system geometries may lead to earlier transition to instability requiring a lower amount of power supplied to the heater. One could consider a double-segment tube with the downstream section having a reduced cross-sectional area (Figure 8). A decreased natural frequency can encourage an early transition to instability, since thermoviscous acoustic losses at the resonator walls increase with frequency. Additionally, a recirculation zone in the mean flow in front of the transition between tube segments may also serve a source of instabilities and encourage early sound onset. Variable-area resonators are also known to ensure earlier transition to instability in other types of thermoacoustic devices that do not involve mean flow, such as standing-wave thermoacoustic engines (Swift, 2002).
1.3. Standing wave thermoacoustic engine

Thermoacoustic engines are energy-conversion devices that produce acoustic power using heat flowing from a high-temperature source to a low-temperature sink. The benefit of a thermoacoustic engine is that they can be made without any moving parts may use a variety of gases as the working fluid. A simple thermoacoustic engine consists of a resonator with one end open and the other end closed, and a piece of porous material, known as a stack, placed inside the resonator at a specified location. The temperature gradient on the stack is maintained externally (Figure 3). A stack position is selected to supply heat to oscillating gas parcels at the moment of their compression and to remove heat at the moment of their rarefaction. Then, according to Rayleigh’s criterion, acoustic power should be generated. The motion of gas parcels is defined by acoustic modes of the engine resonator. The gas parcel experiences temperature changes as it oscillates along the axis of the channel. This is due to adiabatic compression and expansion of the gas by sound pressure and by heat exchange with the solid wall of the channel.
A quasi-one-dimensional theory for low-amplitude thermoacoustic systems has been previously developed and validated by Swift (2002). A variety of thermoacoustic systems can be applied for electricity generation, air-conditioning, refrigeration, and gas mixture separation (Swift, 2002). Thermoacoustic engines can operate with various heat sources; including waste heat, combustion of fuels, solar energy, and radioactive isotopes. Construction and testing of a simple standing-wave demonstrator is described in this work.
1.4. Baffled tube with mean flow

Vortex shedding inside resonators with mean flow can lead to excitation of high-amplitude tones corresponding to resonator acoustic modes. This phenomenon occurs in many practical systems such as duct networks (Harris, 1998), rocket motors (Culick, 2006), and whistles (Wilson et al., 1971). While this sound is often undesirable in most engineering devices, the generated acoustic energy can be harnessed and converted into electricity for powering sensors or other devices, thus eliminating the need for battery replacement. The conversion of acoustic to electric power takes place in electroacoustic transducers. The development of one such transducer, which comprised a piezoelement in a Helmholtz resonator and was intended for harvesting acoustic energy, was described by Liu et al. (2008).

One of the objectives of this study is to demonstrate harvesting of acoustic energy in a simple system with self-sustained acoustic oscillations in the presence of mean air flow. Properly positioned baffles in a pipe produce loud tonal sounds within a certain flow rate range (Nomoto and Culick, 1982). On the upstream baffle, the vortices become detached and impinge themselves on the downstream baffle as shown in Figure 4. Acoustic modes in the pipe consume a portion of the flow energy when the baffles and are placed in the proper location and flow velocity is appropriately selected. The experimental identification of excited acoustic modes and mean flow regimes, corresponding to intensive sound generation, is also accomplished in this work.
Figure 4: Schematic of tube with baffles
Chapter 2

Experimental Systems
2. Experimental Systems

The schematic of the overall general system is shown below in Figure 5. A pipe with a constant or variable cross-sectional area with baffles or heaters placed inside is used as the open-ended acoustic resonator. Specific configurations of the resonator, as well as equipment specific to particular setups are described in the next sections where different operational approaches of the system are discussed. The system elements common to most tests are described in this section.

![Figure 5: Schematic of experimental system](image)

A controlled mean air flow through the resonator is provided by a regulated Jabsco 35440 blower. Custom-built flow meters were used to measure the mean flow rate. To measure low
flow rates, a laminar flow element with a porous material was employed. For measuring high flow rates, an orifice flow meter was utilized. The pressure drop in these devices was detected by Dwyer Magnehelic pressure gages, and temperature of the flow was measured by a type-K thermocouple. Each of the constructed flow meters was calibrated against Dwyer variable-area flow meters.

A settling chamber is placed between the acoustic resonator and the blower in order to minimize interactions between these elements of the system. The damping chamber dimensions are 46 x 46 x 76 cm. The internal surface of the chamber is covered by 2.5 cm thick polyurethane foam with Noise Reduction Coefficient 0.75 in order to reduce sound reflection from the chamber back into the resonator.

An electrically heated nichrome wire with a diameter of 1mm and length of 75cm was used as the heat source in some of the operational modes. The wire was wound on a frame made of either alumina silicate or glass-mica (macor) ceramic to avoid electrical contact with the resonator walls as shown in Figure 6. The electric current was supplied to the heater by a Sorensen DCS33-36E power supply via copper rods with a diameter of 3.2 mm. The heater is inserted inside a resonator via the open tube end. The position of the heat source within the tube can easily be changed without affecting other system elements.
2.1. Modified Rijke tube

The thermoacoustic instabilities that occur in chambers in the presence of mean flow can be conveniently studied using a simplified system by means of using a Rijke tube (Rijke, 1859; Raun et al., 1993). A permeable heat source is typically placed inside the resonator towards the upstream portion of the tube as shown in Figure 7 and Figure 8. Heated gauzes or flames are common heat sources for Rijke tubes. The system used in our case consists of an electrical heater, where the supplied heat is accurately regulated. Acoustic modes can be excited in the tube within certain ranges of the three main variable parameters: mean flow rate, heater location, and supplied heat. In vertically oriented Rijke tubes mean flow is typically produced by means of natural convection, this type of system is commonly used as educational demonstrations. In such configurations however, there is a coupling between mean flow rate and heat added to the
system. To decouple these variables and significantly expand the range of conditions with generated sound, a Rijke tube can be orientated horizontally and a blower can be used to create a mean flow in the system. This configuration is implemented in the present system and shown in Figure 5.

![Figure 7: Constant-area Rijke tube schematic](image)

The constant-area Rijke tube configuration consists of a straight aluminum tube of length 90cm, with an internal diameter of 5.3 cm and wall thickness 4 mm. The heat source described in the

![Figure 8: Segmented-area Rijke tube schematic](image)
previous section (Figure 6) was placed inside the tube. The Rijke tube was placed 5mm into the
damping chamber and the connection between the tube and the chamber wall was sealed shown
in Figure 9.

The structure of the segmented Rijke tube apparatus is shown in Figure 9. The segmented Rijke
tube in this system consists of an upstream aluminum tube with internal diameter 5.2 cm and
length 59.5 cm and a downstream steel tube with internal diameter 2.4 cm and length 30 cm. In
additional tests, only wide aluminum tube was applied to provide reference data of a classical
single-segment Rijke tube setup.

![Figure 9: Experimental setup for segmented Rijke tube with mean flow and heat addition](image)

Figure 9: Experimental setup for segmented Rijke tube with mean flow and heat addition
2.2. Thermoacoustic engine

The Rijke tube described in the previous section can be easily converted into a thermoacoustic engine in the absence of the mean flow. The classical standing-wave engine is comprised of a tube with one closed end and the other open, and a porous material (referred to as a stack) where the temperature gradient is controlled by external means. If the appropriate stack properties are selected and a sufficiently high temperature gradient is imposed the acoustic modes in the system can become unstable (Swift, 2002).

The configuration for the thermoacoustic setup consisted of a 76 cm long aluminum tube with a diameter of 5.3 cm was utilized along with the same heater used for the Rijke tube described in the previous section, as shown in Figure 10. One of the tube ends was sealed closed using a ceramic plate which had holes to accommodate for two copper rods. The other end of the tube remained open and was not inserted into the damping chamber. A branch pipe for capturing acoustic power and converting it to electricity was positioned 13.5 cm from the open end. In order to monitor acoustic pressure, a microphone 1 was positioned 13.5 cm from the open end. The stack used in this system consisted of a square-pore ceramic comb (Celcor) positioned 15.6 cm from the closed end. The stack had an outer diameter corresponding to the inner diameter of the resonator and a length of 4 cm. The square pore has a dimension of 1 mm x 1 mm and there were 24 pores per inch (25.4 mm). The wall thickness around the pore is 0.203 mm. A cold heat exchanger was not applied to the cold side of the stack (left side in Figure 10). Acoustic streaming may play a role as the heat removal mechanism from the right end of the stack. Two type-K thermocouples were placed at the right and left ends of the stack.
A PZT disk element with a brass back plate (model MFT-41T-1.0A1, manufactured by APC International) was chosen as the acoustic energy harvester in this experiment. The PZT layer had a diameter of 2.3 cm and the diameter of the whole element was 41 mm with a thickness of 0.23 mm. The PZT disk was tightly fixed in place with two acrylic flanges bolted together. The complete PZT fixture with the flanges was then placed on the branched end. The position of the element was intended to be close to the pressure anti-node in the second acoustic mode of the system. The resonance frequency and maximum impedance are 1 kHz and 1000 ohms respectively. A photograph of the PZT configuration is shown in Figure 11 and the complete assembled model is shown in Figure 12.
Figure 11: Photograph of the piezoelement held by two flanges

Figure 12: Photograph of thermoacoustic engine experimental setup
Two wires were soldered onto the PZT layer and the brass plate. The wires were then directly connected to a passive electrical load in order to find the electrical power. The electrical power released in this resistor can be determined as follows,

\[ P_E = \frac{V_{RMS}^2}{R} \]  \hspace{1cm} \text{Equation 1}

where \( V \) is the RMS voltage measured across the resistor by means of a multimeter and \( R \) is the electric resistance. The output power is recognized as the harvested power. Five electric loads with resistances 1, 10, 14.8, 55.2, and 100 k\( \Omega \) were varied in this study.

2.3. Baffled tube with mean flow

The Rijke tube discussed in section 2.1 can be easily converted to a setup with two baffles and mean flow for vertex shedding. This experimental setup consisted of a 60 cm long PVC pipe with an inner diameter of 5.3 cm that was used as the main resonator. The PZT with flange configuration was placed 45 cm from the upstream end, close to the damping chamber. A small portion of the resonator was placed into the damping chamber. The same blower used in Rijke tube was used to create a mean flow in the resonator. The damping chamber was used to create an open condition at the downstream end of the resonator, which in turn reduces the interactions the resonator and the blower may have acoustically (Matveev and Culick, 2003). To measure the mean flow rates of the system a calibrated flow meter was placed in the duct between the damping chamber and the blower.
A pair of sharp-edged, 3 mm thick baffles were installed inside the resonator with a spacing of 2 cm between them. The baffles have an orifice diameter of 2.5 cm and were positioned 30 cm from the upstream end. This specific location was chosen since it is close to the velocity antinode in the second acoustic mode of the system.

The acoustic pressure in the system was monitored at three different locations using PCB 377C10 microphones with PCB 426B03 pre-amplifiers. The microphones were flush-mounted on the pipe walls in order to prevent the microphone tip interfering with the acoustic flow. Microphones 1 and 2 were positioned 20 and 45 cm from the main resonators upstream end respectively. Microphone 3 was placed 2 cm below the piezoelement. The signals from the microphones were stabilized and directed to a PCB 482C05 signal conditioner and then displayed on a Rigol DS1064B digital oscilloscope.

A different sized PZT element (model MFT-50T-1.7A1) was used for this set of experiments. The PZT diameter and thickness were the same in this study as that of the PZT used in the thermoacoustic engine. The only difference between the two being that the total diameter of the element in this study was 50mm. The resonance frequency and maximum impedance of this element were 1.7 kHz and 300 Ω, respectively. The overall schematic of this experimental study is shown below in Figure 13.
Figure 13: Schematic of the mean flow and baffle experimental setup
Chapter 3

Experimental Results and Discussion
3. Results

3.1. Segmented Rijke tube

The theory behind the tone excitation in a constant-area Rijke tube and the influence of main variable parameters and resonator geometry are discussed in this section. The sound generation in a Rijke tube can be explained via Rayleigh criterion (Rayleigh, 1945). This criterion implies that the time-averaged thermoacoustic power conversion $\dot{E}_{TA}$ is proportional to the following integral taken over the chamber volume $V$ and acoustic period $T$ (Culick, 1976), assuming small cycle-to-cycle variation of the acoustic amplitudes,

$$ \dot{E}_{TA} \sim \omega \int_0^T \int_V p'(x, t) q'(x, t) \, dv \, dt, \quad \text{Equation 2} $$

where $p'(x, t)$ and $q'(x, t)$ are spatially and time dependent fluctuations of pressure and heat addition per unit volume. Considering small fluctuations of acoustic velocity $u'$ and heat $q'$, the unsteady convective heat transfer rate from the heat wire to the gas flow can be linearized as follows (Merk, 1957),

$$ \frac{\dot{q}'}{Q_0} = |Tr| \frac{u'(x,t-\tau)}{u_0} \delta(x-s), \quad \text{Equation 3} $$

where $\dot{Q}_0$ is the steady convective transfer rate from the heater to the air flow, $\omega$ is the radian frequency of acoustic oscillations, $|Tr|$ is the absolute value of transfer function between velocity and heat fluctuations (Merk, 1957), $\tau$ is the time lag due to thermal inertia in the heat transfer process, $u_0$ is the mean velocity of the incident flow, and $s$ is the heater location. The
delta function in Equation 3 implies that the dominant heat release occurs at the heater, which dimensions are much smaller than the acoustic wave length.

The acoustic pressure in the resonator can be approximated using the idealized wave equation for the longitudinal acoustic oscillations. Assuming low Mach number, uniform gas properties, and absence of loss or excitation mechanisms the idealized wave equation becomes,

\[
\frac{\partial^2 p'}{\partial t^2} - a^2 \frac{\partial^2 p'}{\partial x^2} = 0, \quad \text{Equation 4}
\]

where \(a\) is the average speed of sound. The boundary conditions can be simplified if the ends of the tube are considered to be pressure release terminations (Blackstock, 2000),

\[
p'(0, t) = p'(L, t) = 0 \quad \text{Equation 5}
\]

The acoustic modes of the tube are found by combining and solving Equations 4 and 5.

The dependence of the lowest-frequency mode, sometimes referred to as the fundamental mode, on the spatial coordinate and time is as follows,

\[
p'(x, t) = P_A \sin(kx) \cos(\omega t), \quad \text{Equation 6}
\]

where, \(k = \frac{\pi}{L}\) is the wavenumber, \(\omega = \frac{\pi a}{L}\) is the natural radian frequency of the lowest frequency mode, and \(P_A\) is the maximum pressure amplitude. Since higher modes experience substantially
higher damping in Rijke tubes, they are not considered here. Although higher modes are not considered here, one must note that under special conditions higher modes may become unstable (Matveev, 2003). The acoustic velocity in the fundamental mode can be found using the acoustic momentum equation (Blackstock, 2000)

\[ u'(x,t) = -\frac{\rho}{\rho_a} \cos(kx) \sin(\omega t), \]  
\[ \text{Equation 7} \]

where \( \rho \) is the averaged density. Combining Equations 1, 2, 5 and 6 yields a qualitative dependence of thermoacoustic power conversion on the main variable parameters,

\[ \dot{E}_{TA} \sim \frac{Q_0}{u_0} |Tr| \sin\left(\frac{2\pi s}{L}\right) \sin(\varphi), \]  
\[ \text{Equation 8} \]

where the phase delay, \( \varphi = \omega \tau \), is introduced instead of the time lag. The unsteady convective heat transfer phase delay from a cylinder decreases with an increase in \( u_0 \) from a finite value below \( 90^\circ \) at low mean flow velocities to \( 0^\circ \) at high velocities (Matveev, 2003). Considering the sign in Equation 8, the heater must be placed in the upstream section of the tube to ensure a positive thermal-to-acoustic energy conversion of the fundamental acoustic mode. The range the heater can be positioned for sound generation is shorted due to the presence of acoustic losses, which are weakly dependent on \( u_0, Q_0, \) and \( s \). The convective heat transfer rate \( Q_0 \) increases with supplied electric power with a fixed flow rate and heater location, therefore increasing \( \dot{E}_{TA} \) and expanding the instability domain.
The dependence of $\dot{E}_{TA}$ on the mean flow velocity is more complex. At high mean flow rates, $\varphi$ approaches zero, while $|Tr|$ approaches the quasi-static value $1/2$ for the case of the flow around a cylinder (Lighthill, 1954). Thus, at sufficiently high $u_0$ the thermoacoustic power conversion will decrease. At low mean flow rates, $sin(\varphi)$ approaches a positive constant, while $|Tr|$ decreases (Merk, 1957; Kwon and Lee, 1985).

The total dissipated thermoacoustic power, $\dot{E}_{TA}$, is shown in equation 9,

$$\dot{E}_{TA} = \frac{1}{4} \left( \frac{P_A^2}{\rho_m a^2} \right) \omega \pi R L \left[ \delta_k (\gamma - 1) \left( 1 + \frac{2R}{L} \right) + \delta_v \right] \text{ Equation 9}$$

where $R$ is the radius of the tube, the thermal penetration depth $\delta_k = \sqrt{2 k / \omega}$, and the fluid’s viscous penetration depth $\delta_v = \sqrt{2 \nu / \omega}$ ($\nu$ is kinematic viscosity). The thermoacoustic energy-conversion efficiency, $\eta_s$, can be estimated as follows,

$$\eta_s = \frac{\dot{E}_{TA}}{\dot{E}_H} \text{ Equation 10}$$

where $\dot{E}_H$ is the power supplied to the heater. Evaluating out system with a mean flow rate of 1.6 g/s and 400 W supplied to the heater we get a thermoacoustic power of 6.4 W with a thermoacoustic energy-conversion efficiency of 0.016.

Several test sequences were conducted with the segmented Rijke tube. The first test sequence was with the single-segment, 5.2 cm diameter tube, as shown in Figure 8. The purpose of this test was to obtain reference data in a classical Rijke tube configuration. Preliminary tests were
first conducted to establish a range of heater positions favorable for sound generation within the resonator. The heater was placed 12.5 cm from the upstream end of the tube to conduct further tests in the tube. Various tests were conducted with different mean flow rate settings and in each test the electric power supplied to the heater was slowly increased with a rate of 2 W/min. Audible tonal sound appeared at certain levels of power, which corresponds to the transition to thermoacoustic instability. The single segment tube stability boundary is shown in Figure 14. The lowest value of power required to produce sound is found at an optimal flow rate. The threshold power increases by increasing or decreasing the mean flow rate, which is similar to stability boundaries previously observed in Rijke tubes (Raun et al., 1993, Matveev and Culick, 2003). The frequency of acoustic oscillations measured by an external microphone at the mean flow rate 0.95 g/s is approximately 310 Hz, which is consistent with the natural frequency of the fundamental mode in the studied system.
The most favorable position of the heater for sound generation in a double-segment Rijke tube was experimentally found to be at 35 cm from the tube upstream end. The threshold power values recorded at the sound onset in this configuration are presented in Figure 15. The minimum power required for the transition to instability drops nearly twice in comparison with a single-segment tube (Figure 14), while the range of mean flow rates where instability is possible at available heat power expands significantly. A secondary local minimum in the threshold power data is observed at the flow rate of 1.53 g/s. The sound frequency measured after the sound onset at the flow rate 0.95 g/s is about 165 Hz. A tube with the constant diameter of 5.2 cm and length of 90 cm (similar to the total length of the double-segment tube) was also tried. However, it was discovered that the lowest power necessary for the sound appearance (about 300 W) exceeded
that of the short tube, while the frequency of the excited sound was about 200 Hz. This confirms a strong effect of tube segmentation on the required heating power to achieve thermoacoustic instability.

![Graph showing power required to produce sound in double-segment Rijke tube.](image)

**Figure 15: Power required to produce sound in double-segment Rijke tube.**

3.2. Thermoacoustic engine

The sound generation in the standing-wave thermoacoustic engine can be qualitatively explained by the Rayleigh criterion. A gas parcel oscillating inside the stack in a fundamental acoustic mode receives heat in a compressed state (i.e., it is shifted to the hotter side of the stack in Figure 10). This gas parcel expands and simultaneously rejects heat to the stack, when it is located in the colder stack section (i.e., when it is shifted to the left in Figure 10). This type of heat exchange can occur only if the stack temperature variation in the longitudinal direction is more
significant than the temperature change of a gas parcel caused by an oscillating acoustic pressure. In such cases, the thermoacoustic power conversion is positive in accordance with Equation 2.

For numerical calculations of sound excitation in thermoacoustic engines, both simplified and advanced theories developed by Swift (1988, 2002) can be used. One of the principal theoretical results is the acoustic power generation $\dot{W}_2$ in case of inviscid ideal gas oscillating at two sides of a solid plate with mean temperature gradient $\nabla T$ (Swift, 1988),

$$\dot{W}_2 = \frac{1}{2} \delta_k B L \omega \frac{\gamma - 1}{\gamma} \left( \frac{\nabla T}{\nabla T_{cr.id}} - 1 \right), \tag{Equation 11}$$

where $B$ and $L$ are the beam and length of the plate, respectively, $\omega$ is the radian frequency, $\gamma$ is the specific heat ratio, $p_1$ is the acoustic pressure amplitude, and $p_m$ is the mean pressure. The thermal penetration depth $\delta_k$ and ideal critical temperature gradient $\nabla T_{cr.id}$ are defined as follows,

$$\delta_k = \sqrt{\frac{2 \kappa}{\omega}}, \tag{Equation 12}$$

$$\nabla T_{cr.id} = \frac{\omega p_1}{\rho_m c_p u_1}, \tag{Equation 13}$$

where $\kappa$ is the thermal diffusivity, $\rho_m$ is the mean density, $c_p$ is the specific heat at constant pressure, and $u_1$ is the acoustic amplitude in the gas. The thermal penetration of depth $\delta_k$
corresponds to the distance where heat can travel in the gas during one acoustic cycle. The critical temperature gradient \( \nabla T_{cr,\text{iq}} \) is the threshold for sound onset in the idealized system. Since systems do not operate in ideal settings it should be noted that \( \nabla T_{cr} \) is usually two to three times greater than the ideal \( \nabla T_{cr} \).

Equation 8 gives only a rough idea about acoustic power generated in the stack. More accurate values can be obtained accounted for viscosity and particular stack geometry (Swift, 1998). The critical temperature gradient corresponding to the sound onset in real systems can be several times greater than that given by Equation 12. To calculate the actual \( \nabla T_{cr} \), one would need to balance acoustic power production in the stack with the sum of all acoustic losses in the entire system (Swift, 1988; Jung and Matveev, 2010).

Shown in Figure 16, Figure 17 and Figure 18, are the harvested electric power, acoustic pressure, and temperature difference across the stack respectively. The data taken for all of these tests was taken at the moment when the maximum value for the harvested electric power was established. The maximum electric power harvested by the piezoelement is 0.446 mW at a resistance of 14.8 k\( \Omega \) which is shown in Figure 16. A general trend is noticed that at higher heating powers, higher electric power is harvested for all resistances.

Shown in Figure 17 is the magnitude of acoustic pressure measured by microphone 1. From Figure 18, one can see that there were no substantial pressure changes between 14.8 k\( \Omega \) and 100 k\( \Omega \) loads. Figure 18 shows that a similar trend to the harvested electric power, where the temperature differences of the greater heating power are higher than those induced by the lower
heating power. A result that was not presented but should be noted is that the frequency differences between the resistances were negligible and the frequency at each was approximately 120 Hz. Although our system is not optimized for energy power conversion, it should be noted that optimized thermoacoustic engines may reach up to 20-30% energy conversion efficiency.

Figure 16: Harvested electric power in the thermoacoustic engine for two different powers (solid circle refers to 14.8 kΩ)

Figure 17: Acoustic pressure in the thermoacoustic engine for two different powers
3.3. Baffled tube with mean flow

The tone generation in a baffled tube with mean flow often occurs when the vortex shedding/impingement frequency is close to the natural frequency of one of the acoustic modes. The natural frequencies of a straight pipe with open ends can be calculated with the following equation,

\[ f_n \approx \frac{na}{2(L+2L_e)}, \]

where \( n \) is the number acoustic node number in the pipe, \( a \) is the speed of sound, and \( L \) is the pipe length, and \( L_e \approx 0.61R \) is the length correction of the unflanged pipe end with \( R \) being the pipe radius (Blackstock, 2000). The additions of branches and baffles may affect the natural frequencies. The vortex shedding frequency typically increases with the mean flow velocity. In
the some ranges of flow rates, the vortex frequency approaches the resonator natural frequencies, which leads to excitation of acoustic modes.

The other set of characteristic frequencies in this setup are related to the frequency of vortex impingement on the downstream baffle. These frequencies can be estimated using a modified, low Mach number form of Rossiter’s equation for the vortex-driven cavity tones (Rossiter, 1964)

\[ f_v = n_2 \frac{u_v}{l}, \]  

Equation 15

where \( n_2 \) is the integer number of vortices present between baffles and \( u_v \) is the effective vortex velocity. This velocity is greater than the mean flow in the tube due to flow contraction caused by baffles. This velocity can be evaluated as below:

\[ u_v = k u_0 \left( \frac{p}{d} \right)^2, \]  

Equation 16

where coefficient \( k \) accounts for the recirculation zone formed between the baffles. This coefficient is usually less than one (Dotson et al., 1997). The vortex impingement frequencies are calculated here using \( k = 0.8 \).
Figure 19: Frequencies of tonal sound excited in the baffled tube. Experimental data: circles, $s = 15\text{cm}$; triangles, $s = 30 \text{ cm}$; open symbols, $l = 2\text{cm}$; filled symbols, $l = 4 \text{ cm}$. Dotted lines correspond to natural frequencies with $n_1 = 1,2,3$. Solid lines represents vortex impingement frequencies with $n_2 = 1,2$ and $l = 2$ and $4 \text{ cm}$.

As shown in Figure 19, high intensity tonal sound is excited near the intersections of the tubes natural frequencies (Equation 17) and the vortex impingement frequencies (Equation 18). This suggests that there are strong interactions between hydrodynamic vortex shedding and acoustic motions. A portion of the energy release is fed into acoustic modes from the vortex impingement on the downstream baffle. Vortex shedding is known to occur at a single (Strouhal) frequency in the steady incident flow with no acoustic oscillations. However, when sound waves of high-intensity are present, the oscillating flow modulates the vortex shedding process. This forces vortex detachments form the upstream baffle to occur at a frequency close to one of the natural frequencies of acoustic modes in the tube. Analyzing the vortex-sound interactions in resonators
in full detail is rather complicated (Hourigan et al., 1990; Howe, 1998), but simplified reduced-order models are available (Matveev, 2005).

Simplified calculations using a straight pipe approximation, Equation 19, suggests that the natural frequency of the second mode is approximately 545 Hz. The measured frequencies, 430-470 Hz, differ from the straight pipe approximation due to the more complicated system geometry and the more complex interactions between the vortex dynamic and acoustic oscillations.

In conducting the baffled tube experiment, several fixed mean flow rates were used in the resonator. The harvested electric powers within a given range of flow rates are shown Figure 20, which correspond to a single loud tone excited by vortices shedding and impinging on the baffles. Amongst the three tested resistance loads, 10 kΩ resistive load resulted in the largest harvested power when evaluating all of the studied flow rates. The maximum power recorded was approximately 0.55 mW with the 10 kΩ resistive load. The maximum harvested power for all electric loads occurred with a mean air flow velocity of approximately 2.6 m/s.
Figure 20: Harvested electric power with varying loads

The averaged magnitudes of the measured acoustic pressure and the frequencies of the excited tone are presented below in Figure 21. Through our experiments it was found that the pressure and frequency values were essentially insensitive to the three electrical loads. High acoustic pressure amplitudes generally correspond to large levels of harvested power, which is represented in Figure 20 and Figure 21(a). The microphone located in the upstream portion of the tube measured the highest pressure amplitudes while the lowest values among the three locations are measured with the microphone located where the pipe is split. One can note that the loudest measured sound in the pipe corresponds to about 135 dB and a much weaker sound was measured outside the pipe. The faster convection of shed vortices between the baffles with increased mean flow rates causes the tone frequency to monotonically increase with increasing mean flow rate, shown in Figure 21 (b).
It is useful to compare the harvested power with the power that has to be added to the fluid flow to overcome additional baffle resistance. Using simplified expressions for the orifice head loss, the additional power is found to be in the range from 0.15 – 0.5 W, which gives a power conversion factor of 0.001. Although this is low, the main application would be to produce electricity in configurations where the flow-excited sound is already present, so efficiency is not an issue.
Chapter 4

Concluding Remarks
4. Concluding remarks

Three modular systems, a segmented Rijke tube, thermoacoustic engine, and a baffled tube with mean flow have been developed for studying and demonstrating tonal sound excitation in resonators with mean flow and heat addition. Experiments conducted with a segmented Rijke tube demonstrate that heat required for sound generation in the resonator was half the amount required by the classical single segment Rijke tube. The range of flow rates suitable for sound generation is also significantly greater, which range from 0.2 g/s to 1.7 g/s for the segmented compared to classical Rijke with the range of 0.4 g/s to 1.1 g/s. The reduction of the acoustic losses in the modified system and more effective thermoacoustic power conversion at the heat source may explain the observed effects. This implies that when analyzing practical systems prone to these sorts of instabilities, sufficient amount of detail about system geometries must be taken into consideration. Since the modified segmented Rijke tube requires much less heat to produce sound, a possible implication of the obtained results may be for simplifying Rijke tube lecture demonstrations.

Acoustic energy harvesting has been demonstrated in two simple systems with self-sustained acoustic oscillations in the presence of heat addition (thermoacoustic engine) and mean flow (baffled tube with mean flow). For the thermoacoustic engine, heat was supplied to the resonator and tonal sound was created by a thermoacoustic effect. The thermoacoustic engine generated a maximum electric power of 0.446 mW at a resistance of 14.8 kΩ. In the baffled tube with mean flow, the power generation was more than 0.5 mW of electric power at a resistance of 10 kΩ and mean flow velocity of approximately 2.6 m/s.
Based on the results of the last two experiments, it can be expected that piezoelements will become a viable source of alternative energy for powering sensors or other devices in acoustically, thus eliminating a need for batteries. To further improve the systems, accurate mathematical modeling and more detailed measurements should be taken.
References


