ENERGY-BASED MODELING OF DOWEL-TYPE CONNECTIONS IN WOOD-PLASTIC COMPOSITE HOLLOW SECTIONS

By

WILLIAM ROSSE PARSONS

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN CIVIL ENGINEERING

WASHINGTON STATE UNIVERSITY
Department of Civil and Environmental Engineering

August 2001
To the Faculty of Washington State University:

The members of the Committee appointed to examine the thesis of WILLIAM ROSSE PARSONS find it satisfactory and recommend that it be accepted.

________________________
Chair
ACKNOWLEDGMENTS

First of all I would like to thank my advisor, Don Bender, for all his time, advise, and mentorship throughout my graduate work and the start of my engineering career. I would also like to thank the members of my committee, Mike Wolcott, Dave Pollock, and John Hermanson for providing their unique prospectives and devoting many hours to this project. I would also like to thank Office of Naval Research for providing the project funding.

I would also like to thank the entire staff of the Wood Materials and Engineering Laboratory; everyone at the lab had a part in helping me complete this project. I would especially like to thank Dave Dostal and Scott Lewis for their help extruding the wood-plastic composite material. Thanks also to Bob Duncan and Scott Lewis for helping with sample preparation, testing, and constantly moving my wobbly pallets of material. I would also like to thank all the WSU structural graduate students for their hours of consultation and entertainment. Specifically, Jeff Linville and Vikram Yadama for their tutelage on basic wood science concepts and Brian Tucker for his constant advise, snowboarding trips, golf outings, and late-night concerts. I would also like to thank Chris Brandt, Kristin Meyers, Casey McNeese, Melissa Verwest, and Sara Minier for being a constant source of distraction and great friends.

Finally, I would like to thank my parents, Roger and Rose Marie Parsons, for their moral and financial support. Dad, thanks for all the hours of working extra math problems that provided me with the math skills needed to conduct this research and be an engineer. Mom, thank you for all care packages and for always making sure we did not miss “the Sunday phone call.” I would also like to thank my brothers, Brian and Ben. Brian, thank you for always providing me with an interesting story to tell and a little bit of home when I needed it. Ben, thank you for always keeping the phone calls short by talking as little as possible.
The goal of this research was to develop a rational method of designing dowel-type connections for hollow wood-plastic composite sections. Additionally, a method of predicting the load-displacement behavior of a connection with hollow members was developed for use in energy-based design and deformation calculations.

A yield model consisting of six controlling yield modes was found to govern the hollow section connection behavior. A model for predicting load-displacement behavior of connections with hollow members was derived for the six controlling modes of the hollow section yield model. The models were validated with double-shear unconstrained bolted connection tests using two wood-plastic composite formulations, three wall thicknesses, and three dowel diameters. Input parameters were also quantified through dowel bearing tests and bending yield strength tests. Dowel bearing tests were completed for each combination of WPC formulation, wall thickness, and dowel diameter. Significant variation in dowel bearing strength with dowel diameter and wall thickness was observed.
The hollow section yield model performed well when using a maximum load basis; the average percent difference between the theoretical maximum load and tested maximum load was 5.7%. The maximum connection loads were compared to the theoretical load calculated by entering the dowel bearing strength based on maximum load and a bending yield strength based on the stress in the dowel at the displacement of maximum connection load. Design for WPC hollow section connection maximum loads was based on maximum dowel bearing strength and the 5% diameter offset bending yield strength.

The load-displacement behavior model was validated by comparing the predicted and actual work done by the connections to a displacement of 0.11 inches. The Mode Im work prediction was 4.7% less than the actual value. The Mode IV and Mode IIIx equations under-predicted the actual work by an average of 7.6% and 13.2 %, respectively. All Mode IIIx and Mode IV predicted curves were sensitive to the location parameters of the dowel rotation and dowel yielding.
TABLE OF CONTENTS

ACKNOWLEDGMENTS ............................................................................................................. iii

ABSTRACT ................................................................................................................................ iv

TABLE OF CONTENTS ............................................................................................................... vi

LIST OF TABLES ......................................................................................................................... ix

LIST OF FIGURES ....................................................................................................................... xi

CHAPTER 1 : INTRODUCTION .................................................................................................. 1

Objectives .................................................................................................................................... 2

CHAPTER 2 : LITERATURE REVIEW ....................................................................................... 3

Connection Design Models ......................................................................................................... 3
Load-Displacement Behavior Modeling ..................................................................................... 5
Dowel Bearing Testing................................................................................................................ 5
Defining Yield ............................................................................................................................. 6

CHAPTER 3 : MODEL DEVELOPMENT ................................................................................... 7

Equivalent Specific Gravity......................................................................................................... 7
EYM-Based Yield Model for Allowable Stress Design .............................................................. 8
Load-Displacement Curve Model for Energy-Based Design .................................................... 16

CHAPTER 4 : EXPERIMENTAL METHODS ........................................................................... 21

Wood-Plastic Composite Material............................................................................................. 21
Steel Dowels .............................................................................................................................. 23
Dowel Bearing Strength, $F_e$ ................................................................................................... 23
Bending Yield Strength, $F_{yb}$ ................................................................................................. 25
Connection Testing .................................................................................................................... 27

CHAPTER 5 : RESULTS AND DISCUSSION........................................................................... 31

Dowel Bearing Strength Tests................................................................................................... 31
Bending Yield Strength Tests.................................................................................................... 37
Connection Tests ....................................................................................................................... 41
Model Validation ....................................................................................................................... 48
  Hollow Section Yield Model .................................................................................................. 48
  Load-Displacement Yield Model .......................................................................................... 50
  Load-Displacement Model Sensitivity ................................................................................ 57
Design Procedures for Connections with Hollow Members ..................................................... 60

CHAPTER 6 : SUMMARY AND CONCLUSIONS ................................................................... 61

Summary .................................................................................................................................... 61
Conclusions ............................................................................................................................... 64
Suggestions for Further Research ............................................................................................. 66
## Summary of Derived Equations for the Hollow Section Yield Model

147

## APPENDIX D: COMPUTER PROGRAM TO SIMPLIFY HSYM

148

- **Overview** ................................................................. 149
- **Range of Strength and Section Properties** .................. 149
- **Program Variables** ...................................................... 150
- **Program Code** ............................................................. 151

## APPENDIX E: DERIVATION OF THE LOAD-DISPLACEMENT BEHAVIOR MODEL

157

- **Overview** ................................................................. 158
- **General Procedure** .................................................... 158
- **Derivation of Mode III** .............................................. 162
- **Equation Summary** ..................................................... 168
- **Mode IV Closed-Form Derivation** ............................... 169

## APPENDIX F: TEST DATA

170

- **Dowel Bearing Strength Data** .................................. 171
- **Bending Yield Strength Data** .................................... 184
- **Connection Test Data** .............................................. 189
LIST OF TABLES

Table 3-1: Controlling Hollow Section Yield Mode Equations ................................................... 15
Table 3-2: Energy Terms by Mode .................................................................................................. 18
Table 3-3: Coefficients for Equation 3-6 ...................................................................................... 19
Table 3-4: Load-Displacement Equations for Single Shear Connections ................................. 20
Table 4-1: Number of Dowel Bearing Tests Conducted .............................................................. 25
Table 4-2: Connection Configuration ........................................................................................... 29
Table 5-1: Dowel Bearing Strength Data (yield based on 5% offset) ........................................... 31
Table 5-2: Dowel Bearing Strength Data (based on maximum load) ........................................... 33
Table 5-3: Bending Yield Strength Based on 5% Offset .............................................................. 37
Table 5-4: Bending Yield Strength of Individual Rod Groups ..................................................... 37
Table 5-5: Bending Yield Strength Based on Maximum Load .................................................... 38
Table 5-6: Bending Yield Strength Based on Displacement of the Max Load of Connection Test ........................................................................................................................... 40
Table 5-7: Connection Test Data Based on 5% Offset Method ................................................... 41
Table 5-8: Connection Test Data Based on Maximum Load ....................................................... 42
Table 5-9: Connection Test Results and Predicted Yield (5% offset based) ............................... 48
Table 5-10: Connection Test Results and Predicted Yield (maximum load based dowel bearing strength and bending yield strength based on displacement at maximum connection load) ........................................................................................................................... 49
Table 5-11: Difference between Connection Tests and Load-Displacement Model ................. 51
Table 5-12: Coefficients of Fitted Curves .................................................................................... 53
Table 5-13: Locations used to validate load-displacement model ............................................. 53
Table 5-14: Results of Model Sensitivity Study ....................................................................... 58
Table 5-15: Sensitivity of Mode III, to the location of dowel rotation (x_s) .............................. 59
Table 5-16: Sensitivity of Mode III, to the location of dowel yielding (x_m) ............................ 59
Table 5-17: Sensitivity of Mode IV to the location of dowel yielding (x) ................................... 59
Table 5-18: Connection Test Results and Predicted Yield (maximum load based dowel bearing strength and bending yield strength based on 5% diameter offset method) ............. 60
Table A-1: Yield modes................................................................................................................ 71
Table A-2: Input Parameters ....................................................................................................... 72
Table A-3: Derivation Parameters ............................................................................................ 72
Table A-4: European Yield Model equations ............................................................................. 92
Table A-5: Factors for European Yield Model equations........................................................... 92
Table B-1: Yield Modes................................................................................................................ 94
Table B-2: Input Parameters ....................................................................................................... 95
Table B-3: Derivation Parameters ............................................................................................ 95
Table B-4: European Yield Model equations ............................................................................. 119
Table B-5: Factors for European Yield Model equations........................................................... 119
Table C-1: Input Parameters ....................................................................................................... 122
Table C-2: Derivation Parameters ............................................................................................ 122
Table C-3: Double Shear Equations ......................................................................................... 145
Table C-4: Double Shear Equations For Symmetric Yield Modes......................................... 146
Table C-5: Hollow Section Yield Model Equations ................................................................. 147
Table C-6: Factors for Hollow Section Yield Model................................................................. 147
Table D-1: Range of Strength and Section Properties Used....................................................... 149
Table D-2: Program Variables ................................................................................................. 150
Table E-1: Energy Terms by Mode ......................................................................................... 161
Table E-2: Coefficients for Equation E-2 ................................................................................ 167
Table E-3: Load-Displacement Equations for Single Shear Connections................................. 168
LIST OF FIGURES

Figure 3-1: Double shear connection. Side member and main member have different wall thicknesses. However, the walls are uniform within the individual members. ........ 9

Figure 3-2: Diagram used to develop Mode II - Case 3-3 equation by the virtual displacement method. ..................................................................................................................... 11

Figure 3-3: Assumed yield modes for hollow section yield model. The left member is the side member and the right member is the main member. The shaded region indicates a location of material crushing. .................................................................................... 12

Figure 3-4: Controlling yield modes for hollow section yield model....................................................... 14

Figure 3-5: General internal energy conditions .......................................................................................... 18

Figure 4-1: Triple-box section. Outside dimensions remain constant. However, wall thickness varies between 0.2 inches, 0.3 inches, and 0.4 inches. ...................................................... 22

Figure 4-2: Connection member with dimensions ..................................................................................... 22

Figure 4-3: Dowel bearing test .............................................................................................................. 24

Figure 4-4: The dowel bearing samples were cut from the members after connection testing. ... 25

Figure 4-5: Bending yield test .................................................................................................................... 27

Figure 4-6: Connection test prior to loading. The two LVDTs measure only the displacement of the connection being studied. .................................................................................... 28

Figure 4-7: LVDTs measured the displacement of the connection only. ............................................... 29

Figure 5-1: Typical PVC dowel bearing failures: a) 0.2" wall b) 0.3" wall c) 0.4" wall............. 34

Figure 5-2: Typical HDPE dowel bearing failures: a) 0.2" wall b) 0.3" wall c) 0.4" wall ........... 35

Figure 5-3: Load-displacement curve of PVC 0.4" sample with 3/8" diameter dowel ............. 36

Figure 5-4: Load-displacement curve of HDPE 0.4" sample with 3/8" diameter dowel ............. 36

Figure 5-5: Typical bending yield strength test (3/8" diameter)............................................................. 38

Figure 5-6: Bending yield test diagram ................................................................................................. 39

Figure 5-7: Typical load-displacement curve of Mode I_m connection test - PVC ......................... 43

Figure 5-8: Typical load-displacement curve of Mode I_m connection test - HDPE ..................... 43
Figure 5-9: Typical load-displacement curve of Mode III_s connection test - PVC ................. 44
Figure 5-10: Typical load-displacement curve of Mode III_s connection test - HDPE............. 44
Figure 5-11: Typical load-displacement curve of Mode IV connection test – PVC ............... 45
Figure 5-12: Typical load-displacement curve of Mode IV connection test – HDPE............. 45
Figure 5-13: Confinement in Mode I_m connection tests a) HPDE b) PVC.............................. 46
Figure 5-14: HDPE Mode III_s connections a) Entire connection ........................................ 46
Figure 5-15: PVC Mode III_s connections a) Entire connection ........................................... 46
Figure 5-16: HDPE Mode IV connections a) Entire connection b) Location of dowel yielding in
side member c) Approximate location of dowel and walls during testing ......................... 47
Figure 5-17: PVC Mode IV connections a) Entire connection b) Dowel yielding in side member
c) Approximate location of dowel and walls during testing ....................................... 47
Figure 5-18: PVC Mode I_m Connection Tests with Predicted Curve .................................. 54
Figure 5-19: HDPE Mode I_m Connection Tests with Predicted Curve............................... 54
Figure 5-20: PVC Mode III_s Connection Tests with Predicted Curve............................... 55
Figure 5-21: HDPE Mode III_s Connection Tests with Predicted Curve......................... 55
Figure 5-22: PVC Mode IV Connection Tests with Predicted Curves............................... 56
Figure 5-23: HDPE Mode IV Connection Tests with Predicted Curves............................... 56
Figure A-1: General dowel loading conditions..................................................................... 73
Figure A-2: Single shear connection models......................................................................... 75
Figure A-3: Double shear connection models ...................................................................... 76
Figure A-4: Mode I_m and Mode I_s connection models ...................................................... 77
Figure A-5: Mode II connection model .............................................................................. 78
Figure A-6: Detailed Mode II shear and bending moment diagram.................................... 81
Figure A-7: Mode III_m connection model.......................................................................... 82
Figure A-8: Mode III_s connection model .......................................................................... 85
Figure A-9: Mode IV connection model.............................................................................. 88
Figure A-10: Double shear connection free-body diagrams ................................................................. 90
Figure B-1: Single shear connection models ......................................................................................... 96
Figure B-3: Mode II connection model .................................................................................................. 98
Figure B-4: Mode III_m connection model ............................................................................................ 103
Figure B-5: Mode III_s connection model ............................................................................................. 108
Figure B-6: Mode IV connection model ................................................................................................. 113
Figure C-1: Wrapping Examples ........................................................................................................... 121
Figure C-2: Single shear connection models. Boxes highlight controlling yield modes .................. 124
Figure C-3: Mode I_s and Mode I_m connection model ........................................................................... 125
Figure C-4: Mode II: Case 3-3 connection model ................................................................................ 126
Figure C-5: Mode III_s: Case 3-1 connection model ............................................................................ 133
Figure C-6: Mode III_m: Case 1-3 connection model ........................................................................... 139
Figure C-7: Mode IV: Case 1-1 connection model ................................................................................ 141
Figure C-8: Double shear yield modes due to symmetry ..................................................................... 146
Figure E-1: General internal energy conditions .................................................................................... 161
Figure E-2: Mode III_s connection model ............................................................................................... 162
Figure E-3: Relationship of integration variables .................................................................................. 163
CHAPTER 1: INTRODUCTION

In 1949, Johansen published the basis for the European Yield Model (EYM), which is the current yield model used for dowel-type connection design in wood structures in the U.S., Canada, and Europe. Since the development of the EYM, other methods of modeling bolted connections have been researched ranging from a beam on an elastic foundation model (Kuenzi, 1955) to three-dimensional finite element models (Patton-Mallory et al., 1998). These models either provide crude approximations of connection behavior (beam on elastic foundation models) or more accurate connection behavior but a process that may not be practical for design engineers (finite element models). The EYM represents a reasonable compromise between complexity and accuracy for use by design professionals.

Over the years, the EYM has gone through several revisions and interpretations; however, the basic concepts remain the same. Connection yield strength is based on the geometry of its components (dowel and members), the dowel bearing strength of the member material, and the bending yield strength of the dowel. Experimental research has shown the EYM to be sufficient for the design of timber connections (e.g. Wilkinson, 1978; Soltis and Wilkinson, 1987; McLain and Thangjitham, 1983; Aune and Patton-Mallory, 1986b). Balma (1999) validated the EYM for the design of wood-plastic composite (WPC) members with solid cross-sections.

In timber engineering, the majority of structural components have solid cross-sections. However, many WPC members are extruded in hollow cross-sections, and dowel-type connections may be used to fasten these members. While a majority of WPC products are used as decking, structural framing members will be used in the future and are currently being developed. Another example of hollow sections are structural insulated panels (SIPs) that
consist of a foam core sandwiched between two oriented strand board sheets. SIPs may be considered a hollow cross-section for lateral connection design if the bearing strength of the foam is ignored. The current dowel-type connection yield model (EYM) was developed for members with solid cross-sections. The EYM needs to be modified to accommodate hollow sections.

**Objectives**

The overall goal of this research was to develop rational design procedures for dowel connections using members consisting of wood-plastic composite hollow sections. This goal was achieved by meeting the following objectives:

- Develop a method to predict connection capacity for hollow sections using similar assumptions and derivation procedures as the existing EYM.
- Develop a method to predict the entire load-displacement curve for dowel-type connections in hollow WPC sections
- Validate the yield model and load-displacement model through laboratory testing of bolted connections over a range of WPC formulations, bolt diameters, and wall thicknesses.
CHAPTER 2: LITERATURE REVIEW

Connection Design Models

In 1949, Johansen published the basis for the yield model that is used today to design laterally loaded timber connections in North America and Europe. Over the years, this model has gone through several revisions and interpretations; however, the basic concepts remain the same. Connection yield strength is based on the geometry of its components (dowel and members), the dowel bearing strength of the member material, and the bending yield strength of the dowel. This model is commonly referred to as the European Yield Model (EYM). Experimental research has shown the EYM to be sufficient for the design of timber connections (e.g. Wilkinson, 1978; Soltis and Wilkinson, 1987; McLain and Thangjitham, 1983; Aune and Patton-Mallory, 1986b). Balma (1999) validated the EYM for the design of WPC members with solid cross-sections.

The 1991 National Design Specification for Wood Construction (NDS) was the first NDS edition to include the EYM. Prior to 1991, connection design methods were empirically based. The EYM equation solutions are now arranged in design tables that allow engineers to quickly design common connection configurations. ASTM D5456-98a Annex A2 discusses determination of the design strength of a structural composite lumber connection using the NDS tables. The method described is commonly referred to as the equivalent specific gravity (ESG) method. The equivalent specific gravity method uses the original EYM equations to compute connection strength. Dowel bearing tests are used to compute an equivalent specific gravity. The equivalent specific gravity is then used to design the connection as if it were made of the common material that it most closely resembles. The ESG method is a simplified procedure that enables designers to easily design structural composites using the NDS tables. Johnson and
Woeste (1999) demonstrated several examples that apply this concept to design problems. Bilunas (2000) used a similar procedure to design screw connections in structural insulated panels. The ESG method still relies on a yield model based on solid cross-sections, and does not apply to connecting members with hollow cross-sections.

Several papers have outlined the derivation procedures for the development of the EYM. The American Forest and Paper Association (1999) published Technical Report 12 (TR12) which discusses the static equilibrium-based derivation of the EYM equations. An energy-based derivation of the EYM was developed by Aune and Patton-Mallory (1986a) using the method of virtual displacement to develop connection yield equations. Peyer (1995) expanded the model to include a gap between members at the shear plane. The equations derived produce the same yield load as the equations derived by the static equilibrium-based approach in TR12.

Other methods have also been used to model connection behavior. In an early model, a dowel was modeled as a beam on a finite elastic foundation (Kuenzi, 1955). Wilkinson (1971 and 1972) further refined the modeling equations found by Kuenzi. Wilkinson’s simplifications enabled the beam on elastic foundation concepts to be used by design engineers. However, the connection yield point was still defined empirically. The proportional limit slip of a connection was determined through testing to occur at approximately 0.011 inches (Wilkinson, 1971). However, the load-displacement behavior for most timber connections is nonlinear. Therefore, the beam on elastic foundation equations are only useful for predicting the initial stiffness of a connection (Foschi, 1974).

Three-dimensional finite element models have also been used to model connection behavior (Patton-Mallory et al., 1998). After the finite element model was verified, a maximum stress failure criterion was used to investigate the distribution of critical stress along the dowel in
a connection (Patton-Mallory et al., 1998). However, this type of analysis has not been applied to connection design.

**Load-Displacement Behavior Modeling**

The load-displacement behavior of a connection is important because connection rigidity may contribute significantly to the overall deformation of the structure (Foschi and Bonac, 1977). Researchers have used an empirical model to describe load-displacement behavior in nailed connections (Pellicane et al., 1991; Sá Ribeiro and Pellicane, 1992). Foschi and Bonac (1977) used a finite element approximation of the load displacement behavior of nailed connections with limited success. Aune and Patton-Mallory (1986a) and Peyer (1995) used the general form of the virtual displacement method equation and a fourth-root curve to predict the load displacement behavior of nailed connections.

**Dowel Bearing Testing**

Dowel bearing strengths of most timber species used in construction have been established (Wilkinson, 1991; AF&PA, 1997). Dowel bearing test procedures are outlined in ASTM D5764-97. Balma (1999) found a significant rate of load effect when conducting dowel bearing tests on two formulations of WPCs made from low- and high-density polyethylenes. Presently, no modifications to ASTM D5764-97 have been made for wood-plastic composites. Therefore, it was important to take special care to ensure that the testing procedures used in this research recognize the load rate effect found in WPC.

Several studies have shown that dowel bearing strength is dependent on dowel diameter. Research by Wilkinson (1991) has shown for bolts that in solid cross-sections of timber members the dowel diameter affects the dowel bearing strength in perpendicular to the grain loading.
Balma (1999) found that in two WPC formulations orientation with respect to the extruded direction did not significantly affect dowel bearing strength using 0.5-inch diameter bolts.

**Defining Yield**

In North America, the standard technique for defining the yield point of connection tests, dowel bearing tests, and bending yield strength is the 5% diameter offset method (ASTM D1575, ASTM D5764, and ASTM D5652). A line is fit to the initial linear region of the load-displacement curve of a test. The line is then offset by 5% of the dowel diameter in the positive displacement direction. The yield point is defined as the intersection of the offset line and the load-displacement curve. In some WPC formulations and nailed timber connections, the 5% offset method become cumbersome because there is no definite initial linear region (Balma, 1999; Theilen et al., 1998). When common methods of defining yield prove ineffective, Balma (1999) worked from a basis of maximum load when comparing WPC connection tests with EYM predicted loads.
CHAPTER 3: MODEL DEVELOPMENT

Several design methodologies were investigated. Two models were derived for members with hollow sections. One model predicts the yield point, and is formulated for the allowable stress design of laterally loaded connections with hollow members. The other model predicts the entire load-displacement behavior of laterally loaded connections with hollow members, and will be useful for energy-based design approaches and structural deformation calculations.

**Equivalent Specific Gravity**

The equivalent specific gravity (ESG) method is a simplified procedure that enables designers to easily design connections in structural composites using the NDS tables. The first step of the ESG method is to conduct several dowel bearing tests on a structural composite. Next, the average dowel bearing strength is computed. Then, the equivalent specific gravity is found using the appropriate formula. For bolts, the following equations are used for parallel-to-grain loading and perpendicular-to-grain loading respectively:

\[
ESG_{\parallel} = \frac{F_{el}}{11200} \\
ESG_{\perp} = \left( \frac{F_{el} \sqrt{D}}{6100} \right)^{0.6897}
\]

Next, the NDS dowel bearing strength tables are consulted (e.g. NDS Table 8A for bolted connections). A timber species grouping is found that has a specific gravity less than or equal to the equivalent specific gravity of the structural composite. Connections are then designed assuming that the structural composite has the same dowel bearing strength as the assumed timber species grouping. Because of the hollow geometries of the cross-sections used in this research, the NDS tables were not directly applicable. Therefore, an equivalent specific gravity was not found. Instead, the EYM equations were evaluated using the average dowel bearing
strength. By evaluating the equations instead of using the NDS tables, the yield mode was also identified.

**EYM-Based Yield Model for Allowable Stress Design**

In connections with members with a solid cross-section, the dowel is supported continuously throughout the connection. In a connection with hollow members, the dowel is supported only at the walls which limits the potential locations of dowel rotation and dowel yielding; thus, the number of possible yield modes is increased. In this research, only hollow sections with two walls will be investigated.

Prior to deriving a yield model for hollow sections, the current design model for timber connections, the European Yield Model (EYM), derivation was examined. Two previous modeling approaches were investigated: a static equilibrium-based approach in AF&PA (1999) and an energy-based approach in Aune and Patton-Mallory (1986a). Both papers only included partial derivations. Therefore, the EYM was rederived using the guidelines set forth by the previous work. The complete derivation of the EYM using the static equilibrium-based approach is given in APPENDIX A. The energy-based derivation of the EYM is given in APPENDIX B. Although the two derivations are similar, the energy-based approach is more straightforward and enables the derivation of entire load-displacement curves. Therefore, an energy-based derivation method was chosen to derive the hollow section yield model.

Several simplifying assumptions were deemed necessary for hollow sections. First, members are assumed to have two walls with equal wall thickness and dowel bearing strength (Figure 3-1). The side member and main member can have different wall thicknesses and dowel bearing strengths, but the wall properties are constant within members. Also, identically to the EYM, double shear connections must be symmetric (identical side members).
End fixity of the dowel, tension forces in the dowel, and friction between members are conservatively ignored. Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel. All materials are assumed to exhibit perfect elastic/plastic behavior.

![Figure 3-1: Double shear connection. Side member and main member have different wall thicknesses. However, the walls are uniform within the individual members.](image)

Like the EYM, the input parameters into the hollow section yield model are size of the connection components, dowel bearing strength of the members, and the bending yield strength of the dowel. For the derivation of equations, the dowel bearing strength is converted to dowel bearing resistance (a line load) by multiplying by the dowel diameter and the bending yield strength is converted to the moment resistance of the dowel (a moment) by multiplying by the plastic section modulus.

The energy-based derivation procedures use the virtual displacement method. External work and internal work are set equal to each other as a connection undergoes a unit deformation. The general equation for this energy balance is Equation 3-3.

$$W = F \cdot 1 = \int f_e \cdot \eta \cdot d\xi + \sum (M_y \theta)$$  

Equation 3-3
where:

\[ F = \text{yield load, lb} \]
\[ F_e = \text{dowel bearing strength, psi} \]
\[ F_{yb} = \text{bending yield strength of dowel, psi} \]
\[ D = \text{dowel diameter, in} \]
\[ f_e = F_e D = \text{dowel bearing resistance, lb/in} \]
\[ M_y = F_{yb} \left( \frac{D^3}{6} \right) = \text{moment resistance of the dowel, lb-in} \]
\[ \theta = \text{angle of rotation of the dowel} \]
\[ \eta, \zeta = \text{integration variables for the area crushed by the dowel} \]

The general equation can be simplified to Equation 3-4.

\[ F = \sum (f_e \cdot A) + \sum \left( \frac{M_y}{a} \right) \quad \text{Equation 3-4} \]

where:

\[ A = \text{area of material crushed, in}^2 \]
\[ a = \text{distance from the point of dowel rotation or dowel yielding in the side member (x_s)} \]
\[ \text{to the point of dowel rotation or dowel yielding in the main member (x_m)} \]
\[ = x_s + x_m = \frac{1}{\tan \theta} = \frac{1}{\theta} \quad \text{(small rotation assumed)} \]

The simplified equation is now evaluated in the following manner to determine the yield equations for the hollow section model:

1. The dowel and member in a single-shear connection after undergoing the unit displacement are drawn (Figure 3-2). Note: the dimensions defined in Figure 3-2 apply to every yield mode. Wall thickness is defined as \( t \) and the void width is defined as \( v \). The subscript identifies if the variable corresponds to a dimension in the main or side member.
Figure 3-2: Diagram used to develop Mode II - Case 3-3 equation by the virtual displacement method.

The EYM assumes six yield modes in this step. For the hollow section model, the same six yield modes were assumed. Two of the yield modes involve crushing of the entire main or side member so the equations are the same as the EYM equations. However, due to the void space in a hollow section, in Modes II, III_s, III_m, and IV there are several locations where dowel rotation or dowel yielding can occur in each member. Each scenario is given a different case name. Case 1 is when the dowel rotation or yielding occurs in the wall adjacent to the shear plane. Case 2 is when the dowel rotation or yielding occurs in the void space. Note, Case 2 cannot occur unless the void has negligible dowel bearing strength. Case 3 is when the dowel rotation or yielding occurs in the wall farthest from the shear plane. The combinations result in eighteen possible yield modes (Figure 3-3). The subscript on the yield mode number describes which member is being crushed – $s$ for side member and $m$ for...
main member. For example, the yield mode where the dowel rotation occurs in the wall farthest from the shear plane in the side member and dowel yields in the wall closest to the shear plane in the main member is termed Mode IIIc: Case 3-1.

Figure 3-3: Assumed yield modes for hollow section yield model. The left member is the side member and the right member is the main member. The shaded region indicates a location of material crushing.
2. Expressions for $A$ and $a$ are written in terms of the connection dimensions. In all cases,

$$a = x_s + x_m.$$  The terms $x_s$ and $x_m$ define the location of dowel rotation or bending, and they are the only unknown distances in the problem. An expression for the area crushed by the dowel must be found in terms of the horizontal distances only. The crushed area of material is either triangular or trapezoidal in shape.

3. Next, an expression for the embedment stress distribution is written and used to relate the dimensions of the side and main members. It should now be possible to write a function for the yield load, $F$, that is only a function of one unknown variable ($x_s$). This unknown is the location of dowel rotation in Mode II and Mode III connections or the location of the hinge in Mode III and Mode IV connections.

4. The derivative of $F$ with respect to the unknown variable $x$ is now computed and set to zero. The variable $x_s$ is solved for and thus the location where the energy is minimized is located.

5. Finally, the expression for $x_s$ is now substituted back into the function for $F$. The resulting equation is the yield equation in terms of the connection dimensions and dowel bearing strengths only. The equation is then reduced to a design format.

This procedure was completed for all eighteen yield modes. The equations were further simplified for use with the quadratic formula. Additionally, four double shear yield modes due to symmetry were included. However, these yield modes could not control connection design because the imposed boundary conditions only increased the energy from the assumed yield modes. The complete derivation of the hollow section yield model can be found in APPENDIX C.

A computer program was written to evaluate the yield equations over the complete range of reasonable property values and connection geometries. The program involves a series of
nested loops that evaluated all eighteen equations for the connection properties of that iteration and records the governing yield mode and case. APPENDIX D lists the ranges of properties included and other details of the computer program used.

The program verified that only six equations controlled connection behavior; therefore, the other twelve equations could be eliminated from the model. The resulting yield modes are Mode I_s, Mode I_m, Mode II: Case 3-3, Mode III_s: Case 3-1, Mode III_m: Case 1-3, and Mode IV: Case 1-1 (Figure 3-4). The controlling yield equations are shown in Table 3-1. A pattern in the controlling cases was observed as all rotation of the dowel occurs about a point in the wall(s) farthest from the shear plane and all dowel yielding occurs in the wall(s) next to the shear plane.

Figure 3-4: Controlling yield modes for hollow section yield model.
Table 3-1: Controlling Hollow Section Yield Mode Equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Single Shear</th>
<th>Double Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_m</td>
<td>( F = 2t_m f_{em} )</td>
<td>( F = 2t_m f_{em} )</td>
</tr>
<tr>
<td>I_s</td>
<td>( F = 2t_s f_{es} )</td>
<td>( F = 4t_s f_{es} )</td>
</tr>
<tr>
<td>II-IV</td>
<td>( F = \frac{-B + \sqrt{B^2 - 4AC}}{2A} )</td>
<td>( F = \frac{-B + \sqrt{B^2 - 4AC}}{A} )</td>
</tr>
</tbody>
</table>

Coefficients for Mode II-IV Equations

<table>
<thead>
<tr>
<th>Mode</th>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode II</td>
<td>3-3</td>
<td>( \frac{1}{4f_{em}} + \frac{1}{4f_{es}} )</td>
<td>( t_s + v_s + t_m + v_m )</td>
<td>(- \left[ f_{es} t_s \cdot (t_s + v_s) + f_{em} t_m (t_m + v_m) \right] )</td>
</tr>
<tr>
<td>Mode III _m</td>
<td>1-3</td>
<td>( \frac{1}{4f_{em}} + \frac{1}{2f_{es}} )</td>
<td>( t_m + v_m )</td>
<td>(- \left[ f_{em} t_m (t_m + v_m) + M_y \right] )</td>
</tr>
<tr>
<td>Mode III _s</td>
<td>3-1</td>
<td>( \frac{1}{2f_{em}} + \frac{1}{4f_{es}} )</td>
<td>( t_s + v_s )</td>
<td>(- \left[ f_{es} t_s \cdot (t_s + v_s) + M_y \right] )</td>
</tr>
<tr>
<td>Mode IV</td>
<td>1-1</td>
<td>( \frac{1}{4f_{em}} + \frac{1}{4f_{es}} )</td>
<td>0</td>
<td>(- M_y )</td>
</tr>
</tbody>
</table>

See Table C-1 and Table C-2 for the definitions of the individual terms. Recall that \( f_e = F_e D \).

The Mode I equations are evaluated directly. The Mode II, Mode III, and Mode IV equations utilize the quadratic formula where each mode has different coefficients.

One check of the hollow section yield model is that when the void width equals zero, the equations produce identical results to the EYM. The model was further verified by testing double shear connections and comparing the results.
Load-Displacement Curve Model for Energy-Based Design

The load-displacement curve of a connection provides information useful to energy-based design and can be incorporated into predictions of the overall deformation of a structure. The derivation techniques of the load-displacement method are identical to the methods used to develop the design model. The modeling relies on the virtual displacement method (Equation 3-3). The difference between the two derivations is that instead of displacing a connection yield mode one unit, a yield mode is displaced by the displacement, $\delta$. The term $\delta$ is no longer mathematically removed from the equations. Rather, the final expression for the yield load, $F$, is a function of the connection displacement, $\delta$. The load-displacement model only includes the six controlling modes and cases found during the development of the design yield model. Complete details of the load-displacement model derivation can be found in APPENDIX E.

One advantage of a model that predicts the entire load-displacement behavior of a connection is that the model output no longer relies on ambiguous yield points from the dowel bearing strength and bending yield strength curves. Instead, the strength of the connection materials are entered as curves fit to the dowel bearing or bending yield test data. For this research, sixth-order polynomials (restricted to pass through the origin) were fit to all the dowel bearing tests and bending yield tests conducted. The general form of the fitted curves is shown in Equation 3-5.

$$F = C_1\delta + C_2\delta^2 + C_3\delta^3 + C_4\delta^4 + C_5\delta^5 + C_6\delta^6$$  

Equation 3-5

Similarly to the hollow section yield model, the load-displacement model uses the strength properties converted to line loads and moments. The load from the dowel bearing tests was divided by the sum of the wall thicknesses to produce dowel bearing resistance. Curves fit to the dowel bearing resistance versus displacement curves for the side members are given the
coefficients of A through F. Curves fit to the dowel bearing resistance versus displacement curves for the main members are given the coefficients of L through Q. For the bending yield strength tests, the load versus displacement curve must be changed to a bending moment versus displacement curve by multiplying the load by one-fourth the span. The displacement of the bending yield test must be converted to the connection test displacement using the angle of rotation of the dowel at midspan prior to entering the bending moment curve into the load-displacement model. Curves fit to the bending moment versus displacement at midspan are given the coefficients of Am through Fm.

The Mode Is and Im load-displacement model equations are simply the curves fit to the dowel resistance data multiplied by the sum of the wall thicknesses. Modes II, III, and IV utilize the method of virtual displacement. The external energy consists of the yield load, \( F \), times the displacement, \( \delta \). The internal energy depends on the yield mode and is a combination of dowel rotation and dowel yielding. Five contributions of internal energy have been defined (Figure 3-5). E1, E2, E3, and E4 relate to energy of material crushing. E1, E2, and E3 relate to energy of material crushing when the dowel rotates about a point in the wall farthest from the shear plane. E4 relates to energy of material crushing when a hinge forms within the wall closest to the shear plane. E5 corresponds to the energy of forming a hinge in the dowel. Table 3-2 defines the types of internal energy present in each yield mode. The total internal energy is the sum of all the applicable E-terms to the side and main members. For example, for Mode III_s: Case 3-1, the internal energy equals \( E_{1s} + E_{2s} + E_{3s} + E_{4m} + E_{5m} \).
Table 3-2: Energy Terms by Mode

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Applicable Internal Energy Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode II: Case 3-3</td>
<td>$E_1, E_2, E_3$</td>
</tr>
<tr>
<td>Mode III_{m}: Case 1-3</td>
<td>$E_4, E_5$</td>
</tr>
<tr>
<td>Mode III_{c}: Case 1-1</td>
<td>$E_1, E_2, E_3$</td>
</tr>
<tr>
<td>Mode IV: Case 1-1</td>
<td>$E_4, E_5$</td>
</tr>
</tbody>
</table>

Predicting an entire load-displacement curve is mathematically intensive and hence a spreadsheet was used. Derived equations are evaluated at specific displacements to produce ordered pairs of displacement and load. In order to utilize a spreadsheet more effectively, separate equations were developed for each component of the internal energy. A general equation has been identified for the internal energy due to material crushing ($E_1, E_2, E_3, \text{ and } E_4$) (Equation 3-6).

$$E_{1,2,3,\text{ or } 4} = \frac{\delta}{840a^7} \left[ 280C_1a^5(i^3 - j^3)\delta^2 + 210C_2a^4(i^4 - j^4)\delta^2 + 168C_3a^3(i^5 - j^5)\delta^3 + 140C_4a^2(i^6 - j^6)\delta^4 + 120C_5ai(i^7 - j^7)\delta^5 + 105C_6(i^8 - j^8)\delta^6 \right]$$

Equation 3-6

where:

$$a = x_s + x_m$$

$C_1, C_2, C_3, C_4, C_5, \text{ and } C_6$ are the coefficients from the fit dowel resistance curves $i$ and $j$ depend on the type of energy (Table 3-3)
Table 3-3: Coefficients for Equation 3-6

<table>
<thead>
<tr>
<th>Internal Energy Type</th>
<th>$i$</th>
<th>$j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E$_1$</td>
<td>$2t + v - x$</td>
<td>0</td>
</tr>
<tr>
<td>E$_2$</td>
<td>$x - t - v$</td>
<td>0</td>
</tr>
<tr>
<td>E$_3$</td>
<td>$x$</td>
<td>$x - t$</td>
</tr>
<tr>
<td>E$_4$</td>
<td>$x$</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: $x$ will either be $x_{s}$ or $x_{m}$ depending on the mode and member being considered

Equation 3-7 gives the expression for energy associated with dowel yielding ($E_5$).

$$E_5 = \frac{M_y(\delta)}{x_s + x_y}$$  \hspace{1cm} \text{Equation 3-7}

As stated earlier, a sixth-order polynomial curve was fit to the bending yield strength load-displacement data (Equation 3-8). This fitted curve cannot be used directly in the $E_5$ equation because the displacement in the bending yield test is different than the displacement in a connection test. The angle of rotation of the dowel was used to relate the two tests and produce Equation 3-9.

$$M_y(\Delta) = A_m \Delta + B_m \Delta^2 + C_m \Delta^3 + D_m \Delta^4 + E_m \Delta^5 + F_m \Delta^6$$  \hspace{1cm} \text{Equation 3-8}

$$M_y(\delta) = A_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right] + B_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^2 + C_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^3 + D_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^4 + E_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^5 + F_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^6$$  \hspace{1cm} \text{Equation 3-9}

\textit{where:}

$\Delta = \text{displacement in bending yield strength test}$

$\delta = \text{displacement in connection test}$

Table 3-4 summarizes the load-displacement predicting equations for single shear connections. The double shear equations are produced by multiplying the single shear equations
by two; except in Mode $I_m$ where the single shear and double shear equations are identical. Only the Mode $I_m$, Mode III, and Mode IV equations were compared to test data.

Table 3-4: Load-Displacement Equations for Single Shear Connections

<table>
<thead>
<tr>
<th>Mode</th>
<th>Load-Displacement Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_s$</td>
<td>$F(\delta) = 2t_s \left( A\delta + B\delta^2 + C\delta^3 + D\delta^4 + E\delta^5 + F\delta^6 \right)$</td>
</tr>
<tr>
<td>$I_m$</td>
<td>$F(\delta) = 2t_m \left( L\delta + M\delta^2 + N\delta^3 + O\delta^4 + P\delta^5 + Q\delta^6 \right)$</td>
</tr>
<tr>
<td>II: Case 3-3</td>
<td>$F(\delta) = \left( E_{1s} + E_{2s} + E_{3s} + E_{1m} + E_{2m} + E_{3m} \right)/\delta$</td>
</tr>
<tr>
<td>III: Case 3-1</td>
<td>$F(\delta) = \left( E_{1s} + E_{2s} + E_{3s} + E_{4m} + E_{5} \right)/\delta$</td>
</tr>
<tr>
<td>III: Case 1-3</td>
<td>$F(\delta) = \left( E_{4s} + E_{5} + E_{1m} + E_{2m} + E_{3m} \right)/\delta$</td>
</tr>
<tr>
<td>IV: Case 1-1</td>
<td>$F(\delta) = \left( E_{4s} + E_{5} + E_{4m} + E_{5} \right)/\delta$</td>
</tr>
</tbody>
</table>

The equations for Mode II, III, and IV shown in Table 3-4 assume that the locations of dowel yielding and/or dowel rotation are known (i.e. $x_s$ and $x_m$). In the hollow section yield model these terms were solved for directly by taking the derivative of the yield load expression, setting it to zero, and solving for $x$. In the derivation of the load-displacement equations, this approach is not practical due to the increased number of terms. Therefore, $x_s$ and $x_m$ must either be assumed or found by other means in order to evaluate the load-displacement equations.

Assuming the values of $x_s$ and $x_m$ remain constant during connection deformation, approximate values may be obtained using the expressions for $x_s$ and $x_m$ found during the development of the yield model (APPENDIX C).

In the Mode IV connections used in this research, the side member and the main member are made of the same material. Therefore, only one set of coefficients are needed and the Mode IV equation may be simplified to Equation 3-10.

$$F(\delta) = \sqrt{M_y(\delta) \left[ \frac{2}{3} A\delta + \frac{1}{4} B\delta^2 + \frac{1}{10} C\delta^3 + \frac{1}{24} D\delta^4 + \frac{1}{56} E\delta^5 + \frac{1}{128} F\delta^6 \right]}$$

Equation 3-10
CHAPTER 4: EXPERIMENTAL METHODS

The predicted connection capacities were validated with the laboratory tests. The tests consisted of double shear bolted connection tests. The testing was limited to connections where the main and side members were loaded parallel to their extruded directions. The model was validated through testing Mode I_m, Mode III_s, and Mode IV connections. Prior to connection testing, dowel bearing tests and bolt yielding tests were conducted. The testing utilized three wall thicknesses and three dowel sizes.

The experimental data were collected and analyzed as outlined in the appropriate ASTM standard (discussed individually below). All tests used the 5% diameter offset method to determine yield points. In cases where the 5% diameter offset intersected the load-displacement curve after the maximum load is reached, the maximum load was used as the yield load.

Wood-Plastic Composite Material

The wood-plastic composite materials used in this research were produced with a parallel counter-rotating, twin-screw extruder (Cincinnati Milacron Atlas 93) with a stranding die at the Wood Materials and Engineering Laboratory. In order to bracket the current range of WPC stiffness, two WPC formulations were used. One formulation consists of 50% Ponderosa Pine flour (AWF #4020) and 50% polyvinyl chloride compound (Georgia Gulf: 3014 nat 00) (PVC formulation). The other formulation consists of 66% Maple flour (AWF #4010), 31% high density polyethylene (Equistar: LB 0100 00), 2% zinc stearate (Ferro Chemicals Synpro DLG-20B) and 1% EBS wax (GE Specialty) (HDPE formulation). Three different cross-sections were used to achieve the necessary yield modes to validate the models. The test specimens consisted of triple-box sections shown in Figure 4-1. The outside dimensions of the triple box section remained constant; however, the average wall thickness varied from 0.2 inches, 0.3 inches, and
0.4 inches, which in turn changes the void distance. The connection members were formed by cutting the WPC material into pieces twenty-inches in length. There were two holes of three-quarter-inch diameter to attach the test members to the testing machine and one hole with a diameter equal to the dowel diameter at the location of the connection being studied. The locations of the holes are shown in Figure 4-2.

![Figure 4-1: Triple-box section. Outside dimensions remain constant. However, wall thickness varies between 0.2 inches, 0.3 inches, and 0.4 inches.](image)

ASTM 5652-95 Section 8.1 discusses sample conditioning and states that wood and wood based products for bolted connection tests should be conditioned to meet the objectives of
the testing. ASTM 1761-88 Section 27.1 suggests that wood should be conditioned at a
temperature of 68 ± 6°F and a relative humidity of 65 ± 3%. ASTM D618-96 deals with
conditioning plastics for testing and suggests that the material should be stored at a temperature
of 73.4 ± 3.6°F and a relative humidity of 50 ± 5%. For this research, the WPC samples were
conditioned in the lab environment where testing occurred. That was at approximately 73°F and
30% relative humidity.

**Steel Dowels**

Three dowel sizes were used in this research with nominal diameters of 3/8”, 1/4”, and
3/16”. The 3/8” and 1/4” diameter samples consisted of zinc-plated A307 bolts obtained from
local hardware vendors. To reduce variability, the bolts of each diameter were selected from the
same manufacturing lot. The 3/16” dowels consisted of mild steel welding rod that was cut into
six-inch pieces from an original length of three-feet. Ten three-foot rods were purchased at the
same time and location.

**Dowel Bearing Strength, \( F_e \)**

The test methods outlined in ASTM D5764-97 were used for the dowel bearing tests. A
dowel bearing test involves compressing a dowel into a specimen (Figure 4-3). The dowel is
placed in a semicircular hole created by first drilling a hole and then cutting the member in half
at the location of the hole. The dowel bearing strength is found by using Equation 4-1. The yield
load is commonly found using the 5% diameter offset method.

\[
F_e = \frac{Yield\ Load}{\sum (Wall\ Thicknesse)[Dowel\ Diameter]} \quad \text{Equation 4-1}
\]

There is no dowel bearing test standard that specifically addresses hollow sections.

ASTM D5764-97 calls for minimum specimen dimensions based on the diameter of the fastener
being embedded. Since WPC are extruded from a die with fixed cross-sectional dimensions,
only the specimen length is variable. However, the outside dimensions of the cross-sections used in this research are large enough to meet the dimension requirements of ASTM D5764-97. The length of the dowel bearing samples was 3.5 inches. This same block size was used for all the cross-sections being studied. The dowel bearing samples were removed from the end of the connection test samples (Figure 4-4).

Figure 4-3: Dowel bearing test

ASTM D5764-97 suggests a displacement rate that results in maximum load in one to ten minutes. Balma (1999) showed that dowel bearing strengths of two WPC formulations vary while testing in this large displacement rate range. For this research, all dowel bearing tests were conducted at a constant crosshead displacement of 0.04 inches per minute which resulted in reaching maximum load in about three minutes for most test groups.
Table 4-1 lists the number of dowel bearing tests conducted for this research. The test matrix is larger than necessary to validate the connection model. Several additional groups were added to determine if the dowel bearing strength varied with bolt diameter. The only deviation from ASTM D5764-97 procedures was that the hole drilled in the member was not oversized to be consistent with the connection tests.

Figure 4-4: The dowel bearing samples were cut from the members after connection testing.

<table>
<thead>
<tr>
<th>Dowel Diameter</th>
<th>HDPE 0.2” wall</th>
<th>HDPE 0.3” wall</th>
<th>HDPE 0.4” wall</th>
<th>PVC 0.2” wall</th>
<th>PVC 0.3” wall</th>
<th>PVC 0.4” wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8”</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>14</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1/4”</td>
<td>13</td>
<td>10</td>
<td>12</td>
<td>5</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>3/16”</td>
<td>0*</td>
<td>0*</td>
<td>16</td>
<td>0*</td>
<td>0*</td>
<td>0*</td>
</tr>
</tbody>
</table>

* Dowel bearing tests only required on HDPE 0.4” wall thickness with 3/16” dowel to enable validation of HDPE Mode IV connection tests.

**Bending Yield Strength, \( F_{yb} \)**

The average bending yield strength of the three diameters of bolts used in the connection testing were measured and used as a model inputs. ASTM F1575-95 outlines the determination of the bending yield strength of nails. There is no ASTM standard specifying procedures for
determining the bending yield strength of bolts. Therefore, ASTM F1575 was utilized as a
guideline for determining the bending yield strength of the bolts in this research. A displacement
rate of 0.25 inches per minute was used.

Figure 4-5 shows a typical bending yield strength test. The test apparatus consists of
two-3/4” diameter high-strength steel dowels spaced at a span of four inches on center that
support the test specimen. Another 3/4” diameter high-strength steel dowel is used to load the
test specimen at midspan. The beam formula for a simply-supported beam with a point load at
midspan is used to calculate the moment in the dowel. The bending yield stress is then found by
dividing the moment in the dowel at yield by the plastic section modulus of the dowel (Equation
4-2). The yield load is commonly found using the 5% diameter offset method.

\[
F_{yb} = \frac{3 (Yield \ Load)(Span)}{2 (Dowel\ Diameter)^3}
\]  
Equation 4-2

Fifteen bending yield tests were conducted for the 1/4” diameter bolts. For the 3/8” bolts,
24 bending yield tests were conducted. The increased sample size was required to study the
variability of the bending yield strengths between several boxes of bolts. This extra check was to
insure that indeed all the bolts had the same bending yield strength. A different procedure had to
be performed on the 3/16” dowels since they were purchased in three-foot lengths. The rods
were cut into six-inch pieces. The original rod number was carefully marked on each piece.
Bending yield strength tests were then conducted on four of the six-inch pieces. The five
weakest rods were used in the connection tests because they would be most likely to form plastic
hinges.
Connection Testing

Double shear connections were tested in tension to validate the theoretical models. The typical connection configuration is shown in Figure 4-6. During all the connection tests, load and displacements measurements were recorded. The load was read using a 5-kip load cell with a resolution of ± 1%. Displacement was measured using two LVDTs with a resolution of ± 0.001 inches mounted on either side of the connection being studied (Figure 4-7). Readings from the two LVDTs were averaged for use in the subsequent data analysis.

ASTM D1761-88 and ASTM D5652-95 specify bolted connection testing for solid timber members. Both standards suggest similar testing procedures and sample sizes of five to ten specimens. For this research, a sample size of five was used for the Mode I<sub>m</sub> connections. A sample size of ten was used for Mode III<sub>s</sub> and Mode IV connections. The small sample size is
justified because preliminary testing has indicated a coefficient of variation (COV) lower than the assumed COV for timber connections (ASTM D1761-Note 6). Using Equation 1 in ASTM D2915, a sample size of ten pertains to a 95% confidence interval and a COV of less than or equal to 7%. For a sample size of five, the COV must be less than or equal to 3%. Table 4-2 contains the test matrix for the connection tests.

Figure 4-6: Connection test prior to loading. The two LVDTs measure only the displacement of the connection being studied.
Figure 4-7: LVDTs measured the displacement of the connection only.

Table 4-2: Connection Configuration

<table>
<thead>
<tr>
<th>WPC Formulation</th>
<th>Mode</th>
<th>Sample Size</th>
<th>Dowel Diameter</th>
<th>Main Member Wall Thickness</th>
<th>Side Member Wall Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE</td>
<td>Iₘ</td>
<td>5</td>
<td>3/8”</td>
<td>0.2”</td>
<td>0.3”</td>
</tr>
<tr>
<td></td>
<td>IIIₑ</td>
<td>10</td>
<td>1/4”</td>
<td>0.4”</td>
<td>0.2”</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>10</td>
<td>3/16”</td>
<td>0.4”</td>
<td>0.4”</td>
</tr>
<tr>
<td>PVC</td>
<td>Iₘ</td>
<td>5</td>
<td>3/8”</td>
<td>0.2”</td>
<td>0.3”</td>
</tr>
<tr>
<td></td>
<td>IIIₑ</td>
<td>10</td>
<td>3/8”</td>
<td>0.4”</td>
<td>0.2”</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>10</td>
<td>1/4”</td>
<td>0.4”</td>
<td>0.4”</td>
</tr>
</tbody>
</table>

The displacement rate for the connection tests was 0.04 inches per minute. This displacement rate was consistent with displacement rates specified in timber connection test standards (ASTM D1761-88; ASTM D5652-95) and previous WPC connection testing (Balma, 1999). The HDPE formulation used in this research was comprised of a majority of wood flour. However, there was still a large percentage of plastic in the HPDE formulation and the PVC formulation was comprised of half plastic. Plastic standards have been consulted to verify that displacement rates specified do not affect test results because of the high plastic content. ASTM D6117-97 deals with mechanical fastener testing in plastic lumber, but bolted connections are not covered. ASTM D953-95 discusses determination of bearing strength in plastics and
suggests a displacement rate of $0.05 \pm 0.01$ inches per minute. Therefore, a displacement rate of 0.04 inches per minute was acceptable for the dowel bearing tests and the connection tests. ASTM D5652-95 Section 9.5 states that maximum load should be reached in not less than five minutes and not more than twenty minutes. This criterion was satisfied using the 0.04 inches per minute displacement rate.

The connection models being validated only take into account bolt embedment, bolt rotation, and bolt yielding. Other factors like friction between the members and tensile forces developed in the bolt are not modeled. Therefore, nuts and washers were not used to constrain the bolt at the location of the hole being studied (Figure 4-6).

ASTM D1761 Note 7 states that all holes in a connection member should be oversized. The 3/4” diameter apparatus holes were oversized as specified by 1/16 of an inch to allow members to fit the test apparatus correctly. However, oversizing the test hole did not meet the parameters of this research. The yield models do not account for hole oversizing. When a hole is oversized in thin-walled member, dowel rotation may occur in the wall farthest from the shear plane without any material crushing. When the hole is drilled the same size as the dowel, any rotation or yielding of the dowel must involve yielding of the member material under the dowel.
CHAPTER 5: RESULTS AND DISCUSSION

Dowel Bearing Strength Tests

A total of thirteen test groups were included in the dowel bearing testing. Seven test groups for the HDPE formulation and six groups for the PVC formulation. A majority of the test configurations were needed as input for validating the yield models. Additional test groups were added to facilitate the investigation of the dependence of dowel bearing strength on dowel diameter and wall thickness.

ANOVA analysis was conducted using SAS and verified that significant differences in mean dowel bearing strengths existed between test groups at a significance level of 5%. Dowel bearing strengths increased as the dowel diameters decreased. The dowel bearing strength also increased within a WPC formulation as the dowel diameter was held constant and the wall thickness increased. The dowel bearing test results found using the 5% offset method to define the yield point are in Table 5-1. The COV of each test group was between 1.1% and 4.1%. The low COVs justifies the small samples sizes as predicted prior to testing and meets the sample size requirements of ASTM D2915.

Table 5-1: Dowel Bearing Strength Data (yield based on 5% offset)

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Wall Thickness (inches)</th>
<th>Dowel Diameter (inches)</th>
<th>Sample Size</th>
<th>Average (psi)</th>
<th>COV (%)</th>
<th>Minimum (psi)</th>
<th>Maximum (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE</td>
<td>0.2</td>
<td>1/4</td>
<td>13</td>
<td>4680</td>
<td>2.7</td>
<td>4430</td>
<td>4830</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>8</td>
<td>4250</td>
<td>3.1</td>
<td>4020</td>
<td>4420</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1/4</td>
<td>10</td>
<td>5090</td>
<td>2.1</td>
<td>4940</td>
<td>5280</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>9</td>
<td>4690</td>
<td>1.1</td>
<td>4590</td>
<td>4740</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>3/16</td>
<td>16</td>
<td>6310</td>
<td>2.5</td>
<td>6060</td>
<td>6660</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/4</td>
<td>12</td>
<td>5310</td>
<td>2.6</td>
<td>5060</td>
<td>5510</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>10</td>
<td>4960</td>
<td>1.8</td>
<td>4810</td>
<td>5140</td>
</tr>
<tr>
<td>PVC</td>
<td>0.2</td>
<td>1/4</td>
<td>5</td>
<td>16800</td>
<td>3.4</td>
<td>16100</td>
<td>17400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>14</td>
<td>13800</td>
<td>4.1</td>
<td>13000</td>
<td>14900</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1/4</td>
<td>8</td>
<td>18800</td>
<td>1.0</td>
<td>18500</td>
<td>19100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>10</td>
<td>17000</td>
<td>1.5</td>
<td>16600</td>
<td>17300</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>1/4</td>
<td>16</td>
<td>20100</td>
<td>2.6</td>
<td>19100</td>
<td>21000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/8</td>
<td>10</td>
<td>18600</td>
<td>1.4</td>
<td>18300</td>
<td>18900</td>
</tr>
</tbody>
</table>
The dowel bearing strength of the PVC formulation was an average of 3.6 times higher than the dowel bearing strength of the HDPE formulation. The failure modes of the two materials was different. The PVC formulation failed brittlely and crumbled as failure progressed (Figure 5-1). The HDPE formation deformed ductilely out of plane (Figure 5-2). These phenomena can also be seen in the load-displacement curves. The PVC formulation takes load rapidly, but as failure occurs, the load drops off quickly as the material crumbles (Figure 5-3). In the HDPE formulation, behavior is more ductile; load is gained and lost at a slower rate. After the sample reaches failure at the maximum load, the load decreases gradually until the test is concluded (Figure 5-4).

Careful sample preparation was critical to producing satisfactory dowel bearing results. Any skew between the hole being loaded and the load head would cause one wall to be loaded before the other wall. When one wall yielded prematurely, the load-displacement curve became non-linear when the wall yielded, and the maximum load was much lower than the samples loaded ideally. Tests that had a wall fail prematurely were not included in the final results because those samples did not truly measure the dowel bearing strength. For members with a solid cross-section, the reducing the amount of skew is important, but the dowel is supported continually so the effect is less dramatic.

As seen in Figure 5-3 and Figure 5-4 when using the 5% offset method to predict yield, the intersection load was either near the maximum load or after the maximum load had already occurred. Common practice is that if the maximum load occurs before the intersection load, the maximum load is used as the yield load. When this dowel bearing strength is then used to predict connection capacity, the model input is really based on maximum load rather than yield load. In order to validate the yield models on a maximum load basis, the dowel bearing strengths
using maximum load instead of the 5% offset load were computed (Table 5-2). The HDPE dowel bearing strengths increase slightly from the 5% offset based strengths. The PVC dowel bearing strengths remain the same as the 5% offset based strengths because the intersection load always occurred after the maximum load had been reached.

Table 5-2: Dowel Bearing Strength Data (based on maximum load)

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Wall Thickness</th>
<th>Dowel Diameter</th>
<th>Sample Size</th>
<th>Average</th>
<th>COV</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(inches)</td>
<td>(inches)</td>
<td>-</td>
<td>(psi)</td>
<td>(%)</td>
<td>(psi)</td>
<td>(psi)</td>
</tr>
<tr>
<td>HDPE 0.2</td>
<td>1/4</td>
<td>13</td>
<td>-</td>
<td>4680</td>
<td>2.7</td>
<td>4450</td>
<td>4830</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>8</td>
<td></td>
<td>4250</td>
<td>3.1</td>
<td>4020</td>
<td>4420</td>
</tr>
<tr>
<td></td>
<td>3/4</td>
<td>10</td>
<td></td>
<td>5170</td>
<td>1.9</td>
<td>5010</td>
<td>5340</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>9</td>
<td></td>
<td>4700</td>
<td>1.1</td>
<td>4590</td>
<td>4760</td>
</tr>
<tr>
<td></td>
<td>3/16</td>
<td>16</td>
<td></td>
<td>6800</td>
<td>2.2</td>
<td>6620</td>
<td>7230</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td>12</td>
<td></td>
<td>5620</td>
<td>2.7</td>
<td>5290</td>
<td>5860</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>10</td>
<td></td>
<td>5050</td>
<td>1.7</td>
<td>4940</td>
<td>5230</td>
</tr>
<tr>
<td>PVC 0.2</td>
<td>1/4</td>
<td>5</td>
<td></td>
<td>16800</td>
<td>3.4</td>
<td>16100</td>
<td>17400</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>14</td>
<td></td>
<td>13800</td>
<td>4.1</td>
<td>13000</td>
<td>14900</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td>8</td>
<td></td>
<td>18800</td>
<td>1.0</td>
<td>18500</td>
<td>19100</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>10</td>
<td></td>
<td>17000</td>
<td>1.5</td>
<td>16600</td>
<td>17300</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td>16</td>
<td></td>
<td>20100</td>
<td>2.6</td>
<td>19100</td>
<td>21000</td>
</tr>
<tr>
<td></td>
<td>3/8</td>
<td>10</td>
<td></td>
<td>18600</td>
<td>1.4</td>
<td>18300</td>
<td>18900</td>
</tr>
</tbody>
</table>

Dowel bearing strength increased as wall thickness increased within a given WPC formulation which was likely due to production parameters and the change in boundary conditions between wall thicknesses. The increase of dowel bearing strength by decreasing the dowel diameter was expected. This type of behavior is common in timber design in the dowel bearing strengths for bolt perpendicular to the grain (AF&PA, 1997). Although insignificant in timber design for small diameters (Wilkinson, 1991), there does appear to be a diameter effect in the WPC hollow sections and dowel sizes studied. The average increase in dowel bearing strength by decreasing the bolt diameter from 3/8” to 1/4” was 11%. A designer cannot assume that the dowel bearing strength will be constant for a formulation. The dowel bearing strength will clearly vary for different cross-sections of the same formulation and when using different dowel diameters within a single cross-section.
Figure 5-1: Typical PVC dowel bearing failures: a) 0.2" wall b) 0.3" wall c) 0.4" wall
Figure 5-2: Typical HDPE dowel bearing failures: a) 0.2" wall b) 0.3" wall c) 0.4" wall
Figure 5-3: Load-displacement curve of PVC 0.4" sample with 3/8" diameter dowel

Figure 5-4: Load-displacement curve of HDPE 0.4" sample with 3/8" diameter dowel
**Bending Yield Strength Tests**

The bending yield strength of three dowel diameters (3/8”, 1/4”, and 3/16”) was determined. The 3/8” and 1/4” diameter dowels were commercially available bolts and the 3/16” diameter dowels were cut from mild steel welding rods. The bending yield strength was found using the 5% diameter offset method (Table 5-3). Low COVs were obtained for the 3/8” and 1/4” diameter bolts because the bolts were purchased from the same manufacturing lot. The relatively high COV of the rod group is because five different rods were used. The COV was lower for each individual rod (Table 5-4). Rod type 5 had a significantly lower yield strength that resulted in the higher than expected group COV. A typical load-displacement curve is shown if Figure 5-5.

### Table 5-3: Bending Yield Strength Based on 5% Offset

<table>
<thead>
<tr>
<th>Dowel Type</th>
<th>Sample Size</th>
<th>Average Diameter (inches)</th>
<th>Average $F_{yb}$ (psi)</th>
<th>COV (%)</th>
<th>Minimum (psi)</th>
<th>Maximum (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A307 3/8</td>
<td>23</td>
<td>0.371</td>
<td>87400</td>
<td>1.1</td>
<td>85100</td>
<td>90000</td>
</tr>
<tr>
<td>A307 1/4</td>
<td>15</td>
<td>0.247</td>
<td>86800</td>
<td>2.1</td>
<td>83700</td>
<td>89700</td>
</tr>
<tr>
<td>rod 3/16</td>
<td>20</td>
<td>0.186</td>
<td>55800</td>
<td>6.4</td>
<td>48300</td>
<td>60200</td>
</tr>
</tbody>
</table>

### Table 5-4: Bending Yield Strength of Individual Rod Groups

<table>
<thead>
<tr>
<th>Dowel Type</th>
<th>Rod Type</th>
<th>Sample Size</th>
<th>Average $F_{yb}$ (psi)</th>
<th>COV (%)</th>
<th>Minimum (psi)</th>
<th>Maximum (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rod 3/16</td>
<td>2</td>
<td>4</td>
<td>57400</td>
<td>2.9</td>
<td>55500</td>
<td>58900</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4</td>
<td>57300</td>
<td>1.7</td>
<td>57100</td>
<td>58300</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
<td>49800</td>
<td>3.2</td>
<td>48700</td>
<td>51700</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4</td>
<td>59400</td>
<td>1.7</td>
<td>58000</td>
<td>60200</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>4</td>
<td>55400</td>
<td>2.5</td>
<td>54200</td>
<td>57300</td>
</tr>
</tbody>
</table>
Due to the problems defining a yield point in the dowel bearing tests noted above, maximum connection loads were be compared predicted maximum loads. Bending yield strength was recalculated on a maximum load basis (Table 5-5). The COV of the 3/16” rods improved dramatically since nearly all the rods approached the same load at a large displacement. However, Table 5-5 is misleading because the maximum loads are extremely arbitrary. The maximum load was always the last point recorded from testing and solely depended on when data collection was terminated.

Table 5-5: Bending Yield Strength Based on Maximum Load

<table>
<thead>
<tr>
<th>Dowel Type</th>
<th>Sample Size</th>
<th>Average Diameter</th>
<th>Average $F_{yb}$</th>
<th>COV</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(inches)</td>
<td>(psi)</td>
<td>(%)</td>
<td>(psi)</td>
<td>(psi)</td>
</tr>
<tr>
<td>A307 3/8</td>
<td>23</td>
<td>0.371</td>
<td>100800</td>
<td>3.0</td>
<td>97800</td>
<td>107300</td>
</tr>
<tr>
<td>A307 1/4</td>
<td>15</td>
<td>0.247</td>
<td>105200</td>
<td>1.2</td>
<td>103900</td>
<td>107500</td>
</tr>
<tr>
<td>rod 3/16</td>
<td>20</td>
<td>0.186</td>
<td>77100</td>
<td>1.8</td>
<td>74400</td>
<td>79900</td>
</tr>
</tbody>
</table>
To determine the bending yield strength that relates to the maximum load in the connection tests, the stress in the dowel must be computed when the connection is at maximum load. The two tests can be related by matching the angle of rotation of the dowel in the connection test with the corresponding displacement in the bending yield test. This is done in the following manner:

1) The displacement, \( \delta \), at the location of maximum load was found for each connection test. The average was computed for each of the connection tests groups.

2) An expression that related the angle for rotation to the displacement of the connection was used during the theoretical development of the hollow section yield theory (Equation 5-1). Equation 5-1 is used here to calculate the angle of rotation of the dowel, \( \theta \), at maximum load. The theoretical expressions for \( x_s \) and \( x_m \) from APPENDIX C were used in the calculation.

\[
\tan(\theta) = \theta = \frac{\delta}{x_s + x_m} \quad \text{(small displacement assumed)}
\]  

Equation 5-1

3) Now knowing the angle of rotation of the dowel in the connection test, the displacement of the dowel, \( \Delta \), required to produce this rotation in the bending yield strength test is back calculated. Equation 5-2 was developed based on the geometry of the bending yield test setup (Figure 5-6).

![Figure 5-6: Bending yield test diagram](image-url)
\[ \Delta = 2 \tan \left( \frac{\theta}{2} \right) \]  

Equation 5-2

4) The sixth-root polynomial fit to each group of bending yield strength tests was used to determine the load at the computed displacement, \( \Delta \).

5) Equation 4-2 was used to calculate the bending yield strength from the load found in Step 4.

The results of each step are given in Table 5-6.

Table 5-6: Bending Yield Strength Based on Displacement of the Max Load of Connection Test

<table>
<thead>
<tr>
<th>Dowel Type</th>
<th>Connection Type</th>
<th>Displacement at maximum load (inches)</th>
<th>Angle of Dowel Rotation (radians)</th>
<th>Corresponding Displacement (inches)</th>
<th>Load on Curve (lbs)</th>
<th>Average ( F_{yb} ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A307 3/8</td>
<td>PVC - Mode IIIa</td>
<td>0.065</td>
<td>0.034</td>
<td>0.034</td>
<td>534</td>
<td>62800</td>
</tr>
<tr>
<td>A307 1/4</td>
<td>HDPE - Mode IIIa</td>
<td>0.157</td>
<td>0.079</td>
<td>0.079</td>
<td>230</td>
<td>91800</td>
</tr>
<tr>
<td></td>
<td>PVC - Mode IV</td>
<td>0.092</td>
<td>0.155</td>
<td>0.156</td>
<td>252</td>
<td>100600</td>
</tr>
<tr>
<td>rod 3/16</td>
<td>HDPE - Mode IV</td>
<td>0.244</td>
<td>0.381</td>
<td>0.386</td>
<td>14266</td>
<td>77100</td>
</tr>
</tbody>
</table>

* For HDPE - Mode IV, the \( F_{yb} \) displacement was larger than fit data. Therefore, the maximum load was used to calculate \( F_{yb} \).

This analysis procedure worked for three of the four connection test groups. In the HDPE - Mode IV connections, the maximum load occurred when data collection was terminated. The large angle of rotation of the dowel at maximum connection load corresponded with a displacement in the bending yield test that was out of the range of the bending yield test and the line fit to those tests. The maximum load based bending yield strength (Table 5-5) was used to predict maximum connection capacity for the 3/16” diameter rods only. For connections with 3/8” and 1/4” diameter dowels the Table 5-6 bending yield strength was used to predict maximum connection capacity.
Connection Tests

Six connection configurations were tested – three yield modes in two wood-plastic composite formulations (Table 4-2). The results are tabulated in Table 5-7. The expected yield mode was observed in all the test groups. COVs ranged from 2.3% to 4.5% and are much lower than other WPC connection research (Balma, 1999). Typical curves load-displacement curves are shown in Figure 5-7 through Figure 5-12. All the load-displacement curves can be found later in “Model Validation” section of this chapter.

Table 5-7: Connection Test Data Based on 5% Offset Method

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Connection Type</th>
<th>Sample Size</th>
<th>Average (lbs.)</th>
<th>COV (%)</th>
<th>Minimum (lbs.)</th>
<th>Maximum (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDPE</td>
<td>Mode I_m</td>
<td>4</td>
<td>656</td>
<td>2.7</td>
<td>635</td>
<td>674</td>
</tr>
<tr>
<td></td>
<td>Mode III_s</td>
<td>9</td>
<td>504</td>
<td>2.3</td>
<td>481</td>
<td>521</td>
</tr>
<tr>
<td></td>
<td>Mode IV</td>
<td>10</td>
<td>617</td>
<td>2.9</td>
<td>594</td>
<td>640</td>
</tr>
<tr>
<td>PVC</td>
<td>Mode I_m</td>
<td>4</td>
<td>1992</td>
<td>3.9</td>
<td>1882</td>
<td>2055</td>
</tr>
<tr>
<td></td>
<td>Mode III_s</td>
<td>10</td>
<td>2330</td>
<td>4.5</td>
<td>2170</td>
<td>2485</td>
</tr>
<tr>
<td></td>
<td>Mode IV</td>
<td>10</td>
<td>2570</td>
<td>2.5</td>
<td>2503</td>
<td>2674</td>
</tr>
</tbody>
</table>

A connection that yields in Mode I_m is similar to a dowel bearing test in that the yielding occurs in only one member. No deformation was observed in the side members in either the PVC or the HDPE Mode I_m tests. Therefore, the load-displacement curve of the connection tests should be similar to the load-displacement curve of the dowel bearing test. In the HDPE Mode I_m connections, the initial slope was higher than the slope in the dowel bearing tests. In the PVC Mode I_m connections, initial slope was lower than the dowel bearing tests and the load did not drop off rapidly after the maximum load is reached as it did in the dowel bearing tests. A more ductile behavior was observed. Also, the load was not maintained smoothly due to brittleness of the PVC. It is speculated that additional load was sustained due to the confinement of the material caused by the side members. Even though no nuts were included on the end of the
bolts, the side members remained in contact with the main member throughout the tests. As the main member yielded, the out of plane deformation was restricted by the side members and thus the yield behavior of the member was altered. In Figure 5-13, confinement in both HDPE and PVC is clearly visible.

The Mode III, and Mode IV tests produced expected results. The observed yield modes were Mode III: Case 3-1 and Mode IV: Case 1-1. For Mode III: Case 3-1, dowel rotation occurred in the side members in the walls farthest from the shear plane and dowel yielding occurred in the main member in the walls closest to the shear plane (Figure 5-15). For Mode IV: Case 1-1, dowel yielding occurred in the main and side members in the walls closest to the shear plane (Figure 5-16 and Figure 5-17). Determinations of the exact locations of dowel rotation and dowel yield were impossible due to the small displacement of the connection when yield occurred. All connection test pictures presented were taken at large displacements (> 0.4”) and do not reflect connection behavior at yield. Additional dowel deformation within the void space and separation of the members occurred after maximum load was reached.

Maximum connection load data has been tabulated to enable the comparison of predicted maximum loads (Table 5-8).

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Connection Type</th>
<th>Sample Size</th>
<th>Average</th>
<th>COV</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>(lbs.)</td>
<td>(%)</td>
<td>(lbs.)</td>
<td>(lbs.)</td>
</tr>
<tr>
<td>HDPE</td>
<td>Mode I&lt;sub&gt;m&lt;/sub&gt;</td>
<td>4</td>
<td>669</td>
<td>1.9</td>
<td>652</td>
<td>679</td>
</tr>
<tr>
<td></td>
<td>Mode III&lt;sub&gt;III&lt;/sub&gt;)</td>
<td>9</td>
<td>635</td>
<td>1.9</td>
<td>618</td>
<td>653</td>
</tr>
<tr>
<td></td>
<td>Mode IV</td>
<td>10</td>
<td>864</td>
<td>1.6</td>
<td>833</td>
<td>880</td>
</tr>
<tr>
<td>PVC</td>
<td>Mode I&lt;sub&gt;m&lt;/sub&gt;</td>
<td>4</td>
<td>1997</td>
<td>4.0</td>
<td>1884</td>
<td>2055</td>
</tr>
<tr>
<td></td>
<td>Mode III&lt;sub&gt;III&lt;/sub&gt;)</td>
<td>10</td>
<td>2394</td>
<td>1.9</td>
<td>2334</td>
<td>2485</td>
</tr>
<tr>
<td></td>
<td>Mode IV</td>
<td>10</td>
<td>2858</td>
<td>2.4</td>
<td>2764</td>
<td>2960</td>
</tr>
</tbody>
</table>
Figure 5-7: Typical load-displacement curve of Mode $I_m$ connection test - PVC

Figure 5-8: Typical load-displacement curve of Mode $I_m$ connection test - HDPE
Figure 5-9: Typical load-displacement curve of Mode III_s connection test - PVC

Figure 5-10: Typical load-displacement curve of Mode III_s connection test - HDPE
Figure 5-11: Typical load-displacement curve of Mode IV connection test – PVC

Figure 5-12: Typical load-displacement curve of Mode IV connection test – HDPE
Figure 5-13: Confinement in Mode $I_m$ connection tests a) HPDE b) PVC

Figure 5-14: HDPE Mode III$_c$ connections a) Entire connection  
b) Approximate location of dowel and walls during testing

Figure 5-15: PVC Mode III$_c$ connections a) Entire connection  
b) Approximate location of dowel and walls during testing
Figure 5-16: HDPE Mode IV connections a) Entire connection b) Location of dowel yielding in side member c) Approximate location of dowel and walls during testing

Figure 5-17: PVC Mode IV connections a) Entire connection b) Dowel yielding in side member c) Approximate location of dowel and walls during testing
Model Validation

Several approaches were used to validate the connection models developed in CHAPTER 3. First, the hollow section yield model was evaluated using the 5% diameter offset method to predict the yield point of the input parameters and the connection tests. Then, the maximum load of the input curves was used to predict maximum connection load. Finally, the load-displacement predicting equations were evaluated for Mode I_m, Mode III_s, and Mode IV.

Hollow Section Yield Model

When the hollow section yield model was evaluated on a 5% offset basis, the model prediction differed from the tested connection 5% offset value by an average of 14.7% (Table 5-9). The dowel bearing values from Table 5-1 and the bending yield strength values from Table 5-3 were used as model inputs. The HDPE Mode I_m 5% offset connection load was under-predicted. Whereas, the remainder of the 5% offset connection loads were over-predicted. The increase in the HDPE Mode I_m connection load has been attributed to confinement from the side members. The PVC Mode I_m connection results were not increased, but the shape of the load-displacement curve was modified. As noted earlier, the 5% offset method did not truly identify the yield point of the dowel bearing tests. Therefore, using the 5% offset method to define the yield point is not a correct assessment of validity of the hollow section yield model.

Table 5-9: Connection Test Results and Predicted Yield (5% offset based)

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>HDPE</th>
<th>PVC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Tested</td>
</tr>
<tr>
<td>-</td>
<td>(lbs)</td>
<td>(lbs)</td>
</tr>
<tr>
<td>I_m</td>
<td>630</td>
<td>656</td>
</tr>
<tr>
<td>III_s</td>
<td>654</td>
<td>504</td>
</tr>
<tr>
<td>IV</td>
<td>752</td>
<td>617</td>
</tr>
<tr>
<td>HDPE average = 18.5</td>
<td>PVC average = 10.9</td>
<td></td>
</tr>
</tbody>
</table>

* Difference calculated by subtracting the predicted value from the test value and then dividing by the test value.
A maximum load basis is more appropriate as an unbiased method of validating the yield model for the WPC material used in this research. As noted earlier, the maximum load of the bending yield strength was defined as the stress in the dowel at maximum connection load. When the bending yield strength based on deflection at maximum connection load (Table 5-6) and maximum dowel bearing strength (Table 5-2) are used as input into the hollow section yield model, the predicted values differ from the maximum connection load values by an average of only 5.7% (Table 5-10).

Table 5-10: Connection Test Results and Predicted Yield (maximum load based dowel bearing strength and bending yield strength based on displacement at maximum connection load)

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>HDPE</th>
<th>PVC</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Tested</td>
<td>Difference</td>
</tr>
<tr>
<td>I&lt;sub&gt;in&lt;/sub&gt;</td>
<td>630</td>
<td>669</td>
<td>-5.9</td>
</tr>
<tr>
<td>III&lt;sub&gt;s&lt;/sub&gt;</td>
<td>669</td>
<td>635</td>
<td>5.3</td>
</tr>
<tr>
<td>IV</td>
<td>917</td>
<td>864</td>
<td>6.2</td>
</tr>
</tbody>
</table>

HDPE average = 5.8  PVC average = 5.5

* Difference calculated by subtracting the predicted value from the test value and then dividing by the test value.

The difference between the predicted values and the tested values can be mainly attributed to variation of input parameters and testing anomalies. One model assumption that has not been accounted for is that the dowel loading is uniform along the length of the dowel. It is probable that this assumption is violated in actual connections and that the force on the dowel in the wall closest to the shear plane is higher than the force in the outside wall. This would explain why the yield model over-predicted the actual connection capacity in nearly every case. If the interior wall was carrying more load, then the yield load would be lower. However, the assumption that the dowel loading is uniform simplifies the model and allows for a close approximation of connection behavior.
Another model assumption violation that contributed to the difference between the predicted and actual connection loads was the assumption of perfect elastic/plastic materials. The load-displacement curves clearly show that neither the dowel bearing tests nor the bending yield tests behave as assumed. A derivative of the elastic/plastic material behavior assumption is that both the dowel and the members yield at the same time. This assumption was violated in the PVC connections. The brittle PVC formulation yielded much sooner than the dowels. A load-displacement model is one way of overcoming the elastic/plastic material behavior assumption.

The amount of error between the predicted and tested values of the hollow section yield is well within the range reported by Aune and Patton-Mallory (1986b) in their study of nailed connections. In that work, predicted loads and the Mode III connection loads varied by 6.5% and the Mode IV connection loads varied by 13%. McLain and Thangjitham (1984) evaluated several large data sets of bolted connection tests to validate their modification of the EYM. However, the performance of the unmodified EYM was also evaluated; the ratio of actual load to predicted load ranged from 0.90 to 1.32. When Balma (1999) validated the EYM for use in solid cross-sections of WPC, the EYM over-predicted connection capacity by an average of 14% using the 5% diameter offset to predict load. Using a maximum load basis, the EYM under-predicted connection capacity by an average of 5%.

**Load-Displacement Yield Model**

The load-displacement model inputs are curves fit to the dowel bearing resistance tests and the dowel bending yield tests. The dowel bearing resistance curves were fit over a displacement range of zero to 0.11 inches. The bending yield strength tests were fit over a displacement range of zero to 0.2 inches. Plots of all the dowel bearing resistance curves and the bending yield strength tests are located in APPENDIX F. The coefficients of the fit curves used in this research are given in Table 5-12.
A summary of the difference between the predicted load-displacement curve and the average of the connection tests was found using two methods (Table 5-11). An analysis was completed based on the absolute value of the percent error between the curves and another was based on the work done to a displacement of 0.11 inches. The percent error was calculated by computing the absolute value of the difference between the average connection load and the predicted load at each displacement (every 0.0005 inches) and then dividing by the average connection load at that displacement; the overall average for the entire curve was then calculated. The percent error method is a measure of how well the shape of the curves matched. The work done by the actual connection curves and predicted connection curves was calculated by finding the areas under the curves using the trapezoidal method at a width of 0.0005 inches. The connection work is relevant since the load-displacement behavior predicting equations could be used in energy-based design. Mode \textit{Im} equations under-predicted the work by an average of 4.7\%, and the Mode \textit{IV} and Mode \textit{III}, equations under-predicted the work by 7.6\% and 13.2\%, respectively.

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Connection Type</th>
<th>Absolute Percent Error</th>
<th>Work to a displacement of 0.11 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(%)</td>
<td>Actual (in-lbs)</td>
</tr>
<tr>
<td>HDPE</td>
<td>Mode \textit{I}_m</td>
<td>18.8</td>
<td>537</td>
</tr>
<tr>
<td></td>
<td>Mode \textit{III}_s</td>
<td>14.1</td>
<td>546</td>
</tr>
<tr>
<td></td>
<td>Mode \textit{IV}</td>
<td>5.7</td>
<td>662</td>
</tr>
<tr>
<td>PVC</td>
<td>Mode \textit{I}_m</td>
<td>13.7</td>
<td>1724</td>
</tr>
<tr>
<td></td>
<td>Mode \textit{III}_s</td>
<td>16.3</td>
<td>2153</td>
</tr>
<tr>
<td></td>
<td>Mode \textit{IV}</td>
<td>13.0</td>
<td>2459</td>
</tr>
<tr>
<td></td>
<td>Average =</td>
<td>13.6</td>
<td></td>
</tr>
</tbody>
</table>

The PVC Mode \textit{I}_m prediction agreed well with the connection test curves (Figure 5-18). After the maximum load, the connection tests sustained higher loads than predicted. As stated earlier, this increase was probably due to the confinement effect of the side members. The
HDPE Mode I\textsubscript{m} prediction varied significantly from the connection tests (Figure 5-19). The main difference was that the predicted curve had a higher slope than the connection tests. Another difference between the test data and predicted curve was the aforementioned behavior after maximum load was reached; the HDPE samples also demonstrated an increased ability to sustain load.

In the Mode III\textsubscript{s} connections, the predicted load-displacement curves under-predicted the tests curves, but captured the shape of the curves well (Figure 5-20 and Figure 5-21). The same is true of the Mode IV results in the HDPE material only (Figure 5-23). The Mode IV PVC curve was incorrectly predicted (Figure 5-22). The initial slope is correctly predicted, but the predicted curve did not yield the same as the tested connections. In the predicted PVC Mode IV curve, the internal energy was heavily weighted toward the behavior of the dowel and only a small contribution was from the brittle PVC behavior causing the difference in yield prediction.

The predicted curves were sensitive to the locations of dowel yielding and dowel rotation (x\textsubscript{s} and x\textsubscript{m}). The equations for x\textsubscript{s} and x\textsubscript{m} from the yield model were used to produce the predicted curves found in Figure 5-20, Figure 5-21, Figure 5-22, and Figure 5-23 (Table 5-13). Excluding the PVC Mode IV tests, nearly exact predictions of the test data can be produced by varying the values of x\textsubscript{s} and x\textsubscript{m} from the theoretical values by a few hundredths of an inch. The model predictions, even with the simplifying assumptions, must have been close to the locations of dowel yielding and dowel rotation that occurred during connection testing.

The Mode IV connections were the only test groups that possessed a theoretical closed-form solution for the load-displacement relationship. However, due to the method used to enter the entire load-displacement behavior of the bending yield tests (Equation 3-9), no closed-form solution was found. The location of dowel yielding is needed to calculate the bending yield
moment at each point. Perhaps, if an alternative function was fit to the test data, a closed-form solution may be determined.

Table 5-12: Coefficients of Fitted Curves

<table>
<thead>
<tr>
<th>Material Property</th>
<th>a (x^1)</th>
<th>b (x^2)</th>
<th>c (x^3)</th>
<th>d (x^4)</th>
<th>e (x^5)</th>
<th>f (x^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Strength</td>
<td>3/16&quot; diameter</td>
<td>1213</td>
<td>14441</td>
<td>-535880</td>
<td>5301137</td>
<td>-22553205</td>
</tr>
<tr>
<td></td>
<td>1/4&quot; diameter</td>
<td>3328</td>
<td>70355</td>
<td>-1958282</td>
<td>17575455</td>
<td>-70005907</td>
</tr>
<tr>
<td></td>
<td>3/8&quot; diameter</td>
<td>16474</td>
<td>131273</td>
<td>-6447049</td>
<td>67910008</td>
<td>-299526836</td>
</tr>
<tr>
<td>HDPE Dowel Bearing</td>
<td>0.2&quot; wall - 3/8&quot;</td>
<td>68459</td>
<td>920060</td>
<td>-90879442</td>
<td>1730243488</td>
<td>-13616864344</td>
</tr>
<tr>
<td></td>
<td>0.2&quot; wall - 1/4&quot;</td>
<td>56619</td>
<td>384634</td>
<td>-64068237</td>
<td>1313230298</td>
<td>-10797582528</td>
</tr>
<tr>
<td></td>
<td>0.4&quot; wall - 1/4&quot;</td>
<td>42510</td>
<td>799332</td>
<td>-52486029</td>
<td>892993302</td>
<td>-6613914480</td>
</tr>
<tr>
<td></td>
<td>0.4&quot; wall - 3/16&quot;</td>
<td>37523</td>
<td>878236</td>
<td>-53685873</td>
<td>918546956</td>
<td>-6910183992</td>
</tr>
<tr>
<td>PVC Dowel Bearing</td>
<td>0.2&quot; wall - 3/8&quot;</td>
<td>126180</td>
<td>5056329</td>
<td>-209742898</td>
<td>2442816519</td>
<td>-10460103167</td>
</tr>
<tr>
<td></td>
<td>0.4&quot; wall - 3/8&quot;</td>
<td>144381</td>
<td>5049568</td>
<td>-144743672</td>
<td>591130636</td>
<td>6739670299</td>
</tr>
<tr>
<td></td>
<td>0.4&quot; wall - 1/4&quot;</td>
<td>105162</td>
<td>644097</td>
<td>38579662</td>
<td>-2098284246</td>
<td>24679590411</td>
</tr>
</tbody>
</table>

Table 5-13: Locations used to validate load-displacement model

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>xs (inches)</th>
<th>xm (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>1.74</td>
<td>0.25</td>
</tr>
<tr>
<td>Mode III (HDPE)</td>
<td>1.73</td>
<td>0.2</td>
</tr>
<tr>
<td>Mode III (PVC)</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Mode IV (HDPE)</td>
<td>0.296</td>
<td>0.296</td>
</tr>
<tr>
<td>Mode IV (PVC)</td>
<td>0.296</td>
<td>0.296</td>
</tr>
</tbody>
</table>
Figure 5-18: PVC Mode $I_m$ Connection Tests with Predicted Curve

Figure 5-19: HDPE Mode $I_m$ Connection Tests with Predicted Curve
Figure 5-20: PVC Mode III\textsubscript{s} Connection Tests with Predicted Curve

Figure 5-21: HDPE Mode III\textsubscript{s} Connection Tests with Predicted Curve
Figure 5-22: PVC Mode IV Connection Tests with Predicted Curves

Figure 5-23: HDPE Mode IV Connection Tests with Predicted Curves
Load-Displacement Model Sensitivity

The sensitivities of the Mode III, and Mode IV load-displacement equations were studied by varying the input parameters. The three input parameters are the bearing resistance of the main and side members and the bending yield strength for the dowel. Each parameter was systematically varied to determine their effect on the load-displacement predictions while the remaining input parameters were held constant. The results of model sensitivity study are given in Table 5-14; the tabulated percent difference values is the percent difference between the original predicted curves and the curve predicted with the increased parameter. Any manipulation of the coefficients of the fit curves results in a linear change in the error. The results of increasing the coefficients by 10% are shown.

The Mode III, results were as expected – i.e., changing the dowel bearing strength of the side member had the most effect on the load-displacement behavior, followed by the bending strength of the dowel, and the strength of the main member. Changing the main member dowel bearing strength had little effect on the load-displacement behavior. Also, the dowel bearing strengths of the members had less effect in the stronger, stiffer PVC formulation than it did in the more ductile HDPE formulation.

For the Mode IV sensitivity study, the connections with the same side member and main member were used. Therefore, only one set of dowel resistance coefficients was used and the location of dowel rotation (x) is the same in both members. As expected, the load-displacement model is more sensitive to strength of the bending yield strength than the bearing resistance of the members. Also, similar to the Mode III, the bending yield strength had more of effect in the stiffer PVC formulation than the ductile HDPE formulation.
Table 5-14: Results of Model Sensitivity Study

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Formulation</th>
<th>Material Property</th>
<th>Increase in Property</th>
<th>Material Property</th>
<th>Increase in Property</th>
<th>Difference from Unmodified Predicted Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Absolute Error Based</td>
<td></td>
<td>Work Based</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(%)</td>
<td></td>
<td>(%)</td>
<td></td>
</tr>
<tr>
<td>Mode IIIₜ</td>
<td>HDPE</td>
<td>$f_{es}$</td>
<td>10</td>
<td></td>
<td>7.6</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{em}$</td>
<td>10</td>
<td></td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$</td>
<td>10</td>
<td></td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>PVC</td>
<td>$f_{es}$</td>
<td>10</td>
<td></td>
<td>7.4</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{em}$</td>
<td>10</td>
<td></td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$</td>
<td>10</td>
<td></td>
<td>2.5</td>
<td>2.6</td>
</tr>
<tr>
<td>Mode IV</td>
<td>HDPE</td>
<td>$f_c$</td>
<td>10</td>
<td></td>
<td>3.4</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$</td>
<td>10</td>
<td></td>
<td>6.6</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>PVC</td>
<td>$f_c$</td>
<td>10</td>
<td></td>
<td>2.9</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$</td>
<td>10</td>
<td></td>
<td>7.1</td>
<td>6.8</td>
</tr>
</tbody>
</table>

The sensitivity of the load-displacement equations to the locations of dowel yielding and dowel rotation ($x_s$ and $x_m$) was also investigated using the formulations and cross-sections of this research (Table 5-15, Table 5-16, and Table 5-17). The entire range of valid values for $x_s$ and $x_m$ in each yield mode were investigated. For Mode IIIₜ: Case 3-1, decreasing $x_s$ and $x_m$ from the theoretical value resulted in increasing the connection work. Increasing $x_s$ and $x_m$ from the theoretical value resulted in a decrease in connection work. Additionally, the movement of $x_m$ had a larger effect than the movement of $x_s$. The same results were observed in both the HDPE and the PVC formulation. For Mode IV: Case 1-1, decreasing the location of yielding, $x$, resulted in an increase in the work. Decreasing $x$ by 30% results in a increase in work of nearly 100% in HDPE and nearly 75% in PVC. Predicting the location of dowel yield and dowel rotation is critical to predicting the correct load-displacement behavior.
Table 5-15: Sensitivity of Mode III, to the location of dowel rotation (xs)

<table>
<thead>
<tr>
<th>Formulation</th>
<th>xs</th>
<th>Change in xs</th>
<th>Difference from Unmodified Predicted Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(inches)</td>
<td>(%)</td>
<td>Absolute Error Based</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(%)</td>
</tr>
<tr>
<td>HDPE:</td>
<td>1.60</td>
<td>-8.0</td>
<td>2.7</td>
</tr>
<tr>
<td>theoretical</td>
<td>1.65</td>
<td>-5.2</td>
<td>1.3</td>
</tr>
<tr>
<td>xₕ = 1.74&quot;</td>
<td>1.70</td>
<td>-2.3</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>0.6</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>1.80</td>
<td>3.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>PVC:</td>
<td>1.60</td>
<td>-7.5</td>
<td>2.7</td>
</tr>
<tr>
<td>theoretical</td>
<td>1.65</td>
<td>-4.6</td>
<td>1.4</td>
</tr>
<tr>
<td>xₕ = 1.73&quot;</td>
<td>1.70</td>
<td>-1.7</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>1.2</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>1.80</td>
<td>4.0</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

Table 5-16: Sensitivity of Mode III, to the location of dowel yielding (xm)

<table>
<thead>
<tr>
<th>Formulation</th>
<th>xm</th>
<th>Change in xm</th>
<th>Difference from Unmodified Predicted Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(inches)</td>
<td>(%)</td>
<td>Absolute Error Based</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(%)</td>
</tr>
<tr>
<td>HDPE:</td>
<td>0.00</td>
<td>-100.0</td>
<td>18.2</td>
</tr>
<tr>
<td>theoretical</td>
<td>0.10</td>
<td>-60.0</td>
<td>9.7</td>
</tr>
<tr>
<td>xₘ = 0.25</td>
<td>0.20</td>
<td>-20.0</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>20.0</td>
<td>-2.4</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>60.0</td>
<td>-5.8</td>
</tr>
<tr>
<td>PVC:</td>
<td>0.00</td>
<td>-100.0</td>
<td>14.2</td>
</tr>
<tr>
<td>theoretical</td>
<td>0.10</td>
<td>-50.0</td>
<td>6.4</td>
</tr>
<tr>
<td>xₘ = 0.20&quot;</td>
<td>0.20</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>50.0</td>
<td>-4.6</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>100.0</td>
<td>-7.3</td>
</tr>
</tbody>
</table>

Table 5-17: Sensitivity of Mode IV to the location of dowel yielding (x)

<table>
<thead>
<tr>
<th>Formulation</th>
<th>x</th>
<th>Change in x</th>
<th>Difference from Unmodified Predicted Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(inches)</td>
<td>(%)</td>
<td>Absolute Error Based</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(%)</td>
</tr>
<tr>
<td>HDPE:</td>
<td>0.10</td>
<td>-68.8</td>
<td>86784.9</td>
</tr>
<tr>
<td>theoretical</td>
<td>0.20</td>
<td>-37.5</td>
<td>93.4</td>
</tr>
<tr>
<td>x = 0.32&quot;</td>
<td>0.30</td>
<td>-6.3</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>25.0</td>
<td>-9.8</td>
</tr>
<tr>
<td>PVC:</td>
<td>0.10</td>
<td>-66.2</td>
<td>66224.8</td>
</tr>
<tr>
<td>theoretical</td>
<td>0.20</td>
<td>-32.4</td>
<td>72.6</td>
</tr>
<tr>
<td>x = 0.296&quot;</td>
<td>0.30</td>
<td>1.4</td>
<td>-0.9</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>35.1</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Design Procedures for Connections with Hollow Members

The hollow section yield model was validated using a maximum load basis that included the theoretical stress in the dowel at maximum connection load. This procedure was necessary to demonstrate the validity of the yield model, however it is impractical for design use. Design engineers will not know the displacement at maximum connection load. Therefore, for use in design, the 5% diameter offset method was used to define yield in the bending yield strength tests. The bending yield strengths codified for use in timber design could be utilized in the design of hollow sections. WPC hollow section design would differ from timber design in that the design basis would be maximum connection load rather than 5% offset yield load. This basis was necessary because of the difficulty defining yield in WPC dowel bearing tests. Any additional offset method used to determine yield was arbitrary and specific to a WPC formulation. Working on a maximum load basis was an unbiased method of quantifying dowel bearing strength. For the connections in this research, the predicted maximum load calculated using the 5% diameter offset method to calculate the bending yield strength and the maximum dowel bearing strength differed from the actual maximum connection capacity by an average of 6.1% (Table 5-18).

Table 5-18: Connection Test Results and Predicted Yield (maximum load based dowel bearing strength and bending yield strength based on 5% diameter offset method)

<table>
<thead>
<tr>
<th>Mode</th>
<th>HDPE</th>
<th>PVC</th>
<th>Difference*</th>
<th>HDPE</th>
<th>PVC</th>
<th>Difference*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Tested</td>
<td>(%)</td>
<td>Predicted</td>
<td>Tested</td>
<td>(%)</td>
</tr>
<tr>
<td>-</td>
<td>(lbs)</td>
<td>(lbs)</td>
<td>(%)</td>
<td>(lbs)</td>
<td>(lbs)</td>
<td>(%)</td>
</tr>
<tr>
<td>I_m</td>
<td>630</td>
<td>669</td>
<td>-5.9</td>
<td>2046</td>
<td>1997</td>
<td>2.4</td>
</tr>
<tr>
<td>III_e</td>
<td>657</td>
<td>635</td>
<td>3.3</td>
<td>2699</td>
<td>2394</td>
<td>12.7</td>
</tr>
<tr>
<td>IV</td>
<td>781</td>
<td>864</td>
<td>-9.6</td>
<td>2936</td>
<td>2858</td>
<td>2.7</td>
</tr>
<tr>
<td>HDPE average = 6.3</td>
<td>PVC average = 6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Difference calculated by subtracting the predicted value from the test value and then dividing by the test value.
CHAPTER 6: SUMMARY AND CONCLUSIONS

Summary
Current timber connection design in the U.S. is based on the European Yield Model (EYM). The basis of the EYM was published by Johansen in 1949 and has been codified in the U.S. since 1991 for timber connections with members of solid cross-section. Many wood-plastic composite members are extruded in hollow cross-sections. No design methodology exists for connections with hollow members. The goal of this research was to develop an EYM-based method of predicting connection capacity in hollow sections and validate with experimental testing. Additionally, a method of predicting the entire load-displacement behavior of a connection with hollow members was desired for use in energy-based design and deformation calculations.

Expressions for the yield load of lateral connections in single and double shear were derived using the virtual work concept. Each member was assumed to have two walls of equal thickness and dowel bearing strength, separated by voids. A hollow section yield model, consisting of 18 single-shear equations, was derived by considering all the possible locations of dowel hinge formation and dowel rotation. A computer program was used to evaluate the equations over a reasonable range of input parameters and cross-section geometries. Only six single-shear equations were found to govern the connection behavior, and they reduced into a practical design format.

The model for predicting load-displacement behavior of connections with hollow members was derived for the six controlling modes and cases of the hollow section yield model. The Mode I_s and I_m load-displacement model equations are simply the curves fit to the dowel resistance data multiplied by the sum of the wall thicknesses. Modes II, III, and IV utilize the
method of virtual displacements. A closed-form of the solution was impractical due to the number of terms introduced by the hollow members. Therefore, the load-displacement equations require the calculation of the locations of dowel yielding and dowel rotation based on expressions from the hollow section yield model. Five general contributions to internal energy were defined and equations were derived for each energy component. A general expression was developed for the energy associated with the four types of material crushing. The load-displacement behavior of the predicted yield mode of a connection was found by adding the relevant internal energy terms and dividing by the connection displacement.

The models were validated by conducting double-shear unconstrained bolted connection tests using two wood-plastic composite formulations, three wall thicknesses, and three dowel diameters. Input parameters were quantified through dowel bearing tests and bending yield strength tests for each combination of WPC formulation, wall thickness, and dowel diameter. The dowel bearing strength of the members, bending yield strength of the dowel, and the connection dimensions were need as input for the hollow section yield model. Sixth-order polynomial functions (restricted to pass through the origin) were fit to the dowel resistance curves and bending yield strength data as input for the load-displacement behavior model.

The COVs of all the test groups were low due to the uniformity of the WPC material and all the dowels originated from the same manufacturing lot. The COV of the connection tests ranged from 1.6% to 4.5%. The COV of the dowel bearing tests ranged from 1.0% to 4.1%. The COV of the bending yield strength tests ranged from 1.1% to 6.4%. The relatively higher COV of the bending yield strength data was due to the 3/16” diameter dowels being cut from more than one steel rod.
The hollow section yield model was evaluated using several methods of interpreting the test data. The standard timber method of defining yield using the 5% offset produced an average percent difference between the yield point of the connection test and the predicted yield load of 14.7%. However, the dowel bearing deformation behavior of the WPC material was such that yield point from the 5% offset method did not coincide with the sample yielding. The hollow section yield model was then evaluated on a maximum load basis. The maximum connection loads were compared to the theoretical load calculated by entering the dowel bearing strength based on maximum load and a bending yield strength based on the stress in the dowel at the displacement of maximum connection load. Using this maximum load basis, the average percent difference between the theoretical maximum load and tested maximum load was 5.7%.

The load-displacement behavior model was validated by comparing the percent difference between the predicted curve and the average of the connection tests and by comparing the work of the connections to a displacement of 0.11 inches. A work analysis was deemed more useful since a primary use of the load-displacement model would be energy design. The Mode I_m prediction performed best by under-predicting the work by an average of 4.7%. The Mode IV and Mode III_s equations under-predicted the work by an average of 7.6% and 13.2 %, respectively.

The Mode I_m load-displacement curve predictions should have been nearly identical to the average of the dowel bearing tests. The major difference in the HDPE Mode I_m was the poor prediction of the initial slope. Both the HDPE and PVC connection showed an improved ability to sustain load after reaching maximum; this is likely due to the material confinement caused by the side members.
Both the HDPE and PVC Mode III, predicted load-displacement curves under-predicted the tests curves, but captured the shape of the curves well. The HDPE Mode IV predicted the test curves well by both capturing the initial slope and general curve shape. The PVC Mode IV test curves were predicted poorly. All Mode III, and Mode IV predicted curves were sensitive to the location of the dowel rotation and dowel yielding that was input. Every Mode III, and Mode IV predicted curve could be fit to the test curves by slightly modifying the location of dowel rotation and dowel yielding. The theoretical location of the dowel rotation and dowel yielding may not have occurred during the connection testing due to the violation of the model assumptions or experimental error.

For design purposes, using the maximum dowel bearing strength and 5% offset bending yield strength to predict maximum connection capacity achieved satisfactory results by predicting the maximum load within an average of 6%.

**Conclusions**

1. A hollow section yield model was derived, reduced to a usable allowable stress design format, and validated through the laboratory testing of Mode I_m, Mode III_s, and Mode IV connections in two wood-plastic composite formulations.
   - The predictions of the hollow section yield model differed from the connection test data by an average of 5.7% (maximum load basis).
   - The 5% offset method was an inappropriate method of defining yield in the two WPC formulation studied. Therefore, maximum dowel bearing strength should be used in design for maximum connection capacity.
   - The 5% diameter offset method should be used to design for maximum connection capacity instead of the maximum bending yield strength.
Confinement of the member material resulted the Mode Im connections performing more ductilely than the dowel bearing tests.

2. A method of predicting the load-displacement behavior of dowel-type connections using hollow sections was developed, reduced to a usable format, and validated through the laboratory testing of Mode I, Mode III, and Mode IV connections in two wood-plastic composite formulations.
   - The work done by the connection tests was under-predicted by an average of 8.5%.
   - The shapes of the predicted curves were similar to the connection tests curves. The average percent difference between the curves was 13.6%.
   - The load-displacement model is sensitive to the locations of dowel rotation and dowel yielding.
   - The virtual displacement model provides a straightforward derivation method for the load-displacement behavior of connections that could be used in energy-based design methods.

3. Dowel bearing strength tests should be conducted on each WPC formulation and cross-section geometry before evaluating connection performance.
   - Dowel bearing strength varied significantly within a wood-plastic composite formulation as the wall thickness changed.
   - Dowel bearing strength varied significantly within each wall thickness as the dowel diameter changed.
Suggestions for Further Research

1. Validate the hollow section yield model with additional materials and cross-sections. Include connections where the main and side member have substantially different dowel bearing behaviors and additional types of structural composites (more formulations and traditional timber composites like oriented strand board). Conduct connection tests of connections between hollow members and solid members (i.e. structural insulated panels). Conduct tests and evaluate the models with other types of fasteners such as screws and nails.

2. Rederive hollow section yield model and load-displacement model using alternative function types fit to dowel bearing tests and bending yield tests. Other types of functions may enable prediction of the load-displacement curve to a larger displacement or produce a closed-form solution.

3. Predict the load-displacement behavior of timber connections (solid cross-section) using the concepts utilized in this research.

4. Change the initial assumptions of the hollow section problem and derive an expanded hollow section yield model. Incorporate more than two walls and vary the dowel bearing strength of the walls.

5. Investigate the variability in dowel bearing strength over a large range of dowel diameters.

6. Evaluate hollow sections for such parameters as end spacing, edge spacing, and multiple fasteners.
REFERENCES


Kuenzi, E.W. 1955. Theoretical design of a nailed or bolted joint under lateral load. Report 1951, Forest Products Lab, Madison, WI.


APPENDIX A: DERIVATION OF EYM EQUATIONS – STATIC EQUILIBRIUM BASED
Overview

This appendix outlines the static equilibrium-based derivation of the European Yield Model (EYM). A partial derivation using these same procedures is outlined in American Forest & Paper Association’s Technical Report 12 (1999). The EYM equations predict connection yield load based on its geometry, dowel bearing strength, and bending yield strength. The EYM uses six possible yield modes for single shear connections and four yield modes for double shear connections. The connection yield load is reached when either the compressive yield load of the member under the dowel is reached or when one or more plastic hinges forms in the dowel. Using simplifying assumptions and static equilibrium of the dowel, a general expression of the lateral yield load of each mode was derived. For a specific connection, the general equation with the lowest value controls the design.

Description of Modes

Table A-1: Yield modes

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Description of Failure</th>
<th>Applicable Connection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&lt;sub&gt;m&lt;/sub&gt;</td>
<td>Main member bearing</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Side member bearing</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>II</td>
<td>Main and side member bearing</td>
<td>Only single shear</td>
</tr>
<tr>
<td>III&lt;sub&gt;m&lt;/sub&gt;</td>
<td>Main member bearing, Dowel yielding in side member</td>
<td>Only single shear</td>
</tr>
<tr>
<td>III&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Side member bearing, Dowel yielding in main member</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>IV</td>
<td>Dowel yielding in main and side member</td>
<td>Both single and double shear</td>
</tr>
</tbody>
</table>

Assumptions

- End fixity of the dowel is ignored.
- Tension forces in the dowel are ignored.
- Friction between the members is ignored
• Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel.
• Perfect elastic/plastic behavior of all materials is assumed.

**Input Parameters**

The only necessary input parameters deal with connection geometry and material strengths as follows:

Table A-2: Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_s )</td>
<td>Side member dowel bearing length, in</td>
</tr>
<tr>
<td>( l_m )</td>
<td>Main member dowel bearing length, in</td>
</tr>
<tr>
<td>( g )</td>
<td>Gap between members, in</td>
</tr>
<tr>
<td>( D )</td>
<td>Dowel diameter, in</td>
</tr>
<tr>
<td>( F_{es} )</td>
<td>Side member dowel-bearing strength, psi</td>
</tr>
<tr>
<td>( F_{em} )</td>
<td>Main member dowel-bearing strength, psi</td>
</tr>
<tr>
<td>( F_b )</td>
<td>Dowel bending strength, psi</td>
</tr>
</tbody>
</table>

The input parameters are used to calculate distributed loads and moments on the dowel:

Table A-3: Derivation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_s )</td>
<td>Side member dowel-bearing resistance, lbs/in</td>
</tr>
<tr>
<td>( q_m )</td>
<td>Main member dowel-bearing resistance, lbs/in</td>
</tr>
<tr>
<td>( M_s )</td>
<td>Side member dowel moment resistance, in-lbs</td>
</tr>
<tr>
<td>( M_m )</td>
<td>Main member dowel moment resistance, in-lbs</td>
</tr>
<tr>
<td>( D_s )</td>
<td>Dowel diameter at max. stress in side member, in</td>
</tr>
<tr>
<td>( D_m )</td>
<td>Dowel diameter at max. stress in main member, in</td>
</tr>
</tbody>
</table>

The above parameters can be calculated in the following manner:

\[
q_s = F_{es} D \\
q_m = F_{em} D \\
M_s = F_b \left( \frac{D_s^3}{6} \right) \\
M_m = F_b \left( \frac{D_m^3}{6} \right)
\]
General Dowel Loading Conditions

Dowel Bearing:
\[ P = V = qa \]
\[ M_{\text{max}} = qa^2 / 2 \]

Dowel Bearing with Rotation:
\[ P = V = qa \]
\[ M_{\text{max}} = qax \]

Dowel Bending:
\[ P = V = qa \]
\[ M_{\text{max}} = M_{\text{dowel}} \]

Next, using the three general dowel loading conditions, connection models are developed.

Note:
The “m” subscript indicates main member bearing.
The “s” subscript indicates side member bearing.
In the following brief derivation, a useful expression for the location “a” is developed for the case of dowel bearing with rotation.

From the shear and bending moment diagram:

\[ M_{\text{max}} = q \cdot a^2 \]

Location of zero shear is defined as \( x \):

\[ x = \frac{1 - 2a}{a} \]

Note: \( x = a \) and \( l = 3a \)

Solve for \( a \):

\[ a = \frac{1 - x}{2} \]

Substitute into moment expression:

\[ M_{\text{max}} = q \left( \frac{1 - x}{2} \right)^2 \]

Also, \( P = qx \) or \( P = qa \):

\[ x = \frac{P}{q}, \quad a = \frac{P}{q} \]

Substituting:

\[ M_{\text{max}} = q \left( \frac{1 - q - P}{2q} \right)^2 \]

Now express moment in terms of "a" and solve:

\[ q \cdot a^2 = q \left( \frac{1 - q - P}{2q} \right)^2 \]

\[ a = \frac{1 - q - P}{2q} \]

In terms of the side and main member, this expression is:

\[ a_s = \frac{1}{2} \frac{s \cdot q_s - P}{q_s} \quad \quad a_m = \frac{1}{2} \frac{m \cdot q_m - P}{q_m} \]
Single Shear Connection Models

Figure A-2: Single shear connection models
Figure A-3: Double shear connection models
Figure A-4: Mode $I_m$ and Mode $I_s$ connection models

**Derivation of Mode $I_m$ and Mode $I_s$ – Single Shear**

Note: The Mode $I_m$ failure is produced by the main member crushing under the dowel. The Mode $I_s$ failure is produced by the side member crushing under the dowel. In both cases, the load causing this failure is $P$. Therefore, the equation governing this type of failure is the dowel bearing resistance multiplied by the member bearing length.

Mode $I_m$:

$$ P = q_m l_m $$

Mode $I_s$:

$$ P = q_s l_s $$
Figure A-5: Mode II connection model
Derivation of Mode II

Equilibrium equation found by summing moments at point C and setting equal to zero:

\[-\frac{g}{2} + \frac{b_s}{2} + \frac{b_s}{2} \cdot q_s a_s + \left(\frac{g}{2} + \frac{b_s}{2}\right) \cdot q_s b_s + \left(\frac{g}{2} + \frac{b_m}{2}\right) \cdot q_m b_m - \frac{g}{2} + \frac{b_m}{2} + \frac{a_m}{2} \cdot q_m a_m = 0\]

First, the equilibrium equation is simplified using expressions relating model variables.

Substituting \(b_s = \frac{2a_s}{2}\) and \(b_m = \frac{2a_m}{2}\). Then, simplifying:

\[-\frac{1}{2} q_s a_s \cdot g - \frac{1}{2} q_s a_s^2 + \frac{1}{2} q_m a_m \cdot g - \frac{1}{2} q_m a_m^2 = 0\]

Substituting \(P = \frac{q_s a_s}{2}\) and \(P = \frac{q_m a_m}{2}\) into the "g" terms:

\[-\frac{1}{2} P \cdot g - \frac{1}{2} q_s a_s^2 + \frac{1}{2} P \cdot g - \frac{1}{2} q_m a_m^2 = 0\]

Combining the "g" terms:

\[-\frac{1}{2} q_s a_s \cdot g - \frac{1}{2} q_m a_m \cdot g = 0\]

Next, using the previously derived expression for "a," the above equation is expressed as a quadratic in \(P\).

Substituting \(a_s = \frac{\frac{1}{2} q_s a_s - \frac{P}{2 q_s}}{2 q_s}\) and \(a_m = \frac{\frac{1}{2} q_m a_m - \frac{P}{2 q_m}}{2 q_m}\). Then, simplifying:

\[-\frac{1}{8} q_s a_s \cdot g - \frac{1}{4} q_s a_s^2 + \frac{1}{4} q_m a_m \cdot g - \frac{1}{4} q_m a_m^2 \cdot \frac{P}{8 q_s} - \frac{1}{8} q_m a_m^2 \cdot \frac{P}{8 q_m} = 0\]

Grouping the "P" terms:

\[-\frac{1}{8 q_s} - \frac{1}{8 q_m} \cdot P^2 + \frac{1}{4} q_s a_s + g + \frac{1}{4} q_m a_m \cdot P - \frac{1}{8} q_m a_m^2 - \frac{1}{8} q_s a_s^2 = 0\]

Multiplying both sides by 2:

\[-\frac{1}{8 q_s} - \frac{1}{8 q_m} \cdot P^2 + \frac{1}{4} m + 2 g + \frac{1}{4} q_s a_s \cdot P - \frac{1}{4} q_m a_m^2 - \frac{1}{4} q_s a_s^2 = 0\]
Now, the expression is in the correct general form. However, the coefficients of the $P^2$ and $P$ terms differ from the TR12 equation. The equation is now manipulated to produce the correct coefficients.

Separating a "Pg" term:

\[
\left[ \frac{-1}{4q_s} - \frac{1}{4q_m} \right] P^2 + \left( \frac{1}{2} l_m + g + \frac{1}{2} l_s \right) P - \frac{1}{4} q_m l_m^2 - \frac{1}{4} q_s l_s^2 + P \cdot g = 0
\]

Substituting $g = \frac{1}{2} a_s + \frac{1}{2} a_m$:

(See Technical Note #1)

\[
\left[ \frac{-1}{4q_s} - \frac{1}{4q_m} \right] P^2 + \left( \frac{1}{2} l_m + g + \frac{1}{2} l_s \right) P - \frac{1}{4} q_m l_m^2 - \frac{1}{4} q_s l_s^2 + P \left( \frac{1}{2} a_s + \frac{1}{2} a_m \right) = 0
\]

Substituting $a_s = \frac{P}{q_s}$ and $a_m = \frac{P}{q_m}$:

\[
\left[ \frac{-1}{4q_s} - \frac{1}{4q_m} \right] P^2 + \left( \frac{1}{2} l_m + g + \frac{1}{2} l_s \right) P - \frac{1}{4} q_m l_m^2 - \frac{1}{4} q_s l_s^2 + P \left( \frac{1}{2} \frac{P}{q_s} + \frac{1}{2} \frac{P}{q_m} \right) = 0
\]

Regrouping "P" terms:

\[
\left[ \frac{1}{4q_s} + \frac{1}{4q_m} \right] P^2 + \left( \frac{1}{2} l_m + g + \frac{1}{2} l_s \right) P - \frac{1}{4} q_m l_m^2 - \frac{1}{4} q_s l_s^2 = 0
\]
Technical Note #1

Where did \( g = \frac{1}{2} a_s + \frac{1}{2} a_m \) come from?

An expression for “g” is developed using the Mode II shear and bending moment diagram. The moment at a point on the moment diagram is equal to the area under the shear diagram up to that point.

![Figure A-6: Detailed Mode II shear and bending moment diagram](image)

From the gap region:

\[
\frac{1}{2} P a_s - P g = \frac{-1}{2} P a_m \\
\frac{1}{2} P a_s + \frac{1}{2} P a_m = P g \\
g = \frac{1}{2} a_s + \frac{1}{2} a_m
\]
Figure A-7: Mode III_{m} connection model
**Derivation of Mode III_m**

Equilibrium equation found by summing moments at point B and setting equal to zero:

\[-M_s - \left( \frac{a_m}{2} + \frac{b_m}{2} \right) - q_m a_m + \left( \frac{a_s}{2} + g + \frac{b_m}{2} \right) - q_s a_s = 0\]

First, the equilibrium equation is simplified using expressions relating model variables.

Substituting \( b_m = (2a_m) \):

\[-M_s - \frac{3}{2} a_m q_m + \left( \frac{1}{2} a_s + g + a_m \right) q_s a_s = 0\]

Substituting \( P = (q_s a_s) \) and \( P = (q_m a_m) \):

\[-M_s - \frac{3}{2} a_m P + \left( \frac{1}{2} a_s + g + a_m \right) P = 0\]

Substituting \( a_s = \frac{P}{q_s} \):

\[-M_s - \frac{3}{2} a_m P + \left( \frac{1}{2} \frac{P}{q_s} + g + a_m \right) P = 0\]

Next, using the previously derived expression for "a," the above equation is expressed as a quadratic in \( P \).

Substituting \( a_m = \left( \frac{1}{2} q_m^2 \right) \):

\[-M_s - \frac{3}{4} \left( \frac{1}{2} q_m^2 \right) \left( \frac{P}{q_m^2} \right) + \left( \frac{1}{2} \frac{P}{q_s} + g + \frac{1}{2} \left( \frac{1}{2} q_m^2 \right) \right) P = 0\]

Collecting "P" terms:

\[ \left[ \frac{1}{4q_m} + \frac{1}{2q_s} \right] P^2 + \left( \frac{-1}{4} q_m^2 + g \right) P - M_s = 0 \]
Now, the expression is in the correct general form. However, the constant term and the coefficient of the P term differ from the TR12 equation.

Using a simple mathematical approach, the correct coefficient of the P term is produced.

Adding and Subtracting \( \frac{1}{2} l m P \):

\[
\left[ \frac{1}{4q_m} + \frac{1}{2q_s} \right] P^2 + \left( -\frac{1}{4} \frac{1}{m} + \frac{g}{s} \right) P - \frac{1}{2} \frac{1}{m} P - \frac{1}{2} \frac{1}{m} P = 0
\]

Combining "\( l m P \)" terms:

\[
\left[ \frac{1}{4q_m} + \frac{1}{2q_s} \right] P^2 + \left( \frac{1}{2} \frac{1}{m} + \frac{g}{s} \right) P - \frac{1}{4} \frac{1}{m} P = 0
\]

The constant term for each equation involving dowel bearing with rotation should be a function of the member length squared. This is done by relating the load, the member bearing resistance, and the member length.

\[
P = q_m a_m
\]

\[
l_m = 3 a_m
\]

\[
P = \left( \frac{1}{m} q_m \right) \left( \frac{1}{3} \right)
\]

This expression can now be used to produce the TR12 equation.

Substituting \( P = \left( \frac{1}{m} q_m \right) \left( \frac{1}{3} \right) \):

\[
\left[ \frac{1}{4q_m} + \frac{1}{2q_s} \right] P^2 + \left( \frac{1}{2} \frac{1}{m} + \frac{g}{s} \right) P - \frac{1}{4} \frac{1}{m} q_m = 0
\]
Figure A-8: Mode III_s connection model
**Derivation of Mode III<sub>s</sub> – Single Shear**

Equilibrium equation found by summing moments at point B and setting equal to zero:

\[-M_m - \left(\frac{a_s}{2} + \frac{b_s}{2}\right)q_s a_s + \left(\frac{a_m}{2} + g + \frac{b_s}{2}\right)q_m a_m = 0\]

First, the equilibrium equation is simplified using expressions relating model variables.

Substituting \(b_s = 2a_s\):

\[-M_m - \left(\frac{a_s}{2} + \frac{b_s}{2}\right)q_s a_s + \left(\frac{a_m}{2} + g + \frac{b_s}{2}\right)q_m a_m = 0\]

Substituting \(p = (q_s a_s)\) and \(p = (q_m a_m)\):

\[-M_m - \frac{3}{2}a_s \cdot p + \left(\frac{1}{2}a_m + g + a_s\right)\cdot p = 0\]

Substituting \(a_m = \left(\frac{p}{q_m}\right)\):

\[-M_m - \frac{3}{2}a_s \cdot p + \left(\frac{1}{2} \cdot \frac{p}{q_m} + g + a_s\right)\cdot p = 0\]

Next, using the previously derived expression for "a," the above equation is expressed as a quadratic in \(p\).

Substituting \(a_s = \left(\frac{1_s q_s - p}{2 q_s}\right)\):

\[-M_m - \frac{3}{4} \left(\frac{1_s q_s - p}{q_s}\right)\cdot p + \left[\frac{1}{2} \cdot \frac{p}{q_m} + g + \frac{1}{2} \cdot \frac{1_s q_s - p}{q_s}\right] \cdot p = 0\]

Collecting "\(p\)" terms:

\[\left[\frac{1}{4 q_s} + \frac{1}{2 q_m}\right] p^2 + \left(-\frac{1}{4} - g\right) p - M_m = 0\]
Now, the expression is in the correct general form. However, the constant term and the coefficient of the P term differ from the TR12 equation.

Using a simple mathematical approach, the correct coefficient of the P term is produced.

Adding and Subtracting \( \frac{1}{2}l_sP \):

\[
\left[ \frac{1}{(4q_s)} + \frac{1}{(2q_m)} \right]P^2 + \left( -\frac{1}{4}l_s + g \right)P - M - \frac{1}{2}l_sP - \frac{1}{2}l_sP = 0
\]

Combining "l_sP" terms:

\[
\left[ \frac{1}{(4q_s)} + \frac{1}{(2q_m)} \right]P^2 + \left( \frac{1}{2}l_s + g \right)P - M - \frac{3}{4}l_sP = 0
\]

The constant term for each equation involving dowel bearing with rotation should be a function of the member length squared. This is done by relating the load, the member bearing resistance, and the member length.

\[ P = q_s \cdot a_s \]
\[ l_s = 3 \cdot a_s \]
\[ P = \frac{l_s q_s}{3} \]

This expression can now be used to produce the TR12 equation.

Substituting \( P = \frac{l_s q_s}{3} \):

\[
\left[ \frac{1}{(4q_s)} + \frac{1}{(2q_m)} \right]P^2 + \left( \frac{1}{2}l_s + g \right)P - M - \frac{1}{2}l_s^2 q_s = 0
\]
Mode IV

Figure A-9: Mode IV connection model
Equilibrium Equation found by summing moments at point C and setting equal to zero:

\[- (M_s + M_m) + \left(\frac{a_m}{2} + \frac{g}{2}\right) q_m a_m + \left(\frac{a_s}{2} + \frac{g}{2}\right) q_s a_s = 0\]

The derivation of the Mode IV equation only involves one substitution to produce the required quadratic form.

Expanding:

\[-M_s - M_m + \frac{1}{2} q_m a_m^2 + \frac{1}{2} q_m a_m g + \frac{1}{2} q_s a_s^2 + \frac{1}{2} q_s a_s g = 0\]

Substituting \(a_s = \frac{P}{q_s}\) and \(a_m = \frac{P}{q_m}\):

\[-M_s - M_m + \frac{1}{(\frac{2}{q_m})} P^2 + g \cdot P + \frac{1}{(\frac{2}{q_s})} P^2 = 0\]

Grouping P terms:

\[
\left[\frac{1}{(\frac{2}{q_m})} + \frac{1}{(\frac{2}{q_s})}\right] P^2 + g \cdot P - M_s - M_m = 0
\]
Double Shear Connections

Mode $I_m$

Mode $I_s$

Mode $III_s$

Mode IV

Figure A-10: Double shear connection free-body diagrams
**Derivation of Double Shear Equations**

Note: Each double shear connection consists of two single shear connections. Each single shear connection transfers a load of $P/2$. The double shear connection equations are derived by using the single shear connection equations and replacing the load, $P$, with $P/2$.

Mode Iₘ:

Note: In the case of main member crushing, the total load causing failure is still $P$. Half the load ($P/2$) comes from each side member.

Single Shear Equation:

$$P = q_m l_m$$

Double Shear Equation:

$$P = \frac{q_m l_m}{2}$$

Mode Iₛ:

Single Shear Equation:

$$P = q_s l_s$$

Double Shear Equation:

$$P = \frac{2q_s l_s}{4}$$

Mode IIIₛ:

Single Shear Equation:

$$1 + \frac{1}{4 q_s} + \frac{1}{2 q_m} \cdot P^2 + \frac{1}{2 l_s + g} \cdot P - M_m - \frac{1}{2} \cdot \frac{2 q_s l_s}{4} = 0$$

Double Shear Equation:

$$\left[ \frac{1}{4 q_s} + \frac{1}{2 q_m} \right] \cdot P^2 + \frac{1}{2 l_s + g} \cdot P - M_m - \frac{1}{2} \cdot \frac{2 q_s l_s}{4} = 0$$

Mode IV:

Single Shear Equation:

$$\left[ \frac{1}{2 q_m} + \frac{1}{2 q_s} \right] \cdot P^2 + g \cdot P - M_s - M_m = 0$$

Double Shear Equation:

$$\left[ \frac{1}{2 q_m} + \frac{1}{2 q_s} \right] \cdot P^2 + g \cdot \frac{P}{2} - M_s - M_m = 0$$
**Summary of Derived EYM Equations**

Table A-4: European Yield Model equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Single Shear</th>
<th>Double Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_m$</td>
<td>$P = q_m l_m$</td>
<td>$P = q_m l_m$</td>
</tr>
<tr>
<td>$I_s$</td>
<td>$P = q_s l_s$</td>
<td>$P = 2q_s l_s$</td>
</tr>
<tr>
<td>II-IV</td>
<td>$P = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$</td>
<td>$P = \frac{-B + \sqrt{B^2 - 4AC}}{A}$</td>
</tr>
</tbody>
</table>

Table A-5: Factors for European Yield Model equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>$\frac{1}{4q_s} + \frac{1}{4q_m}$</td>
<td>$\frac{l_s}{2} + g + \frac{l_m}{2}$</td>
<td>$-\frac{q_s l_s^2}{4} - \frac{q_m l_m^2}{4}$</td>
</tr>
<tr>
<td>III$_m$</td>
<td>$\frac{1}{2q_s} + \frac{1}{4q_m}$</td>
<td>$g + \frac{l_m}{2}$</td>
<td>$-M_s - \frac{q_m l_m^2}{4}$</td>
</tr>
<tr>
<td>III$_s$</td>
<td>$\frac{1}{4q_s} + \frac{1}{2q_m}$</td>
<td>$\frac{l_s}{2} + g$</td>
<td>$-\frac{q_s l_s^2}{4} - M_m$</td>
</tr>
<tr>
<td>IV</td>
<td>$\frac{1}{2q_s} + \frac{1}{2q_m}$</td>
<td>$g$</td>
<td>$-M_s - M_m$</td>
</tr>
</tbody>
</table>
APPENDIX B: DERIVATION OF EYM EQUATIONS – ENERGY BASED
Overview

This appendix outlines the energy-based derivation of the European Yield Model (EYM). This method of deriving the EYM is described in Aune and Patton-Mallory (1986). In that paper, the derivation was outlined briefly. Here, all the general dowel equations will be derived. The virtual displacement method provides a more systematic approach to the derivation than the static equilibrium-based approach (APPENDIX A). The method used in the derivation was the method of virtual displacements and was outlined in CHAPTER 3.

Description of Modes

Table B-1: Yield Modes

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Description of Failure</th>
<th>Applicable Connection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_m</td>
<td>Main member bearing</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>I_s</td>
<td>Side member bearing</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>II</td>
<td>Main and side member bearing</td>
<td>Only single shear</td>
</tr>
<tr>
<td>III_m</td>
<td>Main member bearing, Dowel yielding in side member</td>
<td>Only single shear</td>
</tr>
<tr>
<td>III_s</td>
<td>Side member bearing, Dowel yielding in main member</td>
<td>Both single and double shear</td>
</tr>
<tr>
<td>IV</td>
<td>Dowel yielding in main and side member</td>
<td>Both single and double shear</td>
</tr>
</tbody>
</table>

Assumptions

- End fixity of the dowel is ignored.
- Tension forces in the dowel are ignored.
- Friction between the members is ignored
- Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel.
- Perfect elastic/plastic behavior of all materials is assumed.
**Input Parameters**

The only necessary input parameters deal with connection geometry and strength properties as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_s$</td>
<td>Side member dowel bearing length, in</td>
</tr>
<tr>
<td>$\ell_m$</td>
<td>Main member dowel bearing length, in</td>
</tr>
<tr>
<td>$D$</td>
<td>Dowel diameter, in</td>
</tr>
<tr>
<td>$F_{es}$</td>
<td>Side member dowel bearing strength, psi</td>
</tr>
<tr>
<td>$F_{em}$</td>
<td>Main member dowel bearing strength, psi</td>
</tr>
<tr>
<td>$F_b$</td>
<td>Dowel bending strength, psi</td>
</tr>
</tbody>
</table>

The derivation parameters are used in the yield model to incorporate the input parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{es}$</td>
<td>Side member dowel bearing resistance, lbs/in</td>
</tr>
<tr>
<td>$f_{em}$</td>
<td>Main member dowel bearing resistance, lbs/in</td>
</tr>
<tr>
<td>$M_y$</td>
<td>Moment resistance, in-lbs</td>
</tr>
</tbody>
</table>

The above parameters can be calculated in the following manner:

\[
f_{es} = F_{es} D \\
f_{em} = F_{em} D \\
M_y = F_b \left( \frac{D^3}{6} \right)
\]
Figure B-1: Single shear connection models
The Mode $I_m$ failure is produced by the main member crushing under the dowel. The Mode $I_s$ failure is produced by the side member crushing under the dowel. In both cases, the load causing this failure is $F$. Therefore, the equation governing this type of failure is the dowel bearing resistance multiplied by the member bearing length.

Mode $I_m$:
\[ F = f_{em} l_m \]

Mode $I_s$:
\[ F = f_{es} l_s \]
Figure B-3: Mode II connection model
General Equation:

\[ W = \sum f_e A + \sum (M_y \cdot \theta) \]

Small displacements assumed:

\[ \tan(\theta) = \frac{1}{a} = \theta \]

\[ a = x_s + x_m \]

\[ \sum (M_y \cdot \theta) \] term:

For Mode II there is no yielding in the bolt. Therefore, \( M_y \) equals 0.

\[ \sum (f_e A) \] term:

\[ \sum (f_e A) = \frac{f_{es}}{2-a} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2-a} \left[ (l_m - x_m)^2 + x_m^2 \right] \]

Substituting into the general equation:

\[ W = \sum (f_e A) + \sum (M_y \cdot \theta) \]

\[ W = F \cdot 1 = \frac{f_{es}}{2-a} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2-a} \left[ (l_m - x_m)^2 + x_m^2 \right] \]

Substituting for \( a \):

\[ F = \frac{f_{es}}{2 \cdot (x_s + x_m)} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2 \cdot (x_s + x_m)} \left[ (l_m - x_m)^2 + x_m^2 \right] \]

At this point, the general equation is in terms of two unknown variables, \( x_s \) and \( x_m \). An equation relating \( x_s \) and \( x_m \) is found using a diagram of the bearing stress.

By considering equilibrium in the vertical direction (\( \sum F_y = 0 \)), the following expression is obtained:

\[ f_{es} (l_s - x_s) + f_{em} x_m = f_{es} x_s + f_{em} (l_m - x_m) \]
Solve for $x_m$:

$$x_m = \frac{1}{2} \left( \frac{-f_{es}^{-1}s + 2f_{es}x_s + f_{em}^{-1}m}{f_{em}} \right)$$

Substituting $x_m$ into the expression for $F$ and simplifying:

$$F = \frac{1}{2} \left( \frac{2f_{em}f_{es}^{-1}s^2 - 4x_s f_{em}f_{es}^{-1}s + 4x_s f_{es}^{-1}s x_s - 4f_{es}^{-2}x_s^2 - 4f_{es}^{-2}x_s f_{em}^{-1}m - 4f_{es}^{-2}x_s f_{em}^{-1}m + 2f_{es}^{-1}s f_{em}^{-1}m + f_{em}^{-2}m^2}{2x_s f_{em}^{-1}s + 2f_{es}^{-1}s x_s + f_{em}^{-1}m} \right)$$

This results in an equation with one unknown variable, $x_s$. The expression is minimized by finding the derivative with respect to $x_s$ and setting it equal to zero. An expression for $x_s$ in terms of the known variables can then be found.

Evaluating $\frac{d}{dx_s} (F) = F' = 0$:

$$\frac{d}{dx_s} (F) = 0 = \left[ \left( f_{em} + f_{es} \right) \left( \frac{f_{es}^{-2}s^2 + 4f_{es}^{-2}s x_s - 4f_{es}^{-2}x_s f_{em}^{-1}m - 4f_{es}^{-2}x_s f_{em}^{-1}m + 2f_{es}^{-1}s f_{em}^{-1}m + f_{em}^{-2}m^2}{2x_s f_{em}^{-1}s + 2f_{es}^{-1}s x_s + f_{em}^{-1}m} \right) \right]$$

Solving for $x_s$ and simplifying:

$$x_s = \begin{cases} 
\frac{-1}{2} \left[ f_{es}^{-1}(m f_{em}^{-1}s - f_{es}^{-1}s) + \sqrt{f_{em}^{-1}m f_{es}^{-1}s^2 + f_{es}^{-2}m f_{es}^{-1}s^2 + 2f_{es}^{-1}s f_{em}^{-1}m + f_{em}^{-2}m^2} \right] \\
\frac{-1}{2} \left[ f_{es}^{-1}(m f_{em}^{-1}s - f_{es}^{-1}s) - \sqrt{f_{em}^{-1}m f_{es}^{-1}s^2 + f_{es}^{-2}m f_{es}^{-1}s^2 + 2f_{es}^{-1}s f_{em}^{-1}m + f_{em}^{-2}m^2} \right]
\end{cases}$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative of $F$ and simplifying:

$$\frac{d}{dx_s} (F') = \frac{\left( 4f_{em} + 4f_{es} \right) \left( f_{em}^{-1}m f_{es}^{-1}s^2 + f_{es}^{-2}m f_{es}^{-1}s^2 + 2f_{es}^{-1}s f_{em}^{-1}m + f_{em}^{-2}m^2 \right)}{\left[ f_{es}^{-1}(m f_{em}^{-1}s - f_{es}^{-1}s) \right]^{3}}$$
\( x_s \) must be greater than zero (i.e. use the \( x_s \) expression with the positive root.)

Take the positive \( x_s \):

\[
x_s = -\frac{1}{2} \left[ f_{es} \left( l_m f_{em} - l_s f_{es} \right) - \sqrt{f_{em} f_{es} \left( 2 f_{em} f_{es} + f_{es}^2 l_s^2 + 2 f_{es} l_m f_{em} + 2 f_{em} f_{es} l_s^2 + f_{em}^2 l_m^2 \right)} \right] \frac{f_{es} \left( f_{em} + f_{es} \right)}{}
\]

Let \( R = \sqrt{f_{em} f_{es} \left( 2 f_{em} f_{es} + f_{es}^2 l_s^2 + 2 f_{es} l_m f_{em} + 2 f_{em} f_{es} l_s^2 + f_{em}^2 l_m^2 \right)} \):

\[
x_s = -\frac{1}{2} \left[ f_{es} \left( l_m f_{em} - l_s f_{es} \right) - R \right] \frac{f_{es} \left( f_{em} + f_{es} \right)}{}
\]

Substituting \( x_s \) into the expression for \( F \) and reducing:

\[
F = \frac{1}{2} \frac{\left( 2 f_{em} f_{es} l_s^2 - 4 x_s f_{em} f_{es} l_s + 4 x_s^2 f_{em} f_{es} + f_{em}^2 l_m^2 + f_{es}^2 l_s^2 - 4 f_{es}^2 l_s x_s + 4 f_{es}^2 x_s^2 \right)}{\left( 2 x_s f_{em} - f_{es} l_s + 2 f_{es} x_s + f_{em} l_m \right)}
\]

\[
F = \frac{1}{2} \frac{\left( l_m^2 f_{es} f_{em}^3 + 2 f_{em} f_{es}^2 l_s^2 + 2 f_{em} f_{es}^2 l_m^2 + 2 f_{em} f_{es} l_s^2 l_m + 2 f_{es} l_m f_{em} - 2 f_{em} l_s f_{es} R + R^2 \right)}{\left( f_{em} + f_{es} \right)^2}
\]

\[
F = \frac{1}{2} \frac{R^2 - 2 f_{es} l_m f_{em} - 2 f_{em} l_s f_{es} + f_{em} f_{es} l_s + 2 f_{em} f_{es}^2 l_s^2 + 2 f_{em} f_{es}^2 l_m + 2 f_{em} f_{es}^2 l_m f_{em} + 2 f_{em} f_{es}^2 l_m^2 + f_{em} f_{es} l_s^2}{\left( f_{em} + f_{es} \right)^2}
\]

\[
F = \frac{1}{2} \frac{R^2 - 2 f_{es} l_m f_{em} - 2 f_{em} l_s f_{es} + f_{em} f_{es} l_s + 2 f_{em} f_{es}^2 l_s^2 + 2 f_{em} f_{es}^2 l_m + 2 f_{em} f_{es}^2 l_m f_{em} + 2 f_{em} f_{es}^2 l_m^2 + f_{em} f_{es} l_s^2}{\left( f_{em} + f_{es} \right)^2}
\]
\[
F = \frac{R}{f_{em} + f_{es}} - \frac{f_{em} f_{es} (l_m + l_s)}{f_{em} + f_{es}}
\]

\[
F = \frac{1}{f_{em} + f_{es}} \left[ -f_{em} f_{es} (l_m + l_s) + \sqrt{f_{em} f_{es} (2f_{em} l_m^2 + l_s^2 + 2f_{es} l_s^2 + 2f_{em} f_{es} l_s^2 + l_m^2) + f_{em} f_{es} (l_m + l_s)} \right]
\]

\[
F = \frac{1}{f_{em} + f_{es}} \left[ -f_{em} f_{es} (l_m + l_s) + \sqrt{f_{em} f_{es} \left[ f_{em} l_m^2 f_{es} + f_{es} l_s^2 f_{em} l_m + 2f_{em} f_{es} l_s^2 + f_{em} l_m^2 \right]} \right]
\]

A simpler method to evaluate this formula would be to use the quadratic formula:

\[
F = \frac{-B + \sqrt{B^2 - 4AC}}{2A}
\]

Terms A and B can be found by inspection:

\[
A = \frac{(f_{em} + f_{es})}{2} \quad B = f_{em} f_{es} (l_m + l_s)
\]

Using the previously defined \( R \) to represent the radical:

\[
R = \sqrt{B^2 - 4AC - R^2}
\]

\[
0 = B^2 - 4AC - R^2
\]

\[
C = \frac{1}{4} \left( \frac{B^2 - R^2}{A} \right)
\]

Substituting and simplifying produces:

\[
C = \frac{-1}{2} \left( l_s f_{es} + f_{em} l_m^2 \right) f_{em} f_{es}
\]

The terms A, B, and C can now be reduced further. Since \( A \cdot B^2 + B \cdot P + C = 0 \)

The results of this last simplification will be shown in the equation summary.
Figure B-4: Mode $III_m$ connection model
General Equation:
\[ W = \Sigma (f_{e} \cdot A) + \Sigma (M_{y} \cdot \theta) \]

Small displacements assumed:
\[ \tan(\theta) = \frac{1}{a} = \theta \]
\[ a = x_{s} + x_{m} \]

\[ \Sigma (M_{y} \cdot \theta) \text{ term:} \]
\[ \Sigma (M_{y} \cdot \theta) = \frac{M_{y}}{a} \]

\[ \Sigma (f_{e} \cdot A) \text{ term:} \]
\[ \Sigma (f_{e} \cdot A) = \frac{f_{em}}{2a} \left[ (l_{m} - x_{m})^{2} + x_{m}^{2} \right] + \frac{f_{es}}{2a} \left( x_{s}^{2} \right) \]

Substituting into the general equation:
\[ W = \Sigma (f_{e} \cdot A) + \Sigma (M_{y} \cdot \theta) \]
\[ W = F \cdot 1 = \frac{f_{em}}{2a} \left[ (l_{m} - x_{m})^{2} + x_{m}^{2} \right] + \frac{f_{es}}{2a} \left( x_{s}^{2} \right) + \frac{M_{y}}{a} \]

Substituting for \( a \):
\[ F = \frac{f_{em}}{2\cdot(x_{s} + x_{m})} \left[ (l_{m} - x_{m})^{2} + x_{m}^{2} \right] + \frac{f_{es}}{2\cdot(x_{s} + x_{m})} \left( x_{s}^{2} \right) + \frac{M_{y}}{x_{s} + x_{m}} \]

At this point, the general equation is in terms of two unknown variables, \( x_{s} \) and \( x_{m} \). An equation relating \( x_{s} \) and \( x_{m} \) is found using a diagram of the bearing stress.

By considering equilibrium in the vertical direction (\( \Sigma F_{y} = 0 \)), the following expression is obtained:
\[ f_{em} \cdot (l_{m} - x_{m}) + f_{es} \cdot x_{s} = f_{em} \cdot x_{m} \]
Solve for $x_s$: 

$$x_s = -f_{em} \frac{(1 - 2x_m)}{f_{es}}$$

Substituting for $x_m$ into the expression for $F$ and simplifying:

$$F = \frac{1}{2} \left( f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} - 2f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} + 2f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} - 4f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} + 4f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} + 2M_y \cdot f_{es} \right)$$

This results in an equation with one unknown variable, $x_m$. The expression is minimized by finding the derivative with respect to $x_m$ and setting it equal to zero. An expression for $x_m$ in terms of the known variables can then be found.

Evaluating $\frac{d}{dx_m} (F) = F' = 0$:

$$\frac{d}{dx_m} (F) = 0 = \frac{1}{2} \left( (f_{es} + 2f_{em}) \cdot (2f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} - f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} - 2M_y \cdot f_{es} + 4f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} + f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} - 4f_{em} \cdot f_{es} \cdot x_m \frac{2}{m}) \right)$$

Solving for $x_m$ and simplifying:

$$x_m = \frac{1}{2} \left( \sqrt{f_{em} \cdot (f_{es} + 2f_{em})} \cdot f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} + f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} + 2M_y \cdot f_{es} + 4M_y \cdot f_{em} \right)$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative of $F$ and simplifying:

$$\frac{d}{dx_m} (F) = \frac{(f_{es} + 2f_{em}) \cdot f_{es} \cdot (f_{em} \cdot f_{es} \cdot x_m \frac{2}{m} + f_{es} \cdot f_{em} \cdot x_m \frac{2}{m} + 2M_y \cdot f_{es} + 4M_y \cdot f_{em})}{(f_{es} + 2f_{em}) \cdot x_m \frac{2}{m} - f_{em} \cdot f_{es} \frac{2}{m}}$$
\( x_m \) must be greater than zero (i.e. use the \( x_m \) expression with the positive root).

Take the positive \( x_m \):

\[
x_s = \frac{1}{2} \left[ 2 f_{em}^2 m + \sqrt{2 f_{es} f_{em} \left( f_{em}^2 m^2 + f_{es} f_{em} m^2 + 2 M_y f_{es} + 4 M_y f_{em} \right)} \right]
\]

Let \( R = \sqrt{2 f_{es} f_{em} \left( f_{em}^2 m^2 + f_{es} f_{em} m^2 + 2 M_y f_{es} + 4 M_y f_{em} \right)} \):

\[
x_m = \frac{1}{2} \frac{2 f_{em}^2 m + R}{f_{em} f_{es} + 2 f_{em}}
\]

Substituting \( x_m \) into the expression for \( F \) and reducing:

\[
F = \frac{1}{2} \left[ f_{em} f_{es}^2 m^2 + 2 f_{es} f_{em} - f_{es} M y^2 - 2 f_{es} f_{em} f_{es} - 2 f_{es} f_{em}^2 m^2 + 2 f_{es} f_{em}^2 m^2 + 8 f_{es} f_{em} M y^2 + 2 f_{es} f_{em}^3 m^2 \right]
\]

\[
F = \frac{1}{2} \left[ 2 f_{es} f_{em}^2 m^2 + 4 f_{es}^2 f_{em} M y - 2 f_{es} f_{em}^2 m^2 - R + 8 f_{es}^2 M y^2 f_{es} + 2 f_{es} f_{em}^3 m^2 \right]
\]

\[
F = \frac{1}{2} \left[ f_{es} f_{em}^2 m^2 + 4 f_{es}^2 f_{em} M y + 8 f_{es}^2 M y^2 f_{es} + 2 f_{es} f_{em}^3 m^2 \right]
\]
A simpler method to evaluate this formula would be to use the quadratic formula:

\[ F = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \]

Terms A and B can be found by inspection:

\[ A = \frac{(f_{es} + 2f_{em})}{2} \quad B = f_{em}f_{es}^{-1}m \]

Using the procedure outlined in the Mode II derivation:

\[ C = \frac{-1}{2} \left( f_{em}^{-1}m^2 + 4M_y \right)f_{em}f_{es} \]

The terms A, B, and C can now be reduced further. Since \( A \cdot B^2 + B \cdot P + C = 0 \)

The results of this last simplification will be shown in the equation summary.
Figure B-5: Mode III<sub>s</sub> connection model
General Equation:

\[
W = \sum (f_c \cdot A) + \sum (M_y \cdot \theta)
\]

Small displacements assumed:

\[
\tan(\theta) = \frac{1}{a} = \theta
\]

\[
a = x_s + x_m
\]

\[
\sum (M_y \cdot \theta) \text{ term:}
\]

\[
\sum (M_y \cdot \theta) = \frac{M_y}{a}
\]

\[
\sum (f_c \cdot A) \text{ term:}
\]

\[
\sum (f_c \cdot A) = \frac{f_{es}}{2 \cdot a} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2 \cdot a} (x_m^2)
\]

Substituting into the general equation:

\[
W = \sum (f_c \cdot A) + \sum (M_y \cdot \theta)
\]

\[
W = F \cdot l = \frac{f_{es}}{2 \cdot a} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2 \cdot a} (x_m^2) + \frac{M_y}{a}
\]

Substituting for a:

\[
F = \frac{f_{es}}{2 \cdot (x_s + x_m)} \left[ (l_s - x_s)^2 + x_s^2 \right] + \frac{f_{em}}{2 \cdot (x_s + x_m)} (x_m^2) + \frac{M_y}{x_s + x_m}
\]

At this point, the general equation is in terms of two unknown variables, \(x_s\) and \(x_m\). An equation relating \(x_s\) and \(x_m\) is found using a diagram of the bearing stress.

By considering equilibrium in the vertical direction (\(\Sigma F_y = 0\)), the following expression is obtained:

\[
f_{es} \cdot (l_s - x_s) + f_{em} \cdot x_m = f_{es} \cdot x_s
\]
Solve for $x_m$:

$$x_m = -\frac{f_{es}}{f_{em}} \left( 1_s - 2x_s \right)$$

Substituting for $x_m$ into the expression for $F$ and simplifying:

$$F = \frac{1}{2} \left( f_{em} f_{es} 1_s^2 - 2f_{em} f_{es} 1_s x_s + 2f_{em} f_{es} x_s^2 + f_{es} 2 1_s^2 - 4f_{es} 2 1_s x_s + 4f_{es} 2 x_s^2 + 2M_y f_{em} \right)$$

This results in an equation with one unknown variable, $x_s$. The expression is minimized by finding the derivative with respect to $x_s$ and setting it equal to zero. An expression for $x_s$ in terms of the known variables can then be found.

Evaluating $\frac{d}{dx_s}(F) = F' = 0$:

$$\frac{d}{dx_s}(F) = 0 = \frac{1}{2} \left[ (f_{em} + 2f_{es}) \left( 2f_{em} f_{es} x_s^2 - f_{em} f_{es} 1_s^2 - 2M_y f_{em} + 4f_{es} 2 x_s^2 + f_{es} 2 1_s^2 - 4f_{es} 2 1_s x_s \right) \right]$$

Solving for $x_s$ and simplifying:

$$x_s = \left[ \frac{1}{2} \left[ \frac{2f_{es} 2 1_s^2 f_{em} f_{es} x_s^2 + f_{es} 2 1_s^2 + 2M_y f_{em} + 4M_y f_{es}}{f_{es} f_{em} + 2f_{es}} \right] \right]$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative of $F$ and simplifying:

$$\frac{d}{dx_s}(F') = \frac{\left( f_{em} + 2f_{es} \right) f_{em} \left( f_{es} 2 1_s^2 + f_{em} f_{es} 1_s^2 + 2M_y f_{em} + 4M_y f_{es} \right)}{\left[ \left( f_{em} + 2f_{es} \right) x_s - f_{es} 1_s \right]^3}$$
\( x_s \) must be greater than zero (i.e. use the \( x_s \) expression with the positive root).

Take the positive \( x_s \):

\[
x_s = \frac{1}{2} \left[ \frac{2 f_{es}^2 l_s^2 + \sqrt{2 f_{em} f_{es} \left( f_{es}^2 l_s^2 + f_{em} f_{es} l_s^2 + 2 M_y f_{em} + 4 M_y f_{es}\right)}}{f_{es} \left( f_{em} + 2 f_{es}\right)} \right]
\]

Let \( R = \sqrt{2 f_{em} f_{es} \left( f_{es}^2 l_s^2 + f_{em} f_{es} l_s^2 + 2 M_y f_{em} + 4 M_y f_{es}\right)} \):

\[
x_s = \frac{1}{2} \left( \frac{2 f_{es}^2 l_s^2 + R}{f_{es} \left( f_{em} + 2 f_{es}\right)} \right)
\]

Substituting \( x_s \) into the expression for \( F \) and reducing:

\[
F = \frac{1}{2} \frac{f_{em} f_{es} l_s^2 - 2 f_{em} f_{es} l_s x_s + 2 f_{em} f_{es} x_s^2 + f_{es}^2 l_s^2 - 4 f_{es}^2 l_s x_s + 4 f_{es}^2 x_s^2 + 2 M_y f_{em} \left( x_s f_{em} - f_{es} l_s + 2 f_{es} x_s \right)}{\left( f_{em} + 2 f_{es}\right) \left( f_{em} + 2 f_{es}\right)}
\]

\[
F = \frac{1}{2} \frac{2 f_{em}^2 f_{es}^2 l_s^2 + 4 f_{em}^2 f_{es} M_y - 2 f_{em} f_{es}^3 l_s R + 8 f_{es}^2 M_y f_{em} + 2 f_{em} f_{es}^3 l_s^2 + R^2}{\left( f_{em} + 2 f_{es}\right) \left( f_{em} + 2 f_{es}\right)}
\]

\[
F = \frac{1}{2} \frac{2 f_{em}^2 f_{es}^2 l_s^2 + 4 f_{em}^2 f_{es} M_y - 2 f_{em} f_{es}^3 l_s R + 8 f_{es}^2 M_y f_{em} + 2 f_{em} f_{es}^3 l_s^2}{\left( f_{em} + 2 f_{es}\right) \left( f_{em} + 2 f_{es}\right)}
\]

\[
F = \frac{1}{2} \frac{2 f_{em}^2 f_{es}^2 l_s^2 + 4 f_{em}^2 f_{es} M_y - 2 f_{em} f_{es}^3 l_s R + 8 f_{es}^2 M_y f_{em} + 2 f_{em} f_{es}^3 l_s^2}{\left( f_{em} + 2 f_{es}\right) \left( f_{em} + 2 f_{es}\right)}
\]
A simpler method to evaluate this formula would be to use the quadratic formula:

\[
F = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

Terms A and B can be found by inspection:

\[
A = \frac{f_{em} - 2f_{es}}{2}, \quad B = f_{em}f_{es} - 1s
\]

Using the procedure outlined in the Mode II derivation:

\[
C = \frac{1}{2} \left( f_{es} - 1s \right)^2 + 4M_y f_{em}f_{es}
\]

The terms A, B, and C can now be reduced further. Since \(A \cdot R^2 + B \cdot P + C = 0\) the results of this last simplification will be shown in the equation summary.
Mode IV

Figure B-6: Mode IV connection model
General Equation:

\[ W = \sum (f_e \cdot A) + \sum (M_y \cdot \theta) \]

Small displacements assumed:

\[ \tan(\theta) = \frac{1}{a} = \theta \]

\[ a = x_s + x_m \]

\( \sum (M_y \cdot \theta) \) term:

\[ \sum (M_y \cdot \theta) = \frac{2M_y}{a} \]

\( \sum (f_e \cdot A) \) term:

\[ \sum (f_e \cdot A) = \frac{f_{es}}{2a} \cdot (x_s^2) + \frac{f_{em}}{2a} \cdot (x_m^2) \]

Substituting into the general equation:

\[ W = \sum (f_e \cdot A) + \sum (M_y \cdot \theta) \]

\[ W = F \cdot 1 = \frac{f_{es}}{2a} \cdot (x_s^2) + \frac{f_{em}}{2a} \cdot (x_m^2) + \frac{2M_y}{a} \]

Substituting for a:

\[ F = \frac{f_{es}}{2(x_s + x_m)} \cdot (x_s^2) + \frac{f_{em}}{2(x_s + x_m)} \cdot (x_m^2) + \frac{2M_y}{x_s + x_m} \]

At this point, the general equation is in terms of two unknown variables, \( x_s \) and \( x_m \). An equation relating \( x_s \) and \( x_m \) is found using a diagram of the bearing stress.

By considering equilibrium in the vertical direction (\( \sum F_y = 0 \)), the following expression is obtained:

\[ f_{es} \cdot x_s = f_{em} \cdot x_m \]
Solve for $x_m$:

$$x_m = \frac{f_{es} \cdot x_s}{f_{em}}$$

Substituting for $x_m$ into the expression for $F$ and simplifying:

$$F = 1 \cdot \frac{\left(f_{es} \cdot f_{em} \cdot x_s^2 + f_{es}^2 \cdot x_s^2 + 4 \cdot M_y \cdot f_{em} \right)}{x_s \cdot (f_{em} + f_{es})}$$

This results in an equation with one unknown variable, $x_s$. The expression is minimized by finding the derivative with respect to $x_s$ and setting it equal to zero. An expression for $x_s$ in terms of the known variables can then be found.

Evaluating $\frac{d}{dx_s}(F) = F' = 0$:

$$\frac{d}{dx_s}(F) = 0 = 1 \cdot \frac{\left(f_{es} \cdot f_{em} \cdot x_s^2 + f_{es}^2 \cdot x_s^2 + 4 \cdot M_y \cdot f_{em} \right)}{x_s \cdot (f_{em} + f_{es})}$$

Solving for $x_s$ and simplifying:

$$x_s = \frac{-2 \cdot \sqrt{f_{es} \cdot (f_{em} + f_{es}) \cdot M_y \cdot f_{em}}}{\sqrt{f_{es} \cdot (f_{em} + f_{es})} - 2 \cdot \sqrt{f_{es} \cdot (f_{em} + f_{es}) \cdot M_y \cdot f_{em}} / \sqrt{f_{es} \cdot (f_{em} + f_{es})}}$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative of $F$ and simplifying:

$$\frac{d}{dx_s}(F') = \frac{4 \cdot M_y \cdot f_{em}}{x_s^3 \cdot (f_{em} + f_{es})}$$
xs must be greater than zero (i.e. use the xs expression with the positive root.)

Take the positive xs:

$$xs = 2\sqrt{\frac{f_{em} \pm f_{es}}{f_{es} - (f_{em} + f_{es})} \cdot M \cdot y \cdot f_{em}}$$

Substituting xs into the expression for F and reducing:

$$F = 2M y \cdot f_{em} \cdot f_{es} \sqrt{\frac{f_{em} + f_{es}}{M \cdot y \cdot f_{em} \cdot f_{es}}}$$

A simpler method to evaluate this formula would be to use the quadratic formula:

$$F = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

The B term can be found by inspection:

$$B = 0$$

Leaving:

$$F = \frac{\sqrt{-4AC}}{2A} = \frac{-C}{A}$$

Therefore:

$$A = f_{em} + f_{es}$$

$$C = -4M y \cdot f_{em} \cdot f_{es}$$

The simplified form of the terms A, B, and C will be shown in the equation summary.
Summary of Single Shear Equations

Mode I_m:
\[ F = \frac{f_{\text{em}}}{l_m} \]

Mode I_s:
\[ F = \frac{f_{\text{es}}}{l_s} \]

Mode II:
\[ F = \frac{1}{f_{\text{em}} + f_{\text{es}}} \left[ -f_{\text{em}} f_{\text{es}} (l_m + l_s) + \sqrt{f_{\text{em}} f_{\text{es}} \left[ f_{\text{em}}^2 l_m^2 + f_{\text{es}}^2 l_s^2 + (2 l_m^2 + 2 l_s^2 + 2 l_m l_s)^2 \right]} \right] \]

Mode III_m:
\[ F = \frac{1}{f_{\text{es}} + 2 f_{\text{em}}} \left[ -f_{\text{em}} f_{\text{es}} (l_m + l_s) + \sqrt{2 f_{\text{em}} f_{\text{es}} \left[ f_{\text{em}}^2 l_m^2 (f_{\text{em}} + f_{\text{es}}) + 2 M_y (f_{\text{es}} + 2 f_{\text{em}}) \right]} \right] \]

Mode III_s:
\[ F = \frac{1}{f_{\text{em}} + 2 f_{\text{es}}} \left[ -f_{\text{em}} f_{\text{es}} (l_m + l_s) + \sqrt{2 f_{\text{em}} f_{\text{es}} \left[ f_{\text{es}}^2 l_s^2 (f_{\text{es}} + f_{\text{em}}) + 2 M_y (f_{\text{em}} + 2 f_{\text{es}}) \right]} \right] \]

Mode IV:
\[ F = \frac{4 M_y f_{\text{em}} f_{\text{es}}}{\sqrt{f_{\text{em}}^2 + f_{\text{es}}^2}} \]
**Summary of Double Shear Equations**

The double shear equations are obtained by using the single shear equations where half the load, $F$, is applied per shear plane. The only exception is Mode I$_m$ where the load in the main member remains $F$.

Mode I$_m$:
$$F = f_{em} l_m$$

Mode I$_s$:
$$F = 2f_{es} l_s$$

Mode II:
$$F = \frac{2}{f_{em} + f_{es}} \left[ -f_{em} f_{es} (l_m + l_s) + f_{em} f_{es} \left( f_{em}^2 l_m^2 + f_{es}^2 l_s^2 + (2l_m^2 + 2l_s l_m + 2l_s^2) f_{em} f_{es} \right) \right]$$

Mode III$_m$:
$$F = \frac{2}{f_{es} + 2f_{em}} \left[ -f_{em} f_{es} l_m + f_{em} f_{es} \left( f_{em}^2 l_m^2 + f_{es}^2 (f_{em} + f_{es}) + 2M_y (f_{es} + 2f_{em}) \right) \right]$$

Mode III$_s$:
$$F = \frac{2}{f_{em} + 2f_{es}} \left[ -f_{em} f_{es} l_s + f_{em} f_{es} \left( f_{es}^2 l_s^2 + f_{es}^2 (f_{es} + f_{em}) + 2M_y (f_{em} + 2f_{es}) \right) \right]$$

Mode IV:
$$F = \sqrt{\frac{16M_y f_{em} f_{es}}{f_{em} f_{es}}}$$
Summary of Equations for Use with Quadratic Equation

Table B-4: European Yield Model equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Single Shear</th>
<th>Double Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_m</td>
<td>( F = f_{em} \cdot l_m )</td>
<td>( F = f_{em} \cdot l_m )</td>
</tr>
<tr>
<td>I_s</td>
<td>( F = f_{es} \cdot l_s )</td>
<td>( F = 2f_{es} \cdot l_s )</td>
</tr>
<tr>
<td>II-IV</td>
<td>( F = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} )</td>
<td>( F = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{A} )</td>
</tr>
</tbody>
</table>

Table B-5: Factors for European Yield Model equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>( \frac{1}{2} \cdot f_{es} + \frac{1}{2} \cdot f_{em} )</td>
<td>( l_m + l_s )</td>
<td>( \frac{-1}{2} \left( f_{es} \cdot l_s^2 + f_{em} \cdot l_m^2 \right) )</td>
</tr>
<tr>
<td>III_m</td>
<td>( \frac{1}{2} \cdot f_{em} + \frac{1}{f_{es}} )</td>
<td>( l_m )</td>
<td>( \frac{-1}{2} \left( f_{em} \cdot l_m^2 + 4 \cdot M_y \right) )</td>
</tr>
<tr>
<td>III_s</td>
<td>( \frac{1}{f_{em}} + \frac{1}{2} \cdot f_{es} )</td>
<td>( l_s )</td>
<td>( \frac{-1}{2} \left( f_{es} \cdot l_s^2 + 4 \cdot M_y \right) )</td>
</tr>
<tr>
<td>IV</td>
<td>( \frac{1}{f_{em}} + \frac{1}{f_{es}} )</td>
<td>( 0 )</td>
<td>( -4 \cdot M_y )</td>
</tr>
</tbody>
</table>
APPENDIX C: DERIVATION OF HOLLOW SECTION YIELD MODEL
Overview

This appendix outlines the energy-based derivation of the hollow section yield model (HSYM). The method of virtual displacements was used to develop the equations and the general procedures were outlined in CHAPTER 3. The entire HSYM includes 18 equations. Only the derivation of the six controlling equations will be shown in this appendix. The derivation of the other 12 equations used the identical procedures as those presented here. Recall, that this yield model only applies to sections with two identical walls and a void in the middle (Figure 3-1).

Due to the length of the equations of some intermediate steps of the derivation, the equations were wrapped to fit on the page. The software used has several rules it uses when wrapping equations. Figure C-1 shows several algebraic expressions and how they would be wrapped. In general, the sign shown at the end of the first line only pertains to the first term of the wrapped portion of the equation; the negative sign is not distributed to the entire wrapped portion.

\[
\begin{align*}
a + b + c &\quad a - b + c &\quad a - b - c \\
a + \ldots &\quad a - \ldots &\quad a - \ldots \\
+ b + c &\quad + b + c &\quad + b - c
\end{align*}
\]

Figure C-1: Wrapping Examples
**Input Parameters**

The only necessary input parameters deal with connection geometry and strength properties as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_s$</td>
<td>Average thickness of walls in side member, inches</td>
</tr>
<tr>
<td>$t_m$</td>
<td>Average thickness of walls in main member, inches</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Width of void in side member, inches</td>
</tr>
<tr>
<td>$v_m$</td>
<td>Width of void in main member, inches</td>
</tr>
<tr>
<td>$l_s$</td>
<td>Total width of side member ($l_s = 2t_s + v_s$), inches</td>
</tr>
<tr>
<td>$l_m$</td>
<td>Total width of main member ($l_m = 2t_m + v_m$), inches</td>
</tr>
<tr>
<td>$D$</td>
<td>Dowel diameter, inches</td>
</tr>
<tr>
<td>$F_{es}$</td>
<td>Side member dowel bearing strength, psi</td>
</tr>
<tr>
<td>$F_{em}$</td>
<td>Main member dowel bearing strength, psi</td>
</tr>
<tr>
<td>$F_{yb}$</td>
<td>Dowel bending strength, psi</td>
</tr>
</tbody>
</table>

The derivation parameters are used in the yield model to incorporate the input parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{es}$</td>
<td>Side member dowel bearing resistance, lbs./in.</td>
</tr>
<tr>
<td>$f_{em}$</td>
<td>Main member dowel bearing resistance, lbs./in.</td>
</tr>
<tr>
<td>$M_y$</td>
<td>Moment resistance, in-lbs.</td>
</tr>
</tbody>
</table>

The above parameters can be calculated in the following manner:

\[
f_{es} = F_{es} D \\
f_{em} = F_{em} D \\
M_y = F_{yb} \left( \frac{D^3}{6} \right)
\]
Finding Area of Crushed Material

For each derivation, the area crushed by the dowel must be found. This area is either triangular or trapezoidal in shape. The general procedures to find the areas are summarized below. In all cases, the area crushed will be similar to the a triangle of unit height and base "a".

From the figure on the right and connection yield modes:

\[ \tan \theta = \frac{1}{a} \]

\( A_1 \) (for triangular areas):

From drawing on the right:

\[ \tan \theta = \frac{z}{v} \]

An expression for \( z \) in terms of \( v \) is needed.

By similar triangles:

\[ \frac{1}{a} = \frac{z}{v} \]

\[ z = \frac{v}{a} \]

Find the area of the triangle:

\[ \text{Area} = \frac{1}{2} v z = \frac{v^2}{2a} \]

In all the NSYM derivations, the triangular areas crushed are found in this manner. The triangular area is always equal to the base of the triangle squared divided by the quantity 2 times \( a \).

\( A_2 \) (for trapezoidal areas):

From drawing above:

\[ \tan \theta = \frac{y}{x} = \frac{z}{x - w} \]

An expression for \( y \) and \( z \) in terms of \( x \) and \( w \) is needed.

By similar triangles:

\[ \frac{1}{a} = \frac{y}{x} = \frac{z}{x - w} \]

\[ y = \frac{x}{a} \quad z = \frac{x - w}{a} \]

Find the area of the trapezoid:

\[ \text{Area} = \frac{z + \frac{y}{x}}{2} \cdot x = \frac{z + \frac{y}{x}}{2} \cdot \frac{x - w}{2a} = \frac{(2x - w) \cdot w}{2a} \]

In all the NSYM derivations, the trapezoidal areas crushed are found in this manner.
Figure C-2: Single shear connection models. Boxes highlight controlling yield modes. Only the controlling yield modes will be derived in this appendix.
Mode $I_m$ and Mode $I_s$

The Mode $I_m$ failure is produced by the main member crushing under the dowel. The Mode $I_s$ failure is produced by the side member crushing under the dowel. In both cases, the load causing this failure is $F$. Therefore, the equation governing this type of failure is the dowel bearing resistance multiplied by sum of wall thicknesses.

Mode $I_m$:  

$$F = 2 \cdot t_s \cdot f_{es}$$

Mode $I_s$:  

$$F = 2 \cdot t_m \cdot f_{em}$$
Figure C-4: Mode II: Case 3-3 connection model
General Equation:

$$W = \sum (f_e \cdot A) + \sum (M_y \theta)$$

Small displacements assumed:

$$\tan(\theta) = \frac{1}{a} = \theta$$

$$a = x_s + x_m$$

$$\sum (M_y \theta)$$ term:

For Mode II there is no yielding in the bolt. Therefore, $$M_y$$ equals 0.

$$\sum (f_e \cdot A)$$ term:

$$\sum (f_e \cdot A) = \frac{f_{es}}{2 \cdot a} \left[ (l_s - x_s)^2 + (x_s - t_s - v_s)^2 + (2x_s - t_s \cdot t_s) \right] + \frac{f_{em}}{2 \cdot a} \left[ (2x_m - t_m \cdot t_m + (x_m - t_m - v_m)^2 + (l_m - x_m)^2 \right]$$

Bearing Stress:

$$f_{es} \cdot (l_s - x_s) + f_{em} \cdot t_m + f_{em} \cdot (x_m - t_m - v_m) = f_{es} \cdot (x_s - t_s - v_s) + f_{es} \cdot t_s + f_{em} \cdot (l_m - x_m)$$

Solve for $$x_m$$:

$$x_m = \frac{-l_s \cdot f_{es} - 2f_{es} \cdot x_s - f_{em} \cdot v_m + f_{es} \cdot v_s - f_{em} \cdot l_m}{f_{em}}$$

Substituting into the general equation:

$$W = \sum (f_e \cdot A) + \sum (M_y \theta)$$

$$W = F_1 = \frac{f_{es}}{2 \cdot a} \left[ (l_s - x_s)^2 + (x_s - t_s - v_s)^2 + (2x_s - t_s \cdot t_s) \right] + \frac{f_{em}}{2 \cdot a} \left[ (2x_m - t_m \cdot t_m + (x_m - t_m - v_m)^2 + (l_m - x_m)^2 \right]$$
Substituting for $a$:

$$F = \frac{f_{es}}{2\sqrt{(x_s + x_m)}} \left[ (1 - x_s)^2 + (x_s - t_s - v_s)^2 + (2 \cdot x_s - t_s)^4 \right] + \frac{f_{em}}{2\sqrt{(x_s + x_m)}} \left[ (2 \cdot x_m - t_m)^4 + (x_m - t_m - v_m)^2 + (1 - x_m) \right]$$

Substituting for $x_m$ and simplifying:

$$F = \frac{1}{2} \left[ \frac{-2 \cdot f_{es} \cdot f_{em} \cdot 1^2 \cdot s^2 - 4 \cdot f_{es} \cdot f_{em} \cdot 1 \cdot s \cdot v_s + 4 \cdot f_{es} \cdot f_{em} \cdot v_s^2 + 4 \cdot f_{es} \cdot f_{em} \cdot v_s \cdot m + 2 \cdot f_{es} \cdot f_{em} \cdot v_s^2 \cdot v_m + 2 \cdot f_{es} \cdot f_{em} \cdot v_s \cdot m^2 - 2 \cdot f_{em} \cdot v_m^2 \cdot m^2 + \cdots}{2 \cdot x^2 \cdot s^4 \cdot f_{es} \cdot f_{em} \cdot v_s + 2 \cdot f_{es} \cdot f_{em} \cdot v_s + f_{em} \cdot v_m^2 + f_{em} \cdot v_m^2 + f_{es} \cdot v_s^2 + f_{es} \cdot v_s^2} \right]$$

Substituting $1_s = 2 \cdot t_s + v_s$ and $1_m = 2 \cdot t_m + v_m$. Then, simplifying:

$$F = \frac{-3 \cdot f_{es} \cdot f_{em} \cdot t_s \cdot v_s - 2 \cdot f_{es} \cdot f_{em} \cdot v_s \cdot s \cdot t_s + f_{es} \cdot f_{em} \cdot v_s \cdot s \cdot t_s - 2 \cdot f_{es} \cdot x_s \cdot s \cdot v_s + f_{em} \cdot t_s \cdot m \cdot v_m - 2 \cdot f_{es} \cdot x_s \cdot t_s + \cdots}{(-x_s + f_{es} \cdot f_{em} \cdot t_s + f_{es} \cdot v_s - f_{es} \cdot x_s - f_{em} \cdot v_m - f_{em} \cdot t_m)}$$

Evaluating $\frac{d}{dx_s}(F) = 0$:

$$\frac{d}{dx_s}(F) = 0 = \left[ -\left( f_{em} + f_{es} \right) \left[ f_{em}^2 \cdot t_s \cdot m \cdot v_m + f_{em} \cdot t_s \cdot m^2 + f_{es} \cdot f_{em} \cdot v_s^2 + f_{es} \cdot f_{em} \cdot v_s \cdot t_s \cdot v_m + f_{es} \cdot v_s \cdot t_s \cdot v_m + \cdots \right] \right]$$
Solving for $x_s$ and simplifying:

$$x_s = \frac{-f_{es} \left[ f_{em} \cdot \left( t_m + v_m \right) - f_{es} \cdot \left( v_s + t_s \right) \right] - \sqrt{f_{es} \cdot f_{em} \left[ \left( t_m \cdot v_m + t_m \right)^2 \cdot f_{em}^2 + \cdots \right]}}{f_{es} \cdot f_{em} \left[ f_{em} + f_{es} \right]}$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative and simplifying:

$$\frac{d}{dx_s}(F) = \frac{-2 \cdot (f_{em} + f_{es}) \cdot f_{em} \left[ 2 \cdot f_{em} \cdot t_m \cdot f_{es} + f_{es}^2 \cdot v_s \cdot t_s + 2 \cdot f_{em} \cdot f_{es} \cdot t_s \cdot t_m + f_{es} \cdot f_{em} \cdot v_s^2 + 2 \cdot f_{es} \cdot f_{em} \cdot t_s^2 + f_{em} \cdot t_m \cdot v_m + \cdots \right] + f_{es}^2 \cdot t_s^2 + 2 \cdot f_{em} \cdot f_{es} \cdot v_s \cdot t_m + 3 \cdot f_{es} \cdot f_{em} \cdot t_s \cdot v_s + 2 \cdot f_{es} \cdot f_{em} \cdot v_m + 2 \cdot f_{es} \cdot v_m \cdot f_{em} \cdot v_m + f_{em} \cdot f_{es} \cdot v_m^2 + \cdots + 3 \cdot t_m \cdot f_{em} \cdot v_m \cdot f_{es} + f_{em}^2 \cdot t_m \cdot t_s^2 + f_{es} \cdot f_{em}^2 \cdot t_m \cdot t_s \cdot t_m + f_{es} \cdot f_{em}^3 \cdot t_m \cdot t_s \cdot t_m + \cdots}{\left[ (-f_{es} - f_{em}) \cdot x_s - f_{em} \cdot v_m + f_{es} \cdot t_s + f_{es} \cdot v_s - t_m \cdot f_{em} \right]^3}$$

$x_s$ must be greater than zero (i.e. use the $x_s$ expression with the positive root.)
Take the positive $x_s$:

$$x_s = \frac{-f_{es}\left[ f_{em}\left(t_m + v_m\right) - f_{es}\left(v_s + t_s\right) \right] + \ldots}{f_{es}\left(f_{em} + f_{es}\right)}$$

Let $R = \sqrt{f_{es}\cdot f_{em}\left[ t_m v + t_m^2 \cdot f_{em}^2 + (t_s^2 + v_s^2) \cdot f_{es}^2 + \ldots \right] + \left[ 2 t_s v m + 3 t_m v m + 3 t_s v s + 2 t_s t m + 2 v_s t m + (v_s + v_m)^2 + 2 t_m^2 + 2 t_s^2 \right] f_{em} f_{es}}$

$$x_s = \frac{-f_{es}\left[ f_{em}\left(t_m + v_m\right) - f_{es}\left(v_s + t_s\right) \right] + R}{f_{es}\left(f_{em} + f_{es}\right)}$$

Substituting $x_s$ into the expression for $F$ and reducing:

$$F = \frac{1}{f_{em} + f_{es}} - R \left[ \frac{-2 f_{es}\cdot f_{em}\left(v_m + t_m + v_s + t_s\right)}{f_{em} + f_{es}} + \ldots \right]$$

$$F = \frac{1}{f_{em} + f_{es}} - R \left[ \frac{-2 f_{es}\cdot f_{em}\left(v_m + t_m + v_s + t_s\right)}{f_{em} + f_{es}} + \ldots \right]$$
\[ F = \frac{1}{f_{em} + f_{es}} \cdot R + \frac{-2 \cdot f_{es} \cdot f_{em} \cdot (v_m + t_m + v_s + t_s)}{(f_{em} + f_{es})^2} + \frac{R^2}{(f_{em} + f_{es})^2} \]

\[ F = \frac{2}{f_{em} + f_{es}} \cdot R - 2 \cdot f_{es} \cdot f_{em} \cdot \frac{(v_m + t_m + v_s + t_s)}{(f_{em} + f_{es})^2} \]

\[ F = \frac{2}{f_{em} + f_{es}} \left[ -f_{es} \cdot f_{em} (v_m + t_m + v_s + t_s) + R \right] \]

Substituting for \( R \) produces the general equation:

\[ F = \frac{2}{f_{em} + f_{es}} \left[ -f_{es} \cdot f_{em} (v_m + t_m + v_s + t_s) + R \right] \]

\[ + \left[ \frac{t_m \cdot v_m + t_m^2}{f_{em}^2} \cdot f_{em}^2 + \left( t_s^2 + t_s \cdot v_s \right) \cdot f_{es}^2 + \cdots \right] \]

\[ \sqrt{2 \cdot t_s \cdot v_m + 3 \cdot t_m \cdot v_m + 3 \cdot t_s \cdot v_s + 2 \cdot t_s \cdot t_m + 2 \cdot v_s \cdot t_m + (v_s + v_m)^2 + 2 \cdot t_m^2 + 2 \cdot t_s^2} \cdot f_{em} \cdot f_{es} \]
A simpler method to evaluate this equation would be to use the quadratic formula:

\[
F = \frac{-B + \sqrt{B^2 - 4AC}}{2A}
\]

Terms A and B can be found by inspection:

\[
A = \frac{f_{em} + f_{es}}{4} \quad \quad B = (t_m + t_s + v_s + v_{m}) \cdot f_{em} f_{es}
\]

Using the previously defined R to represent the radical:

\[
R = \sqrt{B^2 - 4AC}
\]

\[
0 = B^2 - 4AC - R^2
\]

\[
C = \frac{B^2 - R^2}{4A}
\]

Substituting and simplifying produces:

\[
C = \left[ f_{es} ' t_s (t_s + v_s) + f_{em} ' t_m (t_m + v_m) \right] f_{es} f_{em}
\]

The terms A, B, and C can now be reduced further. Since \( A \cdot P^2 + B \cdot P + C = 0 \)

The results of this last simplification will be shown in the equation summary.

\[
A = \frac{f_{em} + f_{es}}{4} = \frac{f_{em} + f_{es}}{4 \cdot (f_{em} f_{es})} = \frac{1}{4 \cdot f_{es}} + \frac{1}{4 \cdot f_{em}}
\]

\[
B = (t_m + t_s + v_s + v_m) \cdot f_{em} f_{es} = (t_m + t_s + v_s + v_m)
\]

\[
C = \left[ f_{es} ' t_s (t_s + v_s) + f_{em} ' t_m (t_m + v_m) \right] f_{es} f_{em} = \left[ f_{es} ' t_s (t_s + v_s) + f_{em} ' t_m (t_m + v_m) \right]
\]
Figure C-5: Mode III$	ext{′}$: Case 3-1 connection model
General Equation:
\[ W = \Sigma (f \cdot A) + \Sigma (M \cdot \theta) \]

Small displacements assumed:
\[ \tan(\theta) = \frac{1}{a} = \theta \]
\[ a = x_s + x_m \]

\( \Sigma (M \cdot \theta) \) term:
\[ \Sigma (M \cdot \theta) = \frac{M_y}{a} \]

\( \Sigma (f \cdot A) \) term:
\[ \Sigma (f \cdot A) = \frac{f_{es}}{2a} \left[ \left( l_s - x_s \right)^2 + \left( x_s - t_s - v_s \right)^2 + \left( 2x_s - t_s \right) \cdot t_s \right] + \frac{f_{em}}{2a} \left( \frac{a^2}{2} \right) + \frac{M_y}{a} \]

Bearing Stress:
\[ f_{es} \cdot \left( l_s - x_s \right) + f_{em} \cdot x_m = f_{es} \cdot \left( x_s - t_s - v_s \right) + f_{es} \cdot t_s \]

Solve for \( x_m \):
\[ x_m = \frac{- \left( f_{es} \cdot l_s - 2 \cdot f_{es} \cdot x_s + f_{es} \cdot v_s \right)}{f_{em}} \]

Substituting into the general equation:
\[ W = \Sigma (f \cdot A) + \Sigma (M \cdot \theta) \]
\[ W = F \cdot 1 = \frac{f_{es}}{2a} \left[ \left( l_s - x_s \right)^2 + \left( x_s - t_s - v_s \right)^2 + \left( 2x_s - t_s \right) \cdot t_s \right] + \frac{f_{em}}{2a} \left( \frac{x_m^2}{2} \right) + \frac{M_y}{a} \]
Substituting for a:

\[
F = \frac{f_{es}}{2x_s + 2x_m} \left[ (l_s - x_s)^2 + (x_s - t_s - v_s)^2 + (2x_s - t_s)^2s \right] + \frac{f_{em}}{2x_s + 2x_m} x_m^2 + \frac{M_y}{x_s + x_m}
\]

Substituting for \(x_m\) and simplifying:

\[
F = \frac{1}{2} \left( \frac{f_{em} f_{es} l_s^2 - 2f_{em} f_{es} x_s^2 + 2f_{em} f_{es} x_s \cdot v_s + 2f_{em} f_{es} t_s \cdot v_s + f_{em} f_{es} v_s^2 + f_{es} 2l_s^2 - 4f_{es} l_s x_s + \cdots}{x_s f_{em} - f_{es} l_s + 2f_{es} x_s - f_{es} v_s} \right)
\]

Substituting \(l_s = 2t_s + v_s\) and \(l_m = 2t_m + v_m\). Then, simplifying:

\[
F = \frac{1}{2} \left( \frac{2f_{es} f_{em} t_s^2 + 3f_{es} f_{em} t_s \cdot v_s - 2f_{em} f_{es} l_s x_s + f_{em} f_{es} v_s^2 - 2f_{em} f_{es} v_s x_s + f_{em} f_{es} x_s^2 + 2f_{es} x_s^2 \cdots}{x_s f_{em} - 2f_{es} t_s - 2f_{es} v_s + 2f_{es} x_s} \right)
\]

Evaluating \(\frac{d}{dx_s}(F) = 0\):

\[
\frac{d}{dx_s}(F) = 0 = \left[ \frac{-4f_{es}^3 \cdot v_s^2 - 4f_{es}^3 \cdot t_s^2 - 4f_{es}^3 \cdot x_s^2 + M_y f_{em} + 4f_{em} f_{es}^2 \cdot v_s \cdot x_s + 4f_{em} f_{es}^2 \cdot t_s \cdot x_s + 2f_{es} f_{em}^2 \cdot v_s - \cdots}{x_s f_{em} - 2f_{es} t_s - 2f_{es} v_s + 2f_{es} x_s} \right]
\]
Solving for $x_s$ and simplifying:

$$x_s = \left[ \frac{2 f_{es}^2 (v_s + t_s) - \sqrt{f_{em} f_{es} \left[ (2 t_s^2 + 2 t_s v_s) f_{es}^2 + (f_{em} + 2 f_{es}) M_y + (t_s + v_s) (2 t_s + v_s) f_{es} f_{em} \right]} }{f_{es} (f_{em} + 2 f_{es})} \right]$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative and simplifying:

$$\frac{d}{dx_s} (F') = 2 (f_{em} + 2 f_{es}) f_{em} \frac{\left( M_y f_{em} + 2 f_{es} \ t_s v_s + 2 f_{es} t_s^2 + 2 f_{es} f_{em} f_{es} t_s^2 + f_{em} f_{es} v_s^2 + 3 f_{es} f_{em} t_s v_s + 2 M_y f_{es} \right)}{\left( x_s f_{em} - 2 f_{es} t_s - 2 f_{es} v_s + 2 f_{es} x_s \right)^3}$$

$x_s$ must be greater than zero (i.e. use the $x_s$ expression with the positive root.)

Take the positive $x_s$:

$$x_s = \left[ \frac{2 f_{es}^2 (v_s + t_s) + \sqrt{f_{em} f_{es} \left[ (2 t_s^2 + 2 t_s v_s) f_{es}^2 + (f_{em} + 2 f_{es}) M_y + (t_s + v_s) (2 t_s + v_s) f_{es} f_{em} \right]} }{f_{es} (f_{em} + 2 f_{es})} \right]$$

Let $R = \sqrt{f_{em} f_{es} \left[ (2 t_s^2 + 2 t_s v_s) f_{es}^2 + (f_{em} + 2 f_{es}) M_y + (t_s + v_s) (2 t_s + v_s) f_{es} f_{em} \right]}$

$$x_s = \left[ \frac{2 f_{es}^2 (v_s + t_s) + R}{f_{es} (f_{em} + 2 f_{es})} \right]$$
Substituting $x_s$ into the expression for $F$ and reducing:

\[
F = \frac{2f_{es}\cdot f_{em}\cdot t_s^2 + 3f_{es}\cdot f_{em}\cdot t_s\cdot v_s - 2f_{em}\cdot f_{es}\cdot t_s\cdot x_s + f_{em}\cdot f_{es}\cdot v_s^2 - 2f_{em}\cdot f_{es}\cdot v_s\cdot x_s + f_{em}\cdot f_{es}\cdot x_s^2 + 2f_{es}\cdot x_s^2 - 1\ldots}{x_s\cdot f_{em} - 2f_{es}\cdot t_s - 2f_{es}\cdot v_s + 2f_{es}\cdot x_s}
\]

\[
F = \frac{1}{(f_{em} + 2f_{es})}\cdot R + \left[\frac{-2f_{em}\cdot f_{es}\cdot (t_s + v_s)}{(f_{em} + 2f_{es})}\right] + \left[\frac{f_{em}\cdot f_{es}\cdot (2t_s^2 + 2t_s\cdot v_s)\cdot f_{es}^2 + (f_{em} + 2f_{es})\cdot M_y + (t_s + v_s)\cdot (2t_s + v_s)\cdot f_{es}\cdot f_{em}}{(f_{em} + 2f_{es})\cdot R}\right]
\]

\[
F = \frac{1}{(f_{em} + 2f_{es})}\cdot R + \left[\frac{-2f_{em}\cdot f_{es}\cdot (t_s + v_s)}{(f_{em} + 2f_{es})}\right] + \frac{R^2}{(f_{em} + 2f_{es})\cdot R}
\]

\[
F = \frac{2}{(f_{em} + 2f_{es})}\cdot R + \left[\frac{-1f_{em}\cdot f_{es}\cdot t_s - 1f_{em}\cdot f_{es}\cdot v_s + R}{(f_{em} + 2f_{es})}\right]
\]

\[
F = \frac{2}{(f_{em} + 2f_{es})}\cdot R + \left[\frac{-f_{em}\cdot f_{es}\cdot (t_s + v_s) + R}{(f_{em} + 2f_{es})}\right]
\]

Substituting for $R$ produces the general equation:

\[
F = \frac{2}{(f_{em} + 2f_{es})}\cdot R + \left[\frac{-f_{em}\cdot f_{es}\cdot (t_s + v_s) + \sqrt{f_{em}\cdot f_{es}\cdot (2t_s^2 + 2t_s\cdot v_s)\cdot f_{es}^2 + (f_{em} + 2f_{es})\cdot M_y + (t_s + v_s)\cdot (2t_s + v_s)\cdot f_{es}\cdot f_{em}}}{(f_{em} + 2f_{es})}\right]
\]
A simpler method to evaluate this equation would be to use the quadratic formula:

\[ F = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \]

Terms A and B can be found by inspection:

\[ A = \frac{f_{em} + 2f_{es}}{4} \quad B = f_{em}f_{es}(t_s + v_s) \]

Using the previously defined R to represent the radical:

\[ R = \sqrt{B^2 - 4AC} \quad 0 = B^2 - 4AC - R^2 \]

\[ C = \frac{1}{4} \frac{B^2 - 4AC - R^2}{A} \]

Substituting and simplifying produces:

\[ C = \frac{1}{2} (f_{es}t_s^2 + 4M_y)^{1/2}f_{em}f_{es} \]

The terms A, B, and C can now be reduced further. Since \( A, B, + C = 0 \)

The results of this last simplification will be shown in the equation summary.

\[ A = \frac{f_{em} + 2f_{es}}{4} = \frac{f_{em} + 2f_{es}}{4f_{em}f_{es}} = \frac{1}{4f_{es} + \frac{1}{2f_{em}}} \]

\[ B = f_{em}f_{es}(t_s + v_s) = (t_s + v_s) = \frac{1}{2} \]

\[ C = -[M_y + f_{es}t_s^2(t_s + v_s)]f_{es}f_{em} = -[f_{es}t_s(t_s + v_s) + M_y] = \frac{1}{2} \]
Figure C-6: Mode $III_{m}$: Case 1-3 connection model
The derivation of the Mode III_m equations is identical to the derivation of the Mode III_s equations with the s and m subscripts reversed. Therefore, only the results for Mode III_m will be given.

Location of dowel rotation and dowel yielding:

\[ x_s = \frac{- \left( f_{em} t_m - 2 f_{em} x_m + f_{em} v_m \right)}{f_{es}} \]

\[ x_m = \left[ 2 f_{em}^2 (v_m + t_m) + \sqrt{f_{es} f_{em} \left( (2 t_m)^2 + 2 t_m v_m \right) f_{em}^2 + (f_{es} + 2 f_{em}) M_y + (t_m + v_m) \left( 2 t_m + v_m \right) f_{em} f_{es}} \right] \]

General equation:

\[ F = \frac{2}{f_{es} f_{em} (t_m + v_m)} + \sqrt{f_{es} f_{em} \left( (2 t_m)^2 + 2 t_m v_m \right) f_{em}^2 + (f_{es} + 2 f_{em}) M_y + (t_m + v_m) \left( 2 t_m + v_m \right) f_{em} f_{es}} \]

Reduced quadratic formula terms:

\[ A = \frac{1}{4 f_{em}} + \frac{1}{2 f_{es}} \]

\[ B = (t_m + v_m) \]

\[ C = - \left[ f_{em} t_m (t_m + v_m) + M_y \right] \]
Figure C-7: Mode IV: Case 1-1 connection model
General Equation:
\[ W = \sum (f_e \cdot A) + \sum (M_y \cdot \theta) \]

Small displacements assumed:
\[ \tan(\theta) = \frac{1}{a} = \theta \]
\[ a = x_s + x_m \]

\[ \sum (M_y \cdot \theta) \text{ term:} \]
\[ \sum (M_y \cdot \theta) = \frac{2M_y}{a} \]

\[ \sum (f_e \cdot A) \text{ term:} \]
\[ \sum (f_e \cdot A) = \frac{f_{es}}{2a} (x_s^2) + \frac{f_{em}}{2a} (x_m^2) \]

Bearing Stress:
\[ f_{es} \cdot x_s = f_{em} \cdot x_m \]

Solve for \( x_m \):
\[ x_m = \frac{f_{es} \cdot x_s}{f_{em}} \]

Substituting into the general equation:
\[ W = \sum (f_e \cdot A) + \sum (M_y \cdot \theta) \]
\[ W = F \cdot 1 = \frac{f_{es}}{2a} (x_s^2) + \frac{f_{em}}{2a} (x_m^2) + \frac{2M_y}{a} \]

Substituting for \( a \):
\[ F = \frac{f_{es}}{2} x_s^2 + \frac{f_{em}}{2} x_m^2 + \frac{M_y}{2} \frac{1}{x_s + x_m} \]

Substituting for \( x_m \) and simplifying:
\[ F = \frac{1}{2} \frac{f_{es} \cdot f_{em} \cdot x_s^2 + f_{es}^2 \cdot x_s^2 + 4M_y \cdot f_{em}}{x_s \cdot (f_{em} + f_{es})} \]

Evaluating \( \frac{d}{dx_s} (F) = 0 \):
\[ \frac{d}{dx_s} (F) = 0 = \frac{1}{2} \frac{f_{es} \cdot f_{em} \cdot x_s^2 + f_{es}^2 \cdot x_s^2 - 4M_y \cdot f_{em}}{x_s \cdot (f_{em} + f_{es})} \]
Solving for $x_s$ and simplifying:

$$x_s = \frac{2}{\left(f_{es} f_{em} + f_{es}^2\right)^2} \sqrt{f_{es} \cdot (f_{em} + f_{es}) \cdot M_y \cdot f_{em}}$$

To pick which root to use, look at the second derivative. The second derivative must be greater than zero for a minimum.

Taking the second derivative and simplifying:

$$\frac{d}{dx_s}(F') = 4M_y \cdot \frac{f_{em}}{x_s^3 \cdot (f_{em} + f_{es})}$$

$x_s$ must be greater than zero (i.e. use the $x_s$ expression with the positive root.)

Take the positive $x_s$:

$$x_s = \frac{2}{\left(f_{es} f_{em} + f_{es}^2\right)^2} \sqrt{f_{es} \cdot (f_{em} + f_{es}) \cdot M_y \cdot f_{em}}$$

Substituting $x_s$ into the expression for $F$ and reducing:

$$F = \frac{1}{2} \frac{\left( f_{es} f_{em} x_s^2 + f_{es}^2 x_s^2 + 4M_y f_{em} \right)}{\left[ x_s \cdot (f_{em} + f_{es}) \right]}$$

$$F = \frac{1}{4} \frac{\left( 4 f_{es}^2 f_{em}^2 (f_{em} + f_{es}) \cdot M_y \right)}{\left[ f_{es} f_{em} + f_{es}^2 \right]^2} + \frac{\left( 4 f_{es}^3 (f_{em} + f_{es}) \cdot M_y \cdot f_{em} + 4M_y f_{em} \right)}{\left[ f_{es} f_{em} + f_{es}^2 \right]^2}$$

$$F = \frac{2 f_{em} f_{es} M_y}{\sqrt{f_{es} \cdot (f_{em} + f_{es}) \cdot M_y \cdot f_{em}}}$$
Combining the radicals produces the general equation:

\[ F = \frac{\sqrt{4 f_{em}^2 f_{es}^2 M_y}}{\sqrt{f_{es} (f_{em} + f_{es}) M_y f_{em}}} \]

A simpler method to evaluate this equation would be to use the quadratic formula:

\[ F = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \]

The term B can be found by inspection:

\[ B = 0 \]

The quadratic formula can now be reduced in the following manner:

\[ F = \frac{-4AC}{2A} = \frac{-4AC}{4A^2} = \frac{-C}{A} \]

Therefore:

\[ A = f_{em} + f_{es} \]
\[ C = -\left(4 f_{em} f_{es} M_y\right) \]

The terms A, B, and C can now be reduced further. Since \( A \cdot A^2 + B \cdot P + C = 0 \)

The results of this last simplification will be shown in the equation summary.

\[ A = f_{em} + f_{es} = \frac{f_{em} + f_{es}}{f_{em} f_{es}} = \frac{1}{4f_{es}} + \frac{1}{4f_{em}} \]

\[ B = 0 = 0 = 0 \]

\[ C = -\left(4 f_{em} f_{es} M_y\right) = -M_y = -M_y \]
**Double Shear in Hollow Sections**

The hollow section yield equations have been derived in single shear. Table C-3 describes the conversion of the single shear equations to double shear. Mode I\(_m\) in double shear is two times the single shear connection capacity. However, double shear Mode I\(_m\) remains the same as the single shear equation. The double shear yield mode cannot solely consist of dowel rotation in the main member. Therefore, all cases of Mode II and Mode III\(_m\); Mode III\(_c\): Case 1-3 and Case 3-3; and Mode IV: Case 1-3 and Case 3-3 are not physically possible in double shear. The double shear equations for Mode III\(_c\) and Mode IV for Cases 3-1 and 1-1 are twice the single shear equations.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Case</th>
<th>Double Shear Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode I(_m)</td>
<td>-</td>
<td>Same as single shear</td>
</tr>
<tr>
<td>Mode I(_s)</td>
<td>-</td>
<td>Two times single shear</td>
</tr>
<tr>
<td>Mode II</td>
<td>1-1</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>3-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td></td>
</tr>
<tr>
<td>Mode III(_m)</td>
<td>3-3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td></td>
</tr>
<tr>
<td>Mode III(_c)</td>
<td>3-3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>1-1</td>
<td>Two times single shear</td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td>Two times single shear</td>
</tr>
<tr>
<td>Mode IV</td>
<td>1-1</td>
<td>Two times single shear</td>
</tr>
<tr>
<td></td>
<td>3-3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td>Two times single shear</td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td>N/A</td>
</tr>
</tbody>
</table>

In addition to the single shear yield modes considered, four yield modes specific to double shear equations with hollow members must be considered (Figure C-8). These four yield modes are a result of an additional location of dowel yielding due to the symmetry of the double shear problem. Yield load equations were derived of each yield equation (Table C-4). The resulting equations were in a different form from the rest of the equations. Because the equations are for
strictly hollow sections, the EYM equations are not produced when the void spaces are set to zero. Even though theoretically these cases may occur, the derivation of the equations limits there governing. In order to satisfy equilibrium, both the points of dowel yielding and dowel rotation in the side and main members (x_s and x_m) are restricted to specific locations. Since the locations of dowel yielding and dowel rotation are already determined, energy is no longer minimized during the derivation procedure. This results in the equations failing to ever control connection capacity.

Figure C-8: Double shear yield modes due to symmetry

Table C-4: Double Shear Equations For Symmetric Yield Modes

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Case</th>
<th>Yield Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode III_s</td>
<td>1-2</td>
<td>(F = \frac{(4t_s l_s - 4t_s^2)f_{es}^2 + (2t_m m + 2t_m^2)f_{es}f_{em} + f_{em}^2 t_m^2 + 4M_y f_{es}}{[2t_s + 2t_m + v_m] f_{es} + f_{em}^2 t_m]} )</td>
</tr>
<tr>
<td></td>
<td>3-2</td>
<td>(F = \frac{(v_s l_s - l_s^2 + 4v_s l_s)f_{es}^2 + (2t_m^2 + 2t_m v_m)f_{es}f_{em} + f_{em}^2 t_m^2 + 4M_y f_{es}}{(l_s + v_s + 2t_m + v_m) f_{es} + f_{em}^2 t_m]} )</td>
</tr>
<tr>
<td>Mode IV</td>
<td>1-2</td>
<td>(F = \frac{2[2t_s^2 l_m + v_m] f_{es} + f_{em}^2 t_m^2}{(2t_m + v_m) f_{es} + 2f_{em}^2 t_m]} )</td>
</tr>
<tr>
<td></td>
<td>3-2</td>
<td>(F = \frac{2[2t_s l_m f_{es}^2 + f_{em}^2 l_m + v_m] f_{es} + f_{em}^2 t_m^2}{[2v_s + 2t_m + v_m] f_{es} + 2f_{em}^2 t_m]} )</td>
</tr>
</tbody>
</table>
Summary of Derived Equations for the Hollow Section Yield Model

Table C-5: Hollow Section Yield Model Equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Single Shear</th>
<th>Double Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_m</td>
<td>$F = 2t_m f_{em}$</td>
<td>$F = 2t_m f_{em}$</td>
</tr>
<tr>
<td>I_s</td>
<td>$F = 2t_s f_{es}$</td>
<td>$F = 4t_s f_{es}$</td>
</tr>
<tr>
<td>II-IV</td>
<td>$F = \frac{-B+\sqrt{B^2 - 4A\cdot C}}{2A}$</td>
<td>$F = \frac{-B+\sqrt{B^2 - 4A\cdot C}}{A}$</td>
</tr>
</tbody>
</table>

Table C-6: Factors for Hollow Section Yield Model

<table>
<thead>
<tr>
<th>Mode</th>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode II</td>
<td>1-1</td>
<td>$\frac{1}{4f_{em}}$</td>
<td>$t_s + t_m$</td>
<td>$-\left[f_{es} t_s (t_m + v_m) + f_{em} t_m (t_s + v_s)\right]$</td>
</tr>
<tr>
<td>Mode II</td>
<td>3-3</td>
<td>$\frac{1}{4f_{em}} + \frac{1}{4f_{es}}$</td>
<td>$t_s + v_s + t_m + v_m$</td>
<td></td>
</tr>
<tr>
<td>Mode II</td>
<td>3-1</td>
<td>$\frac{1}{2f_{em}}$</td>
<td>$t_s + v_s + t_m$</td>
<td></td>
</tr>
<tr>
<td>Mode III</td>
<td>1-1</td>
<td>$\frac{1}{4f_{em}}$</td>
<td>$t_m + v_m$</td>
<td>$-\left[f_{em} t_m (t_m + v_m) + f_{es} t_s v_s + M_y\right]$</td>
</tr>
<tr>
<td>Mode III</td>
<td>3-3</td>
<td>$\frac{1}{4f_{em}} + \frac{1}{2f_{es}}$</td>
<td>$t_m + v_m$</td>
<td></td>
</tr>
<tr>
<td>Mode III</td>
<td>3-1</td>
<td>$\frac{1}{2f_{em}}$</td>
<td>$t_m + v_m$</td>
<td></td>
</tr>
<tr>
<td>Mode III</td>
<td>1-3</td>
<td>$\frac{1}{2f_{em}} + \frac{1}{4f_{es}}$</td>
<td>$t_s + v_m$</td>
<td>$-\left[f_{es} t_s (t_s + v_s) + f_{em} t_m v_m + M_y\right]$</td>
</tr>
<tr>
<td>Mode IV</td>
<td>1-1</td>
<td>$\frac{1}{4f_{em}} + \frac{1}{4f_{es}}$</td>
<td>$0$</td>
<td>$-M_y$</td>
</tr>
<tr>
<td>Mode IV</td>
<td>3-3</td>
<td>$\frac{1}{2f_{em}} + \frac{1}{2f_{es}}$</td>
<td>$v_m + v_s$</td>
<td>$-\left(v_m t_m f_{em} + v_s t_s f_{es} + 2M_y\right)$</td>
</tr>
<tr>
<td>Mode IV</td>
<td>3-1</td>
<td>$\frac{1}{2f_{em}} + \frac{1}{2f_{es}}$</td>
<td>$v_s$</td>
<td>$-\left(v_s t_s f_{es} + 2M_y\right)$</td>
</tr>
<tr>
<td>Mode IV</td>
<td>3-1</td>
<td>$\frac{1}{2f_{em}} + \frac{1}{2f_{es}}$</td>
<td>$v_m$</td>
<td>$-\left(v_m t_m f_{em} + 2M_y\right)$</td>
</tr>
</tbody>
</table>
APPENDIX D: COMPUTER PROGRAM TO SIMPLIFY HSYM
**Overview**

This appendix provides information about the computer program utilized to simplify the hollow section yield model. The Fortran program loops over a selected range of input parameters and section geometries and evaluates the yield model. The program can output the results two ways: 1) the values for the independent variables followed by the yield load, yield mode, and yield case (results.txt) and/or 2) only names of the yield modes and cases followed by the number of times that the mode and case controlled (count.txt). The most useful information is the output with the number of times each mode and case controlled. The file of controlling yield loads for each loop becomes unmanageably large with even a small range of input parameters and section geometries. Therefore, the lines that output the entire results have been commented out in the “Program Code” section below (i.e. lines 36, 40, 44, 48, 57, 61, 65, 69, 189, 193, 197, 201, 205, 209, 213, 217, 221, 225, 229, 233, 237, 241, 245, 249, 253, and 257).

**Range of Strength and Section Properties**

Table D-1: Range of Strength and Section Properties Used

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial Value</th>
<th>Final Value</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fb</td>
<td>1,000 psi</td>
<td>200,000 psi</td>
<td>50,000 psi</td>
</tr>
<tr>
<td>Fes1</td>
<td>1,000 psi</td>
<td>50,000 psi</td>
<td>5,000 psi</td>
</tr>
<tr>
<td>Fem1</td>
<td>1,000 psi</td>
<td>50,000 psi</td>
<td>5,000 psi</td>
</tr>
<tr>
<td>D</td>
<td>0.05 in</td>
<td>1.10 in</td>
<td>0.15 in</td>
</tr>
<tr>
<td>ts</td>
<td>0.05 in</td>
<td>1.55 in</td>
<td>0.15 in</td>
</tr>
<tr>
<td>tm</td>
<td>0.05 in</td>
<td>1.55 in</td>
<td>0.15 in</td>
</tr>
<tr>
<td>vs</td>
<td>0.05 in</td>
<td>12.55 in</td>
<td>0.5 in</td>
</tr>
<tr>
<td>vm</td>
<td>0.05 in</td>
<td>12.55 in</td>
<td>0.5 in</td>
</tr>
</tbody>
</table>

Several other intermediate ranges were also evaluated using smaller steps and produced the same controlling yield modes. Table D-1 contains the broadest range of strength values and geometries of the program variables evaluated.
### Program Variables

Table D-2: Program Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nIs, nIm</td>
<td>Counter for Mode I sub s and Mode I sub m; respectively</td>
</tr>
<tr>
<td>nII11, nII33, nII13, nII31</td>
<td>Counter for Mode II: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>nIIIm11, nIIIm33, nIIIm13, nIIIm31</td>
<td>Counter for Mode IIIm: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>nIIIs11, nIIIs33, nIIIs13, nIIIs31</td>
<td>Counter for Mode IIIs: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>nIV11, nIV33, nIV13, nIV31</td>
<td>Counter for Mode IV: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>Fb</td>
<td>Bending yield strength (user input)</td>
</tr>
<tr>
<td>Fes1</td>
<td>Dowel bearing strength of the side member (user input)</td>
</tr>
<tr>
<td>Fem1</td>
<td>Dowel bearing strength of the main member (user input)</td>
</tr>
<tr>
<td>D</td>
<td>Dowel diameter (user input)</td>
</tr>
<tr>
<td>fes</td>
<td>Dowel bearing resistance of the side member</td>
</tr>
<tr>
<td>fem</td>
<td>Dowel bearing resistance of the main member</td>
</tr>
<tr>
<td>My</td>
<td>Moment resistance of the dowel</td>
</tr>
<tr>
<td>ts</td>
<td>Thickness of the side member (user input)</td>
</tr>
<tr>
<td>tm</td>
<td>Thickness of the main member (user input)</td>
</tr>
<tr>
<td>vs</td>
<td>Void width in the side member (user input)</td>
</tr>
<tr>
<td>vm</td>
<td>Void width in the main member (user input)</td>
</tr>
<tr>
<td>fIs, fIm</td>
<td>Calculated yield strength for Mode I sub s and Mode I sub m; respectively</td>
</tr>
<tr>
<td>fII11, fII33, fII13, fII31</td>
<td>Calculated yield strength for Mode II: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>fIIIm11, fIIIm33, fIIIm13, fIIIm31</td>
<td>Calculated yield strength for Mode IIIm: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>fIIIs11, fIIIs33, fIIIs13, fIIIs31</td>
<td>Calculated yield strength for Mode IIIs: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>fIV11, fIV33, fIV13, fIV31</td>
<td>Calculated yield strength for Mode IV: Case 1-1, Case 3-3, Case 1-3, Case 3-1; respectively</td>
</tr>
<tr>
<td>A, B, C</td>
<td>Terms for calculation of the yield strength using the quadratic formula</td>
</tr>
<tr>
<td>iresults</td>
<td>Internal variable to write data to results.txt</td>
</tr>
<tr>
<td>icount</td>
<td>Internal variable to write data to count.txt</td>
</tr>
</tbody>
</table>
Program Code

program hsym
implicit double precision (a-h,o-z), integer(i-n)
real My

!---- This program evaluates the HSYM for single shear
!----
icount=10
iresults=11
open (unit=10,status='new',file='count.txt')
open (unit=11,status='new',file='results.txt')

!---- initialize counters
nIs=0
nIm=0
nII11=0
nII33=0
nII13=0
nII31=0
nIIIm11=0
nIIIm33=0
nIIIm13=0
nIIIm31=0
nIIIs11=0
nIIIs33=0
nIIIs13=0
nIIIs31=0
nIV11=0
nIV33=0
nIV13=0
nIV31=0
ntotal=0

!---- Given Loop
!----
Do 10 i=0,200000,50000
Fb=i
! write(iresults,900)'Fb = ',Fb
! 
Do 20 j=0, 50000, 5000
Fes1=j
! write(iresults,900)'Fes = ',Fes1
! 
Do 30 k=0, 50000, 5000
Fem1=k
! write(iresults,900)'Fem = ',Fem1
! 
Do 40 r=0.05, 1.051, 0.15
d=r
! write(iresults,900)'Dia = ',d
! 
! fes=Fes1*d
fem=Fem1*d
My=Fb*(D**3)/6

Do 50 s=0.05, 1.56, 0.15
ts=s
write(iresults,900)'ts = ',ts
c
Do 60 t=0.05, 1.56, 0.15
tm=t
write(iresults,900)'tm = ',tm
c
Do 70 u=0.05, 12.51, 0.5
vs=u
write(iresults,900)'vs = ',vs
c
Do 80 v=0.05, 12.51, 0.5
vm=v
write(iresults,900)'vm = ',vm
c
---- This section is repeated inside the loop every time

---- Mode Im ----------------------------------------
fIm=2*tm*fem
---- Mode Is ----------------------------------------
fIs=2*ts*fes
---- Mode II ----------------------------------------
A=(1/(4*fem))+(1/(4*fes))
C=-1*(fes*ts*(ts+vs)+fem*tm*(tm+vm))
---- Case 1-1
B=ts+tm
fII11=(-1*B+sqrt(B**2-4*A*C))/(2*A)
c
---- Case 3-3
B=ts+vs+tm+vm
fII33=(-1*B+sqrt(B**2-4*A*C))/(2*A)
c
---- Case 3-1
B=ts+vs+tm
fII31=(-1*B+sqrt(B**2-4*A*C))/(2*A)
c
---- Case 1-3
B=ts+tm+vm
fII13=(-1*B+sqrt(B**2-4*A*C))/(2*A)
c
---- Mode IIIm --------------------------------------
A=(1/(4*fem))+(1/(2*fes))
C=-1*(fes*ts*vs+fem*tm*(tm+vm)+My)
c
---- Case 3-3
B=vs+tm+vm
\[ f_{IIIm33} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Case 3-1
\[ B = vs + tm \]
\[ f_{IIIm31} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]
\[ C = -1*(fem*tm*(tm+vm)+My) \]

--- Case 1-1
\[ B = tm \]
\[ f_{IIIl11} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Case 1-3
\[ B = tm + vm \]
\[ f_{IIIl13} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Mode IIIs ---------------------------------------
\[ A = \frac{1}{(2*fem)} + \frac{1}{(4*fes)} \]
\[ C = -1*(fes*ts*(ts+vs)+fem*tm*vm+My) \]

--- Case 3-3
\[ B = vs + ts + vm \]
\[ f_{IIIi33} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Case 1-3
\[ B = ts + vm \]
\[ f_{IIIi13} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]
\[ C = -1*(fes*ts*(ts+vs)+My) \]

--- Case 1-1
\[ B = ts \]
\[ f_{IIIi11} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Case 3-1
\[ B = vs + ts \]
\[ f_{IIIi31} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]

--- Mode IV ----------------------------------------
\[ A = \frac{1}{(4*fem)} + \frac{1}{(4*fes)} \]
\[ B = 0 \]
\[ C = -1*My \]
\[ f_{IVi11} = \frac{-1*B + \sqrt{B^2 - 4*A*C}}{2*A} \]
\[ A = \frac{1}{(2*fem)} + \frac{1}{(2*fes)} \]

--- Case 3-3
\[ B = vs + vm \]
\[ C = -1*(fes*ts*vs+fem*tm*vm+2*My) \]
\[ f_{IV33} = \frac{-1 \cdot B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \]

\[ f_{IV31} = \frac{-1 \cdot B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \]

**Case 3-1**

\[ B = v_s \]
\[ C = -1 \cdot (f_{es} \cdot t_s \cdot v_s + 2 \cdot M_y) \]

\[ f_{IV31} = \frac{-1 \cdot B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \]

**Case 1-3**

\[ B = v_m \]
\[ C = -1 \cdot (f_{em} \cdot t_m \cdot v_m + 2 \cdot M_y) \]

\[ f_{IV13} = \frac{-1 \cdot B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \]

**Find yield load and yield mode**

\[ \text{yload} = \min(f_{II11}, f_{II13}, f_{II33}, f_{II31}, f_{IV11}, f_{IV13}, f_{IV33}, f_{IV31}, f_{IIIs11}, f_{IIIs13}, f_{IIIs33}, f_{IIIs31}, f_{IIIm11}, f_{IIIm13}, f_{IIIm33}, f_{IIIm31}) \]

\[ \text{if (f}_{Im}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode Im ',yload} \]
\[ \quad n_{Im}=n_{Im}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{Is}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode Is ',yload} \]
\[ \quad n_{Is}=n_{Is}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{II11}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode II - 1-1 ',yload} \]
\[ \quad n_{II11}=n_{II11}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{II33}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode II - 3-3 ',yload} \]
\[ \quad n_{II33}=n_{II33}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{II31}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode II - 3-1 ',yload} \]
\[ \quad n_{II31}=n_{II31}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{II13}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode II - 1-3 ',yload} \]
\[ \quad n_{II13}=n_{II13}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm11}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 1-1 ',yload} \]
\[ \quad n_{IIIm11}=n_{IIIm11}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm33}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 3-3 ',yload} \]
\[ \quad n_{IIIm33}=n_{IIIm33}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm31}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 3-1 ',yload} \]
\[ \quad n_{IIIm31}=n_{IIIm31}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm33}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 3-3 ',yload} \]
\[ \quad n_{IIIm33}=n_{IIIm33}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm31}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 3-1 ',yload} \]
\[ \quad n_{IIIm31}=n_{IIIm31}+1 \]
\[ \text{endif} \]

\[ \text{if (f}_{IIIm13}.eq.\text{yload) then} \]
\[ \quad \text{write(ireults,1000)'Mode IIIm - 1-3 ',yload} \]
\[ \quad n_{IIIm13}=n_{IIIm13}+1 \]
\[ \text{endif} \]
c write(iresults,1000)'Mode IIIm - 1-3 ',yload
nIIIm13=nIIIm13+1
eendif
if (fIIIs11.eq.yload) then
c write(iresults,1000)'Mode IIIs - 1-1 ',yload
nIIIs11=nIIIs11+1
eendif
if (fIIIs33.eq.yload) then
c write(iresults,1000)'Mode IIIs - 3-3 ',yload
nIIIs33=nIIIs33+1
eendif
if (fIIIs31.eq.yload) then
c write(iresults,1000)'Mode IIIs - 3-1 ',yload
nIIIs31=nIIIs31+1
eendif
if (fIV11.eq.yload) then
c write(iresults,1000)'Mode IV - 1-1    ',yload
nIV11=nIV11+1
eendif
if (fIV33.eq.yload) then
c write(iresults,1000)'Mode IV - 3-3    ',yload
nIV33=nIV33+1
eendif
if (fIV31.eq.yload) then
c write(iresults,1000)'Mode IV - 3-1    ',yload
nIV31=nIV31+1
eendif
if (fIV13.eq.yload) then
c write(iresults,1000)'Mode IV - 1-3    ',yload
nIV13=nIV13+1
eendif
c ntotal=ntotal+1
c---- This ends the section of repeated eqn evaluation---------
80 continue
70 continue
60 continue
50 continue
40 continue
30 continue
20 continue
10 continue
c---- Report the number of each mode
write(icount,2000)'Mode Im          ',nIm
write(icount,2000)'Mode Is          ',nIs
write(icount,2000)'Mode II - 1-1    ',nII11
write(icount,2000)'Mode II - 3-3    ',nII33
write(icount,2000)'Mode II - 3-1    ',nII31
write(icount,2000)'Mode IIIm - 1-1  ',nIIIm11
write(icount,2000)'Mode IIIm - 3-3  ',nIIIm33
write(icount,2000)'Mode IIIm - 3-1  ',nIIIm31
write(icount,2000)'Mode IIIm - 1-3 ',nIIIm13
write(icount,2000)'Mode IIIs - 1-1 ',nIIIs11
write(icount,2000)'Mode IIIs - 3-3 ',nIIIs33
write(icount,2000)'Mode IIIs - 3-1 ',nIIIs31
write(icount,2000)'Mode IIIs - 1-3 ',nIIIs13
write(icount,2000)'Mode IV - 1-1 ',nIV11
write(icount,2000)'Mode IV - 3-3 ',nIV33
write(icount,2000)'Mode IV - 3-1 ',nIV31
write(icount,2000)'Mode IV - 1-3 ',nIV13
write(icount,2000)'Total ',ntotal

900 format(a6,f12.3)
1000 format(a17,f10.2)
2000 format(a17,i15.1)

c
end
c
c
APPENDIX E: DERIVATION OF THE LOAD-DISPLACEMENT BEHAVIOR MODEL
Overview
This appendix provides the derivation of the equations that predict the load-displacement curve of a laterally-loaded connection with hollow sections. The goal was to develop a set of equations that would enable a designer to predict the load-displacement behavior of a connection after only conducting dowel bearing tests and bending yield strength tests. The input to the model will be the coefficients of the curves fit to the test data and the dimensions of the dowel and members. Equations have been developed for only the controlling modes of the hollow section yield model (Mode Im, Mode Is, Mode II: Case 3-3, Mode IIIc: Case 3-1, Mode IIIm: Case 1-3, and Mode IV: Case 1-1).

General Procedure
The Mode Is and Im load-displacement model equations are simply the curves fit to the dowel resistance data times the sum of the wall thicknesses. Modes II, III, and IV utilize the method of virtual displacements. In the hollow section yield model development the virtual displacement method was used; external work and internal work are set equal to each other as a connection undergoes a unit slip. To produce the load-displacement equations, energy is conserved as the connection displaced a finite distance. The resulting equation for the yield load is a function of the connection displacement, \( \delta \). The general equation becomes Equation E-1. Derivation of load-displacement behavior expressions requires evaluating the integral and simplifying. However, due to the number of terms introduced from the geometry of the hollow section problem, a closed form solution to the problem is impractical.
\[ W = F \cdot \delta = \int f_e \cdot \eta \cdot d\xi + \sum (M_y \theta) \]

Equation E-1

where:
\[ F = \text{yield load} \]
\[ D = \text{dowel diameter} \]
\[ f_e = \text{dowel bearing resistance} = C_1 \delta + C_2 \delta^2 + C_3 \delta^3 + C_4 \delta^4 + C_5 \delta^5 + C_6 \delta^6 \]
\[ M_y = \text{moment resistance of the dowel} \]
\[ \theta = \text{angle of rotation of the dowel} \]
\[ \eta, \xi = \text{integration variables} \]

The dowel bearing resistance is obtained in this research by fitting sixth-order polynomials (restricted to pass through the origin) to the dowel bearing curves divided by the wall thickness (dowel bearing resistance curves). For derivation purposes, the moment resistance will be assumed to be constant. In the final step to of the derivation, a function for the bending moment in terms of connection displacement may be substituted.

Derivation Steps:
1) Substitute known expressions into Equation E-1 (\( f_e, f_{em} \), and \( \theta \))
2) Simplify with known relationships for \( \eta \) and evaluate integral.
3) Substitute for \( x_m \). Leaving the function with only one unknown variable, \( x_s \).
4) Reduce equation to smallest possible form. Divide by \( \delta \) to leave \( F \) only on the left side of the expression.
5) Take the derivative of \( F \) with respect to \( x_s \) and set to zero.
6) Solve for \( x_s \) to find the location where energy is minimized. Reduce \( x_s \) expression.
7) Substitute the reduced \( x_s \) expression back into \( F \). Reduce expression for \( F \) to final usable form.

The above procedure was attempted for each of the yield modes. Steps 1 through 5 were completed successfully. However, once the derivative with respect to \( x_s \) was computed and set
to zero, it was unfeasible to solve for $x_s$. Numerical methods of minimizing the functions were attempted, but proved to be too computationally demanding. Therefore, the load-displacement equations were developed as functions of $x_s$, $x_m$, and $\delta$. An approximate values for $x_s$ and $x_m$ were found using the expressions found during the derivation of the hollow section yield model. Incorporated into the $x_s$ and $x_m$ expressions is the dowel bearing strength of the members and bending yield strength of the dowel. Using these terms appears to be contradictory to the objective of predicting the entire load-displacement curve (i.e. no longer relying on one arbitrary point from the input curves), but only the ratios of the strength properties to each other is important. For example, using a 5% offset basis to determine these properties would provide a ratio of the strength properties at that location. The assumption is that the $x_s$ and $x_m$ remain constant. Therefore, the values obtained from the yield model equations are sound enough to produce approximate $x_s$ and $x_m$ locations.

Equation E-1 will be evaluated as separate terms because a closed form solution was impossible. The external energy consists of the yield load, $F$, times the displacement, $\delta$. The internal energy depends on the yield mode which is a combination of dowel rotation and dowel yielding. Five general types of internal energy have been defined (Figure E-1). $E_1$, $E_2$, $E_3$, and $E_4$ relate to energy of material crushing. $E_1$, $E_2$, and $E_3$ relate to energy of material crushing when the dowel rotates about a point in the wall farthest from the shear plane. $E_4$ relates to energy of material crushing when a hinge forms within the wall closest to the shear plane. $E_5$ corresponds the energy of forming a hinge in the dowel. Table E-2 defines the types of internal energy present in each yield mode. The total internal energy is the sum of all the applicable $E$-terms to the side and main members. For example, for Mode $III_s$: Case 3-1, the internal energy equals $E_{1s} + E_{2s} + E_{3s} + E_{4m} + E_{5m}$. 

160
Table E-1: Energy Terms by Mode

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Applicable Internal Energy Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Side Member</td>
</tr>
<tr>
<td>Mode II: Case 3-3</td>
<td>$E_1, E_2, E_3$</td>
</tr>
<tr>
<td>Mode III_m: Case 1-3</td>
<td>$E_4, E_5$</td>
</tr>
<tr>
<td>Mode III_s: Case 3-1</td>
<td>$E_1, E_2, E_3$</td>
</tr>
<tr>
<td>Mode IV: Case 1-1</td>
<td>$E_4, E_5$</td>
</tr>
</tbody>
</table>

Predicting an entire load-displacement curve is mathematically intensive and hence a spreadsheet was used. Derived equations are evaluated at specific displacements to produce ordered pairs of displacement and load. Separate equations were developed for each component of the internal energy in order to utilize a spreadsheet more effectively and account for the lack of a closed form solution.
**Derivation of Mode III**

Mode III connections contain all five internal energy types. The derivation of Mode III will be shown in detail. The general equations for each energy type were developed from this one derivation.

![Mode III connection model](image)

*Figure E-2: Mode III connection model.*
Useful relationships from the geometry of the problem:

\[
\tan \theta = \theta = \frac{\delta}{a} = \frac{\delta_m + \delta_s}{x_s + x_m} \quad \tan \theta = \frac{\delta_m}{x_m} \quad \tan \theta = \frac{\delta_s}{x_s}
\]

Therefore:

\[
\frac{\delta_m}{x_m} = \frac{\delta_s}{x_s} \quad \text{so} \quad \delta_m = \frac{\delta_s}{x_s} \cdot x_m
\]

Also,

\[
\delta = \delta_m + \delta_s \quad \text{so} \quad \delta = \delta_s \left( \frac{x_m + 1}{x_s} \right) \quad \text{and} \quad \delta_s = \frac{\delta}{x_m + 1}
\]

From Figure E-3, the following relationships can be developed:

For the side member:

\[
\frac{\eta_s}{\delta_s} = \frac{\xi_s}{x_s} \quad \frac{\eta_m}{\delta_m} = \frac{\xi_m}{x_m}
\]

For the main member:

\[
\eta_s = \frac{\xi_s}{x_s} \cdot \delta_s \quad \eta_m = \frac{\xi_m}{x_m} \cdot \delta_m
\]
From the general equation:

\[ F \ast \delta = E_1 + E_2 + E_3 + E_4 + E_5 \]

Each energy term (E1 through E5) is an integral that uses the following inputs:

\[ fes = a \eta s + b \eta s^2 + c \eta s^3 + d \eta s^4 + e \eta s^5 + f \eta s^6 \]

\[ fem = l \eta m + m \eta m^2 + n \eta m^3 + o \eta m^4 + p \eta m^5 + q \eta m^6 \]

The relationships from Figure E-3 can now be substituted:

\[ \eta s = \frac{\xi s}{xs} \ast \delta s \]

\[ \eta m = \frac{\xi m}{xm} \ast \delta m \]

The general energy terms are now defined and evaluated.

\[ E_1 = \int_0^{2t-v-xs} fes \ast \eta s \, d \xi s \]

Substituting and reducing E1 produces:

\[
\frac{1}{xs} \left( \delta s \left[ \begin{array}{c}
a (2t + v - xs)^3 \delta s \\
b (2t + v - xs)^4 \delta s^2 \\
c (2t + v - xs)^5 \delta s^3 \\
d (2t + v - xs)^6 \delta s^4 \\
e (2t + v - xs)^7 \delta s^5 \\
f (2t + v - xs)^8 \delta s^6 \\
\end{array} \right] \right)
\]

\[ E_2 = \int_0^{xs-t-v} fes \ast \eta s \, d \xi s \]

Substituting and reducing E2 produces:

\[
\frac{1}{xs} \left( \delta s \left[ \begin{array}{c}
a (-t - v + xs)^3 \delta s \\
b (-t - v + xs)^4 \delta s^2 \\
c (-t - v + xs)^5 \delta s^3 \\
d (-t - v + xs)^6 \delta s^4 \\
e (-t - v + xs)^7 \delta s^5 \\
f (-t - v + xs)^8 \delta s^6 \\
\end{array} \right] \right)
\]
\[ E_3 = \int_{\text{x}s-t}^{\text{x}s} \text{fes} \ast \eta \, d \xi \, d s \]

Substituting and reducing \( E_3 \) produces:

\[
\frac{1}{\text{x}s} \left( \delta s \left( \frac{1}{3} a \, \text{x}s^2 \, \delta s - \frac{a \, (-t + \text{x}s)^3 \, \delta s}{3 \, \text{x}s} + \frac{1}{4} b \, \text{x}s^2 \, \delta s^2 - \frac{b \, (-t + \text{x}s)^4 \, \delta s^2}{4 \, \text{x}s^2} + \frac{1}{5} c \, \text{x}s^2 \, \delta s^3 - \frac{c \, (-t + \text{x}s)^5 \, \delta s^3}{5 \, \text{x}s^3} + \right. \\
\left. \frac{1}{6} d \, \text{x}s^2 \, \delta s^4 - \frac{d \, (-t + \text{x}s)^6 \, \delta s^4}{6 \, \text{x}s^4} + \frac{1}{7} e \, \text{x}s^2 \, \delta s^5 - \frac{e \, (-t + \text{x}s)^7 \, \delta s^5}{7 \, \text{x}s^5} + \frac{1}{8} f \, \text{x}s^2 \, \delta s^6 - \frac{f \, (-t + \text{x}s)^8 \, \delta s^6}{8 \, \text{x}s^6} \right) \right)
\]

\[ E_4 = \int_{0}^{\text{x}m} \text{fem} \ast \eta \, m \, d \xi \, d m \]

Substituting and reducing \( E_4 \) produces:

\[
\frac{1}{\text{x}m} \left( \delta m \left( \frac{1}{3} \text{x}m^2 \, \delta m + \frac{1}{4} \text{x}m^2 \, \delta m^2 + \frac{1}{5} \text{x}m^2 \, \delta m^3 + \frac{1}{6} \text{x}m^2 \, \delta m^4 + \frac{1}{7} \text{x}m^2 \, \delta m^5 + \frac{1}{8} \text{x}m^2 \, \delta m^6 \right) \right) \right)
\]

\[ E_5 = \frac{\text{My} \, \delta s}{\text{x}s} \]

The above expressions can now be written in terms of \( \delta \) only by substituting:

\[
\delta m = \frac{\text{x}m \, \delta s}{\text{x}s}
\]

and then substituting:

\[
\delta s = \frac{\delta}{\left(1 + \frac{\text{x}m}{\text{x}s}\right)}
\]

The equations can be simplified further by dividing each term by \( \delta \). The summation of the primed terms will now be equal to the load directly.

\[
F = \frac{E_1 + E_2 + E_3 + E_4 + E_5}{\delta} = E_1' + E_2' + E_3' + E_4' + E_5'
\]
Simplified forms:

E1' =
\[
\frac{a}{3 (x_m + x_s)^2} (2 t + v - x_s)^3 \delta + \frac{b}{4 (x_m + x_s)^3} (2 t + v - x_s)^4 \delta^2 + \frac{c}{5 (x_m + x_s)^4} (2 t + v - x_s)^5 \delta^3 + \\
\frac{d}{6 (x_m + x_s)^5} (2 t + v - x_s)^6 \delta^4 + \frac{e}{7 (x_m + x_s)^6} (2 t + v - x_s)^7 \delta^5 + \frac{f}{8 (x_m + x_s)^7} (2 t + v - x_s)^8 \delta^6
\]

E2' =
\[
\frac{a}{3 (x_m + x_s)^2} (t + v - x_s)^3 \delta + \frac{b}{4 (x_m + x_s)^3} (t + v - x_s)^4 \delta^2 - \frac{c}{5 (x_m + x_s)^4} (t + v - x_s)^5 \delta^3 + \\
\frac{d}{6 (x_m + x_s)^5} (t + v - x_s)^6 \delta^4 - \frac{e}{7 (x_m + x_s)^6} (t + v - x_s)^7 \delta^5 + \frac{f}{8 (x_m + x_s)^7} (t + v - x_s)^8 \delta^6
\]

E3' =
\[
\frac{a}{3 (x_m + x_s)^2} ((t - x_s)^3 + x_s^3) \delta + \frac{b}{4 (x_m + x_s)^3} ((t - x_s)^4 + x_s^4) \delta^2 + \frac{c}{5 (x_m + x_s)^4} ((t - x_s)^5 + x_s^5) \delta^3 + \\
\frac{d}{6 (x_m + x_s)^5} ((t - x_s)^6 + x_s^6) \delta^4 + \frac{e}{7 (x_m + x_s)^6} ((t - x_s)^7 + x_s^7) \delta^5 + \frac{f}{8 (x_m + x_s)^7} ((t - x_s)^8 + x_s^8) \delta^6
\]

E4' =
\[
\frac{1}{3 (x_m + x_s)^2} x_m \delta + \frac{m}{4 (x_m + x_s)^3} x_m^4 \delta^2 + \frac{n}{5 (x_m + x_s)^4} x_m^5 \delta^3 + \frac{o}{6 (x_m + x_s)^5} x_m^6 \delta^4 + \frac{p}{7 (x_m + x_s)^6} x_m^7 \delta^5 + \frac{q}{8 (x_m + x_s)^7} x_m^8 \delta^6
\]

E5' =
\[
\frac{M_y}{x_m + x_s}
\]

A general equation for E' can be found by inspection of the E1', E2', E3', and E4' equations:

\[
E' = \frac{c_1 (i^3 - j^3) \delta}{3 a^2} + \frac{c_2 (i^4 - j^4) \delta^2}{4 a^3} + \frac{c_3 (i^5 - j^5) \delta^3}{5 a^4} + \frac{c_4 (i^6 - j^6) \delta^4}{6 a^5} + \frac{c_5 (i^7 - j^7) \delta^5}{7 a^6} + \frac{c_6 (i^8 - j^8) \delta^6}{8 a^7}
\]

Equation E-2

where:

\[
a = x_s + x_m
\]

\[C_1, C_2, C_3, C_4, C_5, and C_6 are the coefficients from the fit dowel resistance curves\]

\[i and j depend on the type of energy (Table E - 2)\]
Table E-2: Coefficients for Equation E-2

<table>
<thead>
<tr>
<th>Internal Energy Type</th>
<th>$i$</th>
<th>$j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>$2t + \nu - x$</td>
<td>0</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$x - t - \nu$</td>
<td>0</td>
</tr>
<tr>
<td>$E_3$</td>
<td>$x$</td>
<td>$x - t$</td>
</tr>
<tr>
<td>$E_4$</td>
<td>$x$</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: $x$ will either be $x_s$ or $x_m$ depending on the mode and member being considered.

Also note: the $i$ and $j$ terms are the integration limits used at the beginning of the derivation for each energy type.

As stated earlier, during the derivation the dowel bending resistance, $M_y$, was assumed constant. A function for the dowel bending resistance in terms of connection slip, $M_y(\delta)$, may be used when actually evaluating the $E_5$ equation. For this research, a sixth-order polynomial curve was fit to the bending yield strength load-displacement data (Equation E-4). Due to the configuration of the bending yield strength test apparatus, the load was equal to the dowel bending resistance, $M_y$ (see Equation E-3).

$$M_y = \frac{PL}{4} = \frac{P(4)}{4} = P$$  \hspace{1cm} \text{Equation E-3}

The curve fit to the load-displacement data cannot be used directly in the $E_5$ equation because the displacement in the bending yield test is different than the displacement in a connection test. The angle of rotation of the dowel was used to relate the two tests and produce Equation E-5.
\[ M_y (\Delta) = A_m \Delta + B_m \Delta^2 + C_m \Delta^3 + D_m \Delta^4 + E_m \Delta^5 + F_m \Delta^6 \]  
\[ M_y (\delta) = A_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right] + B_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^2 + C_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^3 
\]  
\[ + D_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^4 + E_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^5 + F_m \left[ 2 \tan \left( \frac{\delta}{2(x_s + x_m)} \right) \right]^6 \]  

where:
\[ \Delta = \text{displacement in bending yield strength test} \]
\[ \delta = \text{displacement in connection test} \]

**Equation Summary**

Table 3-4 summarizes the load-displacement predicting equations for single shear connections. The double shear equations are produced by multiplying the single shear equations by two; except in Mode I_m where the single shear and double shear equations are identical.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Load-Displacement Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_s</td>
<td>[ F (\delta) = 2t_s \left( A \delta + B \delta^2 + C \delta^3 + D \delta^4 + E \delta^5 + F \delta^6 \right) ]</td>
</tr>
<tr>
<td>I_m</td>
<td>[ F (\delta) = 2t_m \left( L \delta + M \delta^2 + N \delta^3 + O \delta^4 + P \delta^5 + Q \delta^6 \right) ]</td>
</tr>
<tr>
<td>II: Case 3-3</td>
<td>[ F (\delta) = \left( E_{1_s} ' + E_{2_s} ' + E_{3_s} ' + E_{1_m} ' + E_{2_m} ' + E_{3_m} ' \right) ]</td>
</tr>
<tr>
<td>III: Case 3-1</td>
<td>[ F (\delta) = \left( E_{1_s} ' + E_{2_s} ' + E_{3_s} ' + E_{4_m} ' + E_{5} ' \right) ]</td>
</tr>
<tr>
<td>III_m: Case 1-3</td>
<td>[ F (\delta) = \left( E_{4_s} ' + E_{5} ' + E_{1_m} ' + E_{2_m} ' + E_{3_m} ' \right) ]</td>
</tr>
<tr>
<td>IV: Case 1-1</td>
<td>[ F (\delta) = \left( E_{4_s} ' + E_{5} ' + E_{4_m} ' + E_{5} ' \right) ]</td>
</tr>
</tbody>
</table>

In the Mode IV connections used in this research, the side member and the main member are made of the same material. Therefore, only one set of coefficients are needed and the Mode IV equation may be simplified to Equation E-6 (single shear).

\[ F(\delta) = \sqrt{\left[ M_y (\delta) \right] \left[ \frac{2}{3} A \delta + \frac{1}{4} B \delta^2 + \frac{1}{10} C \delta^3 + \frac{1}{24} D \delta^4 + \frac{1}{56} E \delta^5 + \frac{1}{128} F \delta^6 \right]} \]  

Equation E-6

168
**Mode IV Closed-Form Derivation**

Equation E-6 was derived using the closed-form solution outlined above. The noteworthy steps are summarized below.

Using the general equations for the internal energy for Mode IV:

\[ F = 2E_4' + 2E_5' \]

Substituting the terms:

\[
F = \frac{2M_y}{x_m + x_s} \left( \frac{1}{3} \left( \frac{x_m^2}{(x_m + x_s)^2} \right) + \frac{m x_m^4 \delta}{4 (x_m + x_s)^3} + \frac{n x_m^6 \delta^3}{5 (x_m + x_s)^4} + \frac{o x_m^8 \delta^4}{6 (x_m + x_s)^5} + \frac{p x_m^{10} \delta^5}{7 (x_m + x_s)^6} + \frac{q x_m^{12} \delta^6}{8 (x_m + x_s)^7} \right)
\]

Since the problem is symmetric, \( x_m = x_s \). Substituting and simplifying:

\[
F = \frac{M_y}{x_s} + 2 \frac{x_s \delta (89601 + 3360 m \delta + 1344 n \delta^2 + 560 o \delta^3 + 240 p \delta^4 + 105 q \delta^5)}{107520}
\]

Taking the derivative with respect to \( x_s \):

\[
F' = -\frac{M_y}{x s^2} + 2 \left( \frac{1 \delta}{12} + \frac{m \delta^2}{32} + \frac{n \delta^3}{80} + \frac{o \delta^4}{192} + \frac{p \delta^5}{448} + \frac{q \delta^6}{1024} \right)
\]

Setting \( F' \) equal to zero and solving for \( x_s \) produces:

\[
\left\{ x_s \rightarrow \sqrt{\frac{M_y}{2}} \left( \sqrt{\frac{1}{12} + \frac{m \delta^2}{32} + \frac{n \delta^3}{80} + \frac{o \delta^4}{192} + \frac{p \delta^5}{448} + \frac{q \delta^6}{1024} } \right) \right\}
\]

Substituting \( x_s \) back into \( F \) and reducting:

\[
F = \frac{M_y}{2} \sqrt{\delta (89601 + \delta (3360 m + \delta (1344 n + 5 \delta (112 o + 48 p \delta + 21 q \delta^2)))))}}{8 \sqrt{210}}
\]

\[
F = \frac{\sqrt{M_y \left( \frac{21 \delta}{3} + \frac{m \delta^2}{4} + \frac{n \delta^3}{10} + \frac{o \delta^4}{24} + \frac{p \delta^5}{56} + \frac{q \delta^6}{128} \right)}}{\sqrt{210}}
\]
APPENDIX F: TEST DATA
### Dowel Bearing Strength Data

**HDPE 0.2” wall thickness - 1/4” hole**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>15L</td>
<td>480.8</td>
<td>480.8</td>
<td>480.8</td>
<td>0.192</td>
<td>0.212</td>
<td>4821.34</td>
<td>4821.34</td>
</tr>
<tr>
<td>17L</td>
<td>476.0</td>
<td>475.4</td>
<td>475.4</td>
<td>0.194</td>
<td>0.211</td>
<td>4755.42</td>
<td>4761.42</td>
</tr>
<tr>
<td>16L</td>
<td>473.3</td>
<td>470.9</td>
<td>470.9</td>
<td>0.193</td>
<td>0.210</td>
<td>4733.78</td>
<td>4757.91</td>
</tr>
<tr>
<td>10R</td>
<td>471.9</td>
<td>471.7</td>
<td>471.9</td>
<td>0.195</td>
<td>0.215</td>
<td>4662.84</td>
<td>4662.84</td>
</tr>
<tr>
<td>16R</td>
<td>469.5</td>
<td>469.3</td>
<td>469.3</td>
<td>0.193</td>
<td>0.211</td>
<td>4706.02</td>
<td>4708.02</td>
</tr>
<tr>
<td>12R</td>
<td>476.2</td>
<td>476.2</td>
<td>476.2</td>
<td>0.196</td>
<td>0.212</td>
<td>4728.39</td>
<td>4728.39</td>
</tr>
<tr>
<td>8R</td>
<td>452.1</td>
<td>450.7</td>
<td>450.7</td>
<td>0.198</td>
<td>0.214</td>
<td>4431.75</td>
<td>4445.51</td>
</tr>
<tr>
<td>9L</td>
<td>465.8</td>
<td>464.7</td>
<td>464.7</td>
<td>0.198</td>
<td>0.214</td>
<td>4569.41</td>
<td>4580.22</td>
</tr>
<tr>
<td>14R</td>
<td>471.4</td>
<td>471.1</td>
<td>471.1</td>
<td>0.194</td>
<td>0.214</td>
<td>4677.75</td>
<td>4680.73</td>
</tr>
<tr>
<td>11R</td>
<td>482.4</td>
<td>482.1</td>
<td>482.1</td>
<td>0.194</td>
<td>0.212</td>
<td>4810.56</td>
<td>4813.55</td>
</tr>
<tr>
<td>17R</td>
<td>463.6</td>
<td>463.1</td>
<td>463.1</td>
<td>0.194</td>
<td>0.214</td>
<td>4598.32</td>
<td>4603.28</td>
</tr>
<tr>
<td>15R</td>
<td>484.8</td>
<td>484.3</td>
<td>484.8</td>
<td>0.194</td>
<td>0.213</td>
<td>4825.61</td>
<td>4825.61</td>
</tr>
<tr>
<td>13L</td>
<td>447.2</td>
<td>447.0</td>
<td>447.2</td>
<td>0.194</td>
<td>0.212</td>
<td>4462.31</td>
<td>4462.31</td>
</tr>
</tbody>
</table>

**Average =** 469.9  0.195  0.213  4675.7  4680.9  
**Standard Deviation =** 11.27  0.2038  128.39  126.93  
**COV =** 2.4%  2.7%  2.7%  

**Min =** 447.20  4431.75  4445.51  
**Max =** 484.80  4825.61  4825.61  

---

**Dowel Bearing Resistance Curves**

**Average**

---

**Displacement (inches)**

---

171
### HDPE 0.2" wall thickness - 3/8" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>7a</td>
<td>656.4</td>
<td>653.7</td>
<td>656.4</td>
<td>0.198</td>
<td>0.214</td>
<td>4296.20</td>
<td>4296.20</td>
</tr>
<tr>
<td>7c</td>
<td>643.8</td>
<td>639.7</td>
<td>643.8</td>
<td>0.196</td>
<td>0.215</td>
<td>4223.99</td>
<td>4223.99</td>
</tr>
<tr>
<td>5</td>
<td>665.0</td>
<td>663.4</td>
<td>665.0</td>
<td>0.195</td>
<td>0.211</td>
<td>4416.81</td>
<td>4416.81</td>
</tr>
<tr>
<td>6</td>
<td>646.4</td>
<td>641.9</td>
<td>646.4</td>
<td>0.199</td>
<td>0.214</td>
<td>4220.51</td>
<td>4220.51</td>
</tr>
<tr>
<td>5b</td>
<td>654.0</td>
<td>652.3</td>
<td>654.0</td>
<td>0.199</td>
<td>0.211</td>
<td>4301.38</td>
<td>4301.38</td>
</tr>
<tr>
<td>7</td>
<td>617.4</td>
<td>614.2</td>
<td>617.4</td>
<td>0.199</td>
<td>0.215</td>
<td>4021.42</td>
<td>4021.42</td>
</tr>
</tbody>
</table>

- **Average:** 647.2, 0.198, 0.213, 4246.7, 4246.7
- **Standard Deviation:** 16.42, 0.2055, 131.40, 131.40
- **COV:** 2.5%, 3.1%, 3.1%

- **Min:** 617.40, 4021.42
- **Max:** 665.00, 4416.81

---

#### Dowel Bearing Resistance Curves

- **Average Curve**

---

172
### HDPE 0.3” wall thickness - 1/4” hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>9a</td>
<td>786.6</td>
<td>773.2</td>
<td>773.2</td>
<td>0.323</td>
<td>0.296</td>
<td>5060.41</td>
<td>5148.11</td>
</tr>
<tr>
<td>9a4</td>
<td>796.8</td>
<td>776.9</td>
<td>776.9</td>
<td>0.324</td>
<td>0.297</td>
<td>5068.25</td>
<td>5198.07</td>
</tr>
<tr>
<td>9b</td>
<td>786.0</td>
<td>771.8</td>
<td>771.8</td>
<td>0.325</td>
<td>0.296</td>
<td>5034.98</td>
<td>5127.61</td>
</tr>
<tr>
<td>9b2</td>
<td>767.5</td>
<td>762.4</td>
<td>762.4</td>
<td>0.324</td>
<td>0.297</td>
<td>4973.66</td>
<td>5006.93</td>
</tr>
<tr>
<td>9b3</td>
<td>797.0</td>
<td>776.4</td>
<td>776.4</td>
<td>0.322</td>
<td>0.298</td>
<td>5073.16</td>
<td>5207.76</td>
</tr>
<tr>
<td>9b4</td>
<td>793.0</td>
<td>788.5</td>
<td>788.5</td>
<td>0.322</td>
<td>0.296</td>
<td>5168.89</td>
<td>5198.39</td>
</tr>
<tr>
<td>8a</td>
<td>797.3</td>
<td>794.4</td>
<td>794.4</td>
<td>0.323</td>
<td>0.295</td>
<td>5207.57</td>
<td>5226.58</td>
</tr>
<tr>
<td>8a2</td>
<td>796.5</td>
<td>785.0</td>
<td>785.0</td>
<td>0.324</td>
<td>0.296</td>
<td>5129.35</td>
<td>5204.49</td>
</tr>
<tr>
<td>8a3</td>
<td>817.2</td>
<td>807.8</td>
<td>807.8</td>
<td>0.322</td>
<td>0.298</td>
<td>5278.33</td>
<td>5339.75</td>
</tr>
<tr>
<td>8b</td>
<td>766.2</td>
<td>753.3</td>
<td>753.3</td>
<td>0.322</td>
<td>0.296</td>
<td>4938.15</td>
<td>5022.71</td>
</tr>
</tbody>
</table>

- **average** = 779.0
- **standard deviation** = 15.73
- **COV** = 2.0%

**min**
- 753.30

**max**
- 807.80

---

**Average Dowel Bearing Resistance Curves**

![Dowel Bearing Resistance Curves](image)

- Dowel Bearing Resistance Curves
- Average Curve
### HDPE 0.3" wall thickness - 3/8" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6L</td>
<td>1096.0</td>
<td>1090.0</td>
<td>1090.0</td>
<td>0.325</td>
<td>0.298</td>
<td>4717.93</td>
<td>4743.90</td>
</tr>
<tr>
<td>6R</td>
<td>1093.0</td>
<td>1089.0</td>
<td>1089.0</td>
<td>0.323</td>
<td>0.296</td>
<td>4744.07</td>
<td>4761.49</td>
</tr>
<tr>
<td>25L</td>
<td>1073.0</td>
<td>1071.0</td>
<td>1071.0</td>
<td>0.322</td>
<td>0.295</td>
<td>4680.77</td>
<td>4689.52</td>
</tr>
<tr>
<td>23R</td>
<td>1086.0</td>
<td>1084.0</td>
<td>1084.0</td>
<td>0.323</td>
<td>0.294</td>
<td>4737.59</td>
<td>4746.33</td>
</tr>
<tr>
<td>11R</td>
<td>1059.0</td>
<td>1058.0</td>
<td>1058.0</td>
<td>0.326</td>
<td>0.296</td>
<td>4586.79</td>
<td>4591.12</td>
</tr>
<tr>
<td>11L</td>
<td>1078.0</td>
<td>1074.0</td>
<td>1074.0</td>
<td>0.321</td>
<td>0.298</td>
<td>4678.72</td>
<td>4696.15</td>
</tr>
<tr>
<td>17R</td>
<td>1077.0</td>
<td>1075.0</td>
<td>1075.0</td>
<td>0.325</td>
<td>0.297</td>
<td>4660.49</td>
<td>4669.16</td>
</tr>
<tr>
<td>17L</td>
<td>1074.0</td>
<td>1068.0</td>
<td>1068.0</td>
<td>0.323</td>
<td>0.296</td>
<td>4652.58</td>
<td>4678.72</td>
</tr>
<tr>
<td>25R</td>
<td>1077.0</td>
<td>1075.0</td>
<td>1075.0</td>
<td>0.322</td>
<td>0.292</td>
<td>4721.21</td>
<td>4730.00</td>
</tr>
</tbody>
</table>

#### Average Value

- **Average:** 1076.0
- **Standard Deviation:** 10.30
- **COV:** 1.0%

#### Minimum and Maximum Values

- **Min:**
  - Max Load: 1058.00
  - Int Load: 0.3096
  - DBS: 4586.79
  - Max: 4591.12
- **Max:**
  - Max Load: 1090.00
  - Int Load: 0.323
  - DBS: 4744.07
  - Max: 4761.49

### Diagram

- **Dowel Bearing Resistance Curves**
- **Average Curve**
<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>1048.0</td>
<td>967.5</td>
<td>967.5</td>
<td>0.411</td>
<td>0.410</td>
<td>6326.52</td>
<td>6852.91</td>
</tr>
<tr>
<td>20</td>
<td>1065.0</td>
<td>1020.0</td>
<td>1020.0</td>
<td>0.409</td>
<td>0.413</td>
<td>6661.71</td>
<td>6955.60</td>
</tr>
<tr>
<td>15</td>
<td>1027.0</td>
<td>970.7</td>
<td>970.7</td>
<td>0.409</td>
<td>0.409</td>
<td>6370.72</td>
<td>6740.22</td>
</tr>
<tr>
<td>16L</td>
<td>1057.0</td>
<td>962.7</td>
<td>962.7</td>
<td>0.409</td>
<td>0.410</td>
<td>6310.51</td>
<td>6928.64</td>
</tr>
<tr>
<td>8R</td>
<td>1104.0</td>
<td>990.1</td>
<td>990.1</td>
<td>0.409</td>
<td>0.411</td>
<td>6482.20</td>
<td>7227.90</td>
</tr>
<tr>
<td>25R</td>
<td>1042.0</td>
<td>983.6</td>
<td>983.6</td>
<td>0.410</td>
<td>0.410</td>
<td>6439.64</td>
<td>6821.99</td>
</tr>
<tr>
<td>10R</td>
<td>1026.0</td>
<td>956.2</td>
<td>956.2</td>
<td>0.412</td>
<td>0.413</td>
<td>6222.31</td>
<td>6676.53</td>
</tr>
<tr>
<td>27</td>
<td>1013.0</td>
<td>956.2</td>
<td>956.2</td>
<td>0.409</td>
<td>0.413</td>
<td>6245.02</td>
<td>6615.99</td>
</tr>
<tr>
<td>14R</td>
<td>1021.0</td>
<td>951.4</td>
<td>951.4</td>
<td>0.411</td>
<td>0.413</td>
<td>6198.59</td>
<td>6652.05</td>
</tr>
<tr>
<td>18R</td>
<td>1046.0</td>
<td>978.3</td>
<td>978.3</td>
<td>0.410</td>
<td>0.411</td>
<td>6397.14</td>
<td>6839.83</td>
</tr>
<tr>
<td>21R</td>
<td>1031.0</td>
<td>979.3</td>
<td>979.3</td>
<td>0.407</td>
<td>0.410</td>
<td>6435.03</td>
<td>6774.76</td>
</tr>
<tr>
<td>26L</td>
<td>1030.0</td>
<td>966.4</td>
<td>966.4</td>
<td>0.410</td>
<td>0.411</td>
<td>6319.33</td>
<td>6735.21</td>
</tr>
<tr>
<td>23L</td>
<td>1033.0</td>
<td>966.4</td>
<td>966.4</td>
<td>0.409</td>
<td>0.409</td>
<td>6342.50</td>
<td>6779.60</td>
</tr>
<tr>
<td>11L</td>
<td>1026.0</td>
<td>929.7</td>
<td>929.7</td>
<td>0.411</td>
<td>0.410</td>
<td>6079.34</td>
<td>6709.05</td>
</tr>
<tr>
<td>13L</td>
<td>1020.0</td>
<td>932.3</td>
<td>932.3</td>
<td>0.413</td>
<td>0.413</td>
<td>6059.44</td>
<td>6629.45</td>
</tr>
<tr>
<td>17</td>
<td>1047.0</td>
<td>930.2</td>
<td>930.2</td>
<td>0.408</td>
<td>0.410</td>
<td>6104.92</td>
<td>6871.48</td>
</tr>
</tbody>
</table>

average = 965.1  
standard deviation = 23.52  
COV = 2.4%  
min 929.70  
max 1020.00  

Dowel Bearing Resistance Curves
Average Curve
<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>DBS 1</th>
<th>DBS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1155.0</td>
<td>1073.0</td>
<td>1073.0</td>
<td>0.412</td>
<td>0.415</td>
</tr>
<tr>
<td>7</td>
<td>1159.0</td>
<td>1111.0</td>
<td>1111.0</td>
<td>0.409</td>
<td>0.412</td>
</tr>
<tr>
<td>2</td>
<td>1098.0</td>
<td>1036.0</td>
<td>1036.0</td>
<td>0.414</td>
<td>0.409</td>
</tr>
<tr>
<td>4</td>
<td>1138.0</td>
<td>1076.0</td>
<td>1076.0</td>
<td>0.414</td>
<td>0.412</td>
</tr>
<tr>
<td>16R</td>
<td>1129.0</td>
<td>1071.0</td>
<td>1071.0</td>
<td>0.412</td>
<td>0.410</td>
</tr>
<tr>
<td>7b</td>
<td>1158.0</td>
<td>1080.0</td>
<td>1080.0</td>
<td>0.410</td>
<td>0.409</td>
</tr>
<tr>
<td>12</td>
<td>1083.0</td>
<td>1035.0</td>
<td>1035.0</td>
<td>0.415</td>
<td>0.414</td>
</tr>
<tr>
<td>5b</td>
<td>1127.0</td>
<td>1067.0</td>
<td>1067.0</td>
<td>0.410</td>
<td>0.411</td>
</tr>
<tr>
<td>15</td>
<td>1140.0</td>
<td>1087.0</td>
<td>1087.0</td>
<td>0.409</td>
<td>0.410</td>
</tr>
<tr>
<td>6b</td>
<td>1149.0</td>
<td>1086.0</td>
<td>1086.0</td>
<td>0.410</td>
<td>0.412</td>
</tr>
<tr>
<td>21L</td>
<td>1178.0</td>
<td>1108.0</td>
<td>1108.0</td>
<td>0.407</td>
<td>0.408</td>
</tr>
<tr>
<td>4b</td>
<td>1153.0</td>
<td>1108.0</td>
<td>1108.0</td>
<td>0.411</td>
<td>0.410</td>
</tr>
</tbody>
</table>

- **average:** 1078.2  
- **standard deviation:** 24.95  
- **COV:** 2.3%

**Min**  
- 1035.00  
- 5057.90

**Max**  
- 1111.00  
- 5507.65

---

**Dowel Bearing Resistance Curves**

- **Average Curve**
<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>11R</td>
<td>1536.0</td>
<td>1520.0</td>
<td>1520.0</td>
<td>0.415</td>
<td>0.412</td>
<td>4954.09</td>
<td>5006.24</td>
</tr>
<tr>
<td>9</td>
<td>1545.0</td>
<td>1517.0</td>
<td>1517.0</td>
<td>0.412</td>
<td>0.413</td>
<td>4956.30</td>
<td>5047.78</td>
</tr>
<tr>
<td>24L</td>
<td>1569.0</td>
<td>1534.0</td>
<td>1534.0</td>
<td>0.409</td>
<td>0.410</td>
<td>5048.56</td>
<td>5163.75</td>
</tr>
<tr>
<td>13R</td>
<td>1529.0</td>
<td>1504.0</td>
<td>1504.0</td>
<td>0.412</td>
<td>0.412</td>
<td>4919.79</td>
<td>5001.57</td>
</tr>
<tr>
<td>12</td>
<td>1537.0</td>
<td>1518.0</td>
<td>1518.0</td>
<td>0.415</td>
<td>0.412</td>
<td>4947.57</td>
<td>5009.50</td>
</tr>
<tr>
<td>22</td>
<td>1516.0</td>
<td>1481.0</td>
<td>1481.0</td>
<td>0.406</td>
<td>0.410</td>
<td>4892.05</td>
<td>5007.66</td>
</tr>
<tr>
<td>8L</td>
<td>1511.0</td>
<td>1471.0</td>
<td>1471.0</td>
<td>0.413</td>
<td>0.412</td>
<td>4806.01</td>
<td>4936.70</td>
</tr>
<tr>
<td>22R</td>
<td>1590.0</td>
<td>1563.0</td>
<td>1563.0</td>
<td>0.409</td>
<td>0.411</td>
<td>5137.73</td>
<td>5226.48</td>
</tr>
<tr>
<td>10L</td>
<td>1545.0</td>
<td>1519.0</td>
<td>1519.0</td>
<td>0.410</td>
<td>0.413</td>
<td>4974.90</td>
<td>5060.05</td>
</tr>
<tr>
<td>23</td>
<td>1529.0</td>
<td>1513.0</td>
<td>1513.0</td>
<td>0.410</td>
<td>0.411</td>
<td>4967.32</td>
<td>5019.85</td>
</tr>
</tbody>
</table>

average = 1514.0 0.411 0.412 4960.4 5048.0
standard deviation = 25.66 0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11
COV = 1.7% 1.8% 1.7%

min 1471.00
max 1563.00

Displacement (in) vs Dowel Bearing Resistance (lbs/in)
### PVC 0.2" wall thickness - 1/4" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>9a</td>
<td>1608.1</td>
<td>1556.7</td>
<td>1608.1</td>
<td>0.192</td>
<td>0.207</td>
<td>16327.20</td>
<td>16327.20</td>
</tr>
<tr>
<td>9a2</td>
<td>1713.1</td>
<td>1686.3</td>
<td>1713.1</td>
<td>0.195</td>
<td>0.208</td>
<td>17221.52</td>
<td>17221.52</td>
</tr>
<tr>
<td>9b</td>
<td>1605.6</td>
<td>1559.2</td>
<td>1605.6</td>
<td>0.195</td>
<td>0.209</td>
<td>16100.62</td>
<td>16100.62</td>
</tr>
<tr>
<td>9a3</td>
<td>1740.0</td>
<td>1674.0</td>
<td>1740.0</td>
<td>0.195</td>
<td>0.210</td>
<td>17405.38</td>
<td>17405.38</td>
</tr>
<tr>
<td>9b3</td>
<td>1698.5</td>
<td>1652.0</td>
<td>1698.5</td>
<td>0.195</td>
<td>0.210</td>
<td>16989.80</td>
<td>16989.80</td>
</tr>
</tbody>
</table>

- **Average**: 1673.1
- **Standard Deviation**: 62.27
- **COV**: 3.7%

**Minimum**
- Sample 9a: 1605.61
- Sample 9a3: 1605.61
- Sample 9b: 16100.62
- Sample 9b3: 16100.62

**Maximum**
- Sample 9a: 1740.02
- Sample 9a3: 17405.38
- Sample 9b: 16989.80
- Sample 9b3: 16989.80

---

**Displacement (in) vs q (lbs/in)**

- Dowel Bearing Resistance Curves
- Average Curve
### PVC 0.2" wall thickness - 3/8" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS 1</th>
<th>DBS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>11L</td>
<td>1979.5</td>
<td>1935.5</td>
<td>1979.5</td>
<td>0.193</td>
<td>0.210</td>
<td>13245.47</td>
<td>13245.47</td>
</tr>
<tr>
<td>12R</td>
<td>2155.5</td>
<td>2089.5</td>
<td>2155.5</td>
<td>0.194</td>
<td>0.205</td>
<td>14567.44</td>
<td>14567.44</td>
</tr>
<tr>
<td>15R</td>
<td>2236.1</td>
<td>2157.9</td>
<td>2236.1</td>
<td>0.193</td>
<td>0.212</td>
<td>14888.59</td>
<td>14888.59</td>
</tr>
<tr>
<td>17L</td>
<td>2094.4</td>
<td>1986.8</td>
<td>2094.4</td>
<td>0.192</td>
<td>0.208</td>
<td>14119.14</td>
<td>14119.14</td>
</tr>
<tr>
<td>4</td>
<td>1940.4</td>
<td>1901.3</td>
<td>1940.4</td>
<td>0.195</td>
<td>0.208</td>
<td>12983.84</td>
<td>12983.84</td>
</tr>
<tr>
<td>16L</td>
<td>2106.6</td>
<td>2038.2</td>
<td>2106.6</td>
<td>0.193</td>
<td>0.203</td>
<td>14344.97</td>
<td>14344.97</td>
</tr>
<tr>
<td>7</td>
<td>1930.6</td>
<td>1884.2</td>
<td>1930.6</td>
<td>0.192</td>
<td>0.205</td>
<td>13113.67</td>
<td>13113.67</td>
</tr>
<tr>
<td>18L</td>
<td>2028.4</td>
<td>1942.9</td>
<td>2028.4</td>
<td>0.193</td>
<td>0.210</td>
<td>13572.52</td>
<td>13572.52</td>
</tr>
<tr>
<td>14R</td>
<td>1957.5</td>
<td>1920.9</td>
<td>1957.5</td>
<td>0.192</td>
<td>0.207</td>
<td>13229.61</td>
<td>13229.61</td>
</tr>
<tr>
<td>19L</td>
<td>2040.6</td>
<td>2006.4</td>
<td>2040.6</td>
<td>0.191</td>
<td>0.206</td>
<td>13860.65</td>
<td>13860.65</td>
</tr>
<tr>
<td>5</td>
<td>2084.6</td>
<td>1938.0</td>
<td>2028.4</td>
<td>0.192</td>
<td>0.209</td>
<td>13640.22</td>
<td>13640.22</td>
</tr>
<tr>
<td>13L</td>
<td>2101.7</td>
<td>2067.5</td>
<td>2101.7</td>
<td>0.192</td>
<td>0.211</td>
<td>14063.10</td>
<td>14063.10</td>
</tr>
<tr>
<td>5b</td>
<td>1999.1</td>
<td>1913.5</td>
<td>1999.1</td>
<td>0.191</td>
<td>0.208</td>
<td>13510.39</td>
<td>13510.39</td>
</tr>
<tr>
<td>7b</td>
<td>2087.0</td>
<td>2047.9</td>
<td>2087.0</td>
<td>0.197</td>
<td>0.207</td>
<td>13930.42</td>
<td>13930.42</td>
</tr>
</tbody>
</table>

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
<td>2049.0</td>
<td>0.193</td>
<td>0.208</td>
<td>13790.7</td>
<td>13790.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>86.98</td>
<td>0.2003</td>
<td>567.90</td>
<td>567.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>4.2%</td>
<td>4.1%</td>
<td>4.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>1930.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>max</td>
<td>2236.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Dowel Bearing Resistance Curves](image)

179
PVC 0.3" wall thickness - 1/4" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6a</td>
<td>2837.3</td>
<td>2798.2</td>
<td>2837.3</td>
<td>0.314</td>
<td>0.294</td>
<td>18905.46</td>
<td>18905.46</td>
</tr>
<tr>
<td>7a2</td>
<td>2854.4</td>
<td>2825.1</td>
<td>2854.4</td>
<td>0.316</td>
<td>0.295</td>
<td>18926.06</td>
<td>18926.06</td>
</tr>
<tr>
<td>6a3</td>
<td>2876.4</td>
<td>2861.7</td>
<td>2876.4</td>
<td>0.315</td>
<td>0.294</td>
<td>19134.53</td>
<td>19134.53</td>
</tr>
<tr>
<td>7a3</td>
<td>2781.1</td>
<td>2776.2</td>
<td>2776.2</td>
<td>0.314</td>
<td>0.295</td>
<td>18467.99</td>
<td>18500.50</td>
</tr>
<tr>
<td>7b3</td>
<td>2837.3</td>
<td>2773.8</td>
<td>2837.3</td>
<td>0.314</td>
<td>0.296</td>
<td>18843.47</td>
<td>18843.47</td>
</tr>
<tr>
<td>6b2</td>
<td>2812.9</td>
<td>2793.3</td>
<td>2812.9</td>
<td>0.314</td>
<td>0.292</td>
<td>18804.48</td>
<td>18804.48</td>
</tr>
<tr>
<td>6b4</td>
<td>2844.6</td>
<td>2815.3</td>
<td>2844.6</td>
<td>0.316</td>
<td>0.296</td>
<td>18830.43</td>
<td>18830.43</td>
</tr>
<tr>
<td>7b4</td>
<td>2810.4</td>
<td>2778.7</td>
<td>2810.4</td>
<td>0.313</td>
<td>0.295</td>
<td>18726.34</td>
<td>18726.34</td>
</tr>
</tbody>
</table>

average = 2831.2  0.315  0.295  18829.8  18833.9
standard deviation = 30.77  0.046  189.09  180.35
COV = 1.1%  1.0%  1.0%

min  2776.21  min  18467.99  18500.50
max  2876.41  max  19134.53  19134.53

---

**Dowel Bearing Resistance Curves**

- **Average Curve**

---

Displacement (in)
### PVC 0.3” wall thickness - 3/8” hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>8R</td>
<td>3863.7</td>
<td>3729.3</td>
<td>3863.7</td>
<td>0.316</td>
<td>0.294</td>
<td>17080.07</td>
<td>17080.07</td>
</tr>
<tr>
<td>18L</td>
<td>3797.7</td>
<td>3704.9</td>
<td>3797.7</td>
<td>0.314</td>
<td>0.296</td>
<td>16788.38</td>
<td>16788.38</td>
</tr>
<tr>
<td>8L</td>
<td>3836.8</td>
<td>3753.8</td>
<td>3836.8</td>
<td>0.311</td>
<td>0.292</td>
<td>17158.13</td>
<td>17158.13</td>
</tr>
<tr>
<td>1L</td>
<td>3788.0</td>
<td>3685.3</td>
<td>3788.0</td>
<td>0.313</td>
<td>0.293</td>
<td>16855.69</td>
<td>16855.69</td>
</tr>
<tr>
<td>9R</td>
<td>3902.8</td>
<td>3790.4</td>
<td>3902.8</td>
<td>0.314</td>
<td>0.294</td>
<td>17309.67</td>
<td>17309.67</td>
</tr>
<tr>
<td>1R</td>
<td>3827.1</td>
<td>3682.9</td>
<td>3827.1</td>
<td>0.315</td>
<td>0.294</td>
<td>16945.80</td>
<td>16945.80</td>
</tr>
<tr>
<td>9L</td>
<td>3878.4</td>
<td>3802.6</td>
<td>3878.4</td>
<td>0.312</td>
<td>0.294</td>
<td>17258.06</td>
<td>17258.06</td>
</tr>
<tr>
<td>2L</td>
<td>3731.8</td>
<td>3680.4</td>
<td>3731.8</td>
<td>0.315</td>
<td>0.291</td>
<td>16605.58</td>
<td>16605.58</td>
</tr>
<tr>
<td>2R</td>
<td>3748.9</td>
<td>3653.6</td>
<td>3748.9</td>
<td>0.314</td>
<td>0.293</td>
<td>16654.22</td>
<td>16654.22</td>
</tr>
<tr>
<td>18R</td>
<td>3863.7</td>
<td>3734.2</td>
<td>3863.7</td>
<td>0.314</td>
<td>0.290</td>
<td>17249.74</td>
<td>17249.74</td>
</tr>
</tbody>
</table>

- **Average:** 3823.9, 0.314, 0.293, 16990.5, 16990.5
- **Standard Deviation:** 56.42, 0.3035, 258.00, 258.00
- **COV:** 1.5%, 1.5%, 1.5%, 1.5%

<table>
<thead>
<tr>
<th></th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>3731.76</td>
<td>3902.82</td>
<td></td>
</tr>
<tr>
<td>16605.58</td>
<td>17309.67</td>
<td></td>
</tr>
</tbody>
</table>

![Dowel Bearing Resistance Curves](image-url)
### PVC 0.4" wall thickness - 1/4" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall 1</th>
<th>wall 2</th>
<th>DBS</th>
<th>DBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3935.6</td>
<td>3906.2</td>
<td>3935.6</td>
<td>0.390</td>
<td>0.410</td>
<td>19929.89</td>
<td>19929.89</td>
</tr>
<tr>
<td>24L</td>
<td>4121.6</td>
<td>4053.1</td>
<td>4121.6</td>
<td>0.391</td>
<td>0.410</td>
<td>20845.79</td>
<td>20845.79</td>
</tr>
<tr>
<td>7L</td>
<td>4004.1</td>
<td>3945.4</td>
<td>4004.1</td>
<td>0.390</td>
<td>0.410</td>
<td>20276.93</td>
<td>20276.93</td>
</tr>
<tr>
<td>22</td>
<td>4141.2</td>
<td>4053.1</td>
<td>4141.2</td>
<td>0.390</td>
<td>0.409</td>
<td>20997.25</td>
<td>20997.25</td>
</tr>
<tr>
<td>23R</td>
<td>3769.2</td>
<td>3730.0</td>
<td>3769.2</td>
<td>0.389</td>
<td>0.411</td>
<td>19087.09</td>
<td>19087.09</td>
</tr>
<tr>
<td>25</td>
<td>3955.2</td>
<td>3925.8</td>
<td>3955.2</td>
<td>0.391</td>
<td>0.410</td>
<td>20004.04</td>
<td>20004.04</td>
</tr>
<tr>
<td>20R</td>
<td>4013.9</td>
<td>3974.8</td>
<td>4013.9</td>
<td>0.391</td>
<td>0.410</td>
<td>20301.13</td>
<td>20301.13</td>
</tr>
<tr>
<td>26L</td>
<td>3886.6</td>
<td>3867.1</td>
<td>3886.6</td>
<td>0.391</td>
<td>0.410</td>
<td>19657.44</td>
<td>19657.44</td>
</tr>
<tr>
<td>3</td>
<td>3945.4</td>
<td>3925.8</td>
<td>3945.4</td>
<td>0.391</td>
<td>0.413</td>
<td>19880.07</td>
<td>19880.07</td>
</tr>
<tr>
<td>19R</td>
<td>3935.6</td>
<td>3876.9</td>
<td>3935.6</td>
<td>0.391</td>
<td>0.411</td>
<td>19880.19</td>
<td>19880.19</td>
</tr>
<tr>
<td>4L</td>
<td>4004.1</td>
<td>3965.0</td>
<td>4004.1</td>
<td>0.393</td>
<td>0.412</td>
<td>20150.99</td>
<td>20150.99</td>
</tr>
<tr>
<td>18</td>
<td>4013.9</td>
<td>3965.0</td>
<td>4013.9</td>
<td>0.391</td>
<td>0.410</td>
<td>20301.13</td>
<td>20301.13</td>
</tr>
<tr>
<td>21R</td>
<td>4092.2</td>
<td>4033.5</td>
<td>4092.2</td>
<td>0.392</td>
<td>0.411</td>
<td>20645.70</td>
<td>20645.70</td>
</tr>
<tr>
<td>17</td>
<td>3925.8</td>
<td>3896.4</td>
<td>3925.8</td>
<td>0.392</td>
<td>0.410</td>
<td>19830.74</td>
<td>19830.74</td>
</tr>
<tr>
<td>5R</td>
<td>3965.0</td>
<td>3935.6</td>
<td>3965.0</td>
<td>0.390</td>
<td>0.410</td>
<td>20078.62</td>
<td>20078.62</td>
</tr>
<tr>
<td>2R</td>
<td>3788.7</td>
<td>3739.8</td>
<td>3788.7</td>
<td>0.390</td>
<td>0.411</td>
<td>19162.29</td>
<td>19162.29</td>
</tr>
</tbody>
</table>

- **Average**: 3968.6 0.391 0.411 20064.3 20064.3
- **Standard Deviation**: 102.86 0.4007 520.41 520.41
- **COV**: 2.6% 2.6% 2.6%

**Minimum**
- **Sample**: 3769.17
- **DBS**: 19087.09 19087.09

**Maximum**
- **Sample**: 4141.19
- **DBS**: 20997.25 20997.25

---

**Graph:**
- Dowel Bearing Resistance Curves
- Average Curve

---

182
### PVC 0.4" wall thickness - 3/8" hole

<table>
<thead>
<tr>
<th>Sample</th>
<th>Max Load</th>
<th>Int Load</th>
<th>Yield Load</th>
<th>wall thicknesses</th>
<th>5% offset</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>wall 1</td>
<td>wall 2</td>
<td>DBS</td>
</tr>
<tr>
<td>11</td>
<td>5482.4</td>
<td>5247.5</td>
<td>5482.4</td>
<td>0.392</td>
<td>0.412</td>
<td>18387.80</td>
</tr>
<tr>
<td>13</td>
<td>5453.1</td>
<td>5267.0</td>
<td>5453.1</td>
<td>0.393</td>
<td>0.410</td>
<td>18312.07</td>
</tr>
<tr>
<td>11b</td>
<td>5472.6</td>
<td>5237.7</td>
<td>5472.6</td>
<td>0.391</td>
<td>0.411</td>
<td>18400.74</td>
</tr>
<tr>
<td>14</td>
<td>5619.5</td>
<td>5433.5</td>
<td>5619.5</td>
<td>0.392</td>
<td>0.410</td>
<td>18894.50</td>
</tr>
<tr>
<td>17</td>
<td>5639.1</td>
<td>5413.9</td>
<td>5639.1</td>
<td>0.392</td>
<td>0.413</td>
<td>18889.67</td>
</tr>
<tr>
<td>15</td>
<td>5599.9</td>
<td>5472.6</td>
<td>5599.9</td>
<td>0.391</td>
<td>0.411</td>
<td>18828.66</td>
</tr>
<tr>
<td>12</td>
<td>5619.5</td>
<td>5492.2</td>
<td>5619.5</td>
<td>0.392</td>
<td>0.411</td>
<td>18870.97</td>
</tr>
<tr>
<td>14b</td>
<td>5462.8</td>
<td>5208.3</td>
<td>5462.8</td>
<td>0.393</td>
<td>0.410</td>
<td>18344.95</td>
</tr>
<tr>
<td>12b</td>
<td>5599.9</td>
<td>5482.4</td>
<td>5599.9</td>
<td>0.391</td>
<td>0.411</td>
<td>18828.66</td>
</tr>
<tr>
<td>13b</td>
<td>5551.0</td>
<td>5364.9</td>
<td>5551.0</td>
<td>0.390</td>
<td>0.410</td>
<td>18710.74</td>
</tr>
</tbody>
</table>

**Average**: 5550.0 0.392 0.411 18646.9 18646.9
**Standard Deviation**: 74.62 0.4013 252.02 252.02
**COV**: 1.3% 1.4% 1.4%

**Min**: 5453.05 min 18312.07 18312.07
**Max**: 5639.06 max 18894.50 18894.50

---

**Graph**: Dowel Bearing Resistance Curves

![Graph](image-url)
# Bending Yield Strength Data

3/8" Diameter A307 Bolts

<table>
<thead>
<tr>
<th>Sample</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>ave dia</th>
<th>Max Load</th>
<th>Yield Load</th>
<th>Fyb</th>
<th>Fyb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>0.371</td>
<td>0.374</td>
<td>0.373</td>
<td>0.373</td>
<td>861.52</td>
<td>744.04</td>
<td>86255.7</td>
<td>99875.0</td>
</tr>
<tr>
<td>1-2</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>841.94</td>
<td>734.25</td>
<td>86739.8</td>
<td>99461.7</td>
</tr>
<tr>
<td>1-3</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>832.15</td>
<td>724.46</td>
<td>85122.8</td>
<td>97776.1</td>
</tr>
<tr>
<td>1-4</td>
<td>0.371</td>
<td>0.371</td>
<td>0.370</td>
<td>0.371</td>
<td>837.47</td>
<td>742.19</td>
<td>87440.8</td>
<td>98666.0</td>
</tr>
<tr>
<td>1-5</td>
<td>0.372</td>
<td>0.371</td>
<td>0.372</td>
<td>0.372</td>
<td>849.40</td>
<td>748.20</td>
<td>87439.6</td>
<td>99266.5</td>
</tr>
<tr>
<td>1-6</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>845.60</td>
<td>746.80</td>
<td>87747.3</td>
<td>99356.1</td>
</tr>
<tr>
<td>1-7</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>899.90</td>
<td>748.20</td>
<td>87911.8</td>
<td>105736.2</td>
</tr>
<tr>
<td>1-8</td>
<td>0.372</td>
<td>0.371</td>
<td>0.372</td>
<td>0.372</td>
<td>860.40</td>
<td>754.10</td>
<td>88129.1</td>
<td>100552.0</td>
</tr>
<tr>
<td>1-9</td>
<td>0.372</td>
<td>0.370</td>
<td>0.371</td>
<td>0.371</td>
<td>841.60</td>
<td>747.10</td>
<td>87782.6</td>
<td>98886.1</td>
</tr>
<tr>
<td>2-1</td>
<td>0.372</td>
<td>0.371</td>
<td>0.372</td>
<td>0.372</td>
<td>851.73</td>
<td>744.04</td>
<td>86953.8</td>
<td>99539.2</td>
</tr>
<tr>
<td>2-2</td>
<td>0.371</td>
<td>0.372</td>
<td>0.373</td>
<td>0.372</td>
<td>861.52</td>
<td>744.04</td>
<td>86720.2</td>
<td>100412.9</td>
</tr>
<tr>
<td>2-3</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>890.89</td>
<td>734.25</td>
<td>86739.8</td>
<td>105244.3</td>
</tr>
<tr>
<td>2-4</td>
<td>0.370</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>861.50</td>
<td>764.30</td>
<td>90046.0</td>
<td>101497.6</td>
</tr>
<tr>
<td>2-5</td>
<td>0.371</td>
<td>0.372</td>
<td>0.371</td>
<td>0.371</td>
<td>852.30</td>
<td>754.10</td>
<td>88366.6</td>
<td>99873.9</td>
</tr>
<tr>
<td>2-6</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>849.10</td>
<td>747.40</td>
<td>88292.9</td>
<td>100307.1</td>
</tr>
<tr>
<td>2-7</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>827.60</td>
<td>734.50</td>
<td>86769.0</td>
<td>97767.2</td>
</tr>
<tr>
<td>2-8</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>842.10</td>
<td>746.00</td>
<td>88127.5</td>
<td>99480.2</td>
</tr>
<tr>
<td>3-1</td>
<td>0.370</td>
<td>0.370</td>
<td>0.370</td>
<td>0.370</td>
<td>900.68</td>
<td>734.25</td>
<td>86974.5</td>
<td>106688.7</td>
</tr>
<tr>
<td>3-2</td>
<td>0.370</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>910.47</td>
<td>744.04</td>
<td>87659.4</td>
<td>107267.5</td>
</tr>
<tr>
<td>3-3</td>
<td>0.372</td>
<td>0.370</td>
<td>0.370</td>
<td>0.371</td>
<td>832.15</td>
<td>734.25</td>
<td>86506.0</td>
<td>98040.2</td>
</tr>
<tr>
<td>3-5</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>833.30</td>
<td>738.30</td>
<td>87217.9</td>
<td>98440.6</td>
</tr>
<tr>
<td>3-7</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>0.371</td>
<td>845.60</td>
<td>751.10</td>
<td>88252.6</td>
<td>99356.1</td>
</tr>
<tr>
<td>3-8</td>
<td>0.370</td>
<td>0.371</td>
<td>0.370</td>
<td>0.370</td>
<td>897.20</td>
<td>745.00</td>
<td>88090.4</td>
<td>105989.3</td>
</tr>
</tbody>
</table>

overall average = 0.37091 inches 743.69 87443.71 100846.98 psi
standard deviation = 0.000853 8.58 974.39 3026.71 psi
COV = 0.23% 1.15% 1.11% 3.00%

Min = 85122.8 97767.2
Max = 90046.0 107267.5
Bending Yield Strength Curves

Average Curve
### 1/4" bolts - Blank - 8" long

<table>
<thead>
<tr>
<th>Sample</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>ave dia</th>
<th>Max Load</th>
<th>Yield Load</th>
<th>Fyb</th>
<th>Fyb</th>
</tr>
</thead>
<tbody>
<tr>
<td>b8-1</td>
<td>0.247</td>
<td>0.248</td>
<td>0.248</td>
<td>0.248</td>
<td>271.27</td>
<td>224.83</td>
<td>88799.5</td>
<td>107138.5</td>
</tr>
<tr>
<td>b8-2</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>261.49</td>
<td>215.06</td>
<td>85628.3</td>
<td>104116.2</td>
</tr>
<tr>
<td>b8-3</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>261.49</td>
<td>210.17</td>
<td>83682.2</td>
<td>104116.2</td>
</tr>
<tr>
<td>b8-4</td>
<td>0.246</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>261.49</td>
<td>215.06</td>
<td>85578.9</td>
<td>104538.9</td>
</tr>
<tr>
<td>b8-5</td>
<td>0.247</td>
<td>0.247</td>
<td>0.246</td>
<td>0.247</td>
<td>270.00</td>
<td>218.40</td>
<td>86958.7</td>
<td>107503.9</td>
</tr>
<tr>
<td>b8-6</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>270.00</td>
<td>225.40</td>
<td>89745.8</td>
<td>107105.7</td>
</tr>
<tr>
<td>b8-7</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>269.00</td>
<td>225.40</td>
<td>89745.8</td>
<td>107105.7</td>
</tr>
<tr>
<td>b8-8</td>
<td>0.247</td>
<td>0.247</td>
<td>0.246</td>
<td>0.247</td>
<td>259.90</td>
<td>215.40</td>
<td>86112.4</td>
<td>103902.5</td>
</tr>
<tr>
<td>b8-9</td>
<td>0.247</td>
<td>0.247</td>
<td>0.246</td>
<td>0.247</td>
<td>261.50</td>
<td>218.00</td>
<td>87151.8</td>
<td>104654.2</td>
</tr>
<tr>
<td>b8-10</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>263.60</td>
<td>224.30</td>
<td>89307.9</td>
<td>104955.6</td>
</tr>
<tr>
<td>b8-11</td>
<td>0.247</td>
<td>0.246</td>
<td>0.247</td>
<td>0.247</td>
<td>265.80</td>
<td>219.30</td>
<td>87671.5</td>
<td>106261.2</td>
</tr>
<tr>
<td>b8-12</td>
<td>0.246</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>262.80</td>
<td>210.60</td>
<td>84193.4</td>
<td>105096.1</td>
</tr>
<tr>
<td>b8-13</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>261.20</td>
<td>215.00</td>
<td>85952.5</td>
<td>104422.2</td>
</tr>
<tr>
<td>b8-14</td>
<td>0.246</td>
<td>0.247</td>
<td>0.246</td>
<td>0.246</td>
<td>263.70</td>
<td>221.00</td>
<td>88710.3</td>
<td>105850.2</td>
</tr>
<tr>
<td>b8-15</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>0.247</td>
<td>261.00</td>
<td>214.20</td>
<td>85286.4</td>
<td>103920.4</td>
</tr>
</tbody>
</table>

Overall average = 0.24684 inches  
Standard deviation = 0.000475  
COV = 0.19%  
Min = 83682.2  
Max = 89745.8  

![Bending Yield Strength Curves](image)

![Average Curve](image)
<table>
<thead>
<tr>
<th>Sample</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>ave dia</th>
<th>Max Load</th>
<th>Yield Load</th>
<th>Fyb</th>
<th>Fyb</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>0.186</td>
<td>0.187</td>
<td>0.186</td>
<td>0.186</td>
<td>83.09</td>
<td>63.54</td>
<td>58928.7</td>
<td>77060.6</td>
</tr>
<tr>
<td>2-2</td>
<td>0.187</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>83.09</td>
<td>61.10</td>
<td>56662.2</td>
<td>77060.6</td>
</tr>
<tr>
<td>2-3</td>
<td>0.186</td>
<td>0.187</td>
<td>0.187</td>
<td>0.187</td>
<td>83.27</td>
<td>60.13</td>
<td>55468.0</td>
<td>76813.8</td>
</tr>
<tr>
<td>2-4</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>85.69</td>
<td>62.93</td>
<td>58677.3</td>
<td>79899.2</td>
</tr>
<tr>
<td>4-1</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>83.92</td>
<td>61.21</td>
<td>57073.5</td>
<td>78248.8</td>
</tr>
<tr>
<td>4-2</td>
<td>0.186</td>
<td>0.187</td>
<td>0.187</td>
<td>0.187</td>
<td>85.42</td>
<td>63.25</td>
<td>58346.1</td>
<td>78971.1</td>
</tr>
<tr>
<td>4-3</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>84.08</td>
<td>61.69</td>
<td>57521.1</td>
<td>78398.0</td>
</tr>
<tr>
<td>4-4</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>83.70</td>
<td>60.13</td>
<td>56066.5</td>
<td>78043.7</td>
</tr>
<tr>
<td>5-1</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>81.02</td>
<td>54.12</td>
<td>50462.7</td>
<td>75544.8</td>
</tr>
<tr>
<td>5-2</td>
<td>0.186</td>
<td>0.187</td>
<td>0.187</td>
<td>0.187</td>
<td>81.07</td>
<td>56.05</td>
<td>51704.3</td>
<td>74784.4</td>
</tr>
<tr>
<td>5-3</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>80.91</td>
<td>52.24</td>
<td>48709.7</td>
<td>75442.2</td>
</tr>
<tr>
<td>5-4</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>79.84</td>
<td>51.76</td>
<td>48262.2</td>
<td>74444.6</td>
</tr>
<tr>
<td>6-1</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>81.77</td>
<td>62.23</td>
<td>58024.6</td>
<td>76244.1</td>
</tr>
<tr>
<td>6-2</td>
<td>0.186</td>
<td>0.187</td>
<td>0.187</td>
<td>0.187</td>
<td>83.60</td>
<td>64.05</td>
<td>59084.0</td>
<td>77118.3</td>
</tr>
<tr>
<td>6-3</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>84.03</td>
<td>64.59</td>
<td>60225.1</td>
<td>78351.4</td>
</tr>
<tr>
<td>6-4</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>84.08</td>
<td>64.43</td>
<td>60075.9</td>
<td>78398.0</td>
</tr>
<tr>
<td>9-1</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>81.50</td>
<td>58.63</td>
<td>54667.9</td>
<td>75992.4</td>
</tr>
<tr>
<td>9-2</td>
<td>0.186</td>
<td>0.187</td>
<td>0.187</td>
<td>0.187</td>
<td>83.81</td>
<td>62.17</td>
<td>57349.8</td>
<td>77312.0</td>
</tr>
<tr>
<td>9-3</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>82.74</td>
<td>59.44</td>
<td>55423.2</td>
<td>77148.6</td>
</tr>
<tr>
<td>9-4</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>0.186</td>
<td>82.58</td>
<td>58.15</td>
<td>54220.3</td>
<td>76999.4</td>
</tr>
</tbody>
</table>

overall average = 0.18620 inches 60.09 55847.65 77105.11 psi
standard deviation = 0.0004034 3.89 3570.98 1405.70
 COV = 0.22% 6.48% 6.39% 1.82%

Min = 48262.2 74444.6
Max = 60225.1 79899.2

<table>
<thead>
<tr>
<th>Sample</th>
<th>5% Offset</th>
<th>Max Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Fyb</td>
<td>Std Dev</td>
</tr>
<tr>
<td>2-1</td>
<td>57361.9</td>
<td>1657.4</td>
</tr>
<tr>
<td>2-2</td>
<td>57251.8</td>
<td>949.8</td>
</tr>
<tr>
<td>6-1</td>
<td>49784.7</td>
<td>1593.5</td>
</tr>
<tr>
<td>6-2</td>
<td>59352.4</td>
<td>1019.8</td>
</tr>
<tr>
<td>9-1</td>
<td>55415.3</td>
<td>1381.9</td>
</tr>
</tbody>
</table>
## Connection Test Data

### HDPE - Mode Im

<table>
<thead>
<tr>
<th>Sample</th>
<th>Intersection Load</th>
<th>Yield Load</th>
<th>Max Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>17-6-17</td>
<td>666.7</td>
<td>666.7</td>
<td>679.0</td>
</tr>
<tr>
<td>23-6-23</td>
<td>647.1</td>
<td>647.1</td>
<td>666.7</td>
</tr>
<tr>
<td>11-7-11</td>
<td>634.9</td>
<td>634.9</td>
<td>652.0</td>
</tr>
<tr>
<td>25-5-25</td>
<td>674.1</td>
<td>674.1</td>
<td>679.0</td>
</tr>
</tbody>
</table>

average = 655.7  
standard deviation = 17.94584446  
COV = 2.7%

min 634.90  
max 674.10

### PVC - Mode Im

<table>
<thead>
<tr>
<th>Sample</th>
<th>Intersection Load</th>
<th>Yield Load</th>
<th>Max Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-7-1</td>
<td>1991.7</td>
<td>1991.7</td>
<td>1994.2</td>
</tr>
<tr>
<td>2-5-2</td>
<td>2030.8</td>
<td>2055.3</td>
<td>2055.3</td>
</tr>
<tr>
<td>9-7-9</td>
<td>2038.2</td>
<td>2038.2</td>
<td>2055.3</td>
</tr>
<tr>
<td>18-4-18</td>
<td>1881.8</td>
<td>1881.8</td>
<td>1884.2</td>
</tr>
</tbody>
</table>

average = 1991.8  
standard deviation = 78.07092929  
COV = 3.9%

min 1881.80  
max 2055.30
### HDPE - Mode IIIs

<table>
<thead>
<tr>
<th>sample</th>
<th>Intersection Load</th>
<th>Yield Load</th>
<th>Max Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-2-9</td>
<td>520.5</td>
<td>520.5</td>
<td>640.3</td>
</tr>
<tr>
<td>12-5-12</td>
<td>508.3</td>
<td>508.3</td>
<td>652.5</td>
</tr>
<tr>
<td>14-6-14</td>
<td>493.7</td>
<td>493.7</td>
<td>628.1</td>
</tr>
<tr>
<td>10-4-10</td>
<td>501.0</td>
<td>501.0</td>
<td>652.5</td>
</tr>
<tr>
<td>8-3-8</td>
<td>505.9</td>
<td>505.9</td>
<td>633.0</td>
</tr>
<tr>
<td>11-4-11</td>
<td>515.7</td>
<td>515.7</td>
<td>637.8</td>
</tr>
<tr>
<td>16-7-16</td>
<td>503.4</td>
<td>503.4</td>
<td>618.3</td>
</tr>
<tr>
<td>17-7-17</td>
<td>505.9</td>
<td>505.9</td>
<td>635.4</td>
</tr>
<tr>
<td>15-6-15</td>
<td>481.4</td>
<td>481.4</td>
<td>620.7</td>
</tr>
</tbody>
</table>

- **average** = 504.0, 635.4
- **standard deviation** = 11.52, 12.16
- **COV** = 2.3%, 1.9%
- **min** = 481.40, 618.30
- **max** = 520.50, 652.50

### PVC - Mode IIIs

<table>
<thead>
<tr>
<th>sample</th>
<th>Max Load</th>
<th>Intersection Load</th>
<th>Yield Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-17-18</td>
<td>2363.2</td>
<td>2292.3</td>
<td>2292.3</td>
</tr>
<tr>
<td>17-15-17</td>
<td>2412.1</td>
<td>2397.4</td>
<td>2412.1</td>
</tr>
<tr>
<td>15-11-15</td>
<td>2380.3</td>
<td>2338.8</td>
<td>2338.8</td>
</tr>
<tr>
<td>19-13-19</td>
<td>2404.7</td>
<td>2177.5</td>
<td>2177.5</td>
</tr>
<tr>
<td>16-11-16</td>
<td>2453.6</td>
<td>2448.7</td>
<td>2448.7</td>
</tr>
<tr>
<td>11-12-11</td>
<td>2333.9</td>
<td>2331.4</td>
<td>2333.9</td>
</tr>
<tr>
<td>14-13-14</td>
<td>2358.3</td>
<td>2351.0</td>
<td>2358.3</td>
</tr>
<tr>
<td>10-14-10</td>
<td>2485.4</td>
<td>2485.4</td>
<td>2485.4</td>
</tr>
<tr>
<td>12-14-12</td>
<td>2368.1</td>
<td>2282.6</td>
<td>2282.6</td>
</tr>
<tr>
<td>13-12-13</td>
<td>2385.2</td>
<td>2170.1</td>
<td>2170.1</td>
</tr>
</tbody>
</table>

- **average** = 2394.5, 2327.5, 2330.0
- **standard deviation** = 46.0880751, 103.1357337, 104.544037
- **COV** = 1.9%, 4.4%, 4.5%
- **min** = 2333.90, 2170.10, 2170.10
- **max** = 2485.40, 2485.40, 2485.40
<table>
<thead>
<tr>
<th>Sample</th>
<th>Intersection Load</th>
<th>Yield Load</th>
<th>Max Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-12-11</td>
<td>615.8</td>
<td>615.8</td>
<td>862.7</td>
</tr>
<tr>
<td>13-12-13</td>
<td>620.7</td>
<td>620.7</td>
<td>862.7</td>
</tr>
<tr>
<td>10-9-10</td>
<td>606.1</td>
<td>606.1</td>
<td>870.0</td>
</tr>
<tr>
<td>8-9-8</td>
<td>601.2</td>
<td>601.2</td>
<td>865.1</td>
</tr>
<tr>
<td>14-15-14</td>
<td>620.7</td>
<td>620.7</td>
<td>879.8</td>
</tr>
<tr>
<td>16-15-16</td>
<td>640.3</td>
<td>640.3</td>
<td>877.3</td>
</tr>
<tr>
<td>23-19-22</td>
<td>637.8</td>
<td>637.8</td>
<td>833.4</td>
</tr>
<tr>
<td>26-27-25</td>
<td>637.8</td>
<td>637.8</td>
<td>874.9</td>
</tr>
<tr>
<td>21-20-21</td>
<td>593.9</td>
<td>593.9</td>
<td>860.2</td>
</tr>
<tr>
<td>18-17-18</td>
<td>593.9</td>
<td>593.9</td>
<td>852.9</td>
</tr>
</tbody>
</table>

Average = 616.8, 863.9

Standard Deviation = 17.9, 13.6

COV = 2.9%, 1.6%

Min = 593.90, 833.40

Max = 640.30, 879.80

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rod Type</th>
<th>5% offset</th>
<th>Max Load Based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ave stdev</td>
<td>cov</td>
</tr>
<tr>
<td>11-12-11</td>
<td>6</td>
<td>618.3</td>
<td>3.5</td>
</tr>
<tr>
<td>13-12-13</td>
<td>5</td>
<td>603.7</td>
<td>3.5</td>
</tr>
<tr>
<td>10-9-10</td>
<td>2</td>
<td>630.5</td>
<td>13.9</td>
</tr>
<tr>
<td>8-9-8</td>
<td>9</td>
<td>637.8</td>
<td>0.0</td>
</tr>
<tr>
<td>14-15-14</td>
<td>4</td>
<td>593.9</td>
<td>0.0</td>
</tr>
<tr>
<td>16-15-16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23-19-22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26-27-25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21-20-21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18-17-18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sample</td>
<td>Max Load</td>
<td>Intersection Load</td>
<td>Yield Load</td>
</tr>
<tr>
<td>------------</td>
<td>----------</td>
<td>-------------------</td>
<td>------------</td>
</tr>
<tr>
<td>24-25-24</td>
<td>2874.0</td>
<td>2610.0</td>
<td>2610.0</td>
</tr>
<tr>
<td>7-6-7</td>
<td>2764.0</td>
<td>2519.6</td>
<td>2519.6</td>
</tr>
<tr>
<td>26-25-26</td>
<td>2959.5</td>
<td>2614.9</td>
<td>2614.9</td>
</tr>
<tr>
<td>23-22-23</td>
<td>2930.2</td>
<td>2673.6</td>
<td>2673.6</td>
</tr>
<tr>
<td>21-22-21</td>
<td>2842.2</td>
<td>2548.9</td>
<td>2548.9</td>
</tr>
<tr>
<td>4-3-4</td>
<td>2952.2</td>
<td>2651.6</td>
<td>2651.6</td>
</tr>
<tr>
<td>2-3-2</td>
<td>2771.3</td>
<td>2502.5</td>
<td>2502.5</td>
</tr>
<tr>
<td>20-18-20</td>
<td>2837.3</td>
<td>2514.7</td>
<td>2514.7</td>
</tr>
<tr>
<td>19-17-19</td>
<td>2820.2</td>
<td>2561.2</td>
<td>2561.2</td>
</tr>
<tr>
<td>5-6-5</td>
<td>2832.4</td>
<td>2504.9</td>
<td>2504.9</td>
</tr>
</tbody>
</table>

average = 2858.3  2570.2  2570.2  
standard deviation = 69.7  63.2  63.2  
COV = 2.4%  2.5%  2.5%  

min = 2764.00  2502.50  2502.50  
max = 2959.50  2673.60  2673.60  

PVC - Mode IV